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Proceedings of the American Academy of Arts and Sciences.

Vol. XLIX. No. 1. — May, 1913.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY OF HARVARD UNIVERSITY.

THERMODYNAMIC PROPERTIES OF TWELVE LIQUIDS BETWEEN 20° AND 80° AND UP TO 12000 KGM. PER SQ. CM.

By P. W. Bridgman.

With seven Folders.

Investigations on Light and Heat made and published with Aid from the Rumford Fund.

THERMODYNAMIC PROPERTIES OF TWELVE LIQUIDS BETWEEN 20° AND 80° AND UP TO 12000 KGM. PER SQ. CM.

By P. W. Bridgman.

Presented November 13, 1912. Received December 30, 1912.

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I. Introduction.

The experimental material of this paper consists of direct measurements of the volume of twelve liquids at different temperatures and pressures. The pressure range is from atmospheric pressure to 12000 kgm. per sq. cm., and the temperature range from 20° to 80°. The measurements were made at enough points to determine the volume at any pressure and temperature. These data are presented in the first part of the paper. The second part of the paper contains a discussion of a number of quantities of thermodynamic significance which have been computed from the data of the first part. The discussion is concerned only with the more important thermodynamic properties, namely the isothermal compressibility, thermal expansion, work of compression, heat of compression, change of internal energy, and the specific heats at constant pressure and constant volume.

Apparently the only other work of similar character at even comparatively high pressures is that of Amagat,[1] published in 1893. Amagat measured the volume of twelve liquids up to 3000 kgm. and between 0° and 40° or 50°. Beside the volume he tabulated the compressibility, dilatation, and pressure coefficient for some of the liquids, but the tabulation was by no means systematic or complete.

It is hoped that the material in this paper will afford the means for a renewed attack on the problem of the nature of the mechanism of a liquid. Theoretical speculation has been concerned hitherto chiefly with phenomena of liquids at low pressures, such as the latent heat of evaporization or the surface tension. At high pressures there are effects of another order, just as significantly descriptive of the internal mechanism, but as yet hardly touched by speculation. The data of this paper cover four times any previous pressure range, and should be sufficient to show the general nature of high pressure effects. Furthermore, the systematic presentation of different thermodynamic properties should afford different points of view for the attack.

The actual state of affairs at high pressures was found to be exceedingly complicated, contrary to what we might expect. We might suppose that the molecules would become so closely packed at high pressure as to allow less variety in their response to external changes, so that a liquid would approximate to a solid, in which compressibility and expansion change only slightly with pressure and temperature. A hypothesis of this character, backed up it is true by some experimental evidence, has been made the basis of a recent empirical theory of liquids by Tammann,[2] who assumes that at high pressures variations in thermal expansion due to changes of temperature ought to become vanishingly small. Such, however, is by no means the case, but, on the contrary, at high pressures the thermal expansion varies with temperature in a more complicated way than at atmospheric pressure.

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[1] Amagat, Ann. Chim. et Phys., **29**, 68–208 (1893).

The irregularity of the effects at high pressures makes it evident that a complete theory of liquids must be very complicated. The first step, therefore, toward a theory would be to explain only the general features. With this in mind, the average of the various thermodynamic properties over the entire temperature range has been computed for each liquid. To facilitate comparison, the average of any one property, compressibility for example, is shown on the same diagram for all twelve liquids. One would expect to turn at first to these collected diagrams in seeking light on a theory of liquids.

In this paper no attempt has been made at the very considerable task of developing a quantitative theory to represent the data. The results suggest very strongly in some cases, however, that conceptions of the mechanism of a liquid which may be adequate at low pressures can no longer be adequate at high pressures. For instance, we shall probably have to modify our ideas of the mechanism accounting for pressure and temperature. Some discussion is given of modifications that may be necessary, and in several cases it is shown that the proposed modifications are competent to explain, at least qualitatively, the complicated effects found at high pressures.

A preceding paper on water[3] is very similar to the present one in the scope of its measurements and computations. The hope was expressed in the introduction to that paper that the projected study of twelve liquids (that is, this paper) would show what we might expect of a normal liquid, and that the results for water might then be compared with these results, and yield information about the abnormalities of water. This hope now appears to have been unfounded because no liquid is really normal at high pressures; all show individual peculiarities. It is true, nevertheless, that at high pressures these twelve liquids do become more nearly alike in a general way, and that water becomes increasingly like them. In many cases it will be found instructive, therefore, to compare the properties of water at high pressures with those of the liquids of this paper.

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[2] Tammann, Ann. Phys., **37**, 975–1013 (1912).

[3] Bridgman, These Proceedings, **48**, 307–362 (1912).

The initial abnormalities of water have such a far-reaching effect, however, up to 6000 kgm., that it seemed desirable not to include water directly in the same diagrams with the other liquids.

The apparatus and the experimental method are in large measure the same as were used in the preceding work on water, and will not be described again in any detail. The methods of computation, however, are somewhat different, and are discussed at some length. It is impossible to give the original data because of lack of space. However, a few sample curves of the original data are given, and the average error of the compressibility and the expansion measurements is stated for each liquid.

The liquids used are with two exceptions the same as those used by Amagat. This choice of liquids has two advantages. In the first place, the present method does not give accurate results at the very lowest pressures, so that it is desirable to supplement the measurements with others at low pressures. For this purpose the values of Amagat for the change of volume between atmospheric pressure and 500 kgm. have been used. And in the second place, it was not desired to complicate this study, which is concerned with the liquid only, by the possibility of freezing under pressure. The freezing points of all the liquids used here are very low. It was hoped, therefore, that none of them would freeze under pressure at the temperatures of this investigation. The anticipation of no freezing was justified except in the case of acetone, which, however, froze at such high pressures that it was not necessary to discard it.

Comment should perhaps be made on the very extensive use of diagrams instead of tables. A table is capable of greater accuracy than a diagram, but a diagram has the advantage of presenting a large collection of results in a form immediately grasped. By the use of diagrams with fine rulings, the attempt has been made to combine the general grasp afforded by a diagram with the accuracy of a table. In most cases the numerical values may be read directly from the diagrams within the limits of experimental accuracy. The only exception is for the fundamental data, volume against pressure and temperature, for which tables have been given to four significant figures. The diagrams accompanying these tables are quite secondary in importance, giving merely a general survey of the trend of the results.

A brief indication of the plan of presentation may be helpful to the reader. The apparatus and the experimental method are first briefly described, indicating the points of departure from the previous work on water (pages 7 to 18). Then the methods of computation are taken up in detail, describing in succession the method used for each one of the thermodynamic properties (pages 18 to 37). A somewhat detailed analysis of the data for each liquid is next given, including any special features of the experiment for that liquid, the experimental error, and the source and probable accuracy of data by other observers which have been used in the computations (pages 37 to 74). Here are included the fundamental data for each liquid, that is, the tables and diagrams of volume against pressure and temperature. The various thermodynamic properties are then discussed, with comments on the peculiarities of the individual liquids, and on the general features common to them all (pages 74 to 104). The diagrams for this general discussion are given on folders at the end of the paper, all the diagrams for any one property being collected on one sheet. It is hoped that this arrangement will produce less confusion in the text, and at the same time, permit a more ready general survey of the facts. Finally, there is given on pages 104 to 113, a discussion of the bearing of the results on previous theories, and of the possible effects on our ideas of what the actual mechanism of a liquid may be.

II. EXPERIMENTAL METHOD.

The method is essentially similar to that used in the preceding work on water. As in the earlier work, so here, the liquid under investigation is placed in a cylinder closed by a piston which moves absolutely without leak. The pressure on the liquid may be varied by changing the position of the piston, and the temperature may be varied by altering the temperature of the surrounding bath. The volume is found directly by measuring the position of the piston in the cylinder. The fundamental data to be obtained with the apparatus are the values of the volume as a function of pressure and temperature at a sufficient number of points to allow the calculation of the volume at any temperature and pressure within the range.

In actual use the simple procedure suggested above is complicated by the necessity of introducing the instrument for measuring the pressure into the same cylinder with the liquid under investigation. The pressure in these experiments was measured with a coil of manganin wire, the changes in the resistance of which give the pressure by a previous calibration.[4] Although this manganin gauge is compact and simple, its use introduces complications, because it must come in contact only with a liquid that is an insulator. The water of the preceding paper and nearly all the twelve liquids of this are not insulators. Some auxiliary liquid must be used therefore, to transmit pressure to the manganin coil. Kerosene has shown itself perfectly adapted to the purpose, and has been used in all this work. In the former work on water, the kerosene and water could be allowed to come directly in contact with each other, but in this investigation, nearly all the liquids are more or less miscible with kerosene, so that means had to be provided to prevent the liquid under investigation from coming into contact with the kerosene. This necessitated slight changes in the design of the apparatus, and the use of still a third liquid, mercury, to keep the liquid and the kerosene from direct contact. The modified receptacle for holding the liquid is shown in Figure 1. The liquid is enclosed in a steel bulb A, the stem of which dips into the cup B containing mercury. The cup B is bored out at the lower end so as to protect the manganin pressure gauge, which is shown at C. The bulb, cup, and gauge together occupy the lower part of the pressure cylinder. In order to obtain as large a quantity as possible of the liquid to experiment on the diameter of the bulb etc. was made larger than that of the moving piston ($\frac{11}{16}$ inch against $\frac{1}{2}$ inch). To allow this the cylinder had to be bored out at the lower end to the larger diameter. In other respects the cylinder is similar to that used for water, in fact it is the same cylinder, the only change being the enlargement of the lower part to meet the new requirements.

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[4] Bridgman, These Proceedings, **47**, 319–438 (1911).

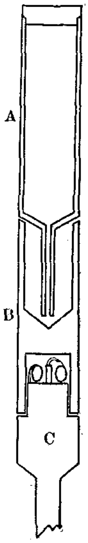

FIGURE. 1. The compressibility bulb with attachments. *A* is the bulb containing the liquid under investigation, *B* the mercury cup, and *C* the insulating plug and the manganin coil with which pressure is measured.

The final form of receptacle shown in Figure 1 was arrived at only after a considerable number of failures. Early attempts to use glass cylinders, which from the point of view of purity would be more desirable, had to be abandoned because of the invariable fracture of the glass. This is doubtless because kerosene under pressure becomes so viscous as not to transmit small changes of pressure hydrostatically immediately after changes in the position of the piston. The enormously increased viscosity both of the kerosene and of the liquid under investigation within the bulb afforded the only satisfactory explanation of many of the capricious misfortunes of the preliminary work. For instance, in the early work the bulb was closed at the upper end by a cap put on with soft solder. Several times after the application of pressure this soft solder was found eaten away by the mercury. The only apparent way in which this could happen would be by the liquid (ethyl alcohol in the preliminary investigations) becoming so viscous as to crack as the volume decreases with rising pressure, thus allowing the mercury to rise through the cracks from below. Another very troublesome effect in the preliminary work was the frequent short circuiting of the manganin coil by small drops of mercury. These were probably forced out of the cup by the viscous motion of the kerosene during changing pressure. The difficulty was avoided by making the lower part of the mercury cup in the form of a protecting cap for the manganin coil, and by enlarging the channels of communication between the cup and the bulb from the kerosene to the mercury. It was found also that it was necessary to give the bulb fairly thick walls. Otherwise, when pressure is relieved, the bulb expands under the viscous motion of the fluid inside and tightly fills the cylinder. Furthermore, the bulb must not fit the cylinder too closely, for otherwise the pressure is not transmitted rapidly enough to the interior of the bulb, which may thereby become collapsed at the upper end. Still again, the upper end of the bulb must be provided with radial grooves to give access from the lower to the upper part of the cylinder, or else when pressure is released, the entire bulb rises with the viscous kerosene against the ledge on the cylinder, acting effectively as a valve which permits only comparatively slow release of pressure, with the result that eventually, when pressure is released, the top is blown off the bulb and forced into the smaller bore of the cylinder above.

It must be a matter of experiment to find the dimensions for any particular piece of apparatus which will avoid all these difficulties. Complete success was attained, however; the last twenty out of a total of twenty-four runs being completed without accident of any kind.

Although the method outlined above is exceedingly simple, yet there are evidently a number of corrections for the distortion of the steel containing vessel and for the auxiliary liquid. These corrections have been discussed in great detail in the preceding paper on water. The corrections to be used in this paper are exactly similar to the former ones, with the exception of the correction for the effect of the mercury. It will be recalled that two sets of measurements are necessary to obtain all the data needed in making the corrections; the first is with the steel bulb filled with the liquid under investigation, and the second with the liquid and bulb replaced by a cylinder of Bessemer steel. The quantities of kerosene are approximately the same in the two sets of measurements. If the quantity of mercury used in the two sets were exactly the same, it would not be necessary to apply any correction for it, but since it was not easy to use exactly the same quantity, a correction for the difference had to be applied. This correction was exceedingly small, being merely the change with pressure and temperature of the small difference, and could be obtained with sufficient accuracy for this purpose from measurements already made on mercury up to 12000 kgm. at 0° and 20°.[4] The variation of compressibility and dilatation with temperature is so small at high pressures that it is perfectly safe to extrapolate to 80°. The quantities of mercury in the two determinations seldom differed by as much as 0.3 gm., in which case the correction can be entirely neglected.

The altered design of the containing vessel made necessary a modification of the procedure in filling the cylinder ready for a run. The bulb was filled by boiling the liquid into it at reduced pressure at room temperature. During the filling the liquid came into contact with a gum stopper for a few seconds. The bulb was then disconnected from the air pump, and gently warmed by holding it in the hands, thus forcing a slight quantity of liquid out of the stem. After the bulb had come to the temperature of the hand, the outside was carefully wiped dry, and the stem immediately inverted into the cup of mercury. In this way the complete exclusion of all air bubbles was ensured. After the bulb had once more come to the temperature of the room, the combined weight of the filled bulb, mercury, and steel cup was found. The weight of the liquid in the bulb can then be found immediately by subtracting the known weight of the mercury, cup, and bulb. The quantity of the liquid used in the majority of the experiments was about 12 c. c. The mercury cup with the bulb still in place was now slipped over the manganin resistance coil, and the coil, mercury cup, and the liquid under investigation were inserted together into the cylinder. This naturally had to be done from below, and for this purpose the cylinder was held vertical during this operation in a special vise, projecting at right angles from the wall. The packing and the retaining screw were also inserted while the cylinder was in this position. The cylinder was now filled with a weighed quantity of kerosene through the open upper end. This filling with kerosene was accomplished in two operations; first, half of the kerosene was poured in, and all air extracted from the lower part of the cylinder by gently exhausting it, and then the remainder of the kerosene was poured in. The moveable plug was then inserted, with special care not to spill any kerosene, and finally the cylinder, kept always upright, was put in position in the lower part of the press. After a run the bulb was cleaned for the next run by heating it nearly to redness. That the cleaning was thorough was shown by the constancy of weight of the bulb, which seldom varied more than one milligram from experiment to experiment.

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[4] Bridgman, These Proceedings, **47**, 319–438 (1911).

The essential idea of the experimental method remains the same as in the work on water, but there are slight modifications. The purpose is to obtain by direct measurements the change of volume with pressure over the entire pressure range at some one constant temperature, and then by another independent set of measurements to obtain the change of volume with temperature over the entire temperature range at a sufficient number of pressures to completely cover the field. This method has a distinct advantage over that employed by Amagat, for example. Amagat obtained the volume as a function of pressure and temperature by measuring the change of volume with pressure isothermally at a number of temperatures. This has the disadvantage that the thermal dilatation can be found only by taking the difference of compressibility determinations at different temperatures. Obviously this makes it necessary to measure the compressibility with a much greater degree of accuracy than the desired final accuracy of the dilatation, and this is a difficult matter to accomplish. But the present method gives both compressibility and dilatation with the accuracy to be expected of direct measurements.

The procedure in measuring the change of volume with pressure at constant temperature was the same as in the work on water. The temperature chosen for this determination was 40°, because it is the same as was used for water. Reference is made to the former paper for the details of manipulation. In order to cover completely the pressure range, it was necessary to make the readings in two series, one at lower pressures up to 2000 kgm., and the second between 2000 kgm. and the maximum, 12000 kgm. A similar method was found necessary in the previous work on water. The reason for this is that at high pressures the moveable piston takes a permanent set, and moves with extreme friction against the sides of the cylinder. The result is that if the pressure has once been pushed to the maximum the piston will not return far enough to reach the low pressures again. The lowest pressure that it was possible to reach after the maximum varied somewhat, but was usually in the neighborhood of 1000 kgm. In order completely to cover the field, therefore, two sets of experiments were necessary, one for the high and the other for the low pressures. To make the results comparable, the low pressure measurements were carried some distance into the high pressure domain, the upper limit of the low pressure measurements being about 2000 kgm.

The chief variation from the method of the earlier work is in finding the thermal expansion. The method of the earlier work involved two steps. The first was to change the temperature at constant volume. This change of temperature, if it were an increase, produced as a secondary effect a rise of pressure. The second step was to lower the pressure to the original value preparatory to the next change of temperature. The withdrawal of the piston necessary to effect this lowering of pressure was always attended by slight irregularities; apparently the piston does not begin its regular march until after some slight motion has taken place. So that although perfect regularity was found in the larger motions of the piston of the compressibility determinations, yet the initial irregularity was sufficient to be disturbing with the smaller motions accompanying the temperature changes. The difficulty was avoided by keeping the piston constant in position during a series of temperature changes, and allowing the pressure to vary accordingly. The variation of pressure for a change of temperature of 60°, from 20° to 80°, was about 1000 kgm. at the highest pressures, and proportionally less at the lower pressures. Temperature measurements were made at six values of the average pressure with the apparatus set up for the higher pressures, above 2000 kgm., and at two mean pressures with the apparatus set up for the lower pressures, below 2000 kgm. Readings were made at 20° intervals, as in the work on water. At 2000 kgm., for example, the temperature was raised from 20° to 40° and the increase of pressure noted, then to 60° and the increase of pressure noted again, and finally from 60° to 80° and once again the increase of pressure noted.

The pressure was then increased at 80° to the next higher value, about 3500 kgm., and the corresponding set of measurements made at 20° intervals, this time with decreasing temperature. The other four sets of readings were made in a similar way. The highest pressure reached, on the last reading at 80°, was about 12500 kgm. From here the temperature was decreased, so that the final reading at 20° was made at a pressure of about 11500 kgm. Here is an incidental advantage of the modified method of making the expansion measurements, because mercury solidifies at 20° at less than 12000 kgm., so that if the former method had been used a smaller value of the mean maximum pressure would have been necessary for the last reading at 20°. Of course the modified method necessitates a slight change in the method of making the computations, which will be described later.

All the measurements on each of the liquids were repeated to ensure greater accuracy. The measurements were made in two series; all twelve liquids were first measured, and, except for the preliminary experiments, no repetitions made until at least one set of readings had been made for each liquid. This has the advantage of separating the measurements on any one liquid by a considerable interval of time, and thereby eliminating any possible effect of temporary variations in the apparatus. During the first extended series of readings the two sets of readings for each liquid at high and low pressures were made with two separate fillings of the apparatus. At the lower pressures the viscosity effects do not play any part, so that it was not necessary to leave so large a space between the bulb and the walls of the cylinder as was necessary for the higher pressures. In this way a larger bulb could be used at the lower than at the higher pressures, so that somewhat greater sensitiveness could be obtained at the lower pressures because of the greater quantity of liquid. But this made it necessary to take the apparatus apart after the set of readings at low pressures, and replace the larger bulb by the smaller one for the run at high pressures. This made considerable trouble, which did not seem adequately compensated by the somewhat greater accuracy, so that in the repetition of the experiment the readings at the high and low pressures were all made with the same filling of the apparatus. The readings at low pressures were made before the readings at high pressures, and so before the piston had been upset.

The final detailed procedure was as follows. The readings at low pressures were made first. It is not necessary to describe them in detail as they were similar to the readings at the higher pressures, the only difference being that no seasoning precautions were taken. At high pressures the thermostat was first adjusted to 40°, and then the cylinder seasoned over the entire pressure range by advancing the pressure to the maximum and releasing it three times. The details of the seasoning process are described in the former paper. The compressibility measurements at 40° were then made. Readings were taken both at increasing and decreasing pressures, so as to correct for hysteresis. The range of these measurements was from 2000 to 12000 kgm., returning after the complete set of readings to the initial pressure of 2000 kgm. Next, the cylinder was seasoned for the thermal expansion readings by advancing the pressure to 11500 kgm. raising the temperature to 80° at constant volume, releasing the pressure to 2000 kgm. at 80°, and finally reducing the temperature to 20°. The thermal expansion measurements were then made by the method already described.

In the later form of procedure, in which the effects at high and low pressures were measured with the same set-up, all the measurements at the low pressures and the isothermal compressibility at 40° were made on the same day; on the next day the thermal dilation at six mean pressures was measured, and the apparatus taken down and set up again with a new liquid ready for another run on the next day.

By the use of large reservoirs of hot water and by nearly emptying the thermostat at every change of temperature, it was possible to make the readings with changing temperature fairly rapidly, the elapsed time between readings at successive temperatures being about fifteen minutes. A much longer time than this would have been necessary to secure temperature equilibrium throughout the mass of the cylinder if a special device had not been adopted. This consisted in running the temperature past the final value, and then returning to it. Thus, let us suppose that the temperature was to be changed from 20° to 40°. Water was drawn from the thermostat and enough hot water poured in to raise the temperature to 45°, and not until the lapse of several minutes, the exact time to be determined by experiment, was the temperature reduced to 40° and the regulator set at this final value. After some practise it was possible to reach temperature equilibrium in little more time than was necessary for the manipulations of drawing water and putting in fresh. Readings were never made, however, until at least three minutes had elapsed without change of pressure.

The actual experiments, after the preliminary work, occupied four months, from February through May, 1912. Many of the early measurements are due to the assistance of Mr. S. L. Gokhale. During the entire time there was no change in the internal diameter of the cylinder of so much as 0.0001 inch. To ensure further that there was no progressive change in the cylinder during the measurements, the comparison measurements with the liquid replaced by Bessemer steel were made at five intervals during this time. The maximum divergence of any of these readings from the mean was only 0.5%, better perhaps than would at first be expected from the method.

The original data are so numerous that it seemed undesirable to give them here in full. Every point recorded involves six readings, two of pressure and four of piston displacement. On the average each liquid involved 140 points, 75 for compressibility and 65 for thermal expansion. This makes a grand total of 12500 readings in the original data. It was thought to be sufficient to give a few sample curves of complete data, and to state for each liquid the average departure from the mean of the two series of compressibility and dilation measurements. The average departure from the mean of the two series of compressibility measurements for the twelve liquids was 0.15% of the maximum, and the departure from the mean of the thermal expansion measurements was 2%. The changes of volume due to changes of pressure are much greater than those due to temperature, so that the compressibility measurements determine the final accuracy of the volume.

In regard to the purity of the liquids, it was not thought necessary to take special precautions, because the properties studied here are not much influenced by the presence of impurities. The compressibility, for example, of a mixture of two liquids of small concentrations of the one in the other is an additive function of the compressibility of the two components. An example of this fact occurred in the preliminary work on ether. The first measurements were made with the ether enclosed in a glass bulb. This bulb broke on the application of pressure because of the great viscosity of the kerosene. The compressibility measurements of this preliminary run were measurements, therefore, on a mixture of ether and kerosene. In spite of the fact that this ether was very much contaminated the result showed that the compressibility of this mixture was only different by 4% from what it was when the measurements were made on the same quantities of ether and kerosene prevented from mixing.

The liquids used were obtained from Eimer and Amend. They were either the purest manufactured by them, or else Kahlbaum's purest. It was to be expected therefore, that only slight impurities were present, and that the errors due to them are beyond the limits of observational error.

Nine months after the completion of the experiments, the remaining samples were subjected to a rough analysis by determining the boiling points. If the liquid is pure, the boiling point should remain constant until the liquid is completely boiled away. The amount by which the boiling point changes during evaporation can be expected to give at least some clue as to the amount of impurity present. [A substance is considered good enough for most *chemical* purposes if it all boils within 1°.] Of course this analysis does not give the nature of the impurity present, nor does it give the impurity at the time the experiment was performed. During the nine months the liquids were left tightly corked, but there can be no question that some deterioration had gone on in this time, so that the liquids were all actually better than is indicated by the analyses. The deterioration with time is much greater for some liquids than for others.

The fractionations were performed by Mr. R. H. Patch at the Chemical Laboratory of Harvard University. The results of his examination are given below. It should be noticed that the temperature readings have not been corrected, so that all they can show is the constancy of the boiling point, not its absolute value.

Methyl Alcohol (*Kahlbaum*).

93% boils between 64.5° and 65° C.
7% " " 65° and 65.8° C.

Free from ethyl alcohol and acetone, the most likely impurities, and was neutral to litmus. Excellent sample. (It would seem that the impurity was probably water, most of which had probably been absorbed while standing, as the stopper was not perfectly tight).

Ethyl Alcohol (*Kahlbaum*).

3.8% boils between 77.3° and 77.8° C.
96.2% " " 77.8° and 78.0° C.

The sample showed some suspended inorganic matter, probably iron from the container. (The sample had been standing for the nine months tightly corked with a cork stopper in the tin vessel in which it came from Kahlbaum). With this exception, and admitting the presence of a small quantity of water the sample was a good one.

Propyl Alcohol (*Kahlbaum*).

4.% boils below 96.° C.
96.% boils between 96.0° and 96.8° C.

A good sample.

Isobutyl Alcohol (Eimer and Amend).

7% boils below 105.8° C.
17% boils between 105.8° and 107° C.
76% " " 107.0° and 107.2° C.
Thus 93% boils within 1.4°, a good sample.

Amyl Alcohol (Kahlbaum, Free from Pyridin).

3% boils below 128.9° C.
89.5% boils between 128.9° and 129.9° C.
7.5 boils between 129.9° and 130.0° C.
Sample free from foreign organic matter and neutral to litmus. An excellent example.

Ethyl Ether (Kahlbaum, "Sp. gr. 0.720").

The whole sample boiled between 34.5° and 35.0° C.
Neutral to litmus, free from aldehydes, and sulphur compounds.
Contained some water, and without doubt some alcohol.

Acetone (Eimer and Amend. Marked "Pure").

70.6% boils between 56° and 57° C.
19.4% boils between 57° and 58° C.
10% boils between 58° and 59° C.
The sample left a dark brown residue in the distillation flask, probably aldehydic in nature. Free from water, and neutral to litmus. A fair sample. (The brown color developed on standing; at the time the experiment was performed, the liquid was perfectly colorless.)

Carbon Bisulphide (From the store room of the Chemical Laboratory).

5% boils between 45.8° and 46.° C.
95 boils at 46.0° C.
The sample was free from hydrogen sulphide, sulphuric and sulphurous acids, but contains some foreign organic sulphur compounds and left a yellowish residue. The latter always results upon allowing the liquid to stand, especially upon exposure to light. (Here again the color had developed upon standing during the nine months. At the time of the experiment the liquid was perfectly colorless.)

Phosphorus Trichloride (Eimer and Amend).

True boiling point is 78.3° C.
Sample showed no constant boiling point.
6.5° boiled below 77° C.
11.2% boiled between 77° and 79° C.
17.9% boiled between 79° and 80° C.

15.9% boiled between 80° and 81° C.
17.9% boiled between 81° and 87° C.
31.4% boiled between 87° and 102° C.
Chief impurity probably phosphorus oxychloride formed by combination of the trichloride with the moisture in the air, also some pentachloride is almost always unavoidably present, due to excess chlorine.

Ethyl Bromide (*Eimer and Amend*).

The whole sample boiled from 38.0° to 38.4° C. and with the exception of not over 2% at 38.4° exactly. Neutral to litmus, an excellent sample.

Analyses could not be made of the ethyl chloride and the ethyl iodide, because these samples were completely used up in the experiment. Both of these were obtained in sealed glass bulbs from Kahlbaum, were used immediately after opening, and were both colorless when used.

These analyses show that probably none of the liquids contained enough impurity to have any perceptible effect, except the phosphorus trichloride, and to a less degree the acetone. The phosphorus trichloride was very noticeably simpler in its behavior at high pressures than the other liquids; probably due to the fact that it is a mixture of different substances, and so can not be expected to show the well marked behavior of a single pure substance.

A few measurements were made on one liquid which are not tabulated here. These were measurements on chloroform, which was found to freeze at fairly low pressures, about 6800 kgm. at 40°, and 10000 kgm. at 80°. Further investigation of this liquid has been postponed until a systematic investigation of freezing curves can be taken up.

III. Methods of Computation.

The methods of computation were slightly different from those used in the work on water, in part because of the somewhat different experimental method. They are, in consequence, here described in detail. As in the computations for water, the first step was to prepare a table giving the volume at regular intervals of pressure and temperature. From this table the other thermodynamic quantities were then computed.

The Table of Volumes. In preparing the table of volumes, the first computation was of the volume as a function of pressure at 40°. The first step was to plot the piston displacement against the displacement of the slider of the bridge wire on which the resistance of the manganin coil was measured. Because of hysteresis effects it was necessary to make two sets of readings, one with increasing and the other with decreasing pressure. Care was taken that any two corresponding readings should be at as nearly as possible the same pressure, so that it should be allowable to take the average of two corresponding displacements as the best value of the displacement at the average of the two corresponding pressures. The next step was to draw a smooth curve through the average points, and from the curve to tabulate the displacement at regular intervals of pressures (5 cm. on the bridge wire, or about 1100 kgm.). The second series of measurements on the same liquid was then treated in the same way. These two series of measurements differed somewhat as to the quantities of materials used, that is, the liquid under investigation, the kerosene, the mercury, and the steel. But the amounts were so nearly the same that it was permissible to take the average displacement of the two series as the displacement that would have been found for the mean between the quantities of material used in the two series. In only a few cases did the amounts of material in the two series differ so much that it was not permissible to do this. In these cases the displacement for the mean quantity had to be determined in a way which need not be described in detail. The average of the displacements obtained in this way were now corrected for the effect of the kerosene, the mercury, the steel, and the distortion of the steel containing vessel. In applying the corrections, the mean of the five auxiliary experiments in which the liquid was replaced by Bessemer steel was used. The correction was applied in essentially the same way as for water. The corrected result gave the motion of the piston due to the compression of only the liquid under investigation. This, with the known cross section of the piston and the weight of the liquid, determined the change of volume at any pressure of 1 gm. of the liquid. Finally, by using the values for the density at 0° deduced from the recent Tables of Kaye and Laby, the results were reduced to the change of volume in c. c. of a quantity of liquid which at 0° C. and atmospheric pressure occupies 1 c. c. This is the unit quantity which is here adopted throughout, and seems to have been most usually used in work of this kind. In particular it is the unit quantity of Amagat.

The computation just described applies only to the measurements at the higher pressures. The results are tabulated as changes of volume from 2000 kgm. as the zero of pressure. If the pressure during the high pressure measurements went lower than this, as it usually did, the corresponding changes of volume were taken as negative. To obtain the volume at lower pressures, the measurements at the lower pressures were reduced in much the same way as has been described for the higher pressures. One difference is that the measurements at the lower pressures were at 20° instead of 40°, because several of the liquids boil at less than 40° at atmospheric pressure. It was not possible to reach entirely to the zero of pressure with the low-pressure measurements, because of the slight friction of the piston even when the pressure had not been pushed so high as to permanently upset the piston. It was not possible to get much nearer to zero than 100 or 200 kgm. after the pressure had once been pushed to 2000 kgm. In order to come still closer to zero, several measurements were made during the very first application of pressure, before the cylinder has been seasoned for hysteresis. During these first readings the pressure was increased to about 1000 kgm. and then seasoned for hysteresis as before. As a result the second set of readings between 200 and 2000 kgm. made after this seasoning process did not make close connection with the first set. The discrepancy was due in part to hysteresis, but also in part to better adaptation of the packing to all the crevices in the steel washers. The direction of the two curves was usually the same, however, within the limits of error. This allowed an extrapolation to zero by combining the results of the first set of readings with those of the second. In this way the change of the unit volume from atmospheric pressure up to 2000 kgm. was determined. It must be remembered, however, that the readings at the very lowest pressures are almost certainly in error, because the instrument is not designed for low pressures. But with increasing pressure the readings become more and more trustworthy, until above 500 kgm. they seem to merit entire confidence, if we can judge from self consistency. To get the total change of volume from atmospheric pressure it is desirable, therefore, to supplement these readings with others made with apparatus especially designed for low pressures. Such measurements are afforded by the data of Amagat, and in some cases by others.

At the time that the computations of this paper were made, the data of Amagat were the best that we had for the compressibility at low pressures. Between the time of computing these results and writing this account, however, there has appeared a paper by Richards,[5] in which the compressibility up to 500 kgm. of several of the liquids used here is determined. It is unfortunate that these values did not appear in time for use in the present computations. They are quoted, however, so as to afford a comparison with the results of Amagat, which are here used as the standard for low pressures. It makes no difference with the essential conclusions of the paper, however, which set of data is accepted as most probably correct. The only effect would be to change the initial values of some of the thermodynamic properties; their magnitudes at high pressures will not be altered.

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[5] Richards, Jour. Amer. Chem. Soc., **34**, 971–993 (1912).

The precise steps in combining into a final result the changes of volume computed from the two sets of readings at high and low pressures were as follows. From the high pressure readings, the changes of volume at 40° (ΔV, cm.^{3}/cm.^{3}) were plotted against the displacement of the slider of the resistance bridge, the zero of pressure being at 10 cm. displacement. The scale of the diagram was large; 2 cm. for 0.1 inch piston displacement, and 55 cm. slider displacement for the maximum pressure. From the low pressure readings ΔV in cm.^{3}/cm.^{3} was found at 20°, and was plotted against slider displacement, the zero being at 5 cm. A smooth curve was drawn through these points. From this curve, knowing the constants of the manganin coil, ΔV was found at 500, 1000, 1500, and 2000 kgm. and 500, 1000, 1500, and 2000 atmos. ΔV in terms of atmos. was now corrected so as to be reckoned from 1000 atmos. as zero. The values of Amagat for ΔV at 20° were next computed with 1000 atmos. as the zero. These two sets of data were compared, and the new values for the lowest interval adjusted so as to be in agreement with Amagat between 1 and 500 kgm. It will be noticed that this preliminary comparison with Amagat does not enter the final result; it was an orienting comparison for obtaining some idea of the accuracy at low pressures. Using Amagat's value for the lower interval, the changes of volume at pressures corresponding to 2.5, 5.0, 7.5, and 10.0 cm. were next determined, reckoned from zero pressure. These values were now corrected to 40° with the data obtained for the thermal dilatation and were recomputed with 10 cm. as the zero of pressure. The changes of volume at 40° obtained from both the low and high pressure sets of measurements were now plotted together on one diagram, and a smooth curve drawn through the points. From this curve the changes of volume with the kilogram as the unit of pressure were read off at 500 kgm. intervals, starting from an origin at 500 kgm. The changes of volume so found were now smoothed to give regular differences. The smoothing was performed in a manner somewhat different from the corresponding operation for water, and in a manner which has the advantage of preserving the irregularities which are shown by experiment actually to exist, but which would have been effaced if the attempt were made to give smooth second differences, as was done for water. In performing this smoothing, the fact was used that the change of volume of nearly all the liquids can be represented by a curve of nearly the same shape. Ethyl chloride is an exception, and a special computation had to be applied to it. As a first approximation the change of volume of the other eleven liquids was found to be reproducible by a formula of the type;

$$\Delta V = \alpha\left(\frac{p-500}{1000}\right)^{.8} + \beta\left(\frac{p-500}{1000}\right)^{.6} + \gamma\left(\frac{p-500}{1000}\right)^{.4} + \delta\left(\frac{p-500}{1000}\right)^{.2}.$$

In order to apply this formula to any one liquid it is necessary simply to multiply all four constants by the same factor. The meaning of this is that to this degree of approximation the chief difference of the liquids with regard to changes of volume is in respect to the absolute, not the relative magnitudes of the change. The constants of the above formula were computed, therefore, so as to apply to the average of the eleven liquids. This was done by finding the average change of volume for the eleven liquids at 1500, 3000, 7000, and 12000 kgm. and determining the four constants of the formula so that the curve should pass through these four points. It will be noticed that in this formula the zero of pressure is at 500 kgm., so that it applies directly to the changes of volume as found above. The formula was now applied to the eleven liquids in succession by multiplying the four constants by the appropriate multiplier for each liquid. The multiplier was so determined that the formula should give the observed change of volume at 7000 kgm. The changes of volume were now calculated with this formula at intervals of 500 kgm. up to 5000 kgm., and beyond 5000 kgm. at intervals of 1000 kgm., and compared with the observed values. The differences between the observed and the computed values were plotted on a large scale and a smooth curve drawn through the points. The values obtained from these smooth curves and the formula were combined to give the final values for the volume at 40°. The advantage of the method is that the final results lie on perfectly smooth curves, and that the curves show the various slight irregularities which correspond to the experimental facts but which would be smoothed out if the second differences were made uniform. The values for ethyl chloride were treated in the same way, except that it was necessary to start from a slightly different formula, to be given on page 67.

It would have been possible to dispense with this finer adjustment by means of a difference curve with very little change in the final result, because the changes of volume obtained from the original smooth curves were almost the same as given by the final computation. Table I shows for one liquid (methyl alcohol, chosen at random) the very slight changes made by this readjustment.

TABLE I.

METHYL ALCOHOL.

DIFFERENCE BETWEEN VOLUME OBTAINED FROM ORIGINAL SMOOTH CURVE AND FINAL COMPUTED VALUE.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Difference.	Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Difference.
500	.0000	5000	−3
1000	−.0000	6000	+2
1500	−1	7000	−3
2000	0	8000	−1
2500	−1	9000	0
3000	+1	10000	0
3500	−2	11000	−1
4000	+1	12000	0
4500	0		

The values obtained by this computation start from 500 kgm. as the zero. The change of volume between atmospheric pressure and 500 kgm. has been taken directly from the results of Amagat in those cases where his data are sufficient. In some cases where Amagat does not give the data, it has been necessary to use the more inaccurate values of the present method. In the detailed presentation of data for each liquid, the values taken from Amagat are given. If it should happen at any future time that more probable values than these of

Amagat are found for the changes of volume at low pressures (which indeed is already the case for those liquids measured by Richards), then the results given here may be corrected by adding a constant to the volumes throughout the tables, except of course at atmospheric pressure. The addition of such a constant to the volumes will not alter the behavior of any of the thermodynamic properties at high pressures; it can at most affect those which involve integrations by a very small constant corrective term.

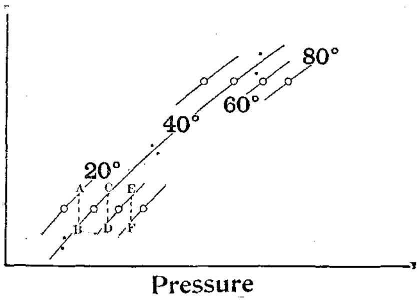

FIGURE 2. Shows a portion of the diagram for determining the compressibility and thermal expansion from the piston displacements. The piston displacements are plotted against pressure. The heavy points are the readings with increasing and decreasing pressure at 40° for the isothermal compressibility. The discrepancy between these points is due to hysteresis. The open circles show the readings at constant volume with changing temperature. The dotted lines, *AB* for example, show the piston displacement which would have been found if the temperature had been changed at constant pressure.

That it was possible to represent the approximate behavior of these twelve liquids by similar formulas is itself a somewhat surprising and significant fact. It seems to suggest that at extremely high pressures all liquids become alike. The greatest differences between different liquids are at the low pressures. The use of a separate formula in the case of ethyl chloride, which might appear to be an exception, was necessitated in fact only by its abnormal compressibility at low pressures.

It was necessary to compute the thermal dilatation also by a method slightly different from that used for water, because only one piston displacement was read at each temperature instead of two. The piston displacements were plotted against pressure on the same diagram as the isothermal compressibility at 40°. (See Figure 2.) Through each of the points a curve was drawn of the same general slope as the curve of pressure and volume at 40°. The slight changes necessary in the slope of this curve at different temperatures could be made graphically with sufficient accuracy. The difference of the piston displacement for every interval of 20° at the mean of the two pressures involved was now read from these curves. Thus in Figure 2, the line AB represents the piston displacement at constant pressure corresponding to a rise of temperature from 20° to 40°, CD from 40° to 60°, and EF from 60° to 80°. These displacements correspond to slightly different values of the mean pressure. The piston displacements at constant pressure obtained from a diagram like Figure 2 were then plotted against pressure on another diagram, the points for the two independent series of measurements on the same liquid being plotted on the same sheet. The quantities of liquid used in the two series were usually so nearly the same that the mean of the piston displacements could be assumed without error to be the piston displacement of the mean quantity of liquid. The mean of the two independent series of readings was found graphically from the diagram by drawing a smooth curve through the two sets of points. From this value of the piston displacement, after corrections had been applied for the kerosene, mercury, and steel, the change of volume per unit quantity for intervals of 20° was computed in a way analogous to the similar computations for the compressibility. Here again the values for the low pressures are most likely to be in error. The change of volume at 20° intervals at atmospheric pressure was taken directly from the tables of Landolt and Börnstein. Finally, the experimental points and the points at atmospheric pressure were plotted together on a single diagram, smooth curves drawn through them, and from these curves the changes of volume for intervals of 20° were obtained which were used in the construction of the tables of volume.

In plotting as above on a single diagram ΔV for 20° intervals, two independent series of measurements, namely those of this paper and those on which the formulas of Landolt and Börnstein are based were therefore brought together. The two sets of data should of course, if consistent, lie on a smooth curve, so that the amount of discrepancy might be expected to afford an indication of the order of accuracy at low pressures. The change in the dilatation is, however, so rapid at the low pressures that it was possible in nearly every instance to make smooth connection between the two sets of points, without departing from either of them. Furthermore, it would be possible in most cases to make just as smooth connection if somewhat different values were used at atmospheric pressure. Slight discrepancies between the smooth curves and the individual points do not therefore, give a reliable indication of the accuracy.

The thermal dilatation is probably not so accurate as are the changes of volume with pressure, because the dilatation is much smaller. The dilatation can be measured with no greater accuracy than the changes of pressure accompanying the changes of temperature at constant volume. These changes were of the order of 200 kgm. for 20° intervals and could be read on the bridge with an accuracy of about $\frac{2}{3}\%$. The agreement between the two independent sets of readings was not in general as good as this, averaging about 2%. This is better than was expected at first could be obtained with the method, and is certainly much better than could have been obtained with the alternative method of determining the difference between isothermal lines for different temperatures. Figure 3 for carbon bisulphide shows a fair average of the order of agreement. A more detailed account of the order of accuracy will be given under the description of the individual liquids.

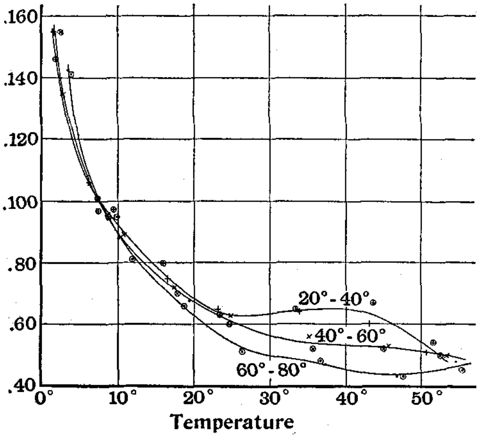

FIGURE 3. A sample set of observations on the change of volume with temperature at constant pressure. The ordinates are piston displacements in inches. Two independent sets of readings are shown on the diagram, those with the circles are the repeated set. The liquid shown here is carbon bisulphide. The accuracy for this is almost exactly the average accuracy for all twelve liquids.

The material was now at hand for the construction of the table of volumes. Up to 5000 kgm. the volume was to be tabulated at intervals of 500 kgm., and above 5000 kgm. at intervals of 1000 kgm. The difference in the length of the pressure steps is desirable because at low pressures the volume changes more rapidly than at high pressures. The tabulation at each pressure was to be made at temperature intervals of 10°. The volume as a function of temperature at atmospheric pressure was first tabulated. This was taken from the formulas of Landolt and Börnstein, or from other sources to be described in detail under the separate liquids. The agreement between different observers, even for atmospheric pressure, is not always as close as could be desired. The next step was to tabulate the volume as a function of pressure over the entire pressure range at 40°, starting from the volume at atmospheric pressure and 40°. These values were then combined with the change of volume for 20° intervals, thus giving the volume for each pressure at 20°, 60° and 80°. To obtain the volume at the intermediate intervals of 10° a device was adopted which at the same time gave the material for determining the thermal dilatation. The change of volume to be expected at 40° and 60° if the change had been linear with temperature over the entire range was calculated from the total change of volume between 20° and 80°. The differences between the actual and the calculated changes of volume were plotted against temperature for each pressure of the table, and smooth curves drawn through these points. From these curves the departure from linearity at the intermediate intervals of 10° was found, and combined with the values computed by the linear relation to give the total change of volume at the desired 10° intervals. This method, as the method for computing the isothermal compressibility, has the advantage of giving smooth curves without smoothing off the second differences.

Thermal Dilatation.—From the table of volumes the next problem was to compute the more significant quantities of thermodynamic interest. The first of these was the thermal dilatation, or $\left(\frac{\partial v}{\partial \tau}\right)_p$. "Dilatation" is perhaps not generally used in this sense, $\frac{1}{v}\left(\frac{\partial v}{\partial \tau}\right)_p$ being more common, but it has the advantage of being the quantity which enters directly into the thermodynamic formulas. The quantity of material to which the former differentiation refers is the unit used throughout this paper, namely the quantity which at 0°C. and atmospheric pressure occupies 1 c.c.

Evidently if the dilatation were uniform over the entire temperature range it could be found from the change of volume between 20° and 80° by dividing by 60. The dilatation is not linear, however, but departs from linearity in a way which can be found from the curves used to determine the change of volume at 10° intervals. The correction to the linear value is obviously to be found from the slope of the difference curve, which can be found graphically from a large drawing with sufficient accuracy. The dilatation was determined in this way at 20° intervals, and was plotted as a function of the pressure for each of the twelve liquids.

The details of the computations for the volume at 10° intervals and of the dilatation are shown more clearly perhaps in Figures 4 and 5. The experimental values of the changes of volume at 20° intervals were first plotted. A represents the change from 20° to 40°, B the change from 20° to 60° (obtained by adding the change 40°–60° to the change 20°–40°) and C the change 20° to 80°. The origin was now connected to C by a straight line (this was done actually by a computation, not graphically) and the differences between the points 0, A, B, and C and this straight line were plotted on another diagram, Figure 5, on a larger scale. A smooth curve was drawn through these four points; from this curve the ordinates were read at the intermediate intervals of 10°, and combined with the straight line values of Figure 4 to give the volume at the temperature in question. The thermal dilatation at any temperature, 40° for example, was found by adding to the slope of the straight line 0C the slope determined graphically at the point A′ of Figure 5.

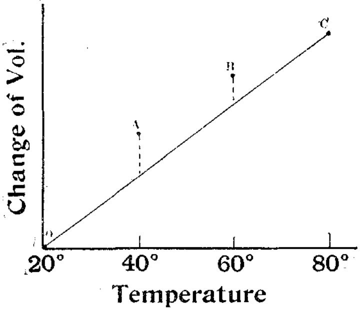

Figure 4. Shows the first step in finding the change of volume at intervals of 10° and the thermal expansion from the readings of the volumes at 20° intervals. The heavy line shows what the volume would be if the relation between volume and temperature were linear.

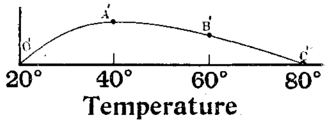

Figure 5. Second step in finding change of volume at 10° intervals and the thermal expansion. The point *A′* is the difference between the point *A* of Figure 4 and the straight line. The ordinates of this curve at intermediate points, when added to the ordinates given by the straight line of Figure 4 at corresponding points, give the volume at intermediate points. The slope at *A′* when added to the slope of the straight line of Figure 4, gives the thermal expansion at 40°, for example.

This method was also applied in determining the dilatation at atmospheric pressure. An alternative method would have been by differentiating the power series of Landolt and Börnstein for volume as a function of temperature. The graphical method was thought preferable, however, because a power series may often reproduce the experimental points with greater fidelity than the slope of the experimental curve.

The dilatation, computed in this way, was transferred directly to tables, and from the tables the curves were drawn which are given later for the dilatation. In order to save space, these tables are not given here. The thermal dilatation enters into many of the other thermodynamic quantities listed in this paper. In computing these quantities the values of the dilatation given in the tables have been used, not the values obtained from the curves given later. The same is true for all the other thermodynamic properties listed in the paper; tables were first computed for all of them before curves had been drawn for any. In this way any progressive error was avoided which might have been introduced by the use of diagrams. Although each diagram shows the property in question with as great an accuracy as is justified experimentally, it might be that if a computation involved the transference of points from one diagram to another several times, the error so introduced might finally mount up to more than the experimental error.

Isothermal Compressibility. — The compressibility, or the quantity $\left(\frac{\partial v}{\partial p}\right)_\tau$, was the next to be determined. This was found by a method somewhat analogous to that for the dilatation. Evidently the compressibility does not vary greatly with temperature. If the compressibility can be found as a function of pressure for one constant temperature, then the compressibility at other temperatures can be found by applying a small correction. The temperature chosen for the direct determination of the compressibility was 40°, since this was the temperature at which the change of volume with pressure had been found. The compressibility was determined graphically from a large scale drawing of the change of volume against pressure. An alternative method would have been to calculate mathematically the slope from the approximate formula for the change of volume, and then to correct this by the graphically determined slope of the difference curve. But this method would fail at the lowest pressure, 500 kgm., and at the higher pressures it did not prove necessary, because the simpler direct graphical method was sufficiently accurate.

It was now possible to correct the compressibility at 40° to the other temperatures of the tables by the use of difference curves. Let us suppose that it was desired to find the compressibility at 60°. Figure 6 represents graphically the operation which was actually performed by a computation. The curve of volume at 60° against pressure was displaced downwards (shown in the dotted line) so as to have the same origin as the curve for 40°. The difference between the curves was plotted on a large scale against pressure, and the graphically determined slope of the difference curve used as a correction to bring the compressibility from 40° to 60°. The process was performed for intervals of 20°, and the results were tabulated and plotted for the twelve liquids.

The compressibility is most likely to be in error at the lower pressures, as was the dilatation. In particular, the compressibility at atmospheric pressure can be found from the method outlined above only by a wide extrapolation, and therefore is not accurate. Another method was adopted, therefore, at atmospheric pressure. Of course, the compressibilities ought to be consistent with the tables of volumes, that is, it ought to be possible to compute from the compressibility to the change of volume given in the table. The compressibility at atmospheric pressure was accordingly computed so that when combined with the compressibility at 500 kgm. it should give the proper change of volume between atmospheric pressure and 500 kgm. It was assumed in the computation that the mean compressibility between 1 and 500 kgm. was the average of the compressibilities at 1 and 500 kgm. This is not quite true, because the compressibility varies rapidly with pressure at low pressures. The value computed in this way is likely to be somewhat low. The discrepancy cannot be large, however, and this method was accepted as the best under the circumstances. The compressibility at atmospheric pressure has also been determined in a number of instances by other observers. There is not always, however, very good agreement between other observers even at atmospheric pressure, so that the compressibility at atmospheric pressure might well be the subject for further experiment in some cases. The actual disagreement at atmospheric pressure and the probable accuracy of the value finally chosen is to be given in the detailed discussion for the separate liquids.

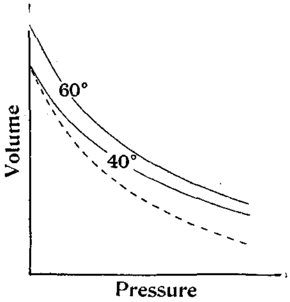

FIGURE 6. Illustrates the method for finding the temperature correction of the compressibility.

The Work of Compression. — The mechanical work of compression was the next quantity of thermodynamic interest to be computed. This is given by the formula $\left(\frac{\partial W}{\partial p}\right)_\tau = -p\left(\frac{\partial v}{\partial p}\right)_\tau$. To find the total quantity of work done from zero up to any given pressure it is evidently necessary to integrate the derivative. This integration was performed mechanically with the integraph of the mathematical department of Harvard University. In performing the integration there are two possible methods. We may either integrate $p\left(\frac{\partial v}{\partial p}\right)_\tau$ as given above, or we may integrate the equivalent expression $\left(\frac{\partial W}{\partial v}\right)_\tau = p$. The first involves an integration with pressure as the independent variable, and the second with volume. The first uses as the integrand the compressibility, which was obtained by computation from the experimental data, and the second uses as the integrand the pressure, which is one of the direct experimental data. It is well known that the derivative of an experimental quantity has considerably greater error than the experimental quantity itself. The second method was adopted, therefore, using the volume as the independent variable of integration. It was a fortunate accident that the method could be used without duplication of effort, because the volume had already been plotted against pressure for another purpose. However, the direct results of the integration were not immediately available because it was necessary to obtain the work of compression as a function of pressure instead of as a function of volume. The change of variable was made with the help of the curves of volume against pressure by reading off the pressures corresponding to the given volumes.

The work of compression was found by the method outlined above at 20° intervals of temperature. It differs only slightly for different temperatures, so slightly that the difference of the work at different temperatures could not be obtained directly from the curves with as great accuracy as was necessary for computing the specific heats. In order to obtain the differences of the work with greater accuracy, an independent integration of the quantity $\Delta p\left(\frac{\partial v}{\partial p}\right)_\tau dp$ was performed. The symbol Δ indicates the difference of the product $p\left(\frac{\partial v}{\partial p}\right)_\tau$ at a given pressure for the interval 20°–40° or 40°–60° or 60°–80. These differences were taken from the tables of compressibility. The integration of the differences was performed with the integrating machine. Tests of the integrating machine showed that the accuracy of this part of the process alone was as high as $\frac{1}{10}$ %. The differences found in this way were now used in finding a better mean value for the total work of isothermal compression. For it is evidently possible with these differences to correct the work of compression at 20° or 60° or 80° back to 40°. If there were no error all these values should agree, but of course complete agreement could not be expected. The agreement of the values corrected to 40° in this way was a few tenths of a per cent. The final value at 40° was taken as the mean of the four corrected values, and is shown in the diagrams. In the lower part of the diagrams the relation between pressure and the difference of the work of compression for 20° intervals is plotted on a larger scale.

The Heat of Compression.— The heat of compression was the next quantity to be computed. It is unfortunate that the expression, "heat of compression" is sometimes used in a sense which is not indicated by the words themselves; namely, as the rise of temperature when the pressure on a substance is increased adiabatically by the unit amount. But by no stretch of the imagination is it possible to identify a temperature with a "heat." A more descriptive name for this effect would seem to be the "temperature effect of compression." The effect was discussed under this name in the previous paper on water. By "heat of compression" we shall mean in this paper what is naturally suggested by the words, namely the heat, Q, which is given out by a unit quantity of a substance when it is compressed isothermally. It may be computed if the dilatation is known, by using the formula $\left(\frac{\partial Q}{\partial p}\right)_\tau = \tau\left(\frac{\partial v}{\partial \tau}\right)_p$. To find the total heat given out by the substance as it is compressed from the initial to the final state it is necessary to integrate this expression. The procedure was exactly analogous to that in computing the work of compression. The integration was performed first for the four temperatures. Then, in order to obtain the differences more accurately, a separate integration was made of the differences of $\tau\left(\frac{\partial v}{\partial \tau}\right)_p$ for intervals of temperature of 20°. With these differences the total heat at any temperature was corrected to 40°, thus giving four values for the heat of compression at 40°, of which the mean was taken for the final value at this temperature. From this final value, the values for the other temperatures were found by computing back again with the differences. The magnitude of the differences is much greater than the differences of the mechanical work, so that it was possible to plot the total heat for each temperature without confusing the diagram.

Change of Internal Energy.— From the heat of compression and the mechanical work of compression we may find at once the change of internal energy when the liquid is compressed isothermally. During compression the liquid receives work from the compressing force and delivers heat. The change of internal energy is the difference between the work received and the heat given out. It was computed in this way and is given in a set of diagrams (Folder 5) for the four regular temperature intervals.

Specific Heat at Constant Pressure.— Other thermodynamic quantities of a simple nature which are usually thought of as characteristic of a liquid are its two specific heats. They also may be found by thermodynamic computation from the data given, but the accuracy is not so great as the accuracy of the other quantities. There are two methods of attack open to us here, but both of them must assume as known the specific heats at atmospheric pressure as a function of temperature. In general, it may be shown that the characteristic equation of a substance is not sufficient in itself to determine the specific heats; we must know in addition the specific heats along some line not an isothermal. Unfortunately, the specific heat of very few of the liquids with which we are concerned is known with accuracy as a function of temperature at atmospheric pressure. The results of different observers are often in essential disagreement. But the characteristic equation can give us the *change* of specific heats along an isothermal. These are the results which will be tabulated in this paper, therefore, leaving for other experimenters the more accurate determination of the specific heats at atmospheric pressure. These future results may then be combined with the differences given here to determine the specific heat at any pressure.

The first method for calculating the specific heat at constant pressure is the method used in the paper on water. It makes use of the formula $\left(\frac{\partial C_p}{\partial p}\right)_\tau = -\tau\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$. Evidently in order to obtain the total change of specific heat at any pressure we must perform an integration. The weakness of the method is that it involves the use of a second derivative, which cannot be determined with great accuracy from measurements of volume. The method would be open to greater error if applied to these twelve liquids than in the case of water, because the dilatation varies more and more irregularly than for water.

The second method uses a cyclic process to determine the amount of heat absorbed in passing from one temperature to another at any constant pressure. Let us imagine a liquid in the condition represented by the point A on the diagram (see Figure 7). The liquid is now to be carried to the neighboring point D at the same pressure but at a higher temperature. The total change of internal energy when we arrive at D is independent of the path which we have traversed. One path from A to D may be described by raising the temperature from t_0 to t_1 at constant pressure. (Path AD in Figure 7.) In this case the liquid does a certain amount of mechanical work against external pressure and also absorbs a quantity of heat which we can compute immediately when we know the specific heat at constant pressure. The external work during this process is simply the product of the constant pressure and the change of volume, and may be computed directly from the table of volume as a function of pressure and temperature. Or we may pass from A to D by a more circuitous route, by lowering the pressure isothermally at t_0 from A to B, then raising the temperature at atmospheric pressure from B to C, and then increasing the pressure isothermally at the final temperature t_1 to the point D. Now the advantage of this longer route is that we know all the quantities of energy which enter the body on the way. The mechanical work of compression along the isothermals from A to B and from C to D we have already computed. We have also found the heat of compression along the lines A–B and C–B. No work is done in the expansion along the line B–C, and the heat absorbed along this line is known if we know the specific heat at atmospheric pressure. By comparing the inflow of energy along these different paths we are in a position to compute either the quantity of heat absorbed along the line A–D at constant pressure, or else the difference between this heat and the heat absorbed along the line B–C. This heat (or else the difference of heat) may be plotted against the difference of temperature between the points A and D. The same process may be performed at the same pressure for a number of temperature intervals, each with t_0 as the lower limit, giving a curve of the quantity of heat absorbed at constant pressure as a function of the temperature. The specific heat at any temperature is the slope of this heat curve at that temperature.

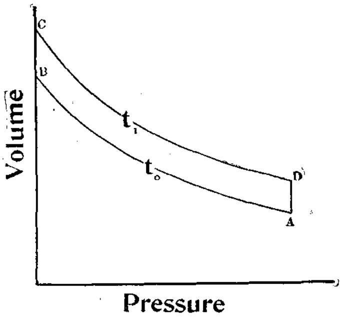

FIGURE 7. Shows the cycles described in finding the specific heat at constant pressure.

The slope was found by a method similar to that for computing the thermal dilatation at constant pressure. At any temperature the difference between the amount of heat actually absorbed and the amount which would have been absorbed if the relation between heat and temperature had been linear was computed and plotted against temperature. The slope of the difference curve was then found graphically, and applied as a correction to the value found on the assumption of a linear relation. The modification if C_p at atmospheric pressure is not known is obvious; a similar procedure, plotting now the difference of heats against temperature gives the difference of the specific heats at the pressure in question and atmospheric pressure. In a few cases, where the liquid boiled at a low temperature at atmospheric pressure so that it was not possible to prolong the curves to the origin of pressure, the difference between the specific heats at the pressure in question and 500 or 1000 kgm. has been given.

It is now obvious why it was necessary to know the differences of the mechanical work and the heat of compression at different temperatures with a greater accuracy than could be found from the curves obtained by a direct integration.

The units in which the specific heats are given should perhaps be mentioned because they are unusual. It is customary to give the specific heat in gm. cal. per gm. of the liquid. But this method of measuring specific heat makes no connection with the thermodynamic formulas, in which the heat is measured in mechanical units corresponding to the units of the other quantities. It was preferred here, therefore, to give the heat in units which are more unusual, but which are consistent with the other quantities, so that it is possible to substitute any of the quantities directly in the formulas without the troublesome work of changing the units. The unit of pressure is the kgm. per sq. cm., and the unit of volume the c.c. Therefore the unit of work which fits the formulas is the kgm. cm., and this is the unit in which the results have been tabulated. It is to be noticed that in making comparison with the usual values of the specific heats, it is not only necessary to change the unit of work, but the unit of quantity as well, because the amount of liquid to which this value of the specific heat is referred is not the gm., as is usual, but is the amount of liquid which at 0° C and atmospheric pressure occupies 1 c.c. In order to convert the usual value of the specific heat into these units, it is necessary to multiply by the density of the liquid at 0° and atmospheric pressure, and by 42.66, the number of kgm. cm. in 1 gm. cal.

As a check on the specific heat at constant pressure found in this way, the same quantity was computed for the first three alcohols, that is for methyl, ethyl, and propyl alcohol, by the alternative method involving the second temperature derivative of the volume which was used in the paper on water. The second derivatives were found graphically from the curves of dilatation against temperature at constant pressure, and were integrated mechanically. The results so found agree fairly well with the values found by the other more accurate method. The magnitude of the discrepancies might be as much as 10%, but all the essential characteristics of the curve as given by one method were reproduced by the other also, such as the maxima and the minima, and the points of inflection. Of course the pressure of maximum or minimum was sometimes displaced, as was to be expected.

Specific Heat at Constant Volume.—From the specific heat at constant pressure it is now possible to compute the specific heat at constant volume by the well known formula for the difference of the two specific heats, namely $C_p - C_v = -\frac{\tau\left(\frac{\partial v}{\partial \tau}\right)_p^2}{\left(\frac{\partial v}{\partial p}\right)_\tau}$. This formula involves only quantities which have already been determined, so that C_v may be found immediately. The values of $\left(\frac{\partial v}{\partial \tau}\right)_p$ and $\left(\frac{\partial v}{\partial p}\right)_\tau$ used in this computation were taken from the tables, not from the diagrams. Just as for the specific heat at constant pressure, the values found in this way are the differences between C_v at atmospheric pressure and the pressure in question. The differences are such that a positive value means that the specific heat is greater at atmospheric pressure than at the pressure in question. A decreasing curve indicates, therefore, that the specific heat is increasing with increasing pressure.

In the paper on water other quantities of thermodynamic interest were plotted. These are the temperature effect of compression and the adiabatic compressibility. They may be easily calculated from the data given in this paper. While they are of interest in themselves, they do not seem to be of such fundamental importance as the quantities already listed in suggesting the possible internal mechanism of the liquid. It was felt, therefore, that to give them would unduly increase the volume of this paper, and they have accordingly not been computed.

A word seems called for as to the general character of the curves. In many cases there are slight irregularities which may very well not correspond to the actual facts, the irregularities being beyond the limit of experimental accuracy of the work. It is true that if each of these quantities were being given for itself alone, without connection with other quantities, it would not have been justifiable to retain all the irregularities which some of the curves show. The reason for retaining the irregularities is that the attempt has been made to present a set of data which should be thermodynamically consistent. Let us suppose, for example, that the compressibility and the dilatation were both determined from the original tables of volumes and that they have been plotted against pressure. Both of these curves show irregularities which may be smoothed off by drawing smoother curves through the points, thus giving values of the compressibility and the dilatation which doubtless in themselves represent with greater probability the actual compressibility and dilatation. But each of these modified values for the compressibility and the dilatation will have a reflex effect on the table of volumes, which has now become inconsistent with the better values of the compressibility and the dilatation, and must therefore be altered slightly so as to be in accord with the new values. The alteration in the table necessary to accomplish this may be produced by changes less than the possible experimental error. But the point is this. Either the revised value of the compressibility or of the dilatation is sufficient of itself to completely revise the table of volume. If we are to adjust the compressibility or the dilatation we must do it so that both have the same reflex action on the table. Furthermore, all seven thermodynamic quantities must be adjusted in the same way. It is evident that this is a task of no small difficulty. To perform it, the only method seems to be a tedious one of trial and error. The labor of such an adjustment would be far beyond the labor of making the measurements with greater accuracy, and the labor had much better be so used in performing new experiments. It must furthermore be remembered that the values in the tables have been smoothed once with respect to both temperature and pressure. Any further changes would amount simply to slight changes in this smoothing; changes which were not justifiable by an examination of the data themselves but are rendered probable only by an examination of certain derived quantities. The choice has been made in this paper, therefore, to present results which may be slightly in error when taken by themselves, but which are nevertheless consistent thermodynamically.

IV. NUMERICAL DETAILS OF EXPERIMENT AND COMPUTATION.

In the detailed discussion and presentation of the results for the twelve liquids which is to follow, there will be given the experimental accuracy to be expected for each liquid (because for some liquids the accuracy is greater than for others), and also the sources and the numerical values of the results of other observers which have been used in computing the results given here. The results taken from other work are the density at atmospheric pressure, the thermal dilatation, the initial compressibility for low pressure ranges, and the specific heats at atmospheric pressure. Unless otherwise specified, the values for the density at atmospheric pressure have been taken from the recent tables of Kaye and Laby, and the values for the thermal dilatation have been deduced from the tables of Landolt and Börnstein. In these tables the volume at any temperature is given in terms of the volume at 0° by a power series of the form $V_t = V_0 (1 + at + bt^2 + ct^3)$. *In reproducing this expression it will* not be necessary to repeat the formula each time, but merely to give the values of the three constants a, b, and c.

It has been mentioned on page 22 that in computing the changes of volume with pressure at 40°, it was found that beyond 500 kgm. the shape of the curves was nearly the same for all twelve liquids, the only difference being in the numerical magnitudes. The constants used in the general pressure-volume formula of page 22 for the average of the twelve liquids were as follows; $\alpha = -0.0029$, $\beta = -0.0546$, $\gamma = +0.2969$, and $\delta = -0.1804$. To pass from this general formula to any one of the twelve liquids each of these four constants is to be multiplied by the same factor. This factor will be given in the following under the name of the "reduction factor."

The discussion is to be one of merely the numerical details of the measurements and the computations. The discussion of the general character of the results and their significance will be reserved until the data have all been presented.

Methyl Alcohol.— Three sets of measurements were made on this substance with three different fillings of the apparatus, the last being separated by nearly three months from the earlier two. The first measurement was of the thermal dilatation and compressibility at low pressures with the larger bulb adapted for low pressure work. The next set of measurements, made immediately afterwards, was of the compressibility and dilatation at the higher pressures with the smaller bulb for the high pressure work. The third measurement was with the high pressure bulb, and included the compressibility and dilatation over the entire pressure range, both high and low pressures. The measurements at low pressures were made, as already explained, before the piston had been upset by the higher pressures. The accuracy of the compressibility measurements may be estimated from the fact that the mean discrepancy of the piston displacements in the two sets of high pressure readings was 0.0035 inch, the maximum displacement being 2.07 inches. For the dilatation, the average discrepancy in the piston displacement for a rise of temperature of 20° was 0.0011 inch, the average displacement being 0.070 inch.

In computing the volumes at atmospheric pressure the density at 15° was taken to be 0.7960, from Kaye and Laby. The constants of the dilatation formula from Landolt and Börnstein are as follows; $a = 0.0_2 1186$, $b = 0.0_5 156$, $c = 0.0_8 91$[6]. This gives for the density at 0°, 0.8100. The quantity of methyl alcohol to which the tables and the diagrams refer weighs, therefore, 0.8100 gm. Since the boiling point of methyl alcohol is 64.7°, the volume listed in Table II for 80° and atmospheric pressure is, therefore, merely an extrapolation by means of the formula.

The "reduction factor" by means of which the transition was made from the mathematical formula for volume in terms of pressure at 40° to the experimental curve was 1.009.

The change of volume from 1 to 500 kgm. at 40° was taken as 0.0483, following Amagat. It should be noticed, however, that Amagat gives for the volume at 40° and atmospheric pressure 1.0438, against 1.0483 of the tables of Landolt and Börnstein. In this work the value 1.0483 was taken as the volume at 40°, but Amagat's value for the change of volume 1–500 kgm. was adopted without correction. At low pressures (20°) the present experimental values for the changes of volume were as follows: 1–500 atmos., 0.0530; 500–1000, 0.0294; 1000–1500, 0.0242; 1500–2000, 0.0199. The corresponding values of Amagat are 0.0480, 0.0300, 0.0239, 0.0194. The agreement is fairly good, except for the lowest pressure interval, where as has been pointed out, the present method can only indicate the probable result by an extrapolation. The newly published result of Richards is 0.0430 for the change of volume at 20° for an increase of 500 kgm. of pressure as against 0.0415 listed in the tables of volume.

The volume of methyl alcohol is shown as a function of pressure and temperature in Table II and in Figure 8.

The compressibility, β, of the first five alcohols has been measured by Pagliani and Palazzo,[7] who have collected their results into formulas of the type, $\beta_t = \beta_0 (1 + at + bt^2)$. Their pressure range was 1–4 atmospheres. Within this range the change of compressibility with pressure is negligible. The values found from their formulas (reduction being made from atmos. to kgm.) at 20°, 40°, 60°, and 80°, were 0.0_3113, 0.0_3124, 0.0_3142, and 0.0_3158 respectively. The values found from the present data by the method of computation outlined

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[6] Pierre, Ann. chim. et phys., **15**, 325 (1845).

[7] Pagliani and Palazzo, Mem. R. Acc. Lin., **19**, 279 (1883/84).

TABLE II.

VOLUME OF METHYL ALCOHOL.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0238	1.0361	1.0483	1.0610	1.0737	1.0869	1.1005
500	0.9823	0.9909	1.0000	1.0096	1.0197	1.0404	1.0416
1000	.9530	.9607	0.9684	0.9763	0.9844	0.9929	1.0023
1500	.9276	.9347	.9415	.9481	.9549	.9621	0.9697
2000	.9087	.9151	.9213	.9271	.9331	.9393	.9456
2500	.8930	.8988	.9044	.9098	.9151	.9205	.9260
3000	.8792	.8845	.8897	.8947	.8997	.9047	.9095
3500	.8663	.8712	.8761	.8808	.8854	.8899	.8944
4000	.8551	.8597	.8642	.8687	.8730	.8773	.8814
4500	.8449	.8492	.8535	.8577	.8618	.8657	.8695
5000	.8354	.8395	.8436	.8476	.8515	.8552	.8588
6000	.8192	.8232	.8271	.8307	.8344	.8379	.8412
7000	.8053	.8091	.8129	.8164	.8196	.8228	.8262
8000	.7936	.7972	.8008	.8040	.8070	.8108	.8134
9000	.7827	.7861	.7894	.7924	.7952	.7981	.8013
10000	.7725	.7757	.7788	.7818	.7847	.7876	.7905
11000	.7634	.7663	.7693	.7724	.7756	.7786	.7813
12000	.7559	.7586	.7614	.7647	.7682	.7712	.7738

on page 29 are 0.0_3101, 0.0_3124, 0.0_3137, and 0.0_3147 respectively. The means adopted for this paper are 108, 124, 140, and 152 respectively. There are also values for the compressibility by other observers, but not under conditions so nearly comparable with those here. These values are: 0.0_3104 at 14.7° and 0.0_3221 at 100° between 8.7 and 37 atmos. by Amagat[8]; 0.0_491 at 13.5° and 7.5 atmos. by Grassi[9]; 0.0_3108 at 2.7° and 8 atmos. and 0.0_3120 at 18° and 8 atmos. by Röntgen.[10] The newly published value of Richards for the compressibility at 20° is 0.0_3109 against 0.0_3108 adopted above.

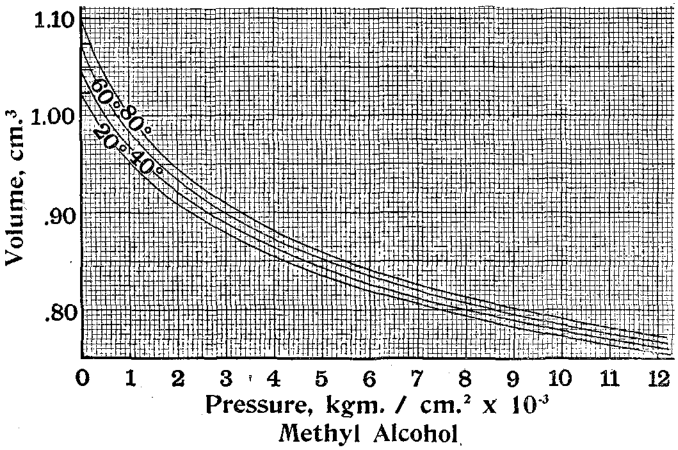

FIGURE 8. Methyl Alcohol. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives the volume at 20°.

There are a few measurements for the specific heat, C_p, at atmospheric pressure; 20.39 between 5° and 10°, 20.28 between 10° and 15°, and 20.76 between 15° and 20° by Regnault[11]; 22.29 between 23° and 43° by Kopp;[12] 21.56 between 5° and 13° by Lecher;[13] and 21.40 between 15.5° and 34.9°, 21.82 between 19.6° and 45°, 22.13 between 18.1° and 50.4°, and 22.77 between 20.5° and 63.2° by von Reis.[14]

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[8] Amagat, Ann. chim. et phys., **11**, 520–549 (1877).
[9] Grassi, Am. chim. et phys., **31**, 437 (1851).
[10] Röntgen, Wied. Ann., **44**, 1 (1891).
[11] Regnault, Ann. chim. et phys., **9**, 322 (1843).
[12] Kopp, Pogg. Ann., **75**, 98 (1848).
[13] Lecher, Wien Ber., **76**, 937 (1877).
[14] von Reis, Wied. Ann., **13**, 447–465 (1881).

These values are in the units of this paper. It will be seen that the results of different observers do not agree within 5%. The results of von Reis, however, do justify us in assuming that C_p increases with rising temperature.

Ethyl Alcohol. More measurements were made on this than on most of the other substances, because it was the liquid with which the preliminary tests of the apparatus were made, but several of the early runs were not carried to completion because of accident. Measurements were made with five fillings of the apparatus. The first of the five fillings was made with the alcohol enclosed in a glass bulb, instead of in a steel one, as in the final experiments. This filling gave all the information desired at the low pressures, and also the thermal dilatation over nearly the entire high pressure range, but was terminated by polarization effects in the manganin. The polarization was found to be due to the breaking of the glass bulb, allowing the alcohol to diffuse to the coil. The readings before the break appeared should be trustworthy. The second of the five sets of measurements was of the compressibility at high pressures, and was completed without accident, but had to be discarded, for reasons that will appear later. The third set was of the dilatation and compressibility at high pressures. This also showed polarization, but not until the very end of the compressibility run. For the second and third runs a steel bulb was used, but the top was put on with soft solder. This soft solder gave way under pressure, allowing the kerosene to mix with the alcohol. The polarization probably did not occur as soon as the solder cracked, because it takes time for the alcohol to diffuse through the kerosene to the coil. The compressibility measurements of the third run are, therefore, more likely to be in error than the dilatation measurements, which were made a considerable time before the polarization appeared. Because the apparatus was the same, the second run is likely to be in error just as the compressibility measurements of the third, the polarization not having time to appear in the second run before the apparatus was taken apart. The early dilatation measurements were retained, therefore, and the early compressibility measurements discarded. The agreement of the early compressibility measurements, which presumably were made on a mixture of kerosene and ethyl alcohol, was good, 0.3%, but they were about 4% higher than the results of the final successful run. The last two of the five runs were carried through without accident, one being of the compressibility and dilatation at low pressures, and the other the corresponding measurements for high pressures. For these, and for all subsequent runs, the top of the bulb was put on with silver solder.

TABLE III.

VOLUME OF ETHYL ALCOHOL.

Pressure. kgm./cm.²	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0212	1.0323	1.0438	1.0557	1.0679	1.0805	1.0934
500	0.9794	0.9873	0.9956	1.0044	1.0135	1.0233	1.0334
1000	.9506	.9570	.9636	0.9707	0.9781	0.9861	0.9944
1500	.9267	.9323	.9380	.9440	.9505	.9572	.9640
2000	.9081	.9131	.9182	.9235	.9291	.9349	.9407
2500	.8923	.8969	.9016	.9064	.9114	.9165	.9216
3000	.8786	.8830	.8874	.8919	.8964	.9010	.9055
3500	.8661	.8702	.8746	.8789	.8831	.8873	.8915
4000	.8545	.8586	.8628	.8668	.8708	.8747	.8787
4500	.8439	.8481	.8521	.8559	.8597	.8634	.8671
5000	.8343	.8383	.8424	.8461	.8498	.8533	.8568
6000	.8178	.8218	.8256	.8291	.8324	.8356	.8387
7000	.8038	.8075	.8110	.8142	.8171	.8200	.8229
8000	.7917	.7952	.7984	.8013	.8038	.8065	.8094
9000	.7807	.7840	.7868	.7893	.7917	.7954	.7973
10000	.7703	.7733	.7760	.7785	.7809	.7835	.7863
11000	.7606	.7633	.7659	.7693	.7713	.7741	.7765
12000	.7521	.7545	.7571	.7600	.7631	.7652	.7682

The average discrepancy of the piston displacement for a rise of temperature of 20° at constant pressure was 0.0016 inch, the mean displacement being about 0.070 inch.

The reduction factor from the mathematical formula for volume in terms of pressure at 40° was 0.9979.

The density at atmospheric pressure and 0° was taken as 0.8063. The constants of the dilatation formula of Landolt and Börnstein were $a = 0.0_2 1022$, $b = 0.0_5 182$, $c = 0$. The values of the volume given by this formula are 1.0212, 1.0138, and 1.0679 at 20°, 40° and 60° respectively. We also have values of Pierre[15], which are 1.0216, 1.0448, and 1.0695 at the same temperatures respectively.

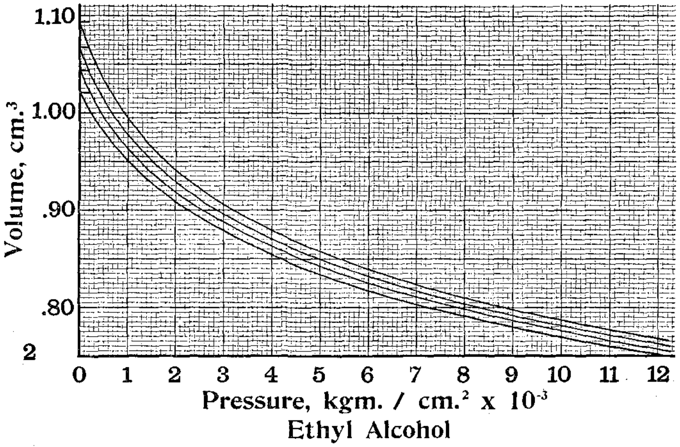

Figure 9. Ethyl Alcohol. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives the volume at 20°.

Amagat gives .0484 for the change of volume from 1 to 500 kgm. His value for the volume at 40° is 1.0442 against 1.0438 above. In the tables, 0.0484 was used as the change of volume 1–500, and 1.0438 as the volume at atmospheric pressure. At 20°, the change of volume between 1 and 500 atmos. was found to be 0.0477 against 0.0438 of Amagat. The numbers for the succeeding 500 atmos. intervals were 0.0287, 0.0236, 0.0193 against 0.0297, 0.0228, and 0.0188 of Amagat.

The volume as a function of pressure and temperature is given in Table III and in Figure 9.

The initial compressibilities at 20°, 40°, 60°, and 80°, computed as described, were found to be $0.0_3 105$, $0.0_3 121$, $0.0_3 138$, and $0.0_3 151$, respectively. The corresponding values of Pagliani and Palazzo are: $0.0_3 102$, $0.0_3 114$, $0.0_3 130$, and $0.0_3 151$. The agreement is as good as could be expected for measurements for this nature. The means shown in the curves are: 0.0_3104, 0.0_3118, 0.0_3135, and 0.0_3151. Comparison may also be made with the values of Amagat,[16] which are

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[15] Pierre, Am. chim. et phys., **19**, 199 (1847).

TABLE IV.

C_p FOR ETHYL ALCOHOL.

Observer.	Temp.	C_p (kgm. cm.).
Regnault [17]	—20°	17.37
	0°	18.81
	40°	22.63
	80°	26.44
Sutherland [18]	80°	24.49
	120°	31.27
Zetterman [19]	20°	31.23
De Heen and Deruyts [20]	40°	20.53
von Reis	15°.7–35°.1	19.93
	20°.7–45°.7	20.78
	18°.4–56°.0	21.32
	19°.8–62°.9	21.79
	20°.5–73°.4	22.44

0.0_4981 at 14° and 0.0_3196 at 99.4° at a mean pressure of 22 atmos. The agreement at the lower temperature is good; the upper temperature is beyond the range.

For C_p at atmospheric pressure we have a number of values which are shown in Table IV. The results are in very bad agreement, as may be seen by plotting them. It is however, perfectly certain that on the whole C_p for ethyl alcohol increases with rising temperature.

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[16] Amagat, l. c. (1877).
[17] Regnault, Mém. Acad., **26**, 262 (1862).
[18] Sutherland, Phil. Mag., **26**, 298 (1888).
[19] Zetterman, Akad. Afh. Helsingfors (1880).
[20] De Heen and Deruyts. Bull de Belg., **15**, 168 (1888).

Propyl Alcohol.— Readings were made on this liquid with two fillings of the apparatus. The first was of the compressibility and the dilatation at the higher pressures, and the second was the complete set, both compressibility and dilatation at both high and low pressures. During the last set of readings, however, the moveable plug

TABLE V.

VOLUME OF PROPYL ALCOHOL.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0173	1.0274	1.0380	1.0493	1.0612	1.0737	1.0865
500	0.9780	0.9864	0.9948	1.0034	1.0121	1.0213	1.0320
1000	.9498	.9571	.9641	0.9710	0.9779	0.9853	0.9934
1500	.9297	.9357	.9415	.9473	.9533	.9594	.9657
2000	.9142	.9192	.9242	.9293	.9344	.9396	.9448
2500	.9011	.9055	.9100	.9145	.9190	.9235	.9282
3000	.8897	.8937	.8979	.9021	.9062	.9103	.9145
3500	.8794	.8833	.8872	.8911	.8949	.8987	.9025
4000	.8700	.8738	.8776	.8813	.8849	.8884	.8919
4500	.8612	.8650	.8688	.8723	.8758	.8791	.8823
5000	.8529	.8567	.8604	.8639	.8671	.8702	.8732
6000	.8390	.8426	.8462	.8494	.8524	.8555	.8579
7000	.8266	.8300	.8333	.8363	.8391	.8416	.8442
8000	.8163	.8193	.8223	.8250	.8277	.8302	.8328
9000	.8069	.8098	.8124	.8150	.8175	.8201	.8230
10000	.7984	.8011	.8037	.8060	.8085	.8112	.8142
11000	.7909	.7934	.7958	.7980	.8004	.8031	.8061
12000	.7840	.7864	.7885	.7905	.7928	.7955	.7982

pinched off at a high pressure because of fatigue, so that there is only one reading for the dilatation at the two highest pressures. The agreement between the two sets at the lower pressures was good enough however, so that it did not seem necessary to set the apparatus up again merely to repeat these last two readings.

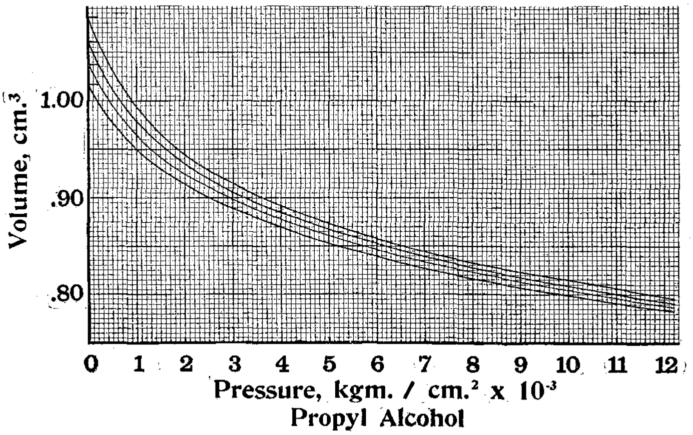

FIGURE 10. Propyl Alcohol. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives the volume at 20°.

The average discrepancy between the piston displacement of the two determinations of the compressibility at high pressures was 0.0026 inch on a total stroke of 2 inches. The average discrepancy in the displacements at constant pressure corresponding to an increase of temperature of 20° was 0.0018 inch on a mean displacement of about 0.070 inch.

The reduction factor from the mathematical formula was 0.8726.

The density at 0° is 0.8179. The constants of the dilatation formula are: $a = 0.0_2774$, $b = 0.0_5497$, $c = -0.0_7141$. These values of a, b, and c are from results of Zahnder,[21] who gives for the density at 0° 0.8177, instead of 0.8179 above. The agreement is virtually perfect. In addition we have data by Naccari and Pagliani[22] who give for the density at 0° 0.8203, and for the volume at 20°, 40°, 60°, and 80°, 1.020, 1.042, 1.064, and 1.090 respectively, against 1.017, 1.038, 1.061, and 1.0865 adopted from Zahnder's formula above.

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[21] Zahnder, Lieb. Ann., **225**, 114–193 (1882).

[22] Naccari and Pagliani. Att. R. Acc. dell. Sc., **16** (Sept. 1881).

Here again it seems as if the agreement between different observers should be better.

The volume of propyl alcohol as a function of pressure and temperature is given in Table V and in Figure 10.

At 40°, Amagat gives for the change of volume between 1 and 500 kgm. 0.0432, which is the value used in the table. He gives, however, 1.0406 for the volume at atmospheric pressure against 1.0380 adopted above. It will be noticed that Amagat's value lies between those of Zahnder and of Naccari and Pagliani. At low pressures and 20° the changes of volume for successive intervals of 500 atmos. were found to be: 0.0407, 0.0245, 0.0202, and 0.0170, against 0.0399, 0.0274, 0.0211, and 0.0176 of Amagat. The agreement for the lowest pressure interval is better than on the average. Richards in his recent paper gives a change of volume between 1 and 500 kgm. considerably smaller than that used here, namely 0.0355 against 0.0393. It should be remembered that the value used in this work for 20° is founded essentially on Amagat's value for 40°; the only difference being a small temperature correction determined from these present data. The disagreement just noted means therefore, that the values of Richards are considerably lower than those of Amagat.

The initial compressibilities at 20°, 40°, 60°, and 80° were as follows; to give the value of ΔV listed in the table 0.0_492, 0.0_3103, 0.0_3118, and 0.0_3130 respectively; the corresponding values of Pagliani and Palazzo are 0.0_490, 0.0_3101, 0.0_3115, and 0.0_3133. The agreement is good. The final values taken as a fair mean were: 0.0_491, 0.0_3102, 0.0_3117, 0.0_3131. Röntgen has also measured the compressibility at atmospheric pressure. His value for 20° would be 0.0_4955, judging from a linear extrapolation from his values at 4° and 18°. Richard's recent value at 20° is 0.0_4873, lower than any other of the values given above.

For C_p at atmospheric pressure we have the following values: – 21° to +12°, 18.02 by Nadejdine[23]; 21° to 23°, 22.99 by Pagliani;[24] 21° to 90°, 23.55 by Lougiunine[25]; and from 16.5° to 42.2°, 20.54, from 20.6° to 53.4°, 21.34, from 20.4°, to 65.2°, 21.99, from 19.5° to 78.5°, 22.63, and from 20.7° to 90.8°, 23.32 by von Reis. These results also indicate a considerable rise of C_p with rising temperature.

Isobutyl Alcohol.— Measurements on this were made with two fillings of the apparatus; the first gave the compressibility and the dilatation over the high pressure range, and the second the compressibility and dilatation over both the high and low pressure ranges. The use of isobutyl instead of normal butyl alcohol was not intended. In ordering the chemicals, normal butyl was not specified, and it was not noticed that the substance sent was isobutyl until all the preparations had been made for a run. This substance has the disadvantage, of not being one of the same series as the four other alcohols. However, it makes little difference so far as the comparison of the results with those of Amagat is concerned, for Amagat did not work with either normal- or iso-butyl alcohol. Furthermore, the use of this substance has proved very instructive in showing that a change in the structural formula changes the properties even at high pressures. It might be expected that high pressures would wipe out variations due to structural differences, but such has not proved to be the case, at least to 12000 kgm.

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[23] Nadejdine, Jour. Russ. Phys. Chem. Ges., **16**, 222 (1884).

[24] Pagliani, N. Cim., **11**, 229 (1882).

[25] Lougiunine, Am. chim. phys., **13**, 289 (1898).

The average discrepancy in the piston displacements of the two determinations of compressibility was 0.0024 inch on a total displacement of 2.0 inches. The mean discrepancy of the displacement for the thermal dilatation for 20° was 0.0008 inch on a mean of about 0.070 inch. The agreement between the two sets of readings for the highest temperature range, 60°–80°, was virtually perfect.

The reduction factor for the mathematical formula was 0.9342.

Landolt and Börnstein's tables do not contain the requisite data for the volume of isobutyl alcohol at atmospheric pressure. The values adopted here were obtained by Naccari and Pagliani, and are apparently the only data which have been published for this liquid. These authors have not expressed their results by a power series, but prefer instead to give the density for a considerable number of temperatures. By interpolation from their results the volumes at 20°, 40°, 60°, and 80° were found to be: 1.0195, 1.0406, 1.0625, and 1.0880. From these results the value of Kaye and Laby for the density at 18° is reduced to 0.8165 at 0°, against 0.8162 of Naccari and Pagliani, virtual agreement.

No measurements of the change of volume of isobutyl alcohol had been made beyond a few kgm. previous to these computations, so the value obtained from the low pressure determinations of the present work was adopted. This was 0.0484 at 40° between 1 and 500 kgm.; not at all an unlikely value, being the same as Amagat's for ethyl alcohol. Compressibility determinations of others at low pressures have shown that isobutyl alcohol has a compressibility considerably higher than that of normal butyl alcohol, so that we are to expect a value higher than we should predict from the behavior of propyl alcohol and a value as large as that of ethyl alcohol does not seem unlikely. The recent work of Richards gives for the change of volume at 20° between 1 and 500 kgm. 0.0355, against 0.0344 used in the tables. The agreement is as close as could be expected when the rough nature

TABLE VI.

VOLUME OF ISOBUTYL ALCOHOL.

Pressure. kgm./cm.²	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0195	1.0300	1.0406	1.0414	1.0625	1.0744	1.0880
500	0.9751	0.9838	0.9922	1.0006	1.0093	1.0184	1.0277
1000	.9486	.9560	.9632	0.9701	0.9768	0.9840	0.9918
1500	.9268	.9330	.9391	.9448	.9505	.9565	.9630
2000	.9097	.9150	.9202	.9252	.9303	.9355	.9410
2500	.8956	.9001	.9048	.9094	.9141	.9187	.9235
3000	.8822	.8867	.8905	.8949	.8994	.9038	.9080
3500	.8705	.8743	.8782	.8823	.8867	.8908	.8947
4000	.8601	.8637	.8673	.8712	.8755	.8794	.8830
4500	.8507	.8541	.8577	.8614	.8655	.8692	.8726
5000	.8409	.8443	.8477	.8513	.8552	.8587	.8619
6000	.8269	.8301	.8335	.8369	.8403	.8433	.8463
7000	.8130	.8163	.8196	.8228	.8260	.8289	.8317
8000	.8028	.8060	.8092	.8123	.8154	.8183	.8210
9000	.7927	.7959	.7990	.8021	.8050	.8079	.8105
10000	.7832	.7863	.7894	.7924	.7953	.7980	.8007
11000	.7742	.7772	.7803	.7833	.7862	.7888	.7913
12000	.7662	.7692	.7722	.7751	.7780	.7805	.7827

of the present determinations at the low pressures is considered; the agreement is better than the agreement of those values which have been taken directly from the work of Amagat.

The volume of isobutyl alcohol as a function of pressure and temperature is shown in Table VI and in Figure 11.

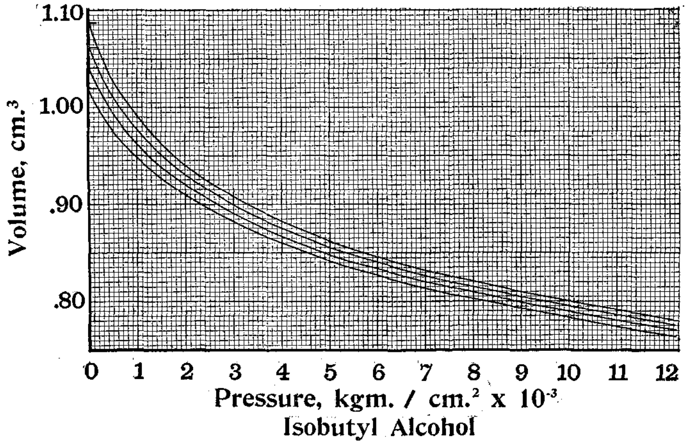

FIGURE 11. Isobutyl Alcohol. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives the volume at 20°.

The compressibility determinations of Pagliani and Palazzo were fortunately made with isobutyl instead of normal butyl alcohol. They give for 20°, 40°, 60°, and 80° the values 0.0_492, 0.0_3103, 0.0_3118, and 0.0_3137. The values required to give the values of ΔV listed in the tables are 0.0_3122, 0.0_3133, 0.0_3144, and 0.0_3164. The discrepancy is large, too large. Instead, however, of taking the average of the discordant results, it was preferred to retain the values consistent with the table, it being understood that the initial values between 1 and 500 kgm., both for the total change of volume and for the compressibility are probably in error. Abnormal variations of compressibility, such as the rapid initial decrease with pressure, may possibly explain part of the discrepancy. We have also a value of Röntgen for the compressibility at 20°, 0.0_496, which is in very much better agreement with the value of Pagliani and Palazzo than the present value. The recent value of Richards is practically the same as Röntgen's.

For C_p we have a larger number of measurements than we should expect from the small number of dilatation measurements. These values are shown in Table VII. When plotted, they show considerable discrepancies. The value at $-5°$ of Nadejdine and that of Pagliani are almost certainly in error. The other points lie roughly on a straight line, such that C_p increases from about 15 at 0° to about 23 at 50°. This is a very considerable increase.

TABLE VII.

C_p FOR ISOBUTYL ALCOHOL.

Observer.	Temperature.	C_p (kgm./cm.)
Longuinine [26]	20°–114°	24.00
	21°–109°	24.94
Nadejdine [27]	−21°– + 10°	17.70
	16°–70°	21.39
	18°–98°	23.24
de Heen and Deruyts [28]	10°	17.49
	40°	22.57
	85°	29.29
Pagliani [29]	26°–30°	23.90

Amyl Alcohol.—Experiments were made on this with three fillings of the apparatus; the first for compressibility and dilatation at low pressures with the large bulb, the second for compressibility and dilatation over the high pressure range, and the third for compressibility and dilatation over both ranges, high and low. The runs were all accomplished without accident of any sort.

The average discrepancy of the piston displacement for compressibility at 40° was 0.0019 inch on about 2 inches. The corresponding discrepancy for changes of temperature of 20° at constant pressure was 0.00206 inch on an mean of about 0.070 inch.

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[26] Louguinine, l. c.

[27] Nadejdine, l. c.

[28] De Heen and Deruyts, l. c.

[29] Pagliani, l. c.

The reduction factor from the mathematical formula was 0.8925, showing that at high pressures amyl alcohol is one of the most incompressible of the twelve liquids.

TABLE VIII.

Volume of Amyl Alcohol.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0181	1.0270	1.0374	1.0476	1.0583	1.0694	1.0814
500	0.9800	0.9880	0.9959	1.0039	1.0122	1.0210	1.0304
1000	.9526	.9593	.9660	0.9724	0.9792	0.9863	0.9936
1500	.9325	.9383	.9440	.9495	.9551	.9608	.9667
2000	.9158	.9210	.9259	.9307	.9354	.9402	.9452
2500	.9015	.9064	.9107	.9149	.9190	.9232	.9277
3000	.8892	.8938	.8979	.9018	.9055	.9094	.9136
3500	.8780	.8823	.8864	.8900	.8935	.8971	.9010
4000	.8682	.8725	.8764	.8800	.8832	.8867	.8903
4500	.8593	.8635	.8673	.8708	.8739	.8772	.8807
5000	.8508	.8548	.8585	.8618	.8648	.8679	.8713
6000	.8373	.8409	.8442	.8471	.8501	.8530	.8560
7000	.8251	.8281	.8310	.8337	.8363	.8390	.8418
8000	.8149	.8176	.8201	.8225	.8250	.8274	.8302
9000	.8044	.8068	.8092	.8114	.8138	.8163	.8190
10000	.7948	.7971	.7994	.8018	.8041	.8066	.8091
11000	.7860	.7886	.7904	.7932	.7954	.7976	.8001
12000	.7782	.7803	.7826	.7854	.7882	.7905	.7926

The density at 0° and atmospheric pressure was taken as 0.8266. The constants of the dilatation formula were: $a = 0.0_{2}89$, $b = 0.0_{6}57$,

$c = 0.0_7118$. This formula of Landolt and Börnstein seems to be taken from Pierre.[30] It gives for the volumes at 20°, 40°, 60°, and 80° the values 1.0181, 1.0374, 1.0583, and 1.0814 respectively. We have also the following values by Pierre and Puchot[31]; 1.0187, 1.0397, 1.0610, and 1.0864. These authors give for the density at 0°, 0.817.

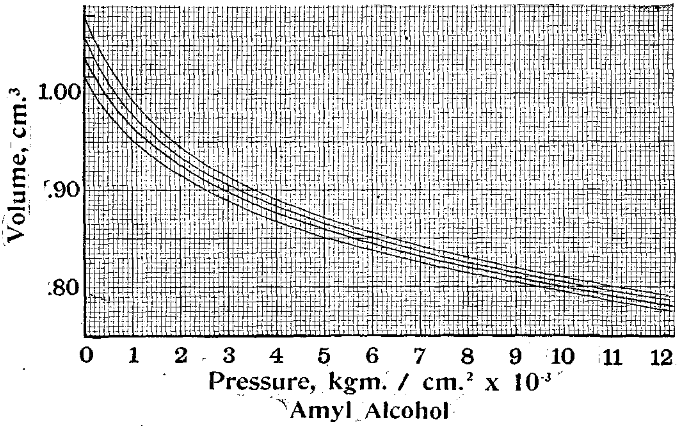

FIGURE 12. Amyl Alcohol. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives the volume at 20°.

Zahnder[32] gives for the density at 0° 0.829, and for the constants of the dilatation formula, $a = 0.0_3919$, $b = -0.0_6461$, and $c = 0.0_7175$? Here again the discrepancies appear to be greater than they should in measurements of this character.

For the change of volume between 1 and 500 kgm. there seem to be no other data as a basis of comparison. Amagat used allyl instead of amyl alcohol for some unknown reason. The only course, therefore, was to accept the value given by this present work for the lower pressure interval, namely 0.0451. That this figure is about correct, however, is spoken for by the rather unusually close agreement of the two measurements of the piston displacement at 20°, 0.389 inch and 0.379 inch, a disagreement of 2.5%.

The volume of amyl alcohol as a function of pressure and temperature is shown in Table VIII and in Figure 12.

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[30] Pierre, l. c. (1847).
[31] Pierre and Puchot, Ann. chim. et phys. (4), **22**, 306?
[32] Zahnder, l. c.

For C_p we have the following values: 19.89 by Kopp[33] for the temperature range 26°–44°; 24.43 by Regnault[34] for the range 10°–117°; 24.51 by Louguinine[35] between 21° and 130°; and 22.97 between 20.5° and 100.1°, 23.64 between 22.2° and 111.6°, and 24.23 between 22.2° and 124.5° by von Reis. These values, which are rather more consistent than usual, show a fairly rapid increase of C_p with temperature.

Ether.—Measurements were made on this liquid with four different fillings of the apparatus. The first two, made before the method had been perfected, were neither complete because of accidents, but between them they give completely the compressibility and the dilatation over the entire high pressure range. The third set of readings was over the low pressure range; this set was repeated without refilling the apparatus. The fourth set, made with the perfected apparatus, was over the high pressure range, and was completed successfully without accident.

There were three sets of piston displacements for the compressibility at 40°. The mean discrepancy of these was $\frac{1}{2}\%$ on the maximum displacement, which is below the average in accuracy. The mean discrepancy of the piston displacements for the thermal dilatation was 0.0022 inch on about 0.070 inch, which is nearly normal.

The reduction factor from the mathematical formula was 1.104, showing that over the entire range ether remains more compressible than any of the other liquids, except ethyl chloride.

The low boiling point of ether at atmospheric pressure, 34.6°, makes it impossible to tabulate the initial properties at the higher temperatures. For this reason many of the curves start at 500 kgm. as the zero instead of atmospheric pressure.

The density of ether at atmospheric pressure and 0° was 0.7382. The three constants of the dilatation formula had the values: $a = 0.0_2 1513$, $b = 0.0_5 236$, and $c = 0.0_7 400$.[36]

The change of volume at 40° between 1 and 500 kgm. was taken from Amagat as 0.0770. Amagat gives for the volume at 40° and atmospheric pressure 1.0672 against 1.0669 of the dilatation formula above. Amagat's value for the volume at 40° and 500 kgm. was corrected, therefore, in accordance with the above. The low pressure measurements of this present work at 20° are in unusually good agreement with Amagat; 0.0665 against 0.0656 for the interval 1–500 atmos.; 0.0370 against 0.0379 between 500 and 1000; 0.0272 against 0.0275 between 1000 and 1500; and 0.0216 against 0.0215 between 1500 and 2000. It may be expected, therefore, that the low pressure values of the various thermodynamic properties are rather more than usually accurate for ether.

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[33] Kopp, l. c.

[34] Regnault, l. c. (1862).

[35] Louguinine, l. c.

[36] Pierre, l. c. (1845).

TABLE IX.

Volume of Ether.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0315	1.0492	1.0669				
500	0.9681	0.9790	0.9899	1.0011	1.0124	1.0247	1.0387
1000	.9363	.9445	.9530	0.9616	0.9707	0.9804	0.9906
1500	.9093	.9153	.9221	.9291	.9364	.9438	.9516
2000	.8871	.8980	.8980	.9038	.9099	.9164	.9223
2500	.8685	.8734	.8785	.8837	.8890	.8943	.8997
3000	.8530	.8576	.8623	.8670	.8718	.8765	.8912
3500	.8395	.8440	.8483	.8526	.8570	.8613	.8654
4000	.8275	.8318	.8359	.8400	.8439	.8478	.8515
4500	.8168	.8209	.8249	.8287	.8324	.8359	.8393
5000	.8071	.8111	.8149	.8186	.8220	.8253	.8284
6000	.7916	.7954	.7989	.8023	.8055	.8085	.8112
7000	.7773	.7806	.7838	.7869	.7899	.7927	.7953
8000	.7645	.7675	.7704	.7732	.7759	.7786	.7813
9000	.7525	.7554	.7580	.7606	.7632	.7658	.7687
10000	.7418	.7444	.7469	.7496	.7520	.7547	.7574
11000	.7312	.7335	.7360	.7388	.7418	.7445	.7469
12000	.7216	.7237	.7261	.7289	.7316	.7342	.7365

The volume of ether as a function of pressure and temperature is shown in Table IX and in Figure 13.

For the initial compressibility at 20° and 40° we have values of Amagat[37]; 0.0_3184, and 0.0_3218 respectively. There are also measurements by Avenarius[38] at 20° and 40°; 0.0_3191, and 0.0_3232, respectively. The values needed to give the changes of volume listed in the table are 0.0_3170, and 0.0_3215. The mean values finally adopted were 0.0_3184, and 0.0_3220. There are also other measurements by Grimaldi[39] and Amagat at temperatures considerably above the normal boiling point, and at accordingly increased pressures, 20 kgm. or so. It was not attempted to make connections with these values.

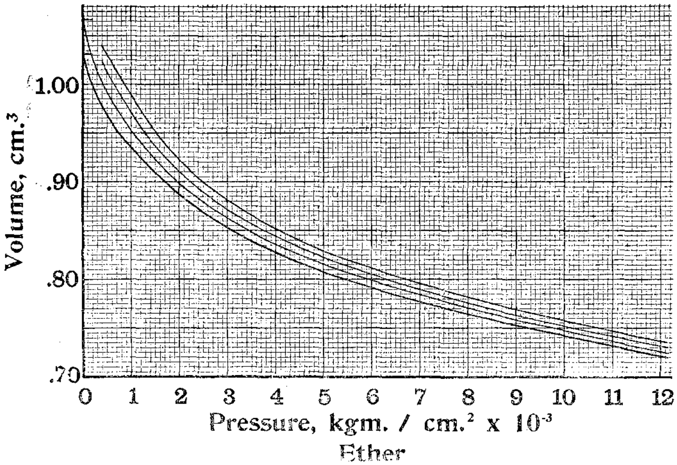

FIGURE 13. Ether. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve gives volume at 20°. The curves for the higher temperatures could not be extended to the origin because of the low boiling point.

C_p has been measured by Regnault[40], who gives 20.97 at 0°, and 21.69 at 30°; by Sutherland[41], whose values are 27.35 at 80°, and 31.86 at 120°; and by de Heen[42], who found 32.59 at 140°, and 41.27 at

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[37] Amagat, l. c., (1877).
[38] Avenarius, Bull. Acc. St. Pet., **10** (1877).
[39] Grimaldi, N. Cim., **19**, 7 (1886).
[40] Regnault, l. c. (1862).
[41] Sutherland, l. c.
[42] de Heen, Bull de Belg., **15**, 522 (1888).

180°. These values all lie roughly on a curve passing through 20.0 at 0°, 23.5 at 40°, 27.0 at 80°, 31.0 at 120°, and 36.5 at 160°. The increase of C_p with temperature is fairly rapid, and becomes more rapid at the higher temperatures.

Acetone.— Two fillings of the apparatus with this substance were made. The high pressure measurements only were made with the first filling, and both high and low pressure readings with the second. Acetone was unique among the liquids used in that it froze under pressure. This was not anticipated nor desired, since, for one thing, it made impossible measurements at the lower temperatures and higher pressures. Furthermore, the separation of the solid phase is apparently accompanied or foreshadowed by complications in the behavior of the liquid, which it was not desired to encounter at the present stage. As a consequence of the freezing, the readings at 20° run only to 8000 kgm. The curves showing the average properties of acetone over the entire temperature range all show a break, therefore, at 8000 kgm. Below 8000 kgm. the average is over the range from 20° to 80°, but above 8000 the range is from 40° to 80°.

No attempt was made to follow out the freezing curve, or to determine accurately the equilibrium pressure at any temperature. It was found, however, that at 40° the freezing pressure is about 10000 kgm. The freezing point of acetone at atmospheric pressure is given by Kaye and Laby at − 95°. This raising of the freezing point by 135° seems to be larger than any previously recorded.

Acetone also showed one other peculiarity. When the liquid was examined at the close of the second run, it was found to be of a slight rose color, and there was a small amount of a fine white precipitate. The rose color deepened in the course of several days to a dirty brown, and the precipitate appeared to increase slightly in quantity. It was thought at the time that this was a chemical reaction brought about by pressure alone, but subsequent investigation showed that the effect was doubtless due to the presence of a slight impurity of phosphorus trichloride, left from the previous run. Phosphorus trichloride when mixed with acetone and allowed to stand at atmospheric pressure was found to produce very slowly the same discoloration and precipitate as observed after exposure to pressure. The effect of pressure apparently is merely to hasten the reaction. In a subsequent experiment, in which every trace of phosphorus trichloride had been carefully removed by prolonged heating, acetone was submitted to the pressures and temperatures of the regular experiment for a day, with absolutely no trace of discoloration. Unfortunately, no examination was made of the condition of the liquid after the end of the first run, so it cannot be told whether the effect was present then or not; probably not. In any event the error so introduced is probably very small, because not more than a very small impurity of PCl_3 could have escaped attention in the weighing. There is, however, a slight possibility that the reaction was catalytic, in which event the error might be greater. The close agreement of the two sets of readings makes this unlikely, however.

The mean discrepancy in the piston readings for compressibility was about 0.005 inch on a total stroke of 2.0 inches. The first compressibility readings were made at 40° The liquid does not freeze until it has been considerably subcooled, so that it was possible to cover the entire pressure range at 40°. But in order to avoid the possibility of the liquid freezing the second time at a less degree of subcooling than at first, the second run was made at 60°, and then reduced to 40° for comparison with the first run. The average discrepancy of the displacements for thermal dilatation was 0.0013 inch on an average of 0.070 inch.

The reduction factor from the mathematical formula was 1.049, showing that acetone is more compressible than the average.

The boiling point of acetone is 56.5°, so that for this reason the initial point of the 80° curve is taken as 1000 kgm. The initial values at 60° were obtained by extrapolation, disregarding the boiling, and strictly apply only to a zero of a few kgm.

The density of acetone at atmospheric pressure and 0° was assumed to be 0.8136. The constants of the dilatation formula were: $a = 0.0_2 1324$, $b = 0.0_5 380$, and $c = -0.0_8 88$. These constants are taken from data of Zahnder, who gives for the density at 0° 0.8125, and for the boiling point 56.3°, values slightly different from those given above.

The change of volume at 40° between 1 and 500 kgm. was taken as 0.0541 from Amagat. His value for the volume at 40° and 1 kgm., however, is 1.0575 against 1.0585 given by the formula above. The probable accuracy of the low pressure measurements of acetone may be judged from a comparison of the values at 20° with those of Amagat. For the successive pressure intervals 1–500, 500–1000, 1000–1500, and 1500–2000 atmos. the present work gave the following changes of volume; 0.0526, 0.0308, 0.0251, 0.0208, while Amagat gives 0.0483, 0.0325, 0.0245, and 0.0196.

The volume of acetone as a function of temperature and pressure is shown in Table X and in Figure 14.

The initial compressibility at 20°, 40°, and 60° may be computed from values of Amagat[43] at 14° and 99°, between 15 and 22 atmos.

TABLE X.

Volume of Acetone.

Pressure. kgm./cm.^{2}	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0279	1.0426	1.0585	1.0752	1.0929		
500	0.9829	0.9931	1.0044	1.0165	1.0297		
1000	.9553	.9638	0.9728	0.9821	0.9924	1.0015	1.0107
1500	.9307	.9385	.9463	.9541	.9619	0.9694	0.9764
2000	.9100	.9173	.9243	.9309	.9374	.9436	.9497
2500	.8927	.8997	.9058	.9116	.9173	.9229	.9285
3000	.8775	.8841	.8897	.8948	.8999	.9051	.9105
3500	.8646	.8707	.8759	.8806	.8853	.8902	.8953
4000	.8532	.8586	.8636	.8681	.8725	.8771	.8819
4500	.8430	.8482	.8528	.8572	.8614	.8657	.8699
5000	.8334	.8380	.8425	.8469	.8510	.8549	.8586
6000	.8175	.8216	.8257	.8299	.8339	.8374	.8403
7000	.8028	.8064	.8103	.8145	.8182	.8215	.8243
8000	.7898	.7933	.7969	.8005	.8039	.8071	.8101
9000			.7847	.7879	.7910	.7942	.7974
10000			.7737	.7767	.7797	.7827	.7857
11000			.7634	.7667	.7697	.7725	.7750
12000			.7546	.7583	.7614	.7638	.7657

by a linear interpolation for temperature and by assuming the variation of compressibility with pressure found here. We obtain in this way from Amagat's data the following values: 0.0_3121, 0.0_3143, and 0.0_3194, respectively. The corresponding values from the present work to give the correct changes of volume are 0.0_3120, 0.0_3146, and 0.0_3167; rather good agreement except at 60°, where for one thing the linear interpolation from Amagat's data would be accountable for

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[43] Amagat, l. c. (1877).

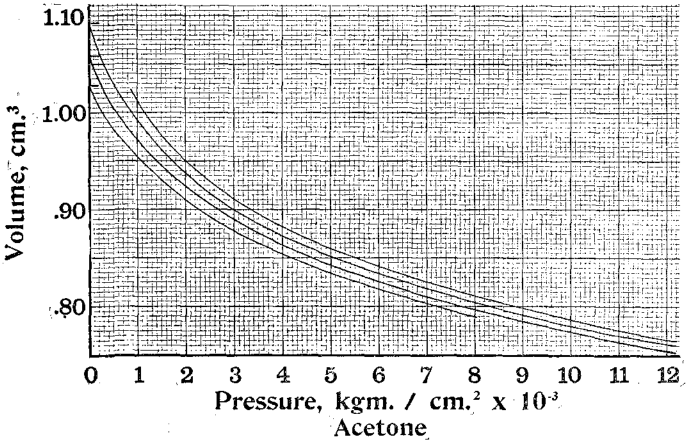

Figure 14. Acetone. Volume at 20°, 40°, 60°, and 80°, against pressure. The lower curve is for 20°. The curve for 20° is terminated at 8000 kgm. because acetone freezes at this point. The curve for 80° starts from 1000 kgm. because acetone boils at atmospheric pressure below 80°.

some of the divergence in the direction shown. The values taken as the average were; 0.0_3120, 0.0_3145, and 0.0_3167.

The only values we have for C_p are by von Reis, who gives 19.43 between 16.4° and 52.6°, 19.65 between 17.6° and 60.3°, 19.68 between 18.9° and 70.2°, and 19.72 between 18.7° and 79.1°. These data show an unusually slight increase of C_p with the temperature.

Carbon Bisulphide.— Three sets of measurements were made on this substance, the first of compressibility and dilatation at low pressures with the larger low pressure bulb, the second of compressibility and dilatation over the high pressure range with the smaller high pressure bulb, and the third of compressibility and dilatation with the smaller bulb over the entire pressure range. All of these runs were made without accident of any sort.

The average discrepancy of the two sets of piston displacements for the isothermal compressibility at 40° was 0.002 inch, on a total stroke of about 2.0 inches. The corresponding discrepancy for the thermal expansion due to a change of temperature of 20° was 0.0013 inch for a mean displacement of about 0.070 inch. As far as self consistency goes, the measurements on carbon bisulphide are among the best of the series.

TABLE XI.

VOLUME OF CARBON BISULPHIDE.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0235	1.0357	1.0490	1.0630	1.0775	1.0928	1.0992
500	0.9865	0.9964	1.0063	1.0158	1.0256	1.0359	1.0473
1000	.9586	.9671	0.9752	0.9829	0.9907	0.9991	1.0083
1500	.9358	.9432	.9504	.9571	.9639	.9709	0.9787
2000	.9173	.9240	.9302	.9362	.9423	.9485	.9552
2500	.9018	.9076	.9133	.9188	.9244	.9299	.9357
3000	.8877	.8928	.8981	.9033	.9084	.9134	.9185
3500	.8756	.8801	.8849	.8897	.8946	.8991	.9035
4000	.8647	.8688	.8732	.8770	.8823	.8855	.8902
4500	.8548	.8586	.8627	.8672	.8714	.8752	.8786
5000	.8453	.8489	.8528	.8570	.8610	.8645	.8676
6000	.8295	.8329	.8367	.8406	.8442	.8472	.8501
7000	.8147	.8184	.8222	.8257	.8290	.8319	.8347
8000	.8022	.8061	.8100	.8131	.8162	.8191	.8220
9000	.7911	.7954	.7989	.8020	.8049	.8078	.8107
10000	.7805	.7844	.7879	.7910	.7940	.7969	.7997
11000	.7715	.7745	.7777	.7809	.7839	.7867	.7894
12000	.7638	.7658	.7682	.7710	.7743	.7772	.7795

The reduction factor from the mathematical formula was 0.9947, showing pretty nearly average compressibility.

The density at 0° and atmospheric pressure was assumed to be 1.292. The three constants of the dilatation formula were as follows: $a = 0.0_21140$, $b = 0.0_5137$, and $c = 0.0_7191$.[44]

Amagat did not measure the volume of CS_2 at 40° at less than 600 atmos. The change of volume between 1 and 500 kgm. was found, therefore, by subtracting the value found here for 500–1000 kgm. from Amagat's value for the change 1–1000 kgm., giving the result 0.0427 between 1 and 500 kgm. Amagat's value for the volume at 40° and 1 kgm. is 1.0484 against 1.0490 given by the formula above. The present low pressure determinations gave results consistently higher than those of Amagat: These values were as follows: 0.0476 (not accurate) for 1–500 atmos., 0.0295 between 500 and 1000, 0.0227 between 1000 and 1500, 0.0198 between 1500 and 2000. For the same pressure intervals Amagat has 0.0387, 0.0277, 0.0222, and 0.0183.

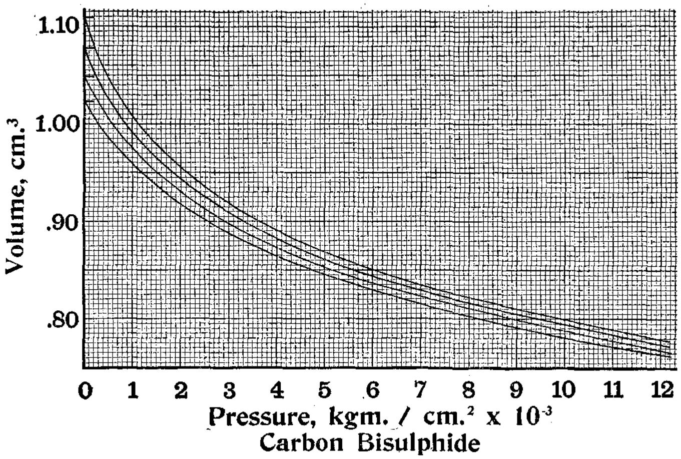

FIGURE 15. Carbon Bisulphide. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve is for 20°.

The volume of carbon bisulphide as a function of pressure and temperature is shown in Table XI and in Figure 15.

The initial compressibilities at 20°, 40°, 60°, and 80° may be found from Amagat to have the values 0.0_490, 0.0_3107, 0.0_3128, and 0.0_3149 respectively. The corresponding values required to give the change of volume listed in the tables are 0.0_492, 0.0_3107, 0.0_3133, and 0.0_3150; good agreement. Röntgen also gives 0.0_487 at 20°. The values shown in the curves are the values to give the correct change of volume, except at 20°, where 0.0_491 was adopted.

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[44] Pierre, l. c. (1845).

There are a number of measurements of C_p for CS_2. Regnault[45] gives 12.68 at −30°, 12.95 at 0°, 13.23 at 30°; Hirn[46] gives 13.13 at 30°; Sutherland[47] 14.33 at 80°, and 15.22 at 120°; and Forch[48] 13.34 at 18°. These results are more consistent than usual, lying on a smooth curve within about 1%. C_p increases with rising temperature, the rate of increase also increasing.

Phosphorus Trichloride.— Two sets of measurements were made on this substance; the first of compressibility and dilatation over the high pressure range, the second over the entire pressure range. Both sets of measurements were made with the smaller high pressure bulb. There was no accident.

The average discrepancy in the piston displacements for the isothermal compressibility at 40° was 0.006 inch on a stroke of about 2.0 inches. The discrepancy in the displacement for the dilatation was 0.0011 inch on 0.070 inch, mean.

The reduction factor from the mathematical formula was 0.9335, showing that PCl_3 is somewhat less compressible than normal.

The density at 0° and 1 kgm. was taken as 1.612. The three constants of the dilatation formula were $a = 0.0_21139$, $b = 0.0_5167$, and $c = 0.0_840$.[49] There are also values for the volume of PCl_3 by Pierre, who gives for 20°, 40°, and 60°: 1.0231, 1.0477, and 1.0747. The corresponding values computed with the above values for the constants are 1.0234, 1.0485, and 1.0752, rather better agreement than we have come to expect.

The change of volume at atmospheric pressure between 1 and 500 kgm. was taken from Amagat as 0.0445. Amagat gives for the atmospheric volume at 40°, 1.0483, in substantial agreement with 1.0485, given by the formula. At 20° and low pressures, the values found for the change of volume for successive intervals of 500 atmos. were, 0.0451, 0.0263, 0.0219, and 0.0187, against the values of Amagat; 0.0396, 0.0282, 0.0224, and 0.0186.

The volume of phosphorus trichloride as a function of pressure and temperature is given in Table XII and in Figure 16.

There seem to be no other determinations of the initial compressibility at atmospheric pressure. Accordingly, the values given in the tables are the values computed in the manner described on page 29 to give the correct changes of volume.

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[45] Regnault, l. c. (1862).
[46] Hirn, Am. d. chim., **10**, 32 (1867).
[47] Sutherland, l. c.
[48] Forch, Ann. d. Phys., **12**, 202 (1903).
[49] Thorpe, Jour. Chem. oc., **63**, 273 (1893).

TABLE XII.

Volume of Phosphorus Trichloride.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0234	1.0358	1.0485	1.0616	1.0752	1.0893	1.1039
500	0.9862	0.9949	1.0040	1.0136	1.0238	1.0346	1.0459
1000	.9593	.9666	0.9739	0.9816	0.9896	0.9980	1.0065
1500	.9382	.9445	.9509	.9575	.9643	.9712	0.9783
2000	.9205	.9262	.9318	.9377	.9437	.9498	.9557
2500	.9057	.9107	.9159	.9212	.9268	.9322	.9375
3000	.8926	.8973	.9022	.9072	.9123	.9171	.9220
3500	.8809	.8853	.8899	.8946	.8994	.9038	.9082
4000	.8705	.8747	.8790	.8836	.8880	.8922	.8962
4500	.8611	.8652	.8693	.8736	.8779	.8819	.8851
5000	.8521	.8560	.8600	.8641	.8682	.8720	.8757
6000	.8375	.8411	.8448	.8486	.8524	.8561	.8596
7000	.8245	.8279	.8313	.8347	.8382	.8416	.8450
8000	.8133	.8165	.8196	.8228	.8260	.8292	.8323
9000	.8029	.8059	.8089	.8120	.8150	.8180	.8210
10000	.7929	.7959	.7989	.8020	.8050	.8080	.8109
11000	.7838	.7866	.7897	.7927	.7957	.7985	.8014
12000	.7761	.7789	.7818	.7847	.7875	.7902	.7928

For C_p there seems to be only one determination, due to Regnault,[50] who finds the mean value 13.67 between 10° and 15°.

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[50] Regnault, l. c. (1843).

Ethyl Chloride.— Two sets of measurements were made on this substance, separated by thirty-six days in time, both with the smaller bulb, and both complete for compressibility and dilatation over the entire pressure range. Both runs were entirely without accident of any kind. The very low boiling point of this substance, 12.5°, and its abnormally high compressibility, made slight changes necessary in the details of the manipulation. The ethyl chloride was furnished by Kahlbaum, in sealed glass bulbs, which were accordingly exposed to an internal pressure greater than atmospheric. The steel compressibility bulb was filled after the ethyl chloride and the bulb had been brought to 0° in an ice bath. The steel bulb was then allowed to warm sufficiently to boil away a slight quantity of the ethyl chloride, when the capillary stem of the bulb was closed by forcing into it a small rubber stopper, considerably too large for it. The friction of the stopper was sufficient to hold it in place against the vapor pressure of the ethyl chloride at room temperature. The first application of a very moderate pressure by the pump was sufficient to drive the stopper into the bulb, where it remained during the rest of the measurements. In this way the filling could be accomplished without the troublesome necessity of cooling the large cylinder below 12° and maintaining it there until pressure could be applied. The small rubber stopper was

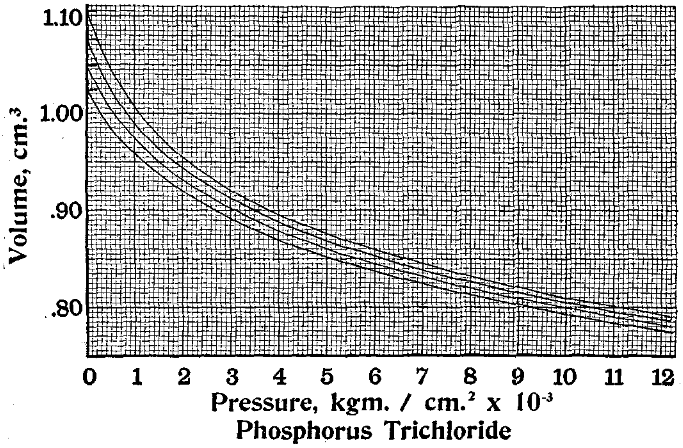

FIGURE 16. Phosphorus Trichloride. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve is for 20°.

weighed, and its weight applied as a correction to the weight of the bulb full of the chloride. The weight was about 0.03 gm.

The piston displacement for isothermal compressibility at 40° showed a mean discrepancy of 0.0016 inch on a total of about 2.1 inches. The discrepancy in the displacements for dilatation averaged 0.002 inch on a mean of about 0.070 inch. The larger discrepancies were at the lower pressures; the mean above 2000 kgm. was 0.001 inch, half as much.

It has already been mentioned that ethyl chloride is so abnormally compressible that it was not possible to use the same formula as for the other eleven liquids to smooth the changes of volume at 40° for the tables. A formula of the same type was used, but with different coefficients (see page 22). The best values of the coefficients for ethyl chloride were found to be, $\alpha = 0.06723$, $\beta = 0.17139$, $\gamma = 0.04030$, and $\delta = -0.06261$. The maximum differences between the observed and the calculated change of volume were $+0.0007$ at 3000 kgm. and -0.0024 at 9000 kgm. The four constants were determined so that the curve passed through the experimental points at 500, 2000, 5000, and 12000 kgm. It should perhaps be mentioned that this formula is merely an empirical expression for the change of volume over the pressure range of the experiment. It has no theoretical significance whatever, and should not be used for purposes of extrapolation. For instance, it is seen immediately that it predicts an impossible behavior at infinite pressure.

The density at 0° was taken as 0.9120. The liquid boils at atmospheric pressure for every temperature within the range of the table. The ordinary dilatation formula would have been valueless, therefore, to fix the volume at any one point of the table. The fiducial point was taken at 40° and 500 kgm. from the data of Amagat,[51] who gives 0.9951 for the volume. The fact that the liquid boils at all temperatures of the table at atmospheric pressure has necessitated starting from 500 or 1000 kgm. as the initial point from which most of the thermodynamic properties have been computed.

The initial compressibilities given in the diagrams for 20° and 40° were taken from Amagat by interpolation and extrapolation from 22 atmos. The values are 0.0_3163, and 0.0_3211; they correspond to pressures somewhat higher than atmospheric.

The volume of ethyl chloride as a function of pressure and temperature is given in Table XIII and in Figure 17.

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[51] Amagat, l. c. (1877).

For C_p we have apparently only one value, again due to Regnault,[52] who gives 16.80 at – 28.4°, a temperature beyond the range of this work.

TABLE XIII.

Volume of Ethyl Chloride.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1							
500	0.9714	0.9831	0.9951	1.0075	1.0105	1.0339	1.0379
1000	.9276	0.9358	.9446	0.9544	0.9647	0.9741	0.9827
1500	.8988	.9059	.9132	.9211	.9296	.9373	.9444
2000	.8774	.8836	.8900	.8967	.9039	.9104	.9264
2500	.8596	.8652	.8709	.8768	.8831	.8888	.8938
3000	.8442	.8492	.8544	.8599	.8654	.8703	.8749
3500	.8311	.8358	.8405	.8456	.8506	.8551	.8591
4000	.8200	.8245	.8289	.8337	.8384	.8426	.8462
4500	.8087	.8129	.8172	.8217	.8262	.8301	.8335
5000	.7994	.8035	.8076	.8118	.8161	.8199	.8230
6000	.7821	.7860	.7900	.7938	.7976	.8010	.8040
7000	.7680	.7718	.7756	.7791	.7825	.7856	.7887
8000	.7561	.7597	.7633	.7666	.7699	.7730	.7762
9000	.7454	.7490	.7522	.7553	.7581	.7611	.7644
10000	.7352	.7385	.7415	.7444	.7473	.7502	.7533
11000	.7259	.7288	.7317	.7347	.7376	.7405	.7432
12000	.7176	.7199	.7225	.7254	.7286	.7314	.7336

Ethyl Bromide.— Two sets of measurements were made on this, both being with the smaller bulb and over the entire pressure range.

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[52] Regnault, l. c. (1862).

An interval of forty days separated the two sets. Both were completed without accident.

The average discrepancy of the piston displacements for isothermal compressibility at 40° was 0.0025 inch on a stroke of 2.05 inches. The mean discrepancy in the displacements for dilatation was 0.0013 inch on a mean of about 0.070 inch.

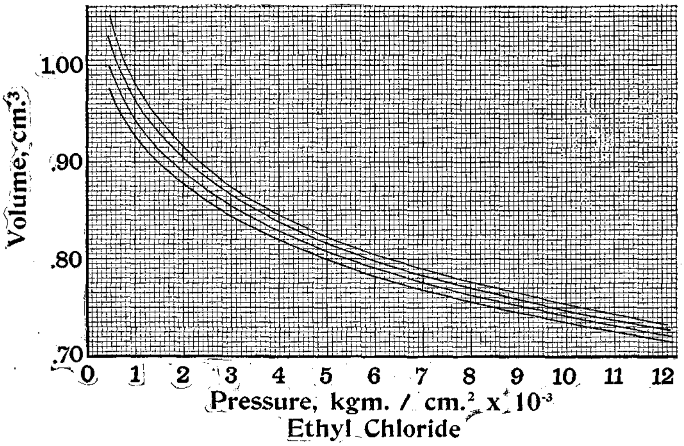

FIGURE 17. Ethyl Chloride. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve is for 20°. The boiling point at atmospheric pressure is at 12°.5, so that it was necessary to take the origin of pressure as 500 kgm.

The reduction factor for passing from the mathematical formula was 1.032, indicating a compressibility somewhat more than normal.

The density at 0° and atmospheric pressure was assumed to be 1.483. The three constants of the dilatation formula were taken as $a_2 = 0.012275$, $b = 0.0_6 4437$, and $c = 0.0_7 0258$.[53] The boiling point of ethyl bromide is 38.4°. The formula gives for the volume at 20° 1.0249, and for the extrapolated value at 40°, 1.0515. Pierre also gives the volumes at 20° and 40°, 1.0275 and 1.0578 respectively. The discrepancies are large, 0.6% at 40°. In this case the preference has been given to the values of Pierre against those of Landolt and Börnstein, because Pierre actually measured the volumes at the temperatures in question, where as the formula of Landolt and Börnstein is directly applicable only at lower temperatures. Furthermore,

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[53] Pierre, l. c. (1845).

Amagat's value at 40°, 1.0583, agrees much more closely with Pierre's than with that given by the formula.

TABLE XIV.

Volume of Ethyl Bromide.

Pressure. kgm./cm.^{2}	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0275	1.0418(?)	1.0578				
500	0.9788	0.9890(?)	1.0004				
1000	.9478	.9557	0.9644	0.9534	0.9824	0.9919	1.0018
1500	.9237	.9309	.9380	.9448	.9517[a]	.9585	0.9654
2000	.9044	.9110	.9175	.9235	.9294	.9350	.9407
2500	.8885	.8950	.9011	.9066	.9120	.9170	.9218
3000	.8776	.8839	.8898	.8951	.9000	.9046	.9090
3500	.8610	.8670	.8725	.8775	.8821	.8864	.8904
4000	.8505	.8556	.8606	.8652	.8696	.8735	.8772
4500	.8410	.8455	.8500	.8543	.8585	.8622	.8657
5000	.8317	.8358	.8399	.8439	.8478	.8514	.8546
6000	.8163	.8201	.8237	.8273	.8307	.8340	.8371
7000	.8020	.8056	.8092	.8125	.8156	.8187	.8220
8000	.7900	.7935	.7968	.7999	.8028	.8059	.8091
9000	.7787	.7821	.7852	.7881	.7911	.7939	.7968
10000	.7686	.7717	.7747	.7777	.7807	.7834	.7858
11000	.7598	.7623	.7653	.7684	.7715	.7741	.7762
12000	.7521	.7546	.7572	.7601	.7633	.7659	.7677

The change of volume at 40° between 1 and 500 kgm. was taken as 0.0573 from Amagat. The values found here at 20° for successive intervals of 500 kgm., beginning at 1 kgm. were 0.0536, 0.0320, 0.0255, and 0.0202, against 0.0492, 0.0322, 0.0248, and 0.0199 of Amagat. The agreement is rather good, except at the lowest pressure, where agreement is not to be expected. The just published value of Richards for the change of volume at 500 kgm. and 20° is 0.0446, against 0.0487 given in the table, which is essentially that of Amagat.

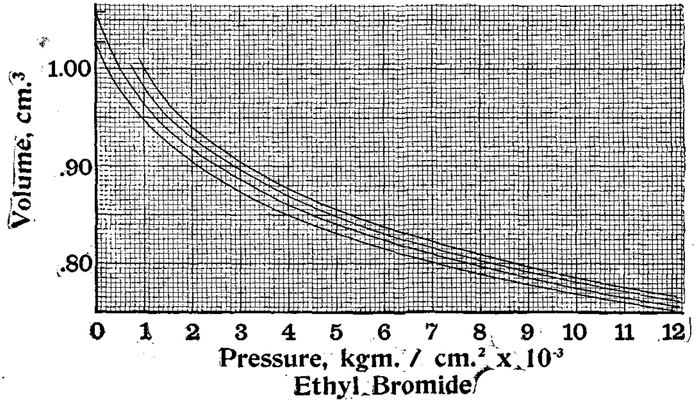

FIGURE 18. Ethyl Bromide. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve is for 20°. The curves for 60° and 80° start from 1000 kgm. because the boiling point at atmospheric pressure is below 60°.

The volume of ethyl bromide as a function of pressure and temperature is given in Table XIV and in Figure 18.

When the computations of this paper were made the only value for the compressibility at low pressures was that of Amagat at 99° and a mean pressure of 20 atmos. This was too far removed from the range of this paper to justify any correction. The initial compressibilities at 20° and 40° were taken, therefore, so as to give the values of the changes of volume listed in the tables; 0.0_31248 and 0.0_31476 respectively. At 60° and 80° the compressibility is not given for pressures lower than 1000 kgm. The recently published data of Richards give for the compressibility at atmospheric pressure and 20° the value 0.0_3106, considerably lower than the value given above.

Regnault[54] gives a few values for C_p at atmospheric pressure; 14.62 between 5° and 10°, 14.42 between 10° and 15°, 14.54 between 15° and 20°. The temperature range is too small and the variations too great to enable us to decide whether C_p really increases with temperature or not.

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[54] Regnault, l. c. (1843).

TABLE XV.

Volume of Ethyl Iodide.

Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Volume.						
	20°.	30°.	40°.	50°.	60°.	70°.	80°.
1	1.0214	1.0324	1.0438	1.0555	1.0677	1.0803	1.0935
500	0.9785	0.9880	0.9979	1.0081	1.0180	1.0276	1.0366
1000	.9502	.9584	.9665	0.9746	0.9825	0.9900	0.9969
1500	.9277	.9345	.9412	.9479	.9544	.9605	.9663
2000	.9092	.9150	.9209	.9266	.9323	.9375	.9425
2500	.8937	.8991	.9043	.9094	.9143	.9188	.9231
3000	.8802	.8851	.8899	.8945	.8988	.9028	.9065
3500	.8684	.8728	.8770	.8811	.8848	.8883	.8917
4000	.8583	.8621	.8659	.8694	.8728	.8759	.8790
4500	.8487	.8522	.8558	.8592	.8624	.8653	.8681
5000	.8394	.8429	.8463	.8496	.8529	.8557	.8581
6000	.8236	.8271	.8306	.8340	.8371	.8398	.8418
7000	.8093	.8129	.8164	.8193	.8220	.8243	.8264
8000	.7968	.8006	.8038	.8065	.8090	.8113	.8134
9000	.7856	.7902	.7922	.7945	.7967	.7989	.8013
10000	.7755	.7789	.7817	.7841	.7862	.7885	.7909
11000	.7665	.7694	.7722	.7747	.7771	.7794	.7817
12000	.7588	.7611	.7638	.7667	.7693	.7717	.7737

Ethyl Iodide.— Two runs were made on this with the smaller high pressure bulb over the entire pressure range, without accident. The mean variation of the displacement readings for compressibility at 40° was 0.0020 inch on a total of 2.05 inches. The agreement is quite perceptibly better than the average. The thermal dilatation measurements show however, by far greater disagreement than any other of the twelve liquids. 0.0030 inch on a mean of 0.070 inch. The discrepancy was greater at the higher temperatures; 0.0012 inch from 20° to 40°, 0.0026 inch from 40° to 60°, and 0.0052 inch from 60° to 80°.

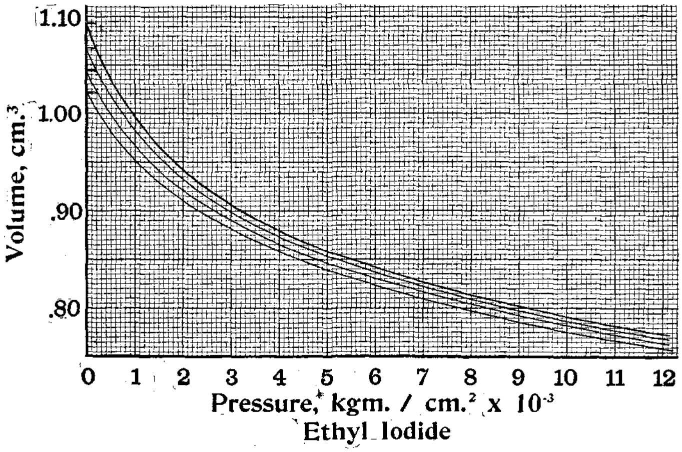

FIGURE 19. Ethyl Iodide. Volume at 20°, 40°, 60°, and 80° plotted against pressure. The lower curve is for 20°.

The reduction factor from the mathematical formula was 0.9817, showing slightly less than normal compressibility.

The density at 0° was assumed to be 1.973. The three constants of the dilatation formula were as follows: $a = 0.0_21054$, $b = 0.0_6636$, and $c = 0.0_71004$.[55] Pierre also gives values for the volume at 20°, 40°, and 60° respectively; 1.0232, 1.0484, 1.0749, against 1.0214, 1.0438, and 1.0677 given by the formula. The agreement should be better. Probably Pierre's values are better, as is shown by the agreement of Amagat's value at 40°, but Pierre does not give the volume at 80°; so the value of the formula was accordingly selected.

The change of volume between 1 and 500 kgm. was taken from Amagat as 0.0459. Amagat gives for the initial volume at 40° 1.0486 against 1.0438 of the formula. The low pressure determinations of the change of volume at 20° gave for successive intervals of 500 atmos. 0.0509, 0.0278, 0.0233, and 0.0195 respectively, against 0.0404, 0.0289, 0.0227, and 0.0184 of Amagat. The agreement is fair, except of course for the first interval. The recent work of Richards gives 0.0370 for the change of volume at 20° and 500 kgm. against 0.0429 given in the tables. It is evident that the work of Richards and of Amagat is here in very essential disagreement.

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[55] Dobriner, Lieb. Ann., **243**, 1–23 (1888).

The volume of ethyl iodide as a function of pressure and temperature is given in Table XV and in Figure 19.

When these computations were made there were no values of the initial compressibility with which to compare the results. Accordingly the value necessary to give the change of volume in the tables was used in every case. The recent data of Richards give 0.0_491 for the initial compressibility at 20°. This is considerably lower than the value shown in the curves, 0.0_4108. This simply means again that Richards finds a much smaller compressibility than Amagat.

C_p for ethyl iodide has apparently been determined only by Regnault[56]. He gives 13.19 at – 30°, 13.60 at 0°, 14.03 at 30°, and 14.44 at 60°. The increase with temperature is linear.

V. Discussion of Thermodynamic Properties.

In the following sections the general characteristics of the several thermodynamic functions will be discussed. The discussion will include suggestions as to what modifications it may possibly be necessary to make in our conceptions of a liquid, or what features that we have neglected at low pressures it may be necessary to emphasize at high pressures. Incidentally in the course of the discussion, suggestions will be made bearing on the theory of liquids, but any detailed examination of the problems that confront us in trying to frame a theory of liquids valid for high pressures will be reserved for section VI.

Volume.— The tables and diagrams of volume as a function of pressure and temperature have already been given, but with little comment.

One of the significant facts about the change of volume is in regard to the volume at infinite pressures, that is, the so-called volume of the molecules themselves, which is one of the quantities entering into nearly every theory of liquids. In particular, Tumlirz[57] and

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[56] Regnault, l. c. (1862).

[57] Tumlirz, Sitz. k. Akd. Wiss. Wien, **118**, 1–39 (1909).

Tammann[58], in their recent theories, give values for the volume at infinite pressure. The values are listed in Table XVI for four of the liquids here investigated, and compared with the volumes found experimentally at 12000 kgm. and 20°. The observed value for ether at 12000 kgm. is actually less than the value predicted by either of these theories for an infinite pressure; and for the other liquids the observed value is close to the predicted minimum. This result serves to emphasize more strikingly a point made in the preceding paper on water; namely, that at high pressures a liquid is more compressible than we might expect from its behavior at low pressures.

TABLE XVI.

Substance.	Volume.		
	Calculated, $p = \infty$		Observed. $p = 12000$.
	Tumlirz.	Tammann.	$t = 20°$
Methyl Alcohol	0.6970	0.7255	0.7559
Ethyl Alcohol	0.7037	0.7380	0.7521
Ether	0.7274	0.7246	0.7216
Carbon Bisulphide	0.6881	0.7246	0.7638

One line of inquiry is worth mentioning which seemed promising before the experiments were performed. The question, suggested by such properties of the atom as the atomic re-fraction, was this; is it possible at the higher pressures to assign to each atom its own specific volume as a function of the pressure, and so compute the volume of a compound at any pressure from its chemical constitution? But an examination of the changes of volumes of the two isomers, ether and isobutyl alcohol, shows that the supposed relation does not hold. For if we compare the volumes of equal weights, that is the volume occupied by the same number of atoms, we shall find that at atmospheric pressure the ratio of the volume of ether to that of isobutyl alcohol is 1.102, and that at 12000 kgm. it has dropped to 1.038. If the above relation were true, this ratio would be unity.

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[58] Tammann, l. c., see also Korber.

The fact that the ratio is approaching unity shows that the atoms are approaching the behavior suggested above, but if they ever reach it, it can only be at pressures considerably beyond those reached here.

In Table XVII are given the average volumes of the twelve liquids between 20° and 80°. This table corresponds to the diagrams for the average between 20° and 80° of the other thermodynamic properties; it will prove useful in plotting any of the average properties against volume, which may in some cases give more significant results than when pressure is used as the independent variable, as here.

Thermal Expansion. — The mechanism ordinarily assumed in explanation of thermal expansion is as follows. Any liquid is continually striving to expand, because of the thermal agitation of its molecules. The tendency to expand is resisted by two forces, the external pressure, and the forces of attraction between the molecules. An increase in temperature means an increase in the expanding force, which results in an increase of volume. This increase of volume would be expected to be greater if the force preventing expansion were less. Now the force preventing expansion becomes less as the volume becomes greater, because the cohesional forces decrease as the volume, or the distance apart of the molecules, becomes greater, The result is that the thermal dilatation increases with increasing temperature, that is with increasing volume. In other words, $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ is positive. Furthermore, as pressure increases, the force resisting expansion increases because of the decreased distance apart of the molecules, so that we are to expect a decreased dilatation at the higher pressures.

An examination of the curves for dilatation against pressure shows that these expectations are much more nearly fulfilled as regards the behavior of the dilatation with respect to pressure than with respect to temperature.

The general tendency of the dilatation of the separate liquids (Folder I, Figures 20 to 31) is to decrease with rising pressure. The decrease is very much more rapid at the lower than at the higher pressures. But beyond this general fact the curves give only an impression of bewildering complexity, crossing and recrossing in apparent disorder at the higher pressures. It is possible to find many instances where the dilatation increases with rising pressure over a range of several thousand kilograms, ultimately, however, to decrease again. One of the most striking examples of this is the 20° curve for carbon bisulphide; other well marked examples are afforded by ace-

TABLE XVII.

AVERAGE VOLUME BETWEEN 20° AND 80°.

Pressure.	Average Volume between 20° and 80°.											
kgm./cm.²	Methyl Alcohol.	Ethyl Alcohol.	Propyl Alcohol.	Isobutyl Alcohol.	Amyl Alcohol.	Ether.	Acetone.	Carbon Bisulphide.	Phosphorus Trichloride.	Ethyl Chloride.	Ethyl Bromide.	Ethyl Iodide.
1	1.0621	1.0573	1.0519	1.0538	1.0498	1.085 †	1.075 †	1.0664	1.0637		1.0755 †	1.0575
500	1.0119	1.0064	1.0050	1.0014	1.0052	1.0034	1.0165	1.0169	1.0161	1.0097	1.0120 †	1.0076
1000	0.9776	0.9725	0.9716	0.9712	0.9731	0.9635	0.9830	0.9835	0.9829	0.9552	0.9748	0.9736
1500	.9486	.9453	.9477	.9449	.9496	.9305	.9536	.9573	.9583	.9216	.9446	.9470
2000	.9271	.9244	.9295	.9254	.9305	.9047	.9299	.9363	.9381	.8969	.9226	.9259
2500	.9095	.9069	.9147	.9096	.9146	.8841	.9106	.9188	.9216	.8767	.9052	.9084
3000	.8943	.8920	.9021	.8951	.9014	.8671	.8943	.9031	.9073	.8596	.8933	.8934
3500	.8803	.8788	.8910	.8826	.8895	.8525	.8800	.8896	.8946	.8451	.8757	.8801
4000	.8682	.8666	.8810	.8716	.8793	.8395	.8676	.8775	.8834	.8331	.8639	.8687
4500	.8572	.8555	.8718	.8617	.8700	.8281	.8565	.8667	.8734	.8211	.8534	.8584
5000	.8471	.8455	.8631	.8514	.8611	.8178	.8460	.8565	.8639	.8112	.8432	.8488
6000	.8302	.8282	.8485	.8366	.8467	.8014	.8289	.8398	.8486	.7931	.8267	.8327
7000	.8157	.8134	.8354	.8224	.8335	.7863	.8136	.8247	.8348	.7784	.8120	.8179
8000	.8035	.8006	.8246	.8119	.8226	.7729	.8000	.8121	.8228	.7662	.7996	.8051
9000	.7920	.7890	.8150	.8016	.8117	.7606	.7911°	.8009	.8120	.7549	.7878	.7935
10000	.7815	.7783	.8063	.7920	.8020	.7496	.7797°	.7901	.8019	.7443	.7772	.7832
11000	.7723	.7695	.7985	.7828	.7931	.7391	.7692°	.7805	.7926	.7346	.7680	.7741
12000	.7668	.7602	.7911	.7745	.7854	.7291	.7602°	.7717	.7845	.7256	.7599	.7663

† Extrapolated to 50°.

° Average between 40° and 80°.

tone and ethyl iodide. The rule, therefore, that dilatation decreases with rising pressure has many exceptions.

As regards the behavior of dilatation with respect to temperature, it is a striking fact that at the higher pressures the dilatation is usually greatest at the lower temperatures, instead of at the higher temperatures, as at atmospheric pressure. Every one of the diagrams shows this. What is more, the reversal of the effect in almost all cases takes place sharply at a definite pressure, the same for all temperatures; or in other words, the curves for the four temperatures, 20°, 40°, 60°, and 80° all cross at approximately the same point. This is exhibited still more strikingly in the curves for the average C_p of the twelve liquids (Figure 99). We have the thermodynamic formula $\left(\frac{\partial C_p}{\partial p}\right)_\tau = -\tau\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$, so that when the average value of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ over the temperature range vanishes C_p will have a maximum. All of the curves show this maximum at the same pressure.

This universal reversal in the sign of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ is a fact of no little interest and importance, and seems not to have been anticipated. In fact, the natural hypothesis of the contrary behavior, namely that at high pressures $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p = 0$, has recently been made the basis of an empirical theory of liquids by Tammann as was pointed out in the introduction. This hypothesis of Tammann is based on very plausible evidence from the data of Amagat, which seem to indicate that at high pressures $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ does vanish. But this apparent evidence from Amagat is founded on an accident, and a rather remarkable accident, as will be evident from an inspection of Figure 99 for C_p. The reversal in the sign of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ takes place for nearly all the twelve liquids at pressures which are in the neighborhood of 3000 kgm., the maximum pressure reached by Amagat. As a matter of fact, Amagat's data do show in some cases the reversal of the effect, but the experimental error was fairly high, and Amagat himself did not credit the reversal as genuine. Only six of Amagat's liquids can yield evidence on this point, because they are the only ones for which readings were made at more then two temperatures; of these six liquids, ether, ethyl alcohol and carbon bisulphide show the reversal at 3000 kgm., while methyl and propyl alcohol and ethyl chloride show a positive $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ over the entire range.

At higher pressures there is in many cases, though not in all, a tendency for the effect to reverse again, that is, for the dilatation to again become greater at the higher temperature. The pressure of this second reversal is in the vicinity of 9000 or 10000 kgm. This fact also is indicated by the C_p curves.

A comparison of the curves for the different liquids shows a few features of interest. In the group of the five alcohols, isobutyl stands out as being the simplest, there being none of the crossing and recrossing which the others exhibit at the higher pressures. In this respect the four normal alcohols are much alike in their high pressure complications. That isobutyl alcohol should be different from the other alcohols was not anticipated before these experiments were made, since it seemed probable that at high pressure the effect of structural differences in the molecule would be eliminated. The effect of structural difference is also shown by a comparison of isobutyl alcohol and ether, since these two have the same formula, $C_4H_{10}O$. The high pressure effects are more complicated for ether. Acetone is peculiar in the wide divergence of the curves at the maximum pressure. It does not show any unusual effects in the neighborhood of the freezing point. It will be seen later that the specific heats are the quantities most susceptible to irregularities at the freezing point. Carbon bisulphide is remarkable for the curve at 20°, which shows a large increase of dilatation between 6000 and 9000 kgm., and also in this region shows a much greater dilatation than the curves for the higher temperatures. Phosphorus trichloride is the only one of the twelve liquids which behaves approximately as had been expected, since it shows little irregularity at the high pressures, and $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ nearly vanishes. The curves for the three ethyl halogen compounds do not show any particular progressive change of character such as one might expect, except with regard to the pressure at which $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ reverses in sign. This is shown best on the diagram for C_p. The pressure for reversal is lower than for most of the other liquids, and becomes less as the molecular weight of the compound increases. This may be because the increased molecular weight produces an increase in the cohesional force of attraction, and a consequent increase in the internal pressure, with the same effect as an increase of the external pressure. Ethyl iodide is remarkable for the low value of the dilatation at 80° at 6000 kgm. The minimum as 0.00017, and is lower than for any of the other liquids at the highest pressures.

It is evident that in order to explain these complicated facts we shall have to give up the simple picture of things that led us to expect the dilatation always to increase with increasing temperature and to decrease with increasing pressure. A natural way of modifying our conceptions so as to make room for these effects would seem to be as follows. We are to think of the molecules as having complicated shapes, or what for our purposes would amount to the same thing, of being surrounded by fields of force different in different directions. This concept does not seem to be a forced or an unlikely one; it must certainly be the fact for liquids which eventually crystallize under pressure, and for liquids which do not crystallize it is hard to conceive how an assemblage of atoms with unlike chemical affinities can coalesce into a molecule identical in all aspects. The effect of pressure on such an assemblage of molecules may be somewhat as follows. As the molecules are crowded closer together, the localized centers of force on the corners and edges play an increasingly individualized part, so that for small volumes we can no longer regard the molecules as centers of force and the cohesional force as a function of the mean distance apart of the molecules, but the orientation of the molecules with respect to each other begins to have its effect. Now we suppose that the natural tendency of the molecules in a liquid under normal conditions is to arrange themselves at haphazard with relation to each other. But as the constraints increase with decreasing volume, the characteristic shape compels an arrangement not entirely at haphazard, but such that the molecules fit into each other to some extent. Now it is evident that with the increasing orderliness of the arrangement of the molecules the relative positions of the localized centers of force will change. What is more, it is conceivable that the change in position of the force centers will be such that on the average the mean attraction between the molecules shall decrease with decreasing volume instead of increasing as is usual. Still more would this be true if there are centers of both attraction and repulsion in the molecule, as seems very likely from our present views of the electrical nature of matter. In this case we should expect the greater dilatation to be at the smaller volumes, because the cohesional force is less. But since the smaller volumes correspond to the lower temperatures, we have the effect already met, namely, that the dilatation is greater at the lower temperatures. The explanation of the sometimes increasing dilatation with rising pressure is similar. With increasing pressure the molecules may be forced to assume positions more regular in arrangement, of less volume, and of less average attractional force also. The number of possible positions which the molecules may assume in this attempt to adapt themselves to the diminished space at their disposal may evidently be very great with molecules of at all complicated shapes, so that there is here the possibility of such very complicated dilatation curves as we actually have.

Another possible explanation which in the end amounts to very much the same thing for some purposes, is that of association. If we suppose that association takes place with decrease of volume, and that the amount of association is increased with increasing pressure, and is decreased with rising temperature, then we have also the possibility of decreasing dilatation with rising temperature and of increasing dilatation with rising pressure. In this case the phenomena are essentially similar to those of solidification under pressure of a mixture of different liquids. Such a case has already been investigated for kerosene, which shows the same general features as above. The phenomena for kerosene are much simpler than for these liquids, however. It is evident that one simple association (such as single to double molecules) is not sufficient of itself to explain the facts, but if we assume several associated molecules of varying degrees of complexity, and that the relative numbers of these change with pressure and temperature, the explanation would account for several of the facts actually found. But reasons will be given in the next section, for supposing that association cannot have a very large part in the phenomena at high pressures.

We now turn our attention to Figure 32, which gives in one diagram the average dilatation between 20° and 80° for all twelve liquids. The origin of each of these curves has been displaced downwards one unit with respect to the one above it. The origin is so located that the value of the thermal dilatation for each of the liquids at 12000 kgm. is between 0.0002 and 0.0003. The scale of the drawing is indicated at the side. The immediately striking feature is that the curves are nearly equi-spaced at the higher pressures. Approximate equal spacing of the curves would of course be a consequence of their all approaching zero, but this is not the entire effect by any means. For instance, the ratio of the dilatation of ether to that of amyl alcohol at low pressure is 1.50 while at 12000 kgm. pressure it has dropped to 1.03. (In the computation for ether, the initial dilatation at 20° was used.) This is only the first example of many we shall meet tending to show that at high pressures liquids lose the individual differences which characterized them at low pressures, and become more alike.

The very gradual change of dilatation with pressure at the high pressures was also a surprise. The grand average for the dilatation at 12000 kgm. for the twelve liquids is 0.00025, about 40% more than that of mercury at atmospheric pressure and much higher than for any solid except possibly gutta-percha. It is ten times as great as that of aluminum, for example. The very slow change of dilatation with pressure is shown in Table XVIII, giving the pressure at which the dilatation has twice its value at 12000 kgm. The table shows that between atmospheric pressure and 2400 kgm. (on the average) the dilatation falls to about 0.4 its initial value, while over a further pressure range four times as large it falls off only an additional 50%. From these data it would appear that the dilatation must remain very considerable for pressures far in excess of those reached here. The approximate equality of the pressures for the twelve liquids at which the dilatation is double its value at 12000 kgm. indicates that the liquids behave similarly over much the greater part of the pressure range.

TABLE XVIII.

PRESSURE AT WHICH THE THERMAL EXPANSION IS TWICE AS LARGE AS IT IS AT 12000 KGM.

Substance.	Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$	Substance.	Pressure. $\frac{\text{kgm.}}{\text{cm.}^2}$
Methyl Alcohol	2150	Acetone	2900
Ethyl "	2000	Carbon Bisulphide	2900
Propyl "	2250	Phosphorus Trichloride	2200
Isobutyl "	1850	Ethyl Chloride	2750
Amyl "	2080	Ethyl Bromide	3100
Ether "	2750	Ethyl Iodide	2400

The curves in general fall with increasing pressure, but all the alcohols show a tendency to become stationary or to rise; while carbon bisulphide shows an extended stationary region between 6000 and 10000 kgm. but beyond 10000 its curve drops with unusual steepness.

The four normal alcohols show a dilatation continually decreasing with increasing molecular weight. But isobutyl alcohol shows the effect of its different structure by a higher dilatation than its position in the series would indicate. A comparison of isobutyl alcohol with ether, its isomer, shows that although initially the dilatation of ether is greater, at 12000 kgm. it has become less. This emphasizes again that the structure of the molecule continues to play a part even at high pressures. It is not to be wondered at in view of the suggested explanation of the complicated nature of the dilatation curves. For molecules of the same atomic formula, but of different structure, are to be thought of as possessing different shapes, and it is at high pressures that the effect of shape is greatest.

Isothermal Compressibility.— The isothermal compressibility is shown in Folder II; the values for the liquids separately at 20° intervals in Figures 33 to 44, and in Figure 45 the average results over the entire temperature range are collected into a single diagram for the twelve liquids.

The curves require a word of explanation. Up to 4000 kgm. the curves for each liquid are drawn for the four different temperatures, but at pressures higher than 4000 kgm. the curves would be so close together, sometimes crossing each other, that it would have been very confusing to draw them on the same scale. Therefore, at higher pressures, the only complete curve given is for 40°, while in the upper part of the diagram are shown on a larger scale the differences between the compressibilities for 20° intervals. The zero of these difference curves is drawn as a heavy line. Negative ordinates of the difference curve 20°–40° indicate that the compressibility is less at 20° than at 40°; positive values for the difference curve 40°–60° indicate a greater compressibility at 60° than at 40°, and similarly positive differences 60°–80° mean a greater compressibility at 80° than at 60°. To find the compressibility at 20°, one adds to the value obtained from the 40° curve the ordinates of the difference curve 20°–40° (this ordinate is usually negative); the compressibility at 60° is found by adding to the ordinate of the 40° curve that of the 40°–60° curve, and the compressibility at 80° by adding to that at 60° (obtained as above) the ordinate of the 60°–80° curve. A larger compressibility at 20° than at 40° is indicated by the difference curve 20°–40° rising from below and crossing the axis, while a smaller compressibility at 60° than at 40° is similarly shown by the curve 40°–60° crossing the axis from above. The mutual crossing of the difference curves, from this point of view, is not especially significant; it is the crossing of the axis that counts. The meaning of the mutual crossing of the curves 40°–60° and 60°–80°, for example, would be that over the temperature range 40°–80°, $\frac{\partial^2}{\partial\tau^2}\left[\left(\frac{\partial v}{\partial p}\right)_\tau\right]_p = 0$.

The general behavior of the compressibility that one is prepared to expect from experiments at low pressures is a decrease with increasing pressure, and an increase with increasing temperature. The familiar conception of a liquid due to van der Waals is competent to explain this. The difference between the free space open to the molecules for their temperature vibrations and the total volume of the liquid is equal to the volume of the molecules themselves, or else to a small multiple of it. When pressure is increased, therefore, the free space diminishes much more rapidly than the total volume. Now we may suppose the pressure exerted by a liquid to be due in large measure to the bombardment of the walls by the temperature agitation of the molecules. The more frequent the collisions, the greater the pressure. Now at constant temperature, the number of collisions is inversely as the free space. At higher pressures, that is smaller volumes, a given diminution of total volume implies a greater diminution of the free volume than at low pressures, and therefore a greater increase of pressure. So that at small volumes (high pressures) a given decrease of volume carries with it a greater increase of pressure than at larger volumes (lower pressures), or in other words, the compressibility decreases with rising pressure. The increase of compressibility at higher temperatures is to be explained in the same way. At higher temperatures (constant pressure) the volume is greater and we expect greater compressibility. This however, is not the only element involved in the change of compressibility with temperature; there is also a temperature effect as such. It was found in the paper on water that, at equal volumes, the compressibility was always less at the higher temperatures. The reason is evidently the more rapid agitation of the molecules. At equal volumes, a given decrease in the total volume, and so of the free volume, will produce the same proportional increase in the number of impacts at high and low temperatures, but at high temperatures each impact involves a greater change of momentum, with the result that at the higher temperatures a given decrease of volume produces a greater increase in pressure, which means a lower compressibility.

A detailed examination of the curves shows that these expectations are justified to a rather greater degree than was the case for the thermal dilatation. The compressibility decreases with rising pressure for all twelve liquids at 40°. A careful analysis of the difference curves will show that this is also the case for the other three temperatures, with a single exception. For carbon bisulphide, between 11000 and 12000 kgm. there is an increase of compressibility from 0.0_3101 to 0.0_3102 at 60°, and from 0.0_3103 to 0.0_3105 at 80°. The change is very small and may well be due to experimental error. The difference curves show as bewildering small variations with temperature as the thermal dilatation curves, so that it is hopeless to try to explain them in detail at this stage of our knowledge. It would seem, however, that an actual reversal of the effect, that is, a smaller compressibility at a higher temperature, does not occur so often as was the case for the dilatation. The reversal of the dilatation was universal for all liquids and took place between 1000 and 3000 kgm. But the reversal of the compressibility, indicated by the difference curve crossing the axis, is sporadic in occurrence, and only once occurs for all temperatures simultaneously.

Comparison of the curves shows some points of interest. The first three alcohols show a slight kink; propyl alcohol at 3000 kgm., ethyl at 1100, and methyl possibly at a somewhat lower pressure. Amyl alcohol apparently has lost the kink. These four alcohols show in rather desultory fashion for at least one temperature the temperature reversal at high pressures. Isobutyl alcohol, as we expect, shows the effect of structural variation by curves of different character from the other four alcohols, in that they show no reversal, but decrease with fair regularity under increasing pressure. Isobutyl alcohol is also remarkable for the extremely rapid initial drop of compressibility with rising pressure. Ether, the isomer of isobutyl alcohol, still further shows the importance of the structure of the molecule; the difference curves for ether show reversal at high pressures, and are also more widely separated than the curves for isobutyl alcohol. The last six liquids show a somewhat greater temperature effect on compressibility than the first six. Acetone shows a strange maximum in the difference curve between 4000 and 5000. Carbon bisulphide shows greater variations at the highest pressures than any other liquid (it will be remembered that the dilatation curves also show abnormally great variations), and a reversal of the effect at 9000 kgm. simultaneous for all temperatures. This simultaneous reversal is shown by no other liquid, and reminds one of the dilatation. Phosphorus trichloride shows no marked features; the compressibility curves are not so simple as the dilatation curves. The halogen compounds do not show any marked similarities. Ethyl chloride is without particular features. Ethyl bromide has a very pronounced separation of the curves for 20° and 40° at 4000 kgm., while ethyl iodide shows a similar, but smaller separation between the 20°–40° and also the 40°–60° curves. The halogens all show reversals at the high pressures.

The occasional reversal of compressibility with temperature, that is, a smaller compressibility at a higher temperature, has ready explanation if we adopt the hypothesis of molecules with shape. We have seen that in this event, because of the varying completeness with which the molecules interlock, it may sometimes happen that the increase of volume with temperature results in a decrease of the free space open to the thermal agitation of the molecules. In this case the compressibility is less at high temperatures than at low. The same argument would admit also the possibility of compressibility increasing with increasing pressure if the effect of pressure is to produce a better fitting together of the molecules and so greater free space. It may be, therefore, that the effect found for carbon bisulphide is genuine, and not to be explained away by experimental error.

As a general rule, the result found for water applies to these twelve liquids also, namely that compressibility when plotted against volume is less at the higher temperatures. There are, however, a few exceptions. Carbon bisulphide and ethyl chloride, for example, show a reverse effect at the higher pressures. A possible explanation of this is to be found in the opposition of two effects. In general we think of the effects of lowering the temperature and of increasing the pressure as the same, namely to increase the degree of interlocking of the molecules. If now we compare two states of the liquid, each occupying the same volume, but one at a higher temperature than the other, we see that the high temperature condition differs from the low in that the molecules are less interlocked so that they have less free space at their disposal. An increase of pressure will tend to produce a smaller change of volume at the high temperature therefore, because of the smaller free space which the molecules possess. But on the other hand, an increase of pressure will tend to produce a greater change of volume because the effect of increased pressure is to increase the amount of interlocking, and so to decrease the volume. As one or the other of these effects predominates, we shall have smaller or greater compressibility at the higher temperatures with constant volume. We have seen that the compressibility is usually smaller.

We turn now to the diagram (Figure 45) in which are collected the average of the results for all twelve liquids. As for the curves of thermal dilatation, the zeros of the successive curves are displaced one square with respect to each other. The origin of each curve is so situated that at 12000 kgm. the compressibility has approached to within less than one unit of zero. Thus, the compressibility of ethyl alcohol at 12000 is 0.0$_5$81.

TABLE XIX.

Changes of Compressibility and Thermal Expansion produced by Pressure.

Liquid.	Compressibility, κ.				Dilatation, δ.			
	$\frac{\kappa_1}{\kappa_{12000}}$	$\frac{\kappa_{1000}}{\kappa_{12000}}$	$\frac{\kappa_{6000}}{\kappa_{12000}}$	κ_{12000}	$\frac{\delta_1}{\delta_{12000}}$	$\frac{\delta_{1000}}{\delta_{12000}}$	$\frac{\delta_{6000}}{\delta_{12000}}$	δ_{12000}
1	18.4	8.2	2.20	0.0$_5$74	4.29	2.76	1.23	0.0$_3$298
2	13.7	7.4	2.02	81	4.50	2.73	1.30	268
3	15.8	7.8	1.94	70	4.80	3.06	1.33	237
4	16.6	6.3	1.68	86	4.15	2.62	1.17	275
5	14.4	7.1	1.88	74	4.40	2.84	1.30	240
6		7.7	1.62	96		3.65	1.32	248
7		7.3	1.85	87		3.27	1.35	282
8	13.8	6.3	1.82	87	5.47	3.16	1.31	262
9	14.2	7.1	1.81	80	4.84	2.83	1.31	278
10		8.4	1.78	90		3.44	1.37	267
11	14.9	8.3	1.87	82		3.46	1.33	260
12	14.9	7.2	1.89	81	4.86	3.15	1.22	248
Water	4.9	3.7	1.64	89	1.00	1.00	1.00	400
Kerosene			1.82	87			1.14	280

The most striking feature is that the curves become nearly equi-spaced at the higher pressures. This appearance of increasing equality of behavior is not an illusion due to the approach of all the curves to zero, but it is real, as shown by the fact that the variation in the ratios of the initial compressibilities of the liquids to each other is greater than the variation in the ratios at 12000 kgm. The increasing equality at high pressures is also made more strikingly visible to the eye by noting on the diagram at what pressure the compressibility is equal to $0.0_4 20$. For all the twelve liquids except propyl, isobutyl, and amyl alcohol this pressure is between 4000 and 5000 kgm., and for the alcohols it is not far removed. This is the same sort of thing that we have seen to hold for the dilatation curves.

The compressibility curves are, however, quite different from the dilatation curves in several respects. The effect of pressure is very much greater in decreasing the compressibility than in decreasing the dilatation. Furthermore, the decrease of compressibility at the higher pressures continues to be more rapid than that of the dilatation; both the initial and the final relative rates of change are more rapid for the compressibility than for the dilatation. This is shown in Table XIX. In the second and sixth columns of this table the relative changes of compressibility and dilatation are given over the entire pressure range. The change is about four times greater for the compressibility than for the dilatation. The fourth and eighth columns show that the change between 6000 and 12000 is greater for the compressibility than for the dilatation. In other words, the dilatation comes much nearer to approaching a finite asymptote than the compressibility. This is not what was expected at first. It was thought that at high pressures the molecules would be squeezed into virtually perfect contact, that the compressibility would be provided for by the compression of the molecules, but that under these conditions the dilatation would practically vanish. The exact reverse has turned out to be the case. The data previously obtained for water and kerosene have also been included in the table. At high pressures kerosene behaves much like the other liquids. Water is nearly normal at high pressures as regards compressibility, but is abnormal over the entire range with respect to temperature effects.

The compressibility of these liquids at 12000 kgm. may be compared with the compressibility of metals under ordinary conditions. For mercury the value is about $0.0_5 39$, and for iron $0.0_6 58$. The average compressibility of these liquids at 12000 kgm. is therefore, about twice the initial compressibility of mercury and about fourteen times that of iron. The average dilatation at 12000 approaches more nearly to that of mercury, being about 40% greater, but is farther removed from the dilatation of iron, being about 20 times greater. The difference between compressibility and dilatation is further accentuated by the fact that the compressibility of iron is nearly as low as that of any solid, while there are a number of solids with a smaller dilatation. There seems, therefore, to be more difference between a solid and a liquid with respect to thermal expansion than with respect to compressibility.

Pressure Coefficient.— No diagrams have been given for this quantity, but it is nevertheless worth some discussion, because of the part it has played in previous theoretical discussions. This so-called "pressure coefficient" is the thermodynamic quantity $\left(\frac{\partial p}{\partial \tau}\right)_v$, the change of pressure when the temperature is raised one degree at constant volume, and is mathematically equivalent to the ratio of dilatation to compressibility $\left(\frac{\partial p}{\partial \tau}\right)_v = -\left(\frac{\partial v}{\partial \tau}\right)_p \Big/ \left(\frac{\partial v}{\partial p}\right)_\tau$. It has been proposed as an empirical law by Ramsay and Young that the pressure coefficient is a function of the volume only. That is, if the pressure coefficient is plotted against volume, the curves for different temperatures will fall together. The experiments of Ramsay and Young covered a wide temperature range, but a comparatively low pressure range, since their chief concern was with the relations between a liquid and its vapor, and their pressures seldom exceeded the critical pressure, a matter of a few hundred atmospheres. Amagat, in his discussion of his own results for liquids up to 3000 atmos., has devoted considerable attention to the pressure coefficient. One of his results was that the pressure coefficient is approximately independent of temperature at constant volume, but does nevertheless show small consistent variations, which Amagat was unwilling to ascribe to experimental errors. The coefficient of different substances may increase or decrease with rising temperature, or show still more complicated variations. The coefficient increases with decreasing volume, that is with increasing pressure. Tammann, however, in his recent empirical theory of liquids for high pressures, has concluded from an examination of Amagat's work that the variations which Amagat found in the pressure coefficient do not exceed the possible experimental errors. Tammann has accordingly taken as one of the fundamental hypotheses of his theory the assumption that the pressure coefficient is a function of the volume only.

The discussion of the pressure coefficient to be given here has for its only purpose to show that at high pressures, whatever the facts may be at low pressures, there is absolutely no ground for the assumption that the coefficient remains independent of the temperature. This was shown to be the fact in the previous paper on water, but the argument lost force because water is abnormal. However, for none of the twelve liquids of this paper is the relation even approximately satisfied. It is not necessary to show curves for all twelve liquids in order to disprove this one point. The data are at hand so that any one may make a complete test for himself. A single diagram, chosen at random, is sufficient to show that the proposed relation breaks down completely, as indeed one would expect it to in view of the complications of compressibility and dilatation. Figure 46 shows this for carbon bisulphide. It speaks for itself.

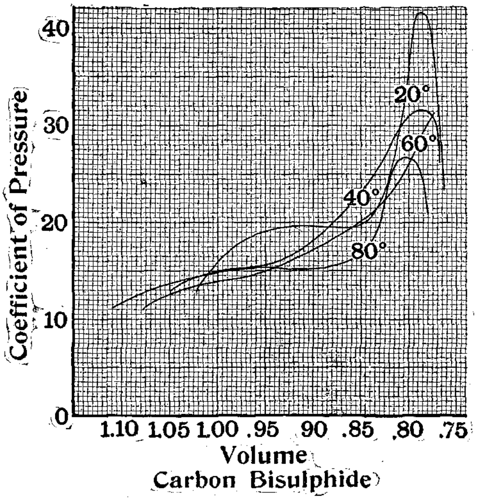

FIGURE 46. The pressure coefficient of carbon bisulphide plotted against volume. The diagram shows that at high pressures the pressure coefficient is a function of temperature as well as of volume.

Work of Compression.— The mechanical work of compression is shown in Folder III; Figures 47 to 58 are for the liquids separately, and Figure 59 shows the average between 20° and 80° of all twelve liquids together. The difference of the work for different temperatures is so nearly the same that the differences could not have been read accurately directly from the curves. The course was adopted, therefore, of plotting the work at 40° only, and in the lower part of the diagram giving on an enlarged scale the difference of the work for intervals of 20°.

The curves for the separate liquids do not require much comment. The difference curves are universally positive; that is, it is always true that more mechanical work is expended when a liquid is compressed to a given pressure at a high temperature than at a lower one. Beyond this, however, there do not seem to be many common features. The curves show irregular and apparently unrelated variations, but the irregularities are not so great as for the dilatation or the compressibility. Of course this was to be expected, because the curves are essentially integral curves.

Three of the alcohols, methyl, ethyl, and amyl, show similar difference curves. Ether and isobutyl alcohol, isomers, are unlike, as we have found them before. The difference curve 20°–40° for carbon bisulphide shows the effect of the abnormally high compressibility at 20°, which we saw previously, and the three ethyl halogens show similarities. Other variations in the difference curves are not particularly illuminating.

Figure 59, combining the average results for the twelve liquids, is of more interest. The similarity in general shape of all the curves is perhaps the most interesting feature. The curves become nearly linear at the higher pressures. This of course is not the usual relation between stress and work for a body like a steel spring, which maintains a stiffness independent of stress. For such bodies the work stored up as potential energy of strain varies as the square of the stress. This is true for liquids also over a pressure range so small that the compressibility may be regarded as constant, and is shown in the initial stages by all the curves, which are tangent to the axis at the origin. The fact that at high pressures the curves tend to become linear, still remaining slightly concave upwards, means that at high pressures the compressibility is becoming less, so that the change of volume, and therefore the work, is less for a given increment of pressure.

If we assume tentatively that the work of compression is linear at high pressures, we have a means of finding the compressibility and the volume at high pressures. For;

$$W = -\int_1^p p\left(\frac{\partial v}{\partial p}\right) dp = a + bp \quad (a \text{ is negative, } b \text{ positive}).$$

Differentiating this equation, we obtain,

$$\left(\frac{\partial v}{\partial p}\right)_t = -\frac{b}{p}$$

whence,

$$\Delta V = \int^{p} \left(\frac{\partial v}{\partial p}\right)_{t} dp = -b \, log \, p + C.$$

This equation cannot be expected to hold for low pressures. The constants may be determined as follows. From the curves for W above 5000 kgm. we find that an approximate value for b is 0.1. If furthermore we assume as a fair average of all twelve liquids that $\Delta V = -0.25$ when $p = 5000$, the equation becomes,

$$\Delta V = -0.1 \, log_{\epsilon} p + 0.6017.$$

The equation evidently cannot hold for infinite pressures, because it demands that ΔV decrease indefinitely, and it is physically impossible that ΔV should become less than -1. But the pressure at which this would happen according to the above formula is 9,060,000 kgm. This is so very far beyond the range of these experiments, that it would probably be safe to apply a similar equation as an approximate expression for any experimental pressures above 5000 kgm.

The mathematical analysis gives us, furthermore, information as to the ultimate behavior of the work of compression. For the actual work must eventually be less than that given by the above formula, which corresponds to a smaller change of volume. It must be, therefore, that at higher pressures the curvature reverses, and the curve becomes convex upwards. Methyl and amyl alcohol show the beginning of this effect, but in view of the extreme remoteness of the vanishing of the volume predicted above, it may well be that this slight change of curvature is merely one of the many local variations.

The maximum amount of work stored up at 12000 kgm. is nearly the same for all twelve liquids; much more nearly equal than the initial differences in the compressibilities would lead us to expect. The variations in the maximum are about 25%, whereas there are initial variations in the compressibility of 100%. If we admit water to our family of curves, as we may because the final work of compression is over 9 kgm. m., the variation in the initial compressibility may be 400%. Initial differences of compressibility have little effect on the work at the maximum pressure, because of the comparatively small amount of work done at low pressures.

The total amount of mechanical work stored up in a liquid is quite considerable. For example the work of compression of ether at 12000 kgm. would suffice to raise it through about 45000 ft., or to give it a velocity of 1700 ft. sec.

Heat of Compression.—The heat of compression, that is the quantity of heat in kgm. m. which flows out of a substance as it is compressed isothermally, is shown in Folder 4; for the twelve liquids separately in Figures 60 to 71, and the average for the twelve liquids collected into a single diagram in Figure 72. The differences of the curves for different temperatures are sufficiently great so that the total heat of compression for each temperature could be plotted and the difference found with sufficient accuracy directly from the curves, without the necessity of drawing difference curves as was the case for the work of compression. The zero of each curve has been displaced upwards one square for the successive temperatures. In the case of those liquids which boil at low temperatures, the zero of the curves for all temperatures has been taken at the same pressure, 500 or 1000 kgm., although it would have been possible to extend the curves to atmospheric pressure for the lower temperatures. In the case of acetone and ethyl chloride the curves for the lower temperatures have been extended backwards from the origin (1000 kgm.) to atmospheric pressure.

The heat of compression is positive, that is, as a substance is compressed isothermally, heat flows out to the surroundings. Examination of the curves in detail shows also that the total heat always increases with increasing pressure. This is a direct consequence of the fact that the thermal expansion is always positive. The curves therefore, show less pronounced irregularities than some, such as those for compressibility for example.

In general, the concavity is toward the pressure axis, that is, the increase of the heat of compression becomes less rapid at the higher pressures. The curves for the different liquids at different temperatures show that the heat is not universally greater at the higher temperatures, although such is generally the case. This is shown by the curves drawing together at high pressures in some cases to within less than the one square which separated them at the origin. An example of this is afforded by amyl alcohol between 20° and 40°, and by ethyl chloride and ethyl iodide between 60° and 80°. In general, a drawing together of the curves with increasing pressure means that the expansion is less at the higher temperature. There are many instances of this, although it is not usual that the drawing together is great enough to bring the curves to within less than the original arbitrary distance of separation.

Figure 72, in which are collected the average results for all twelve liquids, shows again that the twelve liquids are alike in character.

It is true that the curves cross in one or two instances, and are unevenly spaced at the high pressures, but the differences are very much less than one would expect from the differences in the initial direction of the curves. The heat of compression is given by $Q = \int \tau \left(\frac{\partial v}{\partial \tau}\right)_p dp$, so that the relative initial values of $\left(\frac{\partial v}{\partial \tau}\right)_p$, that is the initial slopes

TABLE XX.

Comparison of Thermal Expansion with Heat of Compression.

Liquid.	Expansion.		Heat of Compression	
	Expansion at Atmospheric Pressure.	Ratio to Average.	Heat at 12000 kgm.	Ratio to Average.
1	.00128	.91	18.0	1.04
2	120	.85	16.4	.95
3	115	.82	18.0	1.04
4	114	.81	16.0	.92
5	106	.75	14.9	.86
6	170(?)	1.20	17.4	1.01
7	172(?)	1.22	19.6	1.13
8	143	1.01	18.0	1.04
9	134	.95	17.5	1.01
10	190(?)	1.35	18.5	1.07
11	185(?)	1.31	18.3	1.06
12	120	.85	15.5	.89
Average	141		17.3	

Maximum variation of initial expansion, 1:1.8
Maximum variation of heat at 12000 kgm. 1:1:3

of the curves for Q, give us an idea of what we might expect to be the relative values of Q if the liquids preserved over the entire pressure range their relative initial behavior. Table XX shows this. It gives the initial dilatation and its ratio to the average, the total heat given out at 12000 kgm. and its ratio to the average. The dilatation tabulated for the low boiling liquids was obtained by a linear extrapolation, which gives values too small. Larger values for these liquids would only increase the force of the argument. It is obvious that the two ratio columns show only a general rough agreement; large values of one corresponding to large values of the other. There seems to be no correspondence in the small variations in the ratio column. This shows that the initial behavior of a liquid with respect to the heat of compression does not fix its behavior at the higher pressures. Furthermore, the magnitude of the variations in the ratios of the dilatation are greater than the variations of the heat ratios; a maximum variation in the one column of 1:1.8 against 1:1.3 in the other. This shows again that the liquids become more alike at the high pressures than we should expect from their initial behavior at atmospheric pressure.

The magnitude of the heat of compression is of interest. For the average liquid this is about 17 kgm. m. at 12000 kgm. If we take as a fair average for C_p 25 kgm. cm. this means that the amount of heat flowing out of the average liquid as it is compressed isothermally to 12000 kgm. would raise it through 68° at atmospheric pressure.

Change of Internal Energy. — This quantity is shown on Folder V; for the twelve liquids separately at four temperatures in Figures 73 to 84, and the average results for the twelve liquids plotted against pressure in Figure 85 and against volume in Figure 86. The change of energy plotted in these figures is the internal energy at atmospheric pressure minus the internal energy at the pressure in question. A positive value for the change means, therefore, that the internal energy is less at the higher pressure than it is at atmospheric. The origins of the curves for the separate temperatures have been displaced with respect to each other, so as not to confuse by over lapping. In the cases where the liquid boils at low temperatures the origin of the curve for the higher temperatures has been taken at 500 or 1000 kgm.

The change of internal energy was found by taking the difference between the heat and the work of compression. The curves show nothing, therefore, not already given.

The different substances show irregularities which cannot be dwelt on here. In general, the curves for the higher temperatures tend to draw apart, that is, the decrease of internal energy is greater at the higher temperature, but there are several well pronounced exceptions to this rule. Amyl alcohol, ethyl chloride, and ethyl iodide, for example, are exceptional at the same temperatures where we found exceptional behavior with respect to the work of compression.

The change of internal energy is to be thought of as brought about by the counterplay of two opposing sets of forces, and it is significant because of what it can tell us about these forces. When a substance is compressed, the molecules are brought closer together, the attractive forces between the molecules do work, the potential energy of the attractive forces decreases, and the internal energy decreases. But at the same time, the molecules become compressed by mutual contact, energy is stored up inside the molecule by the external forces in the form of potential energy of strain, and the internal energy increases. Now these two sets of forces play very different roles at different stages of the compression. At low pressures, where the molecules may be thought of as possessing a free path, no potential energy of strain can be permanently stored up in the molecule, because during the motion in the free path it has been entirely converted into translational kinetic or temperature energy. This is what takes place in a gas. But as the volume of the liquid becomes less, the length of the free path rapidly becomes smaller, and at any instant an increasingly large number of molecules is not describing part of any free path at all, but is merely being handed on directly from one collision to the next. At this stage, potential energy of strain can be permanently stored up within the molecule. At still higher compressions, when the molecules are practically in continuous contact, there is still greater possibility of storing up internal energy of strain. The possibility is limitless, provided only that the molecule never becomes incapable of further compression. The loss of internal potential energy by the attractive forces cannot proceed beyond a certain limit, however, imposed by the least distance of approach of the molecules.

We should expect, therefore, that at low pressures the internal energy would decrease with rising pressure, the attractive forces being in the ascendant, but at the higher pressures, where the forces resisting compression have become dominant, that the energy would increase with rising pressure. For the two liquids previously investigated in this respect, mercury and water, the energy was found to continue to decrease over a pressure range of 12000 kgm. The direction of curvature was in each case such as to suggest that this decrease might not continue indefinitely, and it was suggested then that there must ultimately be a reversal of the effect. It seemed surprising that at pressures as high as 12000 kgm. the attractive forces should still do more work than could be stored up as strain by the external forces.

Figure 85, giving the average for the twelve liquids, shows that the anticipation of a reversal in the change of energy is justified for nearly all the liquids; that is, the internal energy, after decreasing for a while, passes through a minimum (on the curves a maximum) and from here increases with rising pressure. The necessary existence of this maximum could of course have been predicted from the curves for the heat and the work of compression, since the one is either linear or concave upwards, while the other is concave downwards.

The change of energy is markedly different for the different liquids, whereas the other thermodynamic properties are similar. Of course the reason is that we are here concerned with the difference of two effects. The position of the maximum of the difference of two functions is very sensitive to slight changes in the functions themselves. Under these circumstances, the mere existence of a maximum is evidence of similarity. The only curves which do not show the maximum are methyl and propyl alcohol. It will be remembered that the work of compression curve for methyl alcohol had a reversed curvature at the upper end, and that the work of compression of propyl alcohol was abnormally low.

It is of interest to plot the change of energy against volume, because it may give information about the attractive forces. If the attractive forces are central forces, functions only of the distance from the centers of the molecules, then the potential energy of the attractive forces will be a function of the volume. Thus if the attractive forces are proportional to the inverse fifth power, as has often been supposed, then the potential energy is inversely as the fourth power of the distance apart of the molecules, or as the inverse four thirds power of the volume. This relation was tried for four of the twelve liquids; for amyl alcohol, ether, phosphorus trichloride, and ethyl iodide. The change of internal energy of these liquids was plotted against $V^{-\frac{4}{3}}$. The diagram was the same in character as the diagram plotting the change of energy against V, except of course that small values of $V^{-\frac{4}{3}}$ correspond to large values of V. Now if the change of internal energy is proportional to $V_0^{-\frac{4}{3}} - V^{-\frac{4}{3}}$, this curve plotted against $V^{-\frac{4}{3}}$ should be linear. The curves were very nearly linear in the initial stages, but of course ultimately diverged greatly from linearity, passing through a maximum. If the straight portion of the curve at the origin represents a region in which the inverse four thirds power law is satisfied, then the tangent to the curve at the origin represents over the entire range of the experiment what the change of energy would have been if the four thirds law had held throughout. The difference between the actual curve and the tangent at the origin is then, according to the above view, equal to the energy which has been stored up as strain inside the molecule. This difference was determined and plotted against volume, in order to find what function of the volume the strain energy might be. For these four liquids, it turned out that the strain energy varies over the entire range approximately as the cube of the change of volume reckoned from a suitable origin. The greatest discrepancy is for ether at low pressures. For PCl_3 a variation of only 0.01 in the arbitrary zero of volume from which the change is reckoned would wipe out the discrepancy; for amyl alcohol a variation in the zero of 0.005 would give perfect agreement; for ethyl iodide a change of 0.01, and for ether a change of 0.04. The variation for ether is all below 3000 kgm.; above this the energy of strain is almost exactly proportional to the cube of the change of volume, taking 1.06 as the origin. The zero of volume for phosphorus trichloride is 1.11, for amyl alcohol 1.07, and for ethyl iodide 1.045. All the curves showed slight consistent variations from the cube law; at low pressures the strain energy varies more rapidly than the cube and less rapidly at the high pressures.

The fact that the internal energy of strain varies as the cube of the change of volume probably does not have very much significance in showing us what the elastic mechanism of the molecule is. If the entire change of volume of the liquid were due to change of volume of the molecules, then we should expect the strain energy to vary as the square of the change of volume, provided the elastic constants of the molecule were unaffected by pressure. If, as is likely, the molecule becomes less compressible at high pressure, then the strain energy would vary less rapidly than the square. But the strain energy was found to vary as the cube. The reason for this is probably that at low pressures strain energy is stored up in only a few of the molecules; those molecules which are describing a free path and are not in contact with other molecules have no strain energy of compression. With increasing pressure the number of molecules in which strain energy is stored up increases rapidly, and the strain energy of each molecule increases at the same time as the square of the strain, so that the total energy of strain would be expected to increase more rapidly than the square of the change of volume, as we found it to. But any more detailed speculation as to the precise way in which the number of molecules taking part in the strain, and the way in which the strain energy of the average molecule varies with the total pressure, would probably be useless because the argument as to the attractive forces breaks down at small volumes. It is probable that the molecules are not really homogeneous spheres, but that there are localities in which the attractive forces are more or less concentrated. Therefore, although we may regard the attraction exerted by the molecule as toward its center, and inversely as the fifth power of the distance when the molecules are separated by wide intervals, we cannot conceive of this law continuing to hold when the molecules are so close as to be in contact. Under these circumstances the force may increase more rapidly than as the inverse fifth power. What is more, the potential energy of the attractive forces will not under these circumstances be a function of the volume only, that is of the mean distance apart of the centers of the molecules, but will also vary with the orientation. We saw that the average orderliness of orientation may be expected to vary with temperature, being on the whole more haphazard for equal volumes at the high temperatures. Even with this picture of what is happening, it would be difficult to say whether the potential energy of attraction should be expected to be greater at the higher or lower temperature. We have seen that the lower temperature usually means a greater space open for occupation, but it may still be that because of the greater approach to order at the low temperatures the localities of intense force are brought closer together, so that the potential of the attractive forces may be less.

The main conclusion to be drawn from the fact that the strain energy of the molecules varies as the cube of the change of volume is, therefore, that at low pressures the greater part of the change of volume is due to the decrease in the distance apart of the molecules, but that at high pressures an increasingly large part of the change of volume is occasioned by the actual change of volume of the molecules themselves.

It is interesting that the initial slopes of the curves of change of energy against volume are very nearly the same for all twelve liquids. This is a little unusual. Previously we have found the twelve liquids to become similar at high pressure, but in respect to the change of energy they appear to be more alike at low pressures.

Specific Heat at Constant Pressure.—The specific heat at constant pressure is shown on Folder VI; the curves for the twelve liquids at four temperatures in Figures 87 to 98, and the collection of the average results in Figure 99. The quantity listed as change of C_p is the specific heat at atmospheric pressure minus the specific heat at the pressure in question. A positive change means, therefore, that the specific heat is less at the pressure in question than at atmospheric pressure. In order not to confuse the curves, the origin for each temperature has been displaced with respect to the neighboring curves. The scale of the drawing is shown at the right hand side.

The twelve liquids show a bewildering variety, so bewildering that speculation as to the cause of all the variations is hopeless. It is to be pointed out nevertheless, that such great variety is to be expected if the molecules take up different positions more or less symmetrical in arrangement with increasing pressure. The process is similar in many ways to a process of association, which is accompanied by much greater changes in the specific heats than in the volume, or compressibility, or dilatation. The curves show some points of similarity, however. It is an almost universal rule that the initial change of C_p at any temperature is a decrease. For the majority of liquids the specific heat on the whole decreases at the high temperature and increases or does not increase so much at the low temperatures. We have seen that as a rule the specific heat at atmospheric pressure is higher at the higher temperatures. For some liquids the temperature effect may be very marked. The change under pressure is in such a direction as to bring the specific heats at high pressures more nearly to equality for all the temperatures. The three halogen compounds are an exception to the rule, however. The very large increase of C_p for ethyl chloride at 80° is very much like that already found in the case of water. It may mean an abnormally high rate of dissociation at the higher pressures. The four normal alcohols show similarities in the abnormally large decrease of C_p at 80°. The decrease evidently cannot go on indefinitely. For methyl and ethyl alcohol, the maximum pressure is sufficient to change the decrease into an increase, as it must eventually, but for propyl and amyl alcohols, the reversal in direction must be at higher pressures than reached here. It seems to be at hand for propyl alcohol.

There is one very rough check which may be applied to the values given here for the specific heats; namely, in no case must the specific heat decrease by an amount more than its original value, for a negative specific heat is impossible. The only case where this condition comes anywhere near making trouble is for amyl alcohol. The curve for 80° shows a decrease of C_p of 21 kgm. cm. The data for atmospheric pressure are discordant, but 25 kgm. cm. seems to be a fair average value at 80°. This would mean that the specific heat at 80° and 12000 kgm. is only 4 kgm. cm., about one sixth its initial value. It is evident that the reversal in the effect must come speedily.

Figure 99 for the average change of C_p over the entire temperature range for the twelve liquids shows much less variation from liquid to liquid than one might expect from the irregular variations at the different temperatures. The curves in general all show the same characteristics; at first a decrease of specific heat and then an increase at higher pressures. The first minimum has already been commented on in another connection. Beyond this minimum in C_p (maximum on the curves) there is in general a continuous increase with rising pressure, although there are several cases where C_p decreases again slightly at the highest pressures. The three halogen compounds are exceptional (and also acetone) for the rather large increase of C_p with rising pressure. The increase is greater for the heavier members of the series.

The data of the previous paper on water show the same behavior, a minimum in C_p for all temperatures (except at 0° where the curve is broken off by the freezing) and then an increase with still further increasing pressure. The pressure for the minimum is higher than for these twelve liquids because of the abnormal behavior of water at low pressures.

So far as is known to the author, these are the only measurements from which an attempt has been made to find the specific heat at high pressures. The probable behavior of the specific heats does not seem to have been suspected. Thus Tumlirz deduces from his empirical equation for liquids that both the specific heats decrease with rising pressure. For water, Tumlirz finds the limiting value of C_p for infinite pressure to be about 0.5 gm. cal. He does not compute it for the other liquids. We see from the above that we are to expect for all liquids an ultimate increase instead of a decrease with rising pressure.

Specific Heat at Constant Volume.—The specific heat at constant volume is shown in Folder VII, the curves of the different liquids at different temperatures in Figures 100 to 111, and the collected averages for all the liquids in Figure 112. The treatment of these curves is the same as for C_p.

The curves are the same in general character as those for C_p. The differences consist in displacements of the pressures of maximum or mimimum, or occasionally in the suppression of small irregularities; but the larger features are the same. The curves for the four normal alcohols retain the same resemblances as before, and isobutyl alcohol is as strikingly not a member of the series. Isobutyl alcohol and ether show general resemblances as to the specific heat, which may, however, be accidental. The halogens are also different from the other liquids with respect to C_v.

The importance of tabulating C_v is that it is a quantity of much greater simplicity than C_p. When we determine C_p by heating the liquid at constant pressure, the liquid expands with rising temperature, and in so doing performs work against both the external pressure and the internal attractive forces. C_p is greater than C_v by this work against external and internal forces. Consequently, it is usual to tacitly assume that C_v contains only the work necessary to raise the temperature energy of the molecules. Evidently the assumption is merely another way of stating the assumption that the potential energy of the attractive forces is a function of the volume only. Furthermore, it is usually assumed in discussions of the significance of C_v that a given increase of temperature corresponds to a given increase of kinetic energy of the molecule, no matter what the pressure of the liquid. It is true that when the kinetic energy of the molecule is to be raised, more total energy must be imparted to the molecule than just sufficient to increase the kinetic energy by this amount, because of the law of the equipartition of energy among the various degrees of freedom, but if the degrees of freedom remain unaltered, we still have the result that to increase the temperature by a given amount requires the same amount of work independent of the volume. The consequence of the hypothesis would be that the specific heat at constant volume is independent of pressure and temperature.

Figure 112 for the average C_v of the twelve liquids, shows that even on the average C_v cannot be independent of pressure, and the preceding diagrams for the separate liquid show that it certainly cannot be independent of temperature. In general the behavior of C_v seems to be at first a decrease, and then with increasing pressure an increase again. Probably the reason is that neither of the simple hypotheses which we discussed above are valid for high pressures. We have seen that at high pressures we may expect the molecules to approach positions of more or less regularity of arrangement, and that the regularity is greater at the low temperatures. Now according to the particular arrangement which the molecules adopt, and the relation of the local centers of force to this arrangement, it is conceivable that the potential of the attractive forces should be either greater or less at the lower temperatures and equal volumes. The likelihood is, however, that the potential will usually be less at the lower temperatures. Similarly, it is usually more likely that the potential of the attractive forces should be less at the higher pressures. As a rule, then, an increase of temperature of one degree at a higher pressure or a lower temperature must provide the energy to do more work against the attractive forces, so that the specific heat at constant volume will be greater at higher pressure and lower temperature. But in those more infrequent cases where the potential energy of position is less at the higher temperature or lower pressure, the specific heat will be less at higher pressures and lower temperatures. It is as a rule true, as we have seen from the curves, that C_v does become greater at the higher pressures and lower temperatures.

The considerations just discussed are somewhat similar to considerations regarding the association of the molecules, but do not in all cases lead to the same results. For instance, if we suppose a liquid of single molecules to associate to one of double molecules, the specific heat of the associated liquid would be one half that of the simple one, if we neglect the effect of the altered number of internal degrees of freedom.

The second hypothesis made above, that a given increase of temperature always corresponds to the same increase of molecular energy, probably breaks down also at high pressures. The difficulty of determining what happens in this case is increased by uncertainty as to what the definition of temperature shall be at high pressures. We may perhaps, however, think of temperature at low pressures as being roughly proportional to the average translational energy of the molecule during its free flight. Now we have seen that as pressure increases, the time of free flight decreases rapidly, and an increasing fraction of the time is spent in collision. During collision the kinetic energy of translation has become potential within the molecule. The result is that as pressure increases, the potential strain energy of the molecules becomes a greater part of the total energy, leaving a smaller residue to become kinetic. Now if temperature corresponds to translational kinetic energy, it is evident that at high pressures more total energy must be imparted to the substance to increase the translational energy a given amount, or in other words, the specific heat will increase with increasing pressure.

The initial decrease of C_v may very possibly be an association effect. A word should be said about this association effect. The reason that an association from single to double molecules, for example, reduces the specific heat to one half, neglecting the effect of the internal degrees of freedom, is that a rise of temperature of one degree corresponds to a definite increase in the kinetic energy of each molecule, and when there are half as many molecules, half the additional energy is needed to increase the energy by an amount corresponding to one degree. This means that association has an effect on specific heat as long as temperature remains a molecular affair. This is true for a gas. But the law of atomic heats for solids suggests very strongly that in solids temperature is no longer an affair of the molecule, but has become a matter of the atom. We naturally expect somewhere a transition from the one state to the other, and the liquid is the natural place to look for it. That is, if a liquid temperature is on its way from being an affair of the molecule to becoming an affair of the atom. It is not likely, therefore, that the existence of large molecular complexes in a liquid, as in the case of association, will modify very much the behavior with respect to temperature, and in particular to the specific heats. Such effect as there is should be looked for at low pressures.

A word should be said about the curves for the average of C_p and C_v over the entire temperature range. The average C_p is the total heat absorbed at constant pressure between 20° and 80° divided by 60. C_v is the average of the four values at 20°, 40°, 60°, and 80°. These two averages are not always equivalent when there are large variations with temperature, but they are always approximately equal. In cases of question, the average C_p corresponding to C_v may be found from the curves for C_p at the four temperatures. To find the average C_v corresponding to the average C_p would involve a more complicated procedure.

GENERAL DISCUSSION OF THE BEARING OF THE RESULTS ON A THEORY OF LIQUIDS FOR HIGH PRESSURES.

It is proposed to discuss here the nature of the problems which confront one at high pressures. No attempt is to be made to develop a new theory, but merely to indicate some of the directions in which the data of this paper suggest modifications of conceptions which have hitherto worked at low pressures.

Perhaps the most far reaching modification is in regard to our idea of the kinetic origin of pressure. We suppose that the pressure of a gas, for instance, is produced by the impact of the molecules against the walls of the vessel; and we compute the magnitude of the pressure from the total change in one second of the momentum of the molecules striking on unit area of the wall. The velocity with which the molecules strike the walls decreases with falling temperature and vanishes at the absolute zero. The pressure exerted by a liquid also is thought of in most attempts at a theory of liquids as exerted by the same mechanism and is computed in the same way. But it is entirely obvious that the molecules may exert pressure on the walls in another way. It is inconceivable that at the absolute zero a substance should not resist an attempt to compress it; yet at this temperature there can be no kinetic effect exerted by the molecules as a whole. Under these conditions the molecules transmit pressure in the same way that a compressed spring does; that is, they behave like elastic solids. It does not concern us here to inquire what the ultimate origin of this elasticity is; it may be kinetic within the atom. The point is simply this; from our point of view, which regards the molecule as a whole, we must recognize two possible functions or modes of action; the molecule may behave like a moving centre of mass with kinetic energy and momentum, or it may behave like an elastic solid. The molecule may exert pressure in virtue of either one of these two modes of action. Under ordinary conditions the momentum effect of its motion as a whole is almost the entire effect. But if we examine the mathematical analysis which justifies us in putting the pressure equal to the total change of momentum in unit time, we shall find that we made certain simplifying assumptions. We assumed that each collision with the wall is unimpeded; that is, the molecule approaches the wall during free flight, has a single encounter and makes a clean get-away. And this assumption was necessary; as soon as the molecule is interfered with during its collision, as it may be by a collision from another molecule from behind, the simple change of momentum relation ceases to give the pressure, and we must treat our molecule during collision as an elastic solid. Now it is evident that as the volume becomes less, that is, as the pressure increases, the total number of collisions with interference will increase, and our kinetic conception of pressure becomes less and less useful.

A very simple model, which may or may not correspond to the physical facts, is instructive in showing how under different conditions we may compute the pressure from the momentum effect alone, or must consider also the elasticity effects. The substance we are to consider consists of a single molecule. The molecule consists of a heavy particle of mass m, with two weightless springs projecting on either side of length l_0, giving a total diameter of the molecule of $2l_0$. (Figure 113.) This molecule travels back and forth in a horizontal line between the two opposite vertical walls of the enclosure. The distance apart of the walls is the volume of the enclosure, the force exerted by the springs on the walls during an encounter gives the pressure exerted by the substance, and the kinetic energy of the particle at its maximum velocity represents temperature.

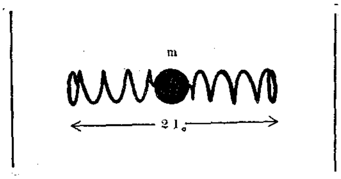

FIGURE 113. Model of a substance consisting of one molecule. The model is to show that at high compressions the pressure is not given by the change of momentum of the molecules striking the walls of the vessel in unit time.

This model is so simple that the entire discussion may be carried through with rigorous mathematical analysis. The pressure exerted on the walls is evidently to be found by taking the time average (over a long interval of time) of the force exerted on the walls by the springs during the encounters. Now in the solution of this problem there are three different cases.

I. The first case is when the distance apart of the walls of the vessel is greater than $2l_0$, the diameter of the molecule. Under these conditions we have collision without interference, which are the only conditions to which the usual analysis applies. In this case it may be proved by a detailed mathematical solution which will not be given here that the time average of the force is exactly equal to the change of momentum in unit time with respect to one of the walls, as it should be. The results of the mathematical analysis for this case may be given in the form of a distinctive equation of the substance,

$$P\,[(v - b) + \alpha\, T^{\frac{1}{2}}] = 2\, T.$$

Here P is pressure, $v =$ volume (distance apart of walls), $b =$ diameter of molecule $(2l_0)$, $\alpha = \pi\sqrt{\frac{2}{k}}$, where k is elasticity of the springs, and $T =$ absolute temperature (kinetic energy of particle). The equation bears a resemblance to van der Waal's equation without the attractive forces. In van der Waal's equation $\alpha = 0$, or $\kappa = \infty$, which means that the time of collision of the particles with the wall may be neglected in comparison with the time of free flight.

II. The distance apart of the walls is less than $2l_0$, the diameter of the molecule, and the kinetic energy of agitation is so small that at no time is the molecule out of contact with *both* walls simultaneously. The detailed mathematical solution shows that under these conditions the momentum effect has no influence whatever, and the pressure is determined simply by the relative magnitude of the volume and the unstressed diameter of the molecule. Under these conditions we obtain as the characteristic equation

$$P = \frac{k}{2}(b-v).$$

It is remarkable that the temperature has disappeared from the characteristic equation, or, in other words, the thermal dilatation is zero. The substance still remains compressible, however. Something like this was expected for the liquids of these experiments; that is, it was expected that the dilatation would tend to vanish more rapidly then the compressibility.

III. Case II passes into this case when the violence of the temperature vibration becomes so great that during part of the vibration the molecule is in contact with only one wall. The critical temperature at which this occurs is when $T = k\left(\frac{b-v}{2}\right)^2$. The mathematical analysis is more complicated for this case, because the motion must be split up into two stages, during which the restoring forces are different functions of the displacement. But just as in Case II, the change of momentum of the molecule in unit time does not give the mean pressure exerted on the wall. The complete equation of state for Case III is complicated, involving antisines, so that it is hardly worth giving. It reduces to one or the other of the other two cases, however, under proper critical conditions. In this case, the pressure computed by the change of momentum is too low, as we should expect it would be, because we have neglected an elasticity term which modifies conditions when the molecule is in contact with both walls.

The sequence of events when we compress a substance at a given temperature from a large volume is first Case I, then Case III, when $v = b$, and then Case II, when the volume has been still further reduced by an amount depending on the temperature. Case I passes smoothly into Case III without discontinuity in either compressibility or dilatation. The difference between Cases I and III is that a higher pressure corresponds to a given volume in Case III than we should expect from the formula of Case I. The pressure at a given volume in Cases I and III depends on temperature, but in Case II, the relation between volume and pressure is the same for every temperature. The only way in which temperature has an effect in Case II is with regard to the critical temperature which determines when Case II passes into Case III. The pressure for a given volume in Case III ultimately becomes less than we would have expected it to be if we extrapolated by the formula of Case I. In other words, the substance is more compressible than we should expect it to be from its behavior at low pressures. This reminds us of the formulas of Tumlirz and Tammann. It should be said however, that the mathematical analysis applied to Case III cannot continue to have physical significance indefinitely, for it was assumed that the springs obey Hooke's law, which is true only for small displacements. The characteristic equation given above for Case III predicts the vanishing of the volume at a finite pressure.

Something similar to the action of the model must take place in a liquid. At any instant there are collisions taking place, some free collisions similar to those of Case I, some collisions with interference like Case III, and some contacts between molecules like Case II, which are not properly called collisions at all. The momentum computation of pressure applies only to Case I. At low pressures, by far the greater number of collisions is of type I, but as the volume decreases and pressure increases, collisions of type II and III become increasingly predominant, until at infinite pressure we may suppose type II only to be present. Under these conditions, the momentum effect has absolutely no connection with pressure.

The ordinary conception of internal pressure must obviously be modified in a similar way. There are many different meanings attached to "internal pressure." One way of defining internal pressure is by constructing an imaginary surface in the interior of a liquid, and finding the momentum of all the molecules which cross this surface in unit time. This process evidently fails to have physical meaning when there are molecules in the liquid in contact with each other for any length of time. The modification of the definition necessary to meet these new conditions would be very complicated. It seems better under the circumstances to give up this conception of internal pressure altogether. Other conceptions may still be useful at high pressures; such, for example, as to regard the internal pressure as the external pressure plus the unbalanced attractive effect of the molecules at the surface of the liquid. But it is a difficult matter to define internal pressure in such a way as to have a physical meaning for one of Maxwell's demons inhabiting the interior of a liquid.

Accordingly, in the previous discussion no use has been made of this "internal pressure."

The usual kinetic conception of temperature must undergo modification at high pressures just as our kinetic conceptions of pressure. We think of the temperature of a gas as proportional to the average kinetic energy of translation of its molecules, the translational energy being the energy of the center of mass during free flight. Now it is inconceivable that at infinite pressure there should be any free flight, that is, there can be no kinetic energy of the molecule as a whole, but it is also inconceivable that at infinite pressures a substance should not possess temperature and be capable of temperature equilibrium with surrounding objects. Our model of the molecule may be helpful to us again. When the volume decreases beyond a certain limit, we saw that the boundaries of the molecule become fixed in position, and that the temperature is represented by the energy of internal agitation. This suggests that temperature changes in character from an affair of the molecule as a whole at low pressures to an affair of agitation within the molecule at high pressures. The behavior of the specific heats of gases and solids also strongly suggests the same thing. We compute the specific heat at constant volume of a gas by supposing that the kinetic energy of translation of each molecule must be increased by a fixed amount to produce a rise of temperature of one degree. But for most solid elements the law of Dulong and Petit holds, which is equivalent to the statement that to increase the temperature of a solid by one degree we must increase the energy of each atom by a fixed amount. Now a microscopic analysis of a solid like iron discloses a crystalline structure of great complexity; we find it hard to think that there are not groups of associated molecules and that the molecules are monatomic. It appears, then, that temperature has become connected somehow with what is going on in the atom. It view of this it would seem that another conception of temperature is desirable. It must be such a conception as not to be at variance with what we suppose to happen in a gas or a solid.

The material for such a conception is at land. It is a property common to the temperature energy of a gas molecule and to the internal energy of our suppositious molecule, that it is constantly in a state of flux, changing during collision from kinetic to potential and back to kinetic again. A natural generalization, then, is as follows: Temperature is proportional to that part of the energy associated with a representative molecule which undergoes periodic changes from kinetic to potential and back to kinetic. In complicated cases where the transformation from kinetic to potential does not take place simultaneously in all parts of the atom, an equivalent generalization is: Temperature is the difference between the maximum and minimum potential energy of a representative molecule. This evidently applies to both the extreme cases above; the one for which temperature is proportional to kinetic translational energy, and the one for which it is proportional to internal energy. In one respect these two extreme cases are not entirely unlike; in accordance with the law of equipartition, only a certain proportion of the total energy communicated to a molecule of a gas becomes kinetic; the rest goes *to the internal degrees of freedom.* So that an increase of temperature always carried with it an increase of the internal energy of the molecule. Whether this internal energy also oscillates in character between kinetic and potential is not obvious.

Of course the justification of this proposed general definition of temperature must be furnished by experiment. It does have the advantage, however, of being applicable at high pressures where the ordinary definition breaks down completely, and it does agree with our physical feeling of what temperature must be in the case of one very simple model of a substance under high pressure.

The effect of this conception of temperature on our conception of the equipartition of energy between the different degrees of freedom is interesting. At high pressures, where the molecules press on each other from all sides, one degree of freedom has been lost (or more properly three) namely, the possibility of motion of the molecule as a whole. We may suppose, if we like, that under these circumstances the total energy is equally divided between the remaining degrees of freedom. Now at any instant in a liquid, there are molecules with varying degrees of freedom, according to the kind of collision in which they are entangled. Furthermore, the same molecule at different stages of its career may enjoy a different number of degrees of freedom. When the degrees of freedom change, there must follow a redistribution of the total energy among the remaining degrees of freedom, and this process takes time. The result is that we cannot ascribe to the average molecule of the substance any definite number of degrees of freedom. The number cannot be an integer in the first place, and in the second place must vary continuously as pressure varies.

The idea that the temperature of a substance need not be proportional to the kinetic energy of translation of the molecules is not new. For instance, there is a recent paper by Brillouin,[59] in which he discusses at length the possible necessity of a change in the usual definition of temperature as the volume changes.

One other very important consideration which we shall probably be obliged to introduce into a theory valid for high pressures has already been mentioned several times in the discussion of the various thermodynamic properties. It is this; in gases we think of the molecules as perfect spheres, but at high pressures we shall undoubtedly have to recognize that they possess more complicated shapes. It is inconceivable that a molecule should not have a characteristic shape; we cannot well imagine the possibility of so fitting together the atoms of a complicated organic compound as not to produce some irregularities in the molecule. Shape becomes increasingly important at high pressures where the molecules are forced together and constrained to adapt themselves as best they can to each others irregularities. Beside the possibility of shape, there is the possibility that there are local centers of force in the molecule; that we cannot regard the molecule as a homogeneous sphere exerting a force towards its own center, but that when we approach too close to the molecule, there are individualized centers of force that begin to act of their own account. This again, seems by no means a forced conception.

Along with the idea of molecules with shape goes the conception that at high pressures these shapes must be forced to more or less adapt themselves to each other; in other words, the molecules must begin to show traces of regular arrangement. The regularity is by no means the thorough going regularity of a crystal in which the molecules are permanently moored to certain mean positions: the molecules of the liquid still circulate about among each other, but as they slide past each other there may be a growing tendency at higher pressures to point the long axes in the direction of relative motion, for example. Just as at a crowded ball room, there is a tendency for the throng of young men making their way to and from the refreshment room to hold their plates out from them in the direction of motion. This increasing order of arrangement seems not only natural, but inevitable at high pressures. It may ultimately terminate in crystallization. We should expect furthermore, at equal volumes, there should be nearer approach to order at lower temperatures where the violence of temperature agitation is less.

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[59] Brillouin, Ann. Chim. et phys., **18**, 387–400 (1909).

Now the combination of these two effects, namely that when the molecules come very close to each other the attractive forces depend on the orientation as well as on the mean distance apart, and that the molecules may assume a greater uniformity of arrangement, has far reaching consequences that provide the possible explanation of all the complicated effects which we have found to exist. Thus, if we consider two possible configurations of the liquid, each having the same volume, but one with a more orderly arrangement of the molecules, we see that the more orderly arrangement involves a greater effective space open to occupation. One consequence is that the more orderly arrangement has the greater compressibility. One striking example of this has been found in the case of mercury. The compressibility of solid mercury has been found to be less than the compressibility which the liquid would have at the same temperature if it could be compressed without freezing to the same volume as the solid.

In the detailed discussion of the thermodynamic properties it has been shown that we have here a possible explanation of many complicated effects. It explains increasing compressibility with rising pressure, decreasing compressibility with rising temperature, increasing thermal expansion with increasing pressure, and decreasing expansion with rising temperature. It is not necessary to go into the details of the argument again. It is to be emphasized, however, that we have here a mechanism capable of explaining a bewildering array of experimental facts. There must be at least some validity in the point of view.

Not much has been said in the explanation above of the results of possible association, because under high pressures, when the molecules find difficulty in adapting themselves to the space at their disposal, it seems unlikely for groups of molecules to unite themselves into very close knit units. The molecules, on the other hand, do apparently preserve their individuality under these high pressures, and do not break up into simpler compounds. It might be expected, for instance, that pressure would transform ether into isobutyl alcohol, a substance of the same atomic constitution, but with a smaller volume. Such was not the case, however. But it may be that association does play an important part at low pressures. In this case it would be capable of explaining various irregularities in very much the same way as suggested above. For instance, if association takes place with decrease of volume, the thermal expansion or compressibility may be greater at lower than at high temperatures. Association has been discussed in greater detail on page 104.

The results have exhibited one striking feature which has been frequently emphasized, namely that at high pressures all twelve liquids become more nearly like each other. This suggests that it might be useful in developing a theory of liquids to arbitrarily construct a "perfect liquid" and to discuss its properties. Certainly the conception of a "perfect gas" has been of great service in the kinetic theory of gases; and the reason is that all actual gases approximate closely to the "perfect gas." In the same way, at high pressures all liquids approximate to one and the same thing, which may be called by analogy the "perfect liquid." It seems to offer at least a promising line of attack to discuss the properties of this "perfect liquid," and then to invent the simplest possible mechanism to explain them.

SUMMARY OF RESULTS.

These measurements have disclosed an unexpectedly complicated state of affairs at high pressures, in many respects the exact opposite of what we would expect from the behavior at low pressures. The compressibility may decrease with increasing temperature, or in a few cases may increase with increasing pressure. The thermal expansion also may decrease with increasing temperature or increase with increasing pressure. The peculiarity of thermal expansion with respect to temperature is possessed by all liquids above 3000 kgm. This has been shown to have a bearing on previous theories. Of the other thermodynamic properties, perhaps the most important is the internal energy. This passes through a minimum and then increases again with increasing pressure. The reason is that beyond a certain pressure the attractive forces do less work than is done by the external forces in compressing the molecule.

Among considerations which would seem to be of importance for a theory of liquids at high pressures, that of the shape of the molecules is worthy of attention. It is inconceivable that the molecules should not have shape, and it is natural to suppose that the shape will play an important part when the molecules are forced into close contact. It is shown in detail that considerations of this sort offer possible explanations of the complicated effects actually found. Other modifications of the ordinary conceptions of liquids that may be necessary have to do with our ideas of the kinetic origin of pressure and temperature. There can be no doubt that at high pressures there are other than kinetic effects involved at least in pressure.

It is a pleasure to acknowledge generous assistance from the Rumford Fund of the American Academy of Arts and Sciences for the purchase of apparatus and supplies, and from the Bache Fund of the National Academy for an assistant in some of the experimental work.

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THE MAXIMUM VALUE OF THE MAGNETIZATION IN IRON.

By B. Osgood Peirce.

With Three Plates.

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The first experiments on the magnetic behavior of soft iron under high excitations were made, more than sixty years ago, upon comparatively short, stout rods, so that the results were affected by the demagnetizing action of the ends of the specimen, but, even under these circumstances, several different observers[1] were able to show that if the magnetizing force to which a piece of iron is exposed be made stronger and stronger, the intensity of the resulting magnetization of the metal usually approaches a definite limit, and that this limit is practically reached in fields of such strength as are frequently used in the laboratory.

The work of Stoletow and Rowland in the early seventies of the last century, upon iron rings or toroids, made the true meanings of H, B, and I in the iron clearer, and since that time many persons[2] have attempted to determine the limiting value (I_∞), of I, as H is made to increase indefinitely. I_∞ is now sometimes called the *specific magnetism* of the material.

From some of his early work, to which he applied a peculiar method of extrapolation, Rowland inferred that in the case of soft iron, I_∞ must be about 1390, whereas Fromme obtained the value 1510 in

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[1] Joule, Phil. Mag., **2**, 1839; Mueller, Pogg. Ann. **79**, 1850; **82**, 1851; Koosen, Pogg. Ann. **85**, 1852; Dub, Pogg. Ann. **90**, 1853; G. Wiedemann, Pogg. Ann. **100**, 1851; **106**, 1859; **117**, 1862.

[2] Rowland, Phil. Mag. **46**, 1873; **48**, 1874; Stefan, Wiener Berichte, 1874; **97**, 1888. Wied. Ann. **38**, 1889; Fromme, Wied. Ann. **13**, 1881; Ewing and Low, B. A. A. S. Report 1887; Phil. Trans. **180**, 1889; H. E. J. G. duBois, Phil. Mag. **29**, 1890; Roessler, Elektrotechnische Zeitschrift, **14**, 1893; Jones, Wied. Ann. **54**, 1895; **57**, 1896; Gumlich, Elektrotechnische Zeitschrift, **30**, 1909; Peirce, These Proceedings, **44**, 1908; Am. Journal of Science, **28**, 1909; Weiss, Journal de Physique, May, 1910; Hadfield and Hopkinson, Jour. Inst. Elect. Eng., **46**, 1911.

1873, and Stefan, 1400, in the following year. In 1881, however, Fromme got the value 1737 for one specimen, and in 1884, Weber, exposing a long rod in a solenoid to a field which had an intensity of only 900 gausses before the iron was introduced, found the corresponding value of I to be 1700.

In 1887, Messrs. Ewing and Low introduced a new and most ingenious method for experimenting upon slender isthmuses of iron and steel under very high excitations and showed that different specimens of soft iron often behaved very differently in very strong fields. For one brand of fine Swedish iron, they found the final value of I to be only 1620, while for a certain kind of Bessemer steel, the value I_∞ was as high as 1770.

Du Bois published in 1890 the results of a series of experiments upon iron in very intense fields the strengths of which he had determined by optical means. In order to obtain, for each brand of material, the Kerr's constant which he needed, he first examined an ellipsoidal test piece of the material in much weaker fields, in a solenoid. In a typical case, the soft iron ellipsoid of revolution was 18 centimeters long and 6 millimeters in diameter at the centre. The solenoid was 30 centimeters long and consisted of 1080 turns of insulated wire wound in twelve layers of about 4 centimeters mean radius. The field intensity at a point 9 centimeters from the centre of the solenoid was about 6% less than at the centre and this introduced a correction into the formula for I. The moment acquired by the bar when it was under excitation was determined, after the effect of the current in the solenoid had been compensated for, by the indications of a magnetometer in Gauss's *A* Position with respect to the specimen. The correction for the ends of the ellipsoid was made by the use of the formula $H' = H - 0.052\ I$. We shall find it convenient to refer to these results later on in this paper and some of them appear in Table I.

TABLE I.

H'.	I.	H'.	I.
100	1410	600	1680
200	1522	800	1695
300	1590	1000	1703
400	1630	1300	1712
500	1661		

In the stronger fields the results were not so regular, for the specimen was magnetized between the poles of a powerful electromagnet and the fields were far from uniform. The final value of I which du Bois obtained lay somewhere between 1700 and 1750.

A paper by Roessler in the Elektrotechnische Zeitschrift for 1893 describes some experiments very like those made by du Bois with the solenoid mentioned above. Roessler's solenoid was 1 meter long and consisted of 16 layers of wire 3 millimeters in diameter. The mean radius of the solenoid was about 5.5 centimeters and the field at a point on the axis 25 centimeters from the centre was about 1% less than at the centre itself. The test piece was an ellipsoid 50 centimeters long and 1 centimeter in diameter. The results which Roessler obtained for a certain specimen of so called "soft iron" are given in Table II.

TABLE II.

H'.	I.	H'.	I.
414	1645	848	1679
481	1663	919	1681
516	1670	991	1681
587	1675	1062	1685
741	1679	1276	1683
777	1681	1312	1687

The values of I_∞ published in 1896 by E. T. Jones, who magnetized a short, slender wire of the material to be tested between the conical pole pieces of an electromagnet of the du Bois form, ranged as high as 1818; and the results of the joint work of du Bois and Jones, printed in 1899, gave values of I_∞ between 1780 and 1850.

Weiss, in 1907 and 1909, experimented upon small ellipsoids of revolution made of iron, nickel, and cobalt, placed symmetrically between the flat pole pieces of a powerful electromagnet. Each ellipsoid was about 9 millimeters long and 3.5 millimeters in diameter. The gap between the pole pieces was about 6 centimeters long and the diameter of the magnet core was 15 centimeters. An excitation of 94000 ampere turns corresponded to a field of about 9000 gausses in the gap centre. The small ellipsoid was suddenly drawn out of the field through a hole in the axis of one of the pole pieces and the flux change in a test solenoid outside the iron was determined. Weiss's values of I_∞ were 1731 and 1706.

Gumlich in 1909 made a series of extremely accurate determinations of the final value of I in soft iron by the Isthmus Method, using an electromagnet of the du Bois form, which was furnished with two soft pole pieces fastened together with the isthmus between them and capable of being rotated together about a horizontal axis perpendicular to the pole axis, so as to reverse suddenly the sign of the magnetization in the test piece. Each specimen was 28 millimeters long and 3 millimeters in diameter. To make sure that the lines of induction in the test piece were throughout parallel to each other, Gumlich sometimes used soft iron rings slipped over the specimen. Gumlich's value of I_∞ was 1725.

In December, 1910, Messrs. Hadfield and Hopkinson printed the results of a very carefully carried out and very elaborate investigation into the question whether in such combinations of iron and less magnetic substances as are in practical use, the specific magnetism of any piece of the material multiplied by the mass of the piece is simply equal to the sum of the products obtained by multiplying the mass of each constituent in the specimen by its specific magnetism. They came to the conclusion that although this rule seems not to hold in certain alloys of iron, nickel, and manganese, it is really fulfilled in many practical cases.

They used a modification of the Isthmus Method very skilfully, employing an electro-magnet like, if not identical with, the magnet which Ewing and Low had, and which was made for the first isthmus experiments, under the direction of W. Low, Esquire, of Balmakewan.

Hadfield and Hopkinson had at command a large number of alloys specially made at the Hecla Works, for research purposes, and the analyses of their test pieces are therefore beyond question. They found that in their annealed iron-carbon steel, where other elements were nearly absent, the specific magnetism was less than for their standard iron by a percentage equal to about six times the percentage of carbon. In such a case they assumed that there are two constituents, pure iron, and iron carbide (Fe_3C) in mechanical mixture, the percentage of the carbide present being 15.5 times that of the carbon in the steel. The "pure iron" used as a standard was a sample of Swedish iron (Maker's mark "S. C. I.") containing less than 0.2 per cent of impurities. Of this they used two specimens: one was 6.26 mm. long, and 3.18 mm. in diameter and weighed 0.385 grammes; the other was 15.92 mm. long and 3.19 mm. in diameter, and its weight was 0.99 grammes. Both yielded the same value (1680) for the specific magnetism. Table III, obtained from measurements of one of the curves given by Hadfield and Hopkinson, reproduces their results sufficiently well. Most of their pieces of steel were slightly less dense than the S. C. I. iron and their final values of I_∞ give the magnetization vector per unit volume of matter of the same density as the iron.

It is now possible to get in the market large pieces of iron which has less than 0.03% of impurities all told, and I have used Norway iron 99.87% pure, as well as many other specimens of nearly this excellence. In some cases the specific magnetism seemed to be about 1740 and in others less than 1700. For these determinations I have usually employed some form of isthmus method.

TABLE III.

Hadfield and Hopkinson's Values of the Specific Magnetism of Iron and Carbon Alloys.

Percentage of Carbon.	Specific Magnetism.
*	1680
0.5	1630
1.0	1580
1.5	1530
2.0	1480
2.5	1430
3.0	1380
3.5	1330

TABLE IV.

Saturation Values of the Magnetization Vector in Iron.

Rowland,	(1873–8)				1390
Stefan	(1874)				1400
Fromme	(1873)				1510
Fromme	(1881)				1737
Weber	(1884)	(H = 900)			1700
Ewing and Low	(1887–9)		1620	to	1740
DuBois	(1890)	(H = 1200)			1710
Roessler	(1893)	(H = 1276)			1688
Jones	(1896)				1818
DuBois and Jones	(1899)		1780	to	1850
Weiss	(1907)				1731
Gumlich	(1909)				1725
Peirce	(1909)		1738	to	1751
Weiss	(1910)				1706
Hadfield and Hopkinson	(1911)				1680

If the lines of magnetic induction in a slender homogeneous cylinder, made of perfectly soft iron, are known to be straight and parallel to the generating lines of the cylinder, we may infer that the induction vector — which in this case must be solenoidal and lamellar in the metal,— has the same intensity throughout the space considered. If, moreover, the lines of force in the air about the cylinder and near it on all sides, seem to be straight, we may believe, since the tangential components of the magnetic force and the normal component of the induction are continuous at the surface of the iron, that the lines of force and induction in the metal are straight and parallel to the lines of the cylinder and to the lines just outside the metal in the air. If, therefore, by means of a test coil of very fine insulated wire wound tightly around the cylinder, and a somewhat larger coaxial coil which does not extend into any portion of the air where the lines of force are not straight, we determine B in the metal and H in the closely surrounding space, the ratio of the two may be supposed to give the value of the permeability in the iron. This is the theory that underlies one form of the "Isthmus Method" of measuring the value of the magnetization vector in the metal at high excitations. If the results are to be satisfactory, great care must be taken to make sure that the magnetic lines just outside the isthmus are really straight in the region to be used, and the dimensions of the test coils must be determined with the aid of trustworthy comparators with great accuracy. The larger test coil must be mounted upon some sort of support, if it is to keep its form unchanged, and the choice of material for a spool is very narrow. No brass or copper that I have ever tried is unmagnetic in very strong fields; paraffine wax and ebonite are often paramagnetic and introduce errors into the readings. Silk insulation for the wire of which the test coils are made is inadmissible without careful examination and even shellack, when dried from an alcoholic solution, is almost always strongly magnetic.

The form of bobbin used by Ewing and Low requires a fresh outer test coil for each specimen, but the little rods inserted at the ends into holes in the pole pieces, as in the work of Gumlich, or the shorter rods butted against the faces of the pole pieces, as in the work of Hadfield and Hopkinson, do not have this disadvantage.

If the lines of force in the air about the isthmus are practically straight for one excitation they often cease to be so when the intensity of the field is much increased. If, with soft iron pole pieces the lines are parallel for a soft iron bobbin, they may not be even approximately so for a bobbin of fairly hard steel, as I have found to my cost in a somewhat large experience.

These and other difficulties lie in the way of anyone who attempts to use the Isthmus Method in its original form, and every modification of it, whatever advantages it has, usually introduces some new problems. Notwithstanding all this, the method is a most useful one and has a much wider application than has usually been given it. No other way that has been proposed of making magnetic measurements at very high excitations is nearly so good, and the test pieces now employed are small and convenient to make.

There is still in many cases some uncertainty in the determinaton of *H*, and Hadfield and Hopkinson discuss the subject in trying to account for the differences between their results[3] and those of Gumlich, obtained at the Reichsanstalt. Moreover, there is sometimes irregularity in the values of *I* measured by the Isthmus Method[4] for a single specimen. For these reasons, it has seemed to me worth while to push the use of the solenoid for magnetizing test pieces farther than has yet been done, to make sure that there are specimens of metal in which *I* is higher than 1700 even in much weaker fields than those which the Isthmus Method furnishes. This is especially desirable if we wish to be able to determine the constitution of a large mass of steel by a quick measurement of the specific magnetism of a small test piece in an electromagnet arranged for the purpose, as has been proposed.

According to the molecular theory of magnetization of Weber and Ewing, the molecules, which lie with their magnetic axes in all directions when the metal is in the neutral state, tend to turn in the direction of any magnetic field to which the iron may be exposed, though they are hindered from doing so by the interaction of the molecules themselves. When, however, the applied field is made strong enough to overcome these intermolecular forces, in large measure, all the axes of the elementary magnets point practically in the same direction. It is evident, therefore, that unless the applied field affects the moments of the elementary magnets of which the metal is made up, the magnetic moment (*I*) of the metal per unit volume should remain nearly constant after the excitation has gone beyond a certain large value. This maximum magnetization is very different in different metals and we may well consider it as characteristic of a material.

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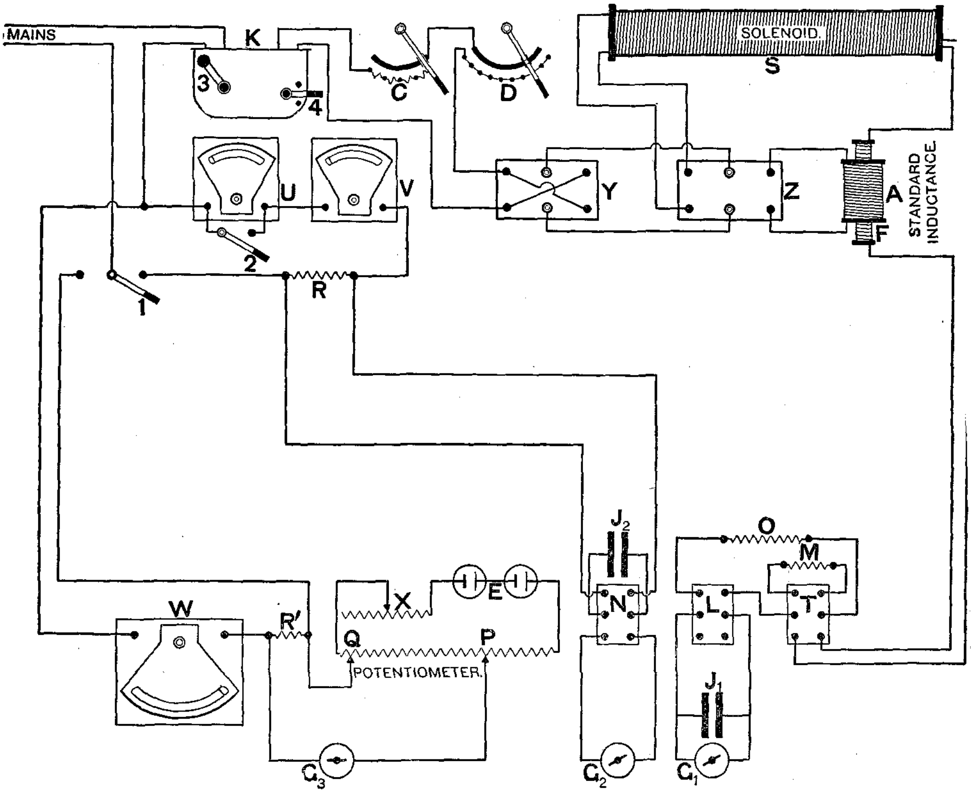
[3] Journal of the Institution of Electrical Engineers, Dec., 1910, p. 253.

[4] Ewing's Magnetic Induction in Iron and Other Metals, Tables XI, XII.

Apparatus and Method of Procedure.

Figure 1 shows diagrammatically the general arrangement of the great array of apparatus used in making the measurements described in this paper. This apparatus was adjusted and some of it constructed by Mr. John Coulson of the scientific staff of the Jefferson Laboratory, who has worked with me at every stage of the investigation, and to whose skill and patience I am deeply indebted. Many details are omitted from the figure. The devices for demagnetizing the specimens to be tested will be described later on.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Q
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POTENTIOMETER
J_1
G_3
G_2
G_1

Figure 1. This Figure shows diagrammatically the general arrangement of some of the apparatus used in making the observations described in this paper. The elaborate devices for demagnetizing the specimens are omitted for simplicity.

It is evident that in such measurements of magnetic flux changes as are necessary in the work described in this paper, it is of fundamental importance that the ballistic galvanometers used be correctly calibrated, and we used a number of standards of mutual inductance, most of them rather larger than those commonly employed for such purposes, since our rather slowly moving galvanometer was not very sensitive. We had in all seventeen mutual inductances for our calibrations. Of these five have been measured for us this year at the United States Bureau of Standards, and seven others are of such forms that their values may be calculated by well known methods. We found one of Doctor Campbell's Variable Standards of Mutual Inductance (which was very kindly lent to us by Professor Kennelly), most useful. It proved to be very accurately calibrated, and it agreed closely at all points with the standards determined for us at Washington.

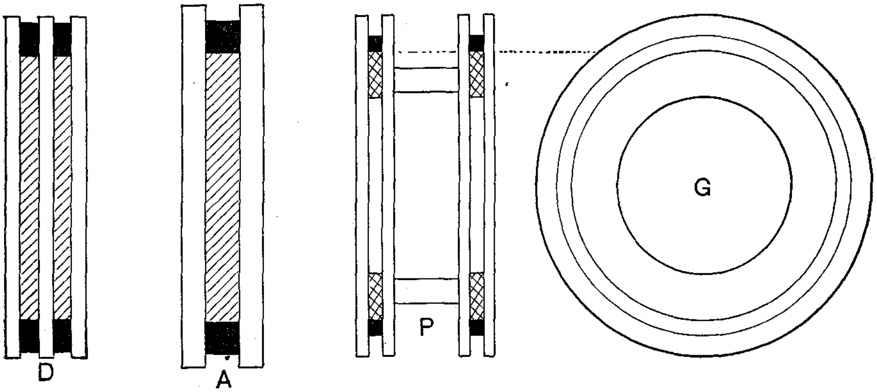

FIGURE 2. Three standards of mutual inductance.

An elaborate series of comparisons of our inductances occupied Mr. Coulson and myself for more than two months, because we found that three or four of those which, according to our computations based upon their geometrical forms, should have certain values, seemed to have slightly different values, though they did not seem to be quite constant. This phenomenon puzzled us at first and gave us much trouble, but we believe, after all our work, that the ebonite used as a core in three of them is very slightly susceptible in a strong magnetic field, that the split thick brass tube used as a core for one of our solenoids is sufficiently paramagnetic to affect the field inside it perceptibly, though another solenoid constructed in a similar manner seems free from this difficulty, and finally, that the white silk triple covering of some of our wire is hygroscopic and that when a closely wound coil of it is damp, there may be a very little leakage from turn to turn through the insulation, under very strong excitation. In any event, we have eliminated all error from these sources, and we believe that the inductances of the standards we have finally used may be depended upon to at least the twentieth of one per cent.

The shapes of three of our standards are shown in Figure 2. In D, the three larger plates (and the shaded cores) are of plate glass about 29 centimeters in diameter. The cores were mounted by Mr. Thompson in an engine lathe, and were ground for about two days, under a constant flow of soda water, by a rapidly turning carborundum wheel fastened to the tool post and driven by its own motor, while the lathe moved slowly. In this way the plates were made very accurately circular. A is also wound upon a plate glass spool, but the two coils are wound together from two spools of wire triply covered with white silk. P consists of two spools with plate glass ends, but the shaded cores are ebonite rings. G shows a side view of P.

The magnetizing solenoid consists of about 300 kilograms of triply covered Number 10 copper wire wound uniformly with great care, by Mr. George W. Thompson, upon a massive brass spool 186.2 centimeters long in inside measurement. The inner coil has 8117 turns in 14 layers, and a resistance at room temperatures of about 7.7 ohms. The outer coil, of slightly different wire, has 5872 turns in 10 layers, and a resistance of about 9.8 ohms. The field intensity at different points of the axis was found for a given current in each layer separately, and it appeared from combining the results, that a current of one ampere sent through the whole inner coil gives rise to a field of intensity 54.71(3) gausses at the centre and 54.60(5) gausses at a point 50 centimeters from the centre, on the axis. A current of one ampere sent through both coils in series creates an electromagnetic field of intensity 94.19(5) gausses at the centre and 93.77(5) gausses at a point 50 centimeters from the centre, a difference of nearly one half percent. The outside diameter of the solenoid is a little less than 20 centimeters.

For currents up to 31.5 amperes, corresponding to a field of about 2900 gausses, the coils were used in series attached to the 550 volt circuit of the Harvard University plant. For stronger fields of 5000 gausses or more, the coils could be attached in parallel to this circuit with a standard amperemeter in each branch. For the preliminary experiments, fields stronger than 4600 gausses were not needed.

The thick, solid-drawn brass tube upon which the wire was wound carried a stream of tap water to keep the specimen at a constant temperature. The test coil was wound upon the test piece after the latter had received a very thin film of varnish. The test coil, after it had been made, was varnished and the whole was then placed for about half an hour in a stream of hot air to harden the coating. The leads were enclosed in a very thin tube of rubber, the test coil was covered with a rubber shield, and melted paraffine wax was then run into the ends of this shield so as to keep the test coil absolutely dry. In this manner all leakage from turn to turn of the triply silk covered wire of which the test coil was made was avoided. In many cases two test coils were wound side by side upon each specimen, but the results, after we had learned how to make the coils properly, were so nearly identical for both coils that we sometimes used only one. In all cases the differences, if there ever were any real differences, were far smaller than the unavoidable errors of ballistic galvanometer reading.

The reversal of a strong current in the circuit of a solenoid with so great an inductance as this one, has to be managed carefully. The main reversing switch, when it was slightly pulled, automatically put the solenoid in parallel with a noninductively wound resistance higher than its own, and, after the handle was raised higher, broke the main circuit so that the discharge from the solenoid could pass through the auxiliary resistance. The process was inverted when the switch handle was pushed down. This switch (Figure 3, Plate 1) was designed and made by Mr. Coulson.

To prove that the field in the solenoid, when a given current passes through the circuit, is really what it should be, according to the calculation, a very carefully made test coil without iron was placed in series with the secondary of a standard of mutual inductance and the field was thus measured. By this means it was shown that there was no appreciable leakage between the turns of the solenoid — a very common fault of the exciting coils of electromagnets — and that there was not enough iron in the brass of the reel to affect the field strength sensibly. So far as we can determine the fact by our many and repeated tests, the solenoid has not been injured by use and is very perfect. It is firmly mounted on a solid oak frame so that its axis is horizontal and perpendicular to the meridian.

The dimensions of the iron test pieces and of the standard inductances were obtained with the help of a set of micrometer screw gauges by Brown and Sharpe. The smallest one of these was used for determining the diameters of the specimens and of the coils wound upon them. The accuracy of this gauge was tested by a comparator (which had a screw by Gaertner), and by another comparator (by Zeiss) which reads directly to microns.

An illustration, the case of a specimen of American Ingot Iron, will show how much error would be introduced into the value of the specific magnetism of the iron by a given error in measurement of its dimensions. The length was 100 cm., the diameter of the bare iron, 1.278(5) cm. and the mean diameter of the test coil, 1.326 cm. The coil consisted of 100 turns of copper wire triply covered with white silk, and as the dimensions show, the flux through it was $128.32\ B + 9.8\ H$. The last term which shows the correction for the air flux linked with the coil, is relatively small at feeble excitations, and even when H rises to 2800 and B becomes about 24500, the whole term is less than 1% of the flux through the iron. Moreover the value of the term may be found to within one twenty-fifth of its value without trouble. An error of 0.001 in measuring the diameter of the iron might make an error of three units in the last place in the value of the specific magnetism and this makes it desirable to use exactly round rods. The piece here described was cut out of a large bar with great skill, at the works of Messrs. Barbour and Stockwell.

At high excitations, the corrections for the effect of the ends of the cylindrical test pieces are, of course, much less than those which according to theory and to the formulas of DuBois and of Shuddemagen are necessary in low fields. I shall hope to discuss this matter at length in another paper, and need only state here that the correction for a piece of the dimensions used was practically negligible in fields of strength above 2000 gausses.

The rods to be annealed were first packed tightly in fine iron filings in a piece of pipe the ends of which were closed by screw caps, and the whole was carefully supported perpendicular to the meridian in a special gas heater where it would be exposed to several hundred flames driven by a power compressor. In this manner a piece 150 centimeters long could be heated very uniformly. After the specimen had been kept for perhaps an hour at a temperature considerably above the critical point of the iron it could be then allowed to cool very slowly *in situ*, protected from magnetic action.

If a slender rod of iron be placed inside a long solenoid which is in the secondary circuit of a powerful open-core transformer, and if while the primary circuit is attached to the alternate current mains, the secondary coil of the transformer be slowly drawn off the core and the primary coil, by help of some mechanical device, it is possible to send through the solenoid a long series of currents alternating in direction and gradually decreasing in intensity and thus to demagnetize the iron rod very well. We had an apparatus of this kind permanently connected with our apparatus, but it was not shown in Figure 1 lest the diagram be too complex.

When the direction of a strong electrical current in the circuit of the large solenoid (S) in which the iron rods to be tested were magnetized, was suddenly reversed, some time was needed to establish the new current in its full value, and the change in the magnetic flux through the test coils wound upon the rods was not complete until after several seconds. This fact, due to the large inductance in the circuit, made it unsafe to employ a ballistic galvanometer of ordinary type for measuring this flux change, and we had recourse to a long period instrument of a kind which has been used for a number of years in the Jefferson Laboratory. The particular galvanometer (G) we chose, had a period of 156 seconds which was quite long enough for our purposes, but we had a much more slowly moving instrument at hand in case of need. Any fairly long throw of G could be determined with an error of less than one tenth of one per cent, and we could do better than this by careful repetition. G is shown in Figure 4, Plate **1**.

The main currents in the solenoid circuit were measured with the help of a series of Weston Amperemeters (two of which are shown diagrammatically as U and V in Figure 1) properly arranged for the special intensity ranges we needed, but the accurate determination of large currents was made by aid of a potentiometer (Figure 5) with standard cadmium cells, which measured the potential drop across a standard one hundredth of an ohm resistance (R) by Crompton, which had been tested against another standard by Wolff. The largest currents we used could not very well be allowed to run very long through the coils because the amount of heat set free in the circuit was enormous. Indeed, with an energy expenditure of more than fifty kilowatts, the heating problem, in spite of running water in the core of the solenoid needed careful consideration. As a matter of fact, the only difficulty we finally encountered was a slight falling off of our largest currents with repeated throws, owing to a little increase in the resistance of the circuit, and this came at a place where the flow of inductance through the test coil changed very slowly with H. To save time we arranged a standard condenser (Elliot Brothers,

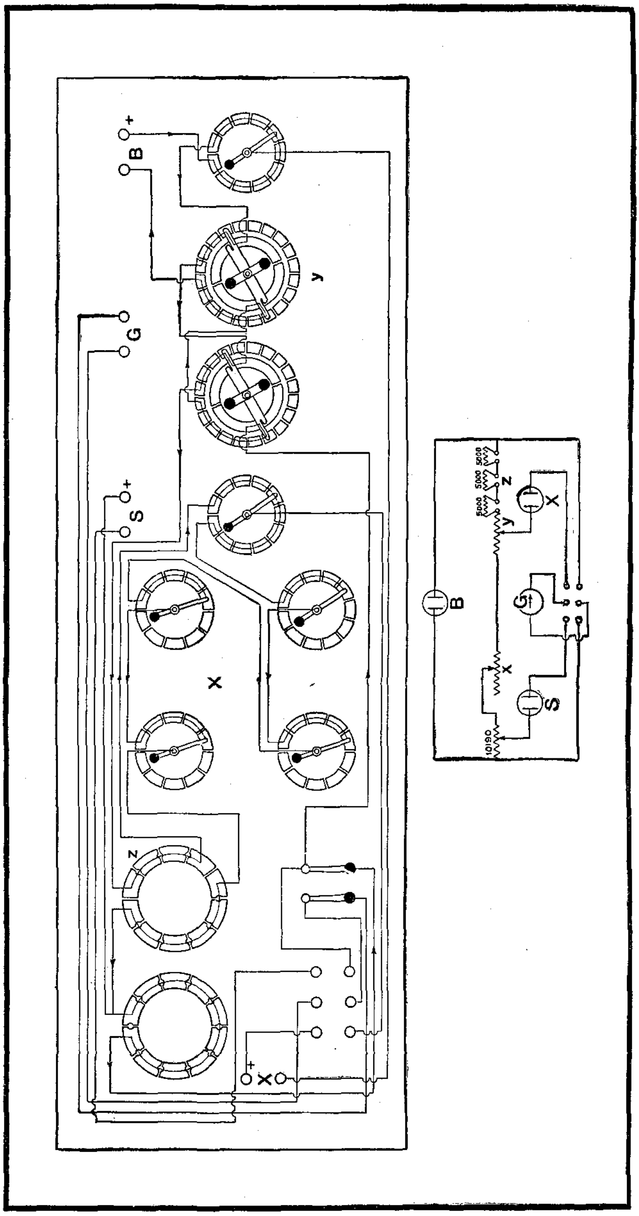
B
+
G
S
+
y
x
z
X
+
B
G
S
X
x
y
z
5000
5000
5000
10190

FIGURE 5. The standard potentiometer.

No. 72) so that it became automatically charged at the terminals of *R* just as the main switch was reversed, and the charge could then be measured at our ease four or five seconds after the switch had been thrown over. By these means we avoided the delay which would have resulted if we had been obliged to read the amperemeters before the reversal.

Besides this slowly moving ballistic galvanometer, we used three other mirror galvanometers, one for the condenser throws, one for the potentiometer, and one for the accurate comparison of our inductances, and in addition, a large standard laboratory amperemeter (*W*), by Weston, which could be checked at any instant against the potentiometer. This beautiful instrument has an engine divided scale 31 cms. long.

At very high excitations, the reversal of any switch of ordinary construction gives rise to a very unpleasant explosion, and we often made use of a large controller (*K*) constructed by the General Electric Company for use upon electric cars. This was very kindly lent to us by Mr. F. W. Lieberknecht, and served an excellent purpose. We do not need to describe a large number of auxiliary amperemeters and galvanometers used in our work.

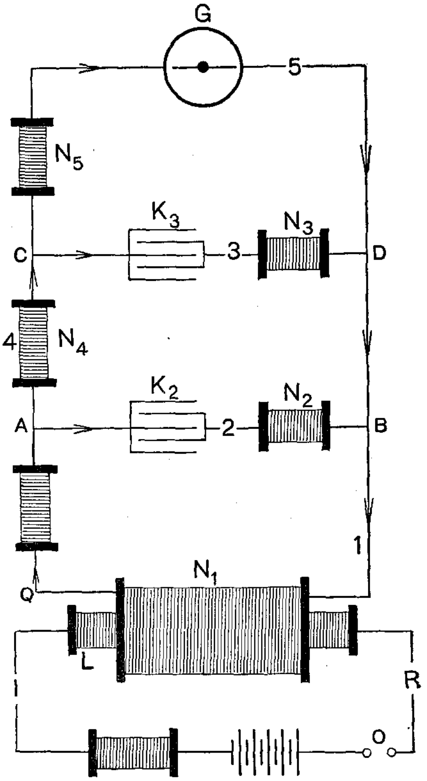

FIGURE 6.

THE USE OF CONDENSERS IN THE INDUCTIVE SECONDARY CIRCUIT.

The inductance in the secondary circuit, which contained the test coil or coils, the secondary coils of the inductance standards, and the coil of the large ballistic galvanometer, was usually considerable and the strain upon the insulation of the wire was sometimes large when a powerful current in the primary circuit was suddenly reversed. Occasionally, there seemed to be some leakage in this circuit, so we introduced a number of condensers into the circuit in the attempt to reduce the stress. What the exact effect of such condensers in a complex circuit will be when the breaking arc in the primary circuit is oscillatory, it is usually impossible to predict, because some necessary data are wanting or because the literal equations are of too high a degree to be solved, but certain general facts are clear. The following analysis treats some questions, as applied to a circuit taken for illustration, which are really, perhaps, too elementary to need any discussion.

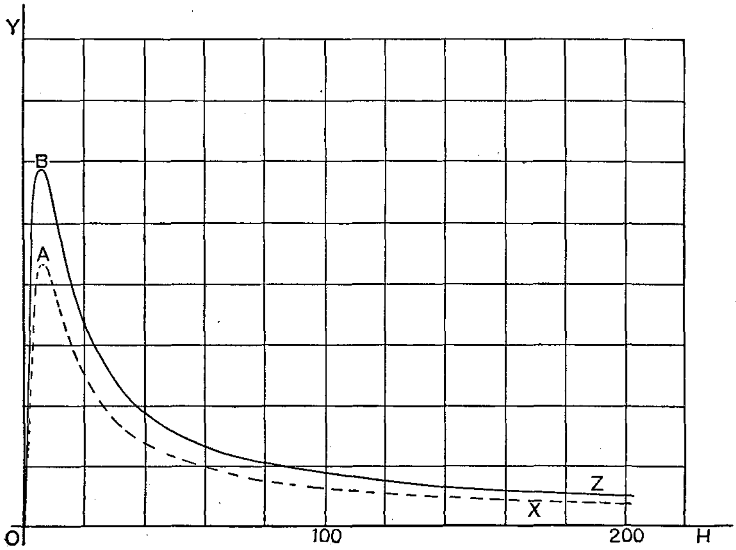

FIGURE 7. This figure shows the forms of the curves obtained by plotting the permeability and the susceptibility of a certain kind of soft iron against the exciting field. These curves, OBZ, OAX, are drawn to different scales.

Figure 6 represents two neighboring circuits:—

(*a*) A primary circuit of total resistance R, and total self inductance L, which contains a constant battery of voltage, V, and carries a current I. This circuit is furnished with a gap (O) which may be closed or opened at pleasure.

(*b*) A secondary circuit of several branches, which has no battery, but which is linked with the primary circuit by the mutual inductance, M. The branch AQB or (1), which is directly coupled with the primary, has a resistance R_1, a total self inductance N_1, and carries a current I_1. The branches AB, CD, or (2), (3), contain condensers of capacities K_2 and K_3 respectively. Their resistances are R_2 and R_3, their self inductances, N_2 and N_3 and they carry currents I_2 and I_3. The branches (4) and (5) have no condensers. Their resistances are R_4 and R_5, their self inductances, N_4, N_5, and their currents, I_4 and I_5. The current in DB, which has a negligible resistance, is, of course, I_4.

If accents are used to denote differentiations with respect to the time, an easy application of Kirchhoff's Laws to these two circuits leads to the equations:—

$$\begin{aligned}
&V - L \cdot I' - M \cdot I'_1 = R \cdot I, \\
&- M \cdot I' - N_1 \cdot I_1' - N_4 \cdot I_4' - N_5 \cdot I_5' \\
&\qquad\qquad = R_1 \cdot I_1 + R_4 \cdot I_4 + R_5 \cdot I_5, \\
&- M \cdot I' - N_1 (I_2' + I_3' + I_5') - N_2 \cdot I_2' - Q_2/K_2 \\
&\qquad\qquad = R_1 (I_2 + I_3 + I_5) + R_2 \cdot I_2, \\
&- M \cdot I' - N_1(I_2' + I_3' + I_5') - N_4 (I_3' + I_5') - N_3 \cdot I_3' - Q_3/K_3 \\
&\qquad\qquad = R_1 (I_2 + I_3 + I_5) + R_4 (I_3 + I_5) + R_3 \cdot I_3, \\
&I_1 = I_2 + I_4 = I_2 + I_3 + I_5,
\end{aligned} \tag{1}$$

or

$$\begin{aligned}
&(L \cdot I' + R \cdot I) + M \cdot I_2' + M \cdot I_3' + M \cdot I_5' = V, \\
&M \cdot I' + (N_1 \cdot I_2' + R_1 \cdot I_2) + (N_1 \cdot I_3' + N_4 \cdot I_3' + R_1 \cdot I_3 + R_4 \cdot I_3) \\
&\quad + (N_1 \cdot I_5' + N_4 \cdot I_5' + N_5 \cdot I_5' + R_1 \cdot I_5 + R_4 \cdot I_5 + R_5 \cdot I_5) = 0, \\
&M \cdot I'' + (N_1 \cdot I_2'' + N_2 \cdot I_2'' + R_1 \cdot I_2' + R_2 \cdot I_2' + I_2/K_2) \\
&\qquad + (N_1 \cdot I_3'' + R_1 I_3') + (N_1 \cdot I_5'' + R_1 \cdot I_5') = 0, \\
&M \cdot I'' + (N_1 \cdot I_2'' + R_1 \cdot I_2') + (N_1 \cdot I_3'' + N_4 \cdot I_3'' + N_3 \cdot I_3'' \\
&\qquad + R_1 \cdot I_3' + R_4 \cdot I_3' + R_3 \cdot I_3' + I_3/K_3) \\
&\qquad + (N_1 \cdot I_5'' + N_4 \cdot I_5'' + R_1 \cdot I_5' + R_4 \cdot I_5') = 0.
\end{aligned} \tag{2}$$

If, for I we write $I_0 + V/R$, the second number of the first equation becomes zero, while all the equations remain otherwise unchanged in form, and it follows that every one of the currents satisfies a single linear differential equation of the sixth order with constant coefficients and that if a, b, c, d, e, and h are the roots of the equation formed by equating to zero the determinant,

$$\begin{vmatrix} Lx+r & Mx & Mx & Mx \\ Mx & N_1x + R_1 & \left\{\begin{matrix} N_1x + N_4x + R_1 \\ + R_4 \end{matrix}\right\} & \left\{\begin{matrix} N_1x + N_4x + N_5x \\ + R_1 + R_4 + R_5 \end{matrix}\right\} \\ Mx^2 & \left\{\begin{matrix} N_1x^2 + N_2x^2 + R_1x \\ + R_2^x x + 1/K_2 \end{matrix}\right\} & N_1x^2 + R_1x & N_1x^2 + R_1 \\ Mx^2 & N_1x^2 + R_1x & \left\{\begin{matrix} N_1x^2 + N_4x^2 + N_3x^2 \\ + R_1x + R_4x + R_3x \\ + 1/K_3 \end{matrix}\right\} & \left\{\begin{matrix} N_1x^2 + N_4x^2 + R_1x \\ + R_4x \end{matrix}\right\} \end{vmatrix} \quad (3)$$

then

$$\begin{aligned} I &= Ae^{at} + Be^{bt} + Ce^{ct} + De^{dt} + Ee^{et} + He^{ht} + \frac{V}{R} \\ I_2 &= \alpha_2Ae^{at} + \beta_2Be^{bt} + \gamma_2Ce^{ct} + \delta_2De^{dt} + \epsilon_2Ee^{et} + \eta_2He^{ht} \\ I_3 &= \alpha_3Ae^{at} + \beta_3Be^{bt} + \gamma_3Ce^{ct} + \delta_3De^{dt} + \epsilon_3Ee^{et} + \eta_3He^{ht} \\ I_5 &= \alpha_5Ae^{at} + \beta_5Be^{bt} + \gamma_5Ce^{ct} + \delta_5De^{dt} + \epsilon_5Ee^{et} + \eta_5He^{ht} \end{aligned} \quad (4)$$

If these values be substituted in one of the Kirchhoff equations above and the coefficients of the different exponential expressions separately equated to zero, it will appear that the α's, β's, γ's, δ's, ϵ's, and η's are determinate functions of the constants of the circuit and in no way dependent upon the manner in which the currents are managed.

The other six constants (A, B, C, D, E, H) have to be computed from a knowledge of the electrical conditions which determine any problem concerning these two fixed circuits.

If, for instance, there is no current in any branch of the circuits at the outset, and if the gap, O, be suddenly closed at the origin of time, the values of the constants for all positive time satisfy the equations

$$\begin{aligned} A + B + C + D + E + H &= -\frac{V}{R} \\ \alpha_2A + \beta_2B + \gamma_2C + \delta_2D + \epsilon_2E + \epsilon_2H &= 0 \\ \alpha_3A + \beta_3B + \gamma_3C + \delta_3D + \epsilon_3E + \epsilon_3H &= 0 \\ \alpha_5A + \beta_5B + \gamma_5C + \delta_5D + \epsilon_5E + \epsilon_5H &= 0 \end{aligned} \quad (5)$$

and, after these have been solved, it is easy to compute the whole flow of electricity through the galvanometer, for

$$\int_0^\infty I_5\,dt = -\left(\frac{\alpha_5A}{a} + \frac{\beta_5B}{b} + \frac{\gamma_4C}{c} + \frac{\delta_5D}{d} + \frac{\epsilon_5E}{e} + \frac{\eta_5H}{h}\right) \quad (6)$$

If, however, when the primary current has its steady value, V/R, and there are no currents in the secondary circuit, the primary resistance be instantaneously changed from R to R', at the time $t = 0$, there is no sudden change in the current in any branch, but for all subsequent time, the constants are determined by the equations:—

$$\begin{aligned} A + \quad B + \quad C + \quad D + \quad E + \quad H &= \frac{V}{R} - \frac{V}{R'} \\ \alpha_2 A + \beta_2 B + \gamma_2 C + \delta_2 D + \epsilon_2 E + \eta_2 H &= 0 \\ \alpha_3 A + \beta_3 B + \gamma_3 C + \delta_3 D + \epsilon_3 E + \eta_3 H &= 0 \\ \alpha_5 A + \beta_5 B + \gamma_5 C + \delta_5 D + \epsilon_5 E + \eta_5 H &= 0 \end{aligned} \qquad (7)$$

and it is evident that every one of the quantities, A, B, C, D, E, H, I, given by (7) has a value which bears to the corresponding value given by (5) the ratio $(R - R')/R'$, and the same relation holds between the whole discharges through the galvanometer in the two cases. If the gap be instantly opened so that R' is infinite, when the current in the primary circuit is V/R and there are no secondary currents, the galvanometer throw is equal, but opposite in sign, to the throw caused by suddenly closing the gap when all the currents are zero.

The electrokinetic energy for the coupled circuits is

$$\begin{aligned} T = \tfrac{1}{2}L \cdot I^2 + M \cdot I(I_2 + I_3 + I_5) + \tfrac{1}{2}N_1(I_2 + I_3 + I_5)^2 \\ + \tfrac{1}{2}N_2 \cdot I_2^2 + \tfrac{1}{2}N_3 \cdot I_3^2 + \tfrac{1}{2}N_4(I_3 + I_5)^2 + \tfrac{1}{2}N_5 \cdot I_5, \end{aligned} \qquad (8)$$

so that the electrokinetic momenta are

$$\begin{aligned} p &= L \cdot I + M(I_2 + I_3 + I_5) \\ p_2 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_2 \cdot I_2 \\ p_3 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_3 \cdot I_3 + N_4(I_3 + I_5) \\ p_5 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_4(I_3 + I_5) + N_5 I_5 \end{aligned} \qquad (9)$$

If, when I has the value $I = V/R$, and there are no other currents, the gap be instantly opened, I suddenly drops to zero, and I_2, I_3, I_5, which were 0, suddenly acquire initial values which may be determined by the fact that the electrokinetic momenta, p_2, p_3, p_5, which before the change were equal to $P_0 = MV/R$, are not altered by the impulse. After the gap is opened, the currents in the branches obey the system of equations (2), but the initial values of these currents are to be found from the equations

$$\begin{aligned}(N_1 + N_2)I_2 + N_1I_3 + N_1I_5 &= P_0 \\ N_1I_2 + (N_1 + N_3 + N_4)I_3 + (N_1 + N_4)I_5 &= P_0 \\ N_1I_2 + (N_1 + N_4)I_3 + (N_1 + N_4 + N_5)\, I_5 &= P_0\end{aligned} \qquad (10)$$

If then Δ denote the determinant of the coefficients,

$$\Delta = N_1N_2N_3 + N_1N_3N_4 + N_1N_3N_5 + N_1N_2N_5 + N_1N_4N_5 + N_2N_3N_5 + N_2N_3N_4 + N_2N_4N_5,$$

and the values of I_2, I_3 and I_5 just after the gap is opened, are

$$(N_3N_4 + N_3N_5 + N_4N_5)P_0/\Delta,\ N_2N_5P_0/\Delta,\ \text{and}\ N_2N_3P_0/\Delta. \qquad (11)$$

The total amount of electricity carried by the currents I_2, I_3, and I_5 are 0, 0, and Ω; and to find Ω we may integrate the second equation of the system (2) with respect to the time from 0 to ∞ and use the initial values of I_2, I_3, I_5 just found. This procedure leads to the equation:

$$\Omega = \int_0^\infty I_5 \cdot dt = \frac{P_0}{R_1 + R_4 + R_5}, \qquad (12)$$

and this is evidently the same result that would have been obtained for the whole discharge through the galvanometer, if the branches (2) and (3), with their condensers, were removed from the secondary circuit. It is easy to compute the sudden loss of energy when the gap is opened.

Results.

For the purposes of the investigation here described, we used about twenty-five different brands of iron obtained from several different sources. Of these, five gave values of I larger than 1700 for comparatively low excitations of about 2800 gausses.

About ten of our specimens were described by the dealers as "Bessemer" and showed similar micrographs. Most of these were in no way remarkable. For excitations of about 2700 they gave values of I of about 1675 in the average and might be expected to give 1685 for fields of strength 5000. One specimen (No. 10) was quite different from the others. For $H = 2730$ the corresponding value of I was 1727. This result is based upon several different determinations made upon different days, and during the interval the rod was once annealed. A long series of annealings however reduced the permeability so that finally the I corresponding to $H = 2600$ fell to about 1700. One specimen of wrought iron which we have annealed a great number of times shows no permanent change in permeability although at one stage this fell by more than one per cent temporarily, and was restored by the next annealing.

According to my experience during the last few years with a good many pieces of so called "Norway Iron," about one specimen in three of those bought without care in the open market may be expected to have a specific magnetism considerably above 1700. Different portions of the same large bar may have very different permeabilities, however, as one may readily believe after an examination of a series of micrographs which always show a considerable amount of slag. I believe that an occasional small piece such as would be used for an isthmus might be found to have a specific magnetism three or four percent above the best value to be found in a bar. I have myself encountered two isthmuses which gave 1790 and 1751 respectively, in spite of my best efforts to reduce what seemed to me at the time impossibly large values. Some small specimens used by other observers have shown even greater values than this. In the case of a rod a meter long and twelve millimeters in diameter, however, I have never found an average value much above 1740.

Much of the wrought iron to be had in the market, though very useful to blacksmiths, contains such an amount of slag that the continuity of the metal is seriously affected and the permeability of the mass is not very high. Such are the specimens of "Farnley Iron," marked here "F," the "Taylor Iron" and the "Best Refined Iron," which show low values of I in moderate fields. It is possible to get in the open market, "Norway Iron" of great purity. One specimen which I used showed, upon analysis, no nickel, cobalt, manganese or tungsten. It contained less than 0.03% of carbon, less than 0.047% of phosphorus, less than 0.03% of silicon and less than 0.003% of sulphur. This however does not compare in purity with the "American Ingot Iron," which contains less than 0.03% of impurities all told, and shows a very remarkable micrograph.

Our specimens of this iron were very kindly furnished by Doctor Percy W. Bridgman, who has been using this material in some of his experiments upon the behavior of metals under very high pressures. Plate II shows micrographs of two pieces. The first was in the normal state: the second had been exposed by Dr. Bridgman to a hydrostatic pressure of 17000 atmospheres for about 16 hours! The magnification is 120 diameters.

TABLE V.

VALUES OF THE MAGNETIZATION VECTOR IN VARIOUS KINDS OF IRON.

No.	Material.	Diameter.	H	I
1	"Taylor Iron"	0.975	2490	1645
2	"Taylor Iron"	0.998	2685	1654
3	Bessemer	1.266	2695	1663
4	Bessemer	1.269	2675	1687
5	Bessemer	1.269	2790	1677
6	Bessemer	0.950	2880	1671
7	Bessemer	0.950	2825	1666
8	Bessemer	1.269	2370	1685
9	Bessemer	0.634	2800	1673
10	Bessemer	0.632	2730	1727
11	"American Ingot Iron"	1.277	2760	1708
11	"American Ingot Iron"	1.277	4395	1711
12	"American Ingot Iron"	1.279	2780	1725
12	"American Ingot Iron"	1.279	4545	1735
13	Norway	1.078	2865	1686
14	Norway	0.969	2885	1685
15	Norway	1.302	2770	1661
16	Norway	1.280	2770	1727
16	Norway	1.280	4360	1742
17	Norway	0.294	2970	1735
18	"Cold Rolled Shafting"	1.269	2680	1693
19	"Cold Rolled Shafting"	1.269	2740	1678
20	"Best Refined Iron"	1.280	2740	1620
21	"Best Refined Iron"	0.999	2825	1589
22	"Best Refined Iron"	1.002	2690	1611
23	Drill Rod	0.794	2790	1533
24	"R"		2810	1637
25	"F"	0.968	2665	1627

Mr. Herbert M. Boylston, of Messrs. Sauveur & Boylston, who has most kindly examined, under the microscope, the polished and etched specimens of these irons, reports that Nos. 1, 2, 11, 12, 14, 20, 21, 22, and 25 contain little, if any carbon. Of the Bessemers, with which "R" must be reckoned, No. 6 contains only about 0.05% of carbon, while No. 10 which has a high specific magnetism has 0.15% and each of the other pieces about 0.10%. The Drill Rod has about 1.10% of carbon; the specimen, which is of very fine grain, has no slag and shows simply sorbite, pearlite, and some cementite in fine net work.

Nos. 1, 2, 20, 21, 22, 24, and 25 contain considerable slag, and this is in comparatively large masses in portions of the "Refined Iron. The relatively small amount of slag in the Norway Irons is in fine particles distributed through the mass. Nos. 11, and 12 seem to be simply ferrite.

Table VI gives some corresponding values of *H* and *I* for the specimen of American Ingot Iron, known as No. 12.

TABLE VI.

H.	*I.*	*H/I.*
40.5	1343	0.030
68.8	1398	0.049
129.9	1476	0.088
229.8	1570	0.146
271	1597	0.170
399	1660	0.240
457	1684	0.271
871	1715	0.507
1126	1717	0.655
1569	1719	0.913
2288	1724	1.323
2747	1727	1.589
4543	1735	2.620

It is well known that if a relatively stout rod of soft iron be exposed to a strong field in a solenoid and if the magnetizing current be very suddenly broken, the direction of the residual magnetism in the rod may be opposite in sign to what it was when the current was running. If the rod be enclosed in a thick walled copper tube within the solenoid, this reversal never takes place and the sign of the residual magnetism is always normal. The moment of the rod under the new field is often larger when the current is suddenly reversed than when it is slowly reduced to zero through a constantly growing resistance before the switch is thrown over and then gradually brought to its new strength, or when the change is made less violent by eddy currents induced in a thick copper shell around the specimen. Though it seemed possible that the results given in this paper might be slightly affected by this so-called von Waltenhofen phenomenon, we could not discover the least difference in our results whether the rod to be used was or was not surrounded by a thick copper tube, though the tube makes the throws a trifle more regular.

It is well known, also, that under low excitations, the magnetic moment acquired by a rod in a solenoid under a given final excitation may be much increased if the rod be constantly tapped while the magnetic changes are taking place. So far as we can make out this effect is entirely lacking at very high excitations. We used a large electric tapping apparatus made by Mr. Coulson to give many sharp blows per second to a brass rod butted upon the specimen in the solenoid, and Table VII shows characteristic results.

TABLE VII.

Current.	Flux Change when the Iron was undisturbed.	Flux Change when the Iron was tapped.
4.85	939a	938a
8.95	974a	974a
30.2	1013a	1013a

Some years ago I encountered three specimens of very pure Norway Iron, each of which showed a very high specific magnetism, when tested by a modification of the Isthmus Method. Each piece was about 8 cms. long and 1.26 cms. in diameter. They were presumably from different sources.

TABLE VIII.

Specimen.	Exciting Field.	*I*.
1	2500	1733
2	2400	1738
3	2350	1751

Each of these numbers comes from a series of closely consistent values for different field strengths, and I have no reason to think that the determinations were not good, but I consider the probable error somewhat greater in all work I have done with isthmuses than with such experiments as I have made with larger specimens, with the help of a solenoid.

A single slender isthmus cut from the bar from which No. 2 in this table was taken, gave the very large value 1796 for *I* in fields above 6000, but other larger pieces from the same bar showed lower values for *I*. According to my experience very small bits taken from

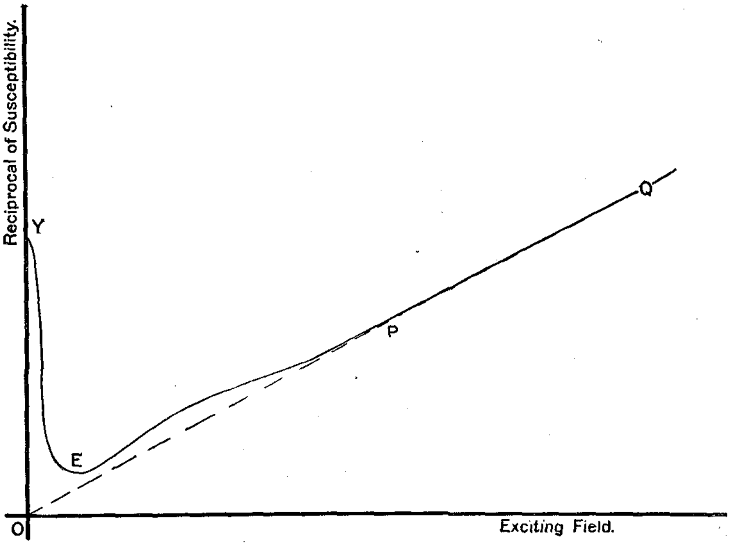

FIGURE 8 shows the form of the curve obtained by plotting the reciprocal of the susceptibility of soft iron against the intensity of the exciting field. The ratio of the abscissa of any point of the curve to the corresponding ordinate is less than the final value of I, and the tangent of the angle which the tangent to the curve makes with the ordinate axis is greater than this value except for small values of H.

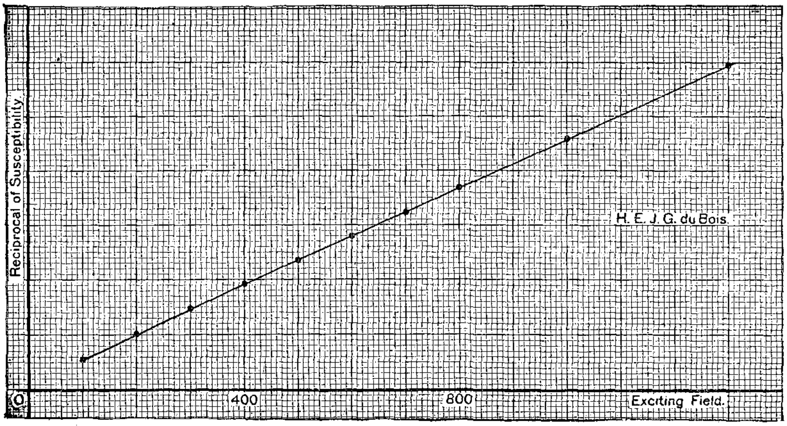

FIGURE 9. This curve shows the results of observations made by DuBois upon an ellipsoidal piece of soft iron 18 cms. long and 0.6 cms. in diameter at the middle.

closely adjacent regions of the same bar may have very different specific magnetisms, owing perhaps to differently arranged inclusions of slag. It is certain that pieces cut across the direction of the rolling often show different permeabilities from those of pieces cut in the same region in the direction of the length of the bar. Gumlich found a piece of soft "Steirisches Eisen" about 3 cms. long and about 3 mm. in diameter which also showed the value 1796 for I.

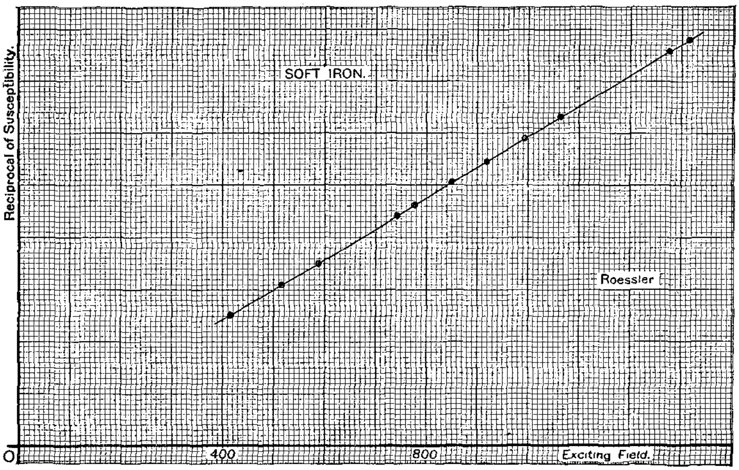

FIGURE 10. This Figure shows the results of observations made by Roessler *upon an ellipsoidal piece of soft iron 50 cms. long and 1 cm. in diameter at the* center.

There seems to be no doubt, therefore, that some specimens of soft iron are to be found which have materially higher maximum values of I than had the specimen used as a standard by Messrs. Hadfield and Hopkinson. Four different observers, using solenoids for magnetizing their test pieces, and seven persons using other methods have thought that they met with such pieces. This fact does not, of course, make the work of Messrs. Hadfield and Hopkinson any the less valuable, but it shows, I think, since some pieces which contain considerable quantities of $Fe_3 \cdot C$, have given values of I above 1720, that material bought in the open market cannot be expected to obey the law which the series of steel pieces from the Hecla Works follow.

Still, the majority, perhaps, of pieces of iron and steel bought at random will have specific magnetisms not very different from the values given as a result of experiments upon these beautiful test pieces.

If a series of observations be made by the Method of Reversals, upon a piece of iron originally in a neutral state, and if the permeability and the susceptibility obtained in this way be plotted against the intensity of the exciting field, those portions of the resulting curves (Figure 7) which correspond to large values of *H* resemble hyperbolas which have the *x* and the *y* axes as asymptotes. A generation ago, therefore, it seems to have occurred to a number of persons at about the same time, that if the reciprocals of the permeability and of the susceptibility were plotted against *H*, the curves must become finally more or less straight. It appeared upon trial that for values of *H* larger than 100, say, the reluctance gives a line only slightly convex upwards, and that the reciprocal of the susceptibility which for comparatively weak fields has the general shape shown in Figure 8 becomes very nearly coincident with a straight line drawn through the origin under high excitation. This last function has been found useful by Professor Kennelly in his paper upon the relation between

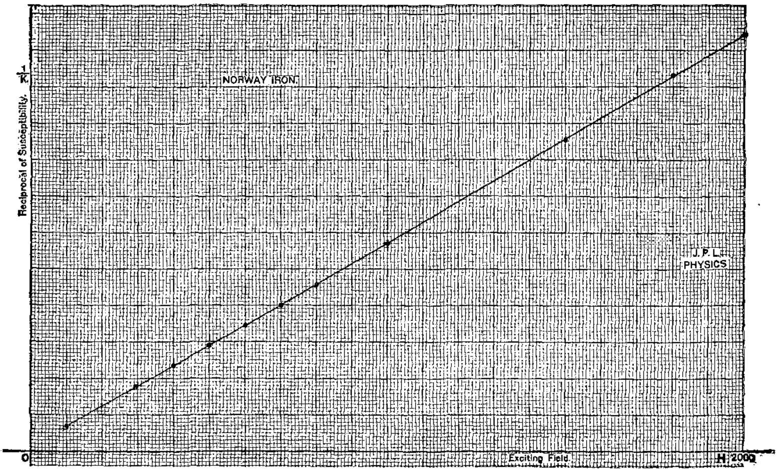

FIGURE 11 shows the results of observations made in the Jefferson Laboratory upon a rod of Norway Iron. For excitations up to about 400, the specimen was magnetized in a solenoid. For more intense fields, the determinations were made by a modification of the Isthmus Method.

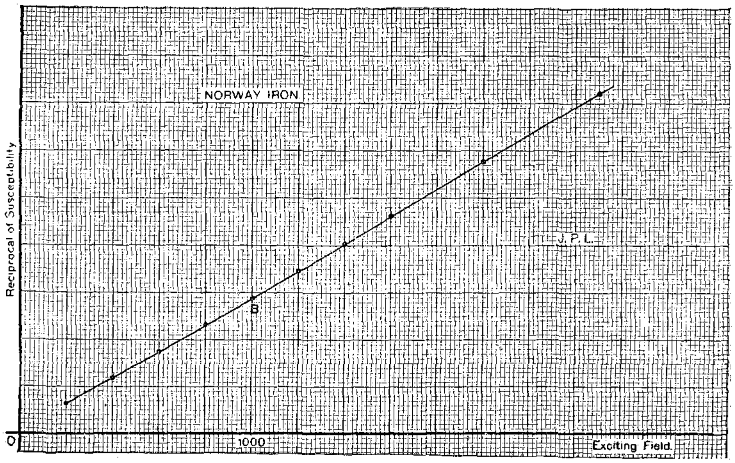

FIGURE 12 represents observations made in the Jefferson Laboratory upon a second specimen of Norway Iron.

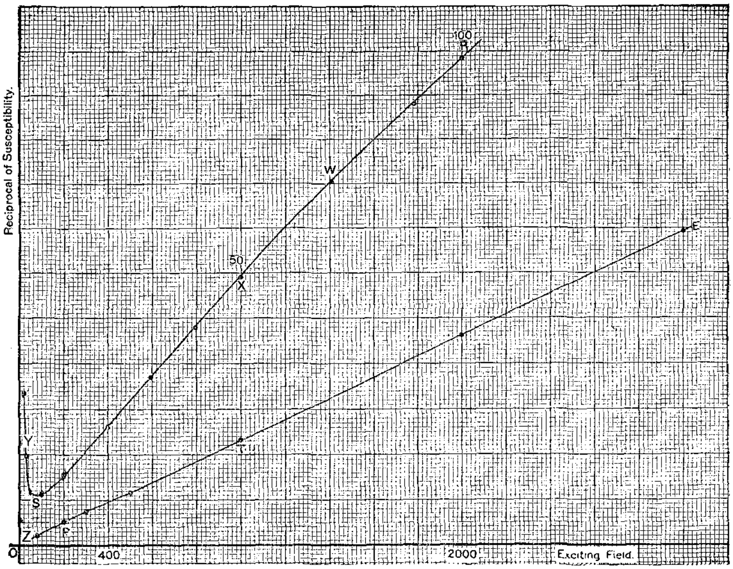

FIGURE 13. This Figure shows results obtained from tests made upon a specimen of Bessemer steel, 8.0 cms. long and 1.26 cms. in diameter. For low excitations the tests were made in a long slender solenoid. For higher fields a modification of the Isthmus Method was used.

B and H in fields[5] of commercial strength. It is clear from the curve in Figure 8 the ordinates of which are $H/I = 1/k$, that the ratio of the abscissa of any point of the curve to its ordinate always yields a value of I somewhat less than the saturation value, whereas the slope against the ordinate axis of the tangent of the curve, after H equals perhaps 200, is always greater than I. Such curves as this are especially useful when one wishes to study the saturation values of the magnetization in iron or steel.

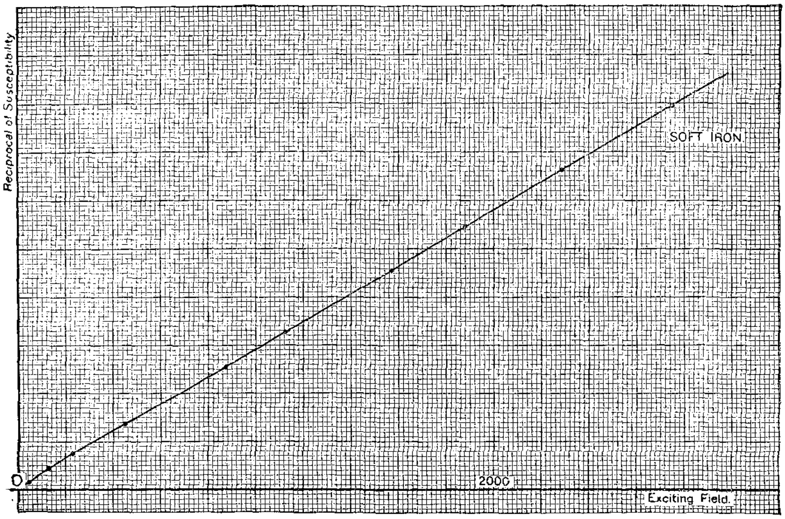

FIGURE 14. This Figure is plotted form the observations made in the Jefferson Laboratory upon a piece of "American Ingot Iron" magnetized in the solenoid. The piece was 100 cms. long and 1.279 cms. in diameter.

Figures 9 and 10 show the results of plotting the reciprocals of the susceptibilities obtained by DuBois, and Roessler in their experiments already described.

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[5] Lenz and Jacobi, Pogg. Ann. **47**, 1839; Mueller, Pogg. Ann. **79**, 1850; Von Waltenhofen, Wiener Berichte, **52**, 1865; Lamont, Handbuch d. Magnetismus, p. 41; Sohncke, Elektrotechnische Zeitschrift, 1883; Ruths, Ueber d. Magnetismus weicher Eisenxylinder, 1876; Froelich, Elektrotechnische Zeitschrift, 1881, 1882, 1894; Kennelly, Trans. Am. Inst. El. Eng. **8**; Steinmetz, Elektrotechnische Zeitschrift, **13**, 1892; Muellendorf, Elektrotechnische Zeitschrift, **22**, 1901; **23**, 1902; Kapp, Electrician, **18**, 1886; Winkelmann's Handbuch der Physik, V, p. 200; Walter, Drude Ann. **14**, 1904; Czermak and Hausmaninger, Wiener Berichte, **98**, 1889; Du Bois, Wied. Ann. **51**, 1894; Fromme, Wied. Ann. **13**, 1881; **33**, 1888.

Figures 11, 12 and 13 reproduce the results of a series of measurements made two or three years ago in the Jefferson Laboratory upon specimens of Bessemer steel and of Norway Iron. For excitations up to about 400 the specimens were magnetized in a slender solenoid about five meters long, but for stronger fields a modification of the Isthmus Method was employed. Figure 14 shows some measurements made lately upon a specimen of American Ingot Iron magnetized in the shorter solenoid described above. Such curves become practically straight for much weaker fields in the case of some irons than in others.

I wish to express my great obligation to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of some of the apparatus used in making the observations mentioned in this paper.

The Jefferson Physical Laboratory,
Cambridge, Mass.

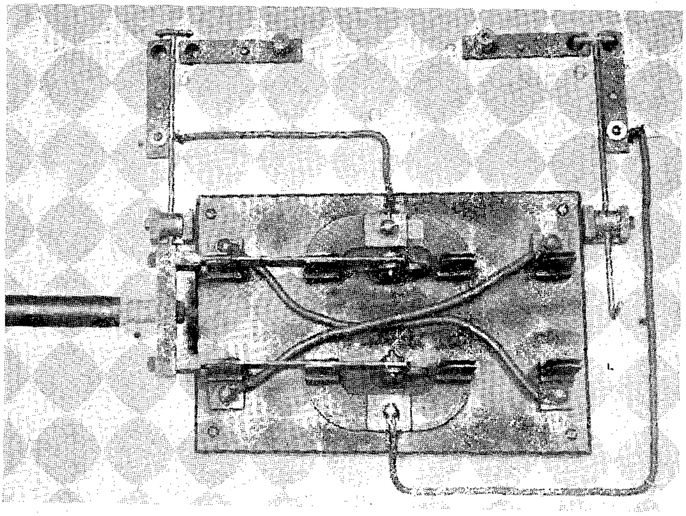


Figure 3. A switch in the main circuit so arranged that if it be suddenly thrown over, the energy in the medium which accompanied the old current is spent largely in heating an auxiliary coil.

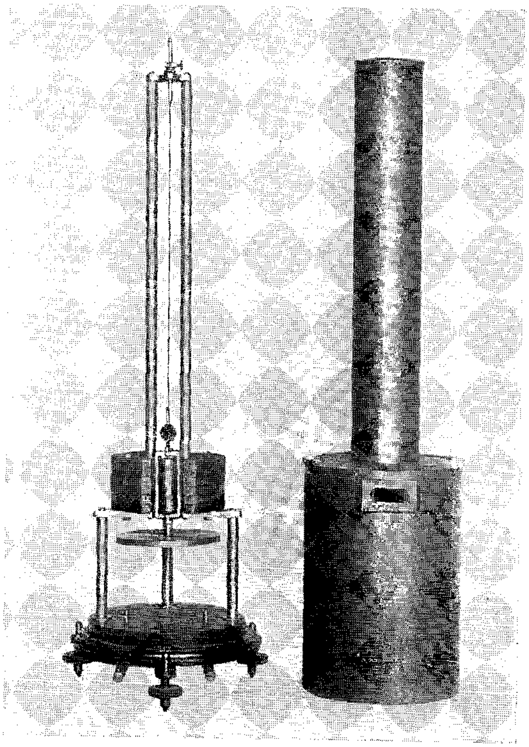
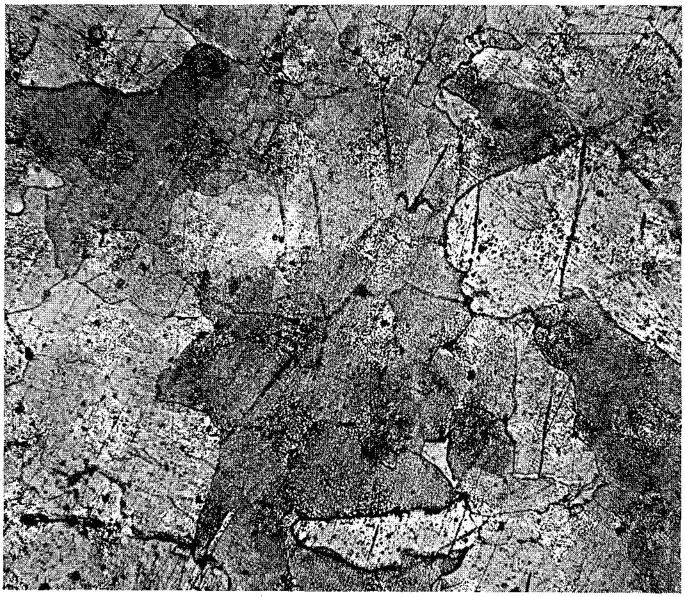
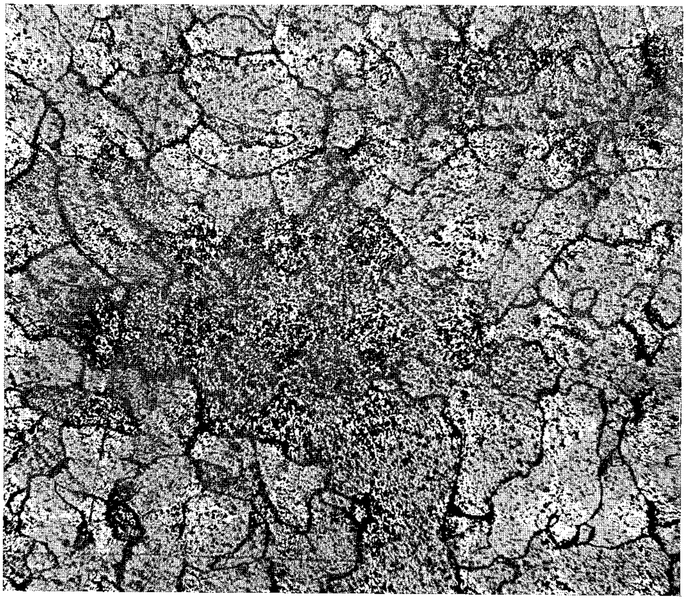


Figure 4. The long period ballistic galvanometer.





Magnification 120.

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BUDDHAGHOSA'S TREATISE ON BUDDHISM, ENTITLED THE WAY OF SALVATION: ANALYSIS OF PART I, ON MORALITY.

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BUDDHAGHOSA'S TREATISE ON BUDDHISM, ENTITLED THE WAY OF SALVATION.
ANALYSIS OF PART I, ON MORALITY.

By Charles Rockwell Lanman.

Presented January 11, 1911. Received January 25, 1913.

Among the great Buddhist writers upon Buddhism, Buddhaghosa stands preëminent. His most important works are the treatise on Buddhism entitled The Way of Salvation or Visuddhi-magga, and his four commentaries, one upon each of the four principal Nikāyas or 'Collections' of the canonical scriptures of Buddhism, the Dīgha-nikāya, the Majjhima, the Saŋyutta, and the Anguttara. Of these his chief works, the Visuddhi-magga stands first in importance as also in order, and is presupposed by the others and referred to by them. It was composed in the sacred language of Buddhism, the Pāli, and was written by him at the Mahā-Vihāra or Great-Monastery of Ceylon in or about 410 A. D., that is, some nine centuries after the death of Buddha. For a millennium and a half it has maintained itself as a masterly and authoritative work. It has been copied upon palm-leaves times without number; and the entire text has recently been edited by the Sayā U Pye, and published in Burmese letters in Rangoon, once in 1901, and again in 1909–1910; and a part of it, books 1–13, has been issued in Cingalese letters by Dhammaratana at Colombo, Ceylon, 1890–1909. It is a lengthy treatise. The Sayā's first edition extends over 618 large octavo pages of 30 lines to the page. It was the hope of my beloved and unforgotten friend and pupil, the late Henry Clarke Warren, to make an edition and translation of the work, and it has fallen to me to endeavor to finish the large and difficult task interrupted by his death.

The Way of Salvation has three principal parts:

I. Morality (books 1–2);
II. Concentration (books 3–13); and
III. The Higher Wisdom (books 14–23).

For the purposes of Occidental students, the native division into twenty-three books is of great value. Unfortunately, there are no original numbered native subdivisions of the books. To supply such subdivisions is a matter of great importance, not only for making it possible to cite the work with convenience and precision, but also for facilitating the study of the work and in particular the comprehension of the logical relation of the different parts. It is extremely desirable that these subdivisions should be made so circumspectly and successfully that future editors will not be tempted to change them and so to defeat the purposes of their predecessors who shall cite the work according to the subdivisions of the first Occidental edition.

The first duty of an editor is to understand his text, and this is true whether he means to subdivide it rightly or not. But the subdivisions cannot rightly be made without such an understanding. And on the other hand, the very making of them forces the editor to come to a clear comprehension of the purpose of each constituent part of his text. This last is accordingly tantamount to making an analytical table of contents, and of such a table the subdivisions are of course an implicit and essential feature. It is indeed not possible to make a logical and orderly analysis of an illogical and disorderly book. Buddhaghosa's discussion is of absolutely logical and orderly sequence, but it is nevertheless at times extraordinarily complicated and difficult. Thus for the editor's purposes during the progress of so large a work, a printed analysis is an almost indispensable necessity. Besides all this, the analysis is valuable to students of Buddhism at large as a provisional report of the contents of a practically inaccessible treatise. It is true that what is here given covers only a portion of the work, but that portion makes a complete whole by itself. For these reasons I have deemed the publication of the Analysis of Part I. to be abundantly justified.

Titles of the several books of the Visuddhi-magga.

PART I.— SĪLA, MORALITY.

Book
1 Sīla-Niddesa
2 Dhutañga-Niddesa

PART II.— SAMĀDHI, CONCENTRATION.

Book
3 Kammaṭṭhānaggahaṇa-Niddesa
4 Pathavīkasiṇa-Niddesa
5 Sesakasiṇa-Niddesa
6 Asubhakammaṭṭhāna-Niddesa
7 Cha-anussati-Niddesa
8 Anussatikammaṭṭhāna-Niddesa
9 Brahmavihāra-Niddesa
10 Āruppa-Niddesa
11 Samādhi-Niddesa
12 Iddhividha-Niddesa
13 Abhiññā-Niddesa

PART III.— PAÑÑĀ, THE HIGHER WISDOM.

14 Khandha-Niddesa
15 Āyatana-Dhātu-Niddesa
16 Indriya-Sacca-Niddesa
17 Paññābhūmi-Niddesa
18 Diṭṭhi-visuddhi-Niddesa
19 Kaṅkhāvitaraṇa-visuddhi-Niddesa
20 Maggāmagga-ñāṇadassana-visuddhi-Niddesa
21 Paṭipadā-ñāṇadassana-visuddhi-Niddesa
22 Ñāṇadassana-visuddhi-Niddesa
23 Paññābhāvanānisaŋsa-Niddesa

Distribution of the 40 Kammaṭṭhānas over Books 4 to 11.

Book		
1	Morality	
2	The pure practices — Appendix to book 1	
3	The undertaking of the [40] businesses or kammaṭṭhānas	
4	The earth-kasiṇa [1]	10
5	The remaining [9] kasiṇas	
6	The [10] unpleasant businesses	10
7	The [first] 6 contemplations: Buddha, dhamma, etc.	10
8	The [4] others: death, body, breathing, nibbāna	
9	The [4] exalted states	10
10	The [4] formless realms	
11	The [1] realization of the repulsiveness of food	
	The [1] analysis into the four elements	
	The five advantages of concentration	
12	The five supernormal powers: 1. iddhi	— Appendix to books 3–11
13	The other 4 (hearing, thought-reading, etc.)	

ANALYSIS OF THE WAY OF SALVATION.

BOOK 1.— MORALITY.

§

1 Stanza, uttered by The Exalted One, which is both text and substance of the whole work

The question, put by some deity, to which that stanza was the answer

2 Brief comment on the text of the deity's question

3 The answer, now in its proper sequence, is repeated

4 Buddhaghosa's preface, in five stanzas, to the Visuddhi-magga

5 Comment on the title, Visuddhi-magga

6 Brief comment on The Exalted One's stanza. It implies six things as requisite for Nibbāna: Morality, Concentration, mother-wit and skill and the Higher Wisdom, and strenuousness

Allegory: the man with the cleaver

7 Since mother-wit does not have to be acquired, and on the other hand since strenuousness merely implies constancy and since skill merely implies heedfulness,— therefore it is under the heads of the three remaining things — Morality and Concentration and Higher Wisdom — that Buddha in this stanza sets forth the way of salvation

8 These three heads form a triad which suggests nine triads akin with it, each also conducive to Nibbāna. Each member of the primary triad — Morality and Concentration and Higher Wisdom — suggests respectively one member of each of the nine triads

9 Triad 1. of higher disciplines: discipline of the Higher Morality, discipline of the Higher Concentration, discipline of the Higher Wisdom

10 Triad 2. of beauties or beneficialnesses: beneficialness of The Religion at the outset, in its course, and in its consummation

11 Triad 3. of bases: Morality is basis of the three vijjās; Concentration is basis of the six supernormal powers; wisdom is basis of the four paṭisambhidās

12 Triad 4. of avoidances and practice: avoidance of two extremes (low pleasures and self-mortification) and practice of the golden mean (middle path)

13 Triad 5. of means of escape: escape from hell, from the world of sense, from all existences

14 Triad 6. of abandonments of evil tendencies: abandonment of their elements, abandonment by temporary stopping, abandonment by extirpation.

15 Triad 7. of forces counteracting the evil tendencies: forces counteracting actual transgression, counteracting the arising of the tendencies, counteracting the last residue of them

16 Triad 8. of cleansings from minor evil tendencies: of behavior, of lusts, of false views

17 Triad 9. of instrumentalities: for attaining the First and Second Path, the Third Path, the Fourth Path

18 Summary of the nine triads and suggestion of others

19 Seven questions which form the frame-work of the discussion of Morality
1. What is morality? [discussed at §20]
2. What is the etymological import of the word morality? [§21]
3. What is its characteristic, its essence, its manifestation, etc. [§22]
4. What are its advantages? [§23–§24]
5. How many kinds of it are there? [§25 – §126][1]
6. How is it defiled? [§128 – §129]
7. How is it kept undefiled? [§130 – §134]

20 Question 1: What is morality? It is the thought and the mental attributes and the restraint and the non-transgression of one who refrains from sin and fulfils his duties. Proof from the Paṭisambhidā-magga

21 Question 2: What is the etymological meaning of morality (sīla)?
It is a being moral (sīlana) in the sense
either of a reposefulness of body and voice and mind
or of a basis of good states of mind

22 Question 3: What is its characteristic, its essence, its manifestation, its immediate basis?
Its characteristic (lakkhaṇa) is a being moral
Its essence (rasa), in the sense of function, is to put an end to bad morals
Its essence (rasa), in the sense of success, is blamelessness
Its manifestation (paccupaṭṭhāna) is purity of body and speech and mind
Its immediate basis (padaṭṭhāna) is self-respect and respect for others

23 Question 4: What are the advantages of morality? They are: absence of remorse; five others (cited from the Dīgha), wealth, good name, self-confidence in public, a serene death, and a happy hereafter; and others
24 The advantages of morality — summary in verse — 10 stanzas

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[1] As appears from these section-numbers and from the sequel, the discussion of this fifth question forms the bulk of book 1; and of this discussion, the treatment of Tetrad 4 (see p. 155 and note) forms by far the largest part, just one half of the whole book.

25–126 Discussion of the fifth question [forming the bulk of book 1]
25 Table of contents of the discussion of the fifth question
Question 5: How many kinds of morality are there? Morality is
Of one kind in one aspect (one monad) [§26]
Of two kinds in each of seven aspects (seven dyads) [§27–§33]
Of three kinds in each of five aspects (five triads) [§34–§39]
Of four kinds in each of four aspects (four tetrads) [§40–§113]
Of five kinds in each of two aspects (two pentads) [§114–§126]

26 Morality is of one kind if viewed —
Monad 1. and sole — as a being moral

27 Morality is of two kinds if viewed —
Dyad 1.— as a conforming with positive rules of conduct (precepts)
or with negative rules of conduct (prohibitions)
28 Dyad 2.— as a conforming with the minor precepts
(of the Khandhaka: that is, Mahāvagga and Cullavagga)
or with the major precepts
(of the Bhikkhu- and Bhikkhunī-vibhañga)
29 Dyad 3.— as abstention from killing etc.
or as all the five kinds of morality except such abstention:
to wit, abandonment, volition, restraint, non-transgression: §125
30 Dyad 4.— as interested or as disinterested
31 Dyad 5.— as undertaken to be practised for a part of life
or as undertaken to be practised until the end of life
32 Dyad 6.— as violable (for the sake of gain etc.) or as inviolable
33 Dyad 7.— as ordinary or as transcendental

34 Morality is of three kinds if viewed —
Triad 1.— as inferior or as middling or as superior
35 Triad 2.— as practised out of respect for one's self or for others or
for the moral law
36 Triad 3.— as interested (that is, motivated by some bad desire or view)
or as disinterested (that is, unmotivated by such desire or view)
or as tranquilized (that is, as being beyond the sphere of such disturbing
influences, because it has reached its goal)
37 Triad 4.— as pure or as impure or as doubtful
38 Triad 5.— as sekkha (that is, as the morality of those who have attained
the four Paths and three Fruits)
or as a-sekkha that is, as the morality of those who have attained Arahatship, the fourth Fruit)
or as the morality of those who have not yet attained even the first Path
39 The Paṭisambhidā-magga speaks of good and bad and indifferent sīla;
but since sīla there means 'disposition,' and since it means 'morality' in
this discussion (which is not about bad morals), therefore this triad is not
brought in here

40 Morality is of four kinds if viewed —
Tetrad 1.— as tending to inferiority
or as tending to a remaining in statu quo
or as tending to some special Attainment
or as tending toward a penetration of the Truths

41 Tetrad 2.— as a keeping of precepts that concern the monks
or as a keeping of precepts that concern the nuns
or as a keeping of precepts that concern the male and female novices
or as a keeping of precepts that concern the lay followers

42 Tetrad 3.— as natural, that is, as not the fruit of training
or as an aggregate of customary practices
or as the ways of the mother of one destined to become a Buddha
or as the fruit of elevated morality in former births

43 Tetrad 4.— as depending on restraints enjoined by the Pātimokkha: [1]
(first kind) pātimokkha-saŋvara-sīla; quotation 1 (from Vbh. 244):
idha bhikkhu pātimokkha-saŋvara-saŋvuto viharati,
ācara-gocara-sampanno
aṇu-mattesu vajjesu bhaya-dassāvī, samādāya sikkhati sikkhāpadesu;

or as depending on the restraint of the senses:
(second kind) indriya-saŋvara-sīla; quotation 2 (from M. 1.180^{27}):
so cakkhunā rūpaŋ disvā, na nimittaggāhī hoti...
manindriye saŋvaraŋ āpajjati (to M. 1.181^{1});

or as ensuring a blameless livelihood:
(third kind) ājīva-pārisuddhi-sīla; quotation 3 (from Vbh. 345):
kuhanā, lapanā, nemittikatā, nippesikatā,
lābhena-lābhaŋ-nijigiŋsanatā;

or as a using of the everyday necessaries of life in the right spirit:
(fourth kind) paccaya-sannissita-sīla; quotation 4 (from M. 1. 10^{5}):
paṭisaŋkhā yoniso cīvaraŋ paṭisevati ... piṇḍapātaŋ ... senāsanaŋ ...
gilānapaccayabhesajjaparikkhāraŋ ... abyābajjha-paramatāya (M. 1.10^{20}).

44–88 Discussion of the four kinds of morality given under **Tetrad 4**, (§43),
consisting of commentary and explanation and stories commingled:
First kind, pātimokkha-saŋvara-sīla, §44–§51
Second kind, indriya-saŋvara-sīla, §52–§57
Third kind, ājīva-pārisuddhi-sīla, §58–§76
Fourth kind, paccaya-sannissita-sīla, §77–§88

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[1] The discussion of the four kinds of morality comprised under the fourth tetrad occupies sections 44 to 112 or just one half of this long first book. Albeit of absolutely logical and orderly sequence, it is nevertheless extraordinarily complicated, and I have therefore made this part of the analysis somewhat ampler than the rest, in order to a quicker and better understanding.

44–51 Discussion of morality viewed as depending on the
restraints enjoined by the rules of the Pātimokkha:
Kind 1, tetrad 4, pātimokkha-saŋvara-sīla:
Cm on idha bhikkhu. . . sikkhāpadesu, quotation 1, §43, from Vbh. 244
44 Cm on idha bhikkhu pātimokkhasaŋvarasaŋvuto viharati
45–49 Cm on ācara-gocara-sampanno
45 Ācara explained by a long quotation from Vbh. (246) itself
46 Gocara explained by a long quotation from Vbh. (246) itself
47 Ācara explained (a second time) by two quotations from Niddesa
describing misbehavior 1. in bodily act and 2. in word
48 Ācara explained (a third time) by Buddhaghosa, following some cm
49 Gocara explained (a second time) in its three varieties:
1. upanissaya; 2. ārakkha; 3. upanibandha
50 Cm on aṇu-mattesu vajjesu bhayadassāvī. . . sikkhāpadesu
51 The Vbh. passage refers literally to the moral man, but virtually to morality

52–57 Discussion of morality viewed as depending on the restraint of the senses:
Kind 2, tetrad 4, indriya-saŋvara-sīla:
Cm on so cakkhunā rūpaŋ disvā · · ·, quotation 2, §43, from M.1.180^{27}–181^{1}
52 Cm on so cakkhunā rūpaŋ disvā. . . nānubyañjanaggāhī
Neither the eye nor the mind sees; the mind sees through the eye
53 Story of Mahā-Tissa of Mount Cetiya
54 Cm on yatvādhikaraṇam. . . cakkhundriye saŋvaraŋ āpajjati
55 Interjected discussion of 'restraint of the sense of sight,' cakkhundriye
saŋvaro
Incidental analysis of the process of cognition
In the eye there is neither restraint nor absence of restraint
But at a certain step (javana) of the process there may be non-restraint
56 The non-restraint is likened to the leaving open the principal gates of a city
Restraint is likened to keeping them well-closed
57 The same discussion applies also to restraint of the other senses
Morality, in so far as it depends on restraint of the senses, characterized

58–76 Discussion of morality viewed as ensuring a blameless livelihood:
Kind 3, tetrad 4, ājīva-pārisuddhi-sīla:
Cm on Buddhaghosa's 16 words (plus 5 quoted from Vbh. 345) at §43:
yā pana ājīvahetu paññattānaŋ. . . idaŋ ājīvapārisuddhi-sīlaŋ
58 Cm on four of the 16 words: ājīvahetu paññattānaŋ channaŋ sikkhāpadānaŋ:
consisting for substance of six precepts from Mahā-vibhaṅga and Bhikkhunī-vibhaṅga [see §76 for the remaining 12 words]
59 Explanation of the 5 words (just mentioned as quoted from Vbh. 345),
kuhanā lapanā nemittikatā nippesikatā
lābhena-lābhaŋ-nijigiŋsanatā,
by means of five quotations from the text (352) of the Vbh. itself
Text of each of these five Vibhaṅga quotations

60–75 Cm on the text of the five Vibhañga quotations of §59

60–66 Cm on the quotation (1. of §59) explaining kuhanā
60 Cm on the clause lābha-sakkāra-siloka-sannissitassa pāpicchassa icchāpakatassa
61 The next clause, beginning paccaya-paṭisevana-saŋkhātena, suggests three kinds of un-straightforwardness, as mentioned in Mahā-niddesa
62 First kind of unstraightforwardness, namely in the way of using the daily necessaries of life, concretely described by Buddhaghosa
63 The same, as described by a long quotation from Nd. 1. 207–208
64 Second kind, namely in the employment of roundabout talk, as described by a long quotation from Nd. 1. 209–210
65 Third kind, namely in general carriage and bearing, as described by a quotation from Nd. 1. 208–209
66 Verbal cm on the rest of the kuhanā-quotation (first of the five of §59)

67–70 Cm on the quotation (2. of §59) explaining lapanā
67 Concrete instance of ālapanā
Lapanā, sallapanā, ullapanā, and samullapanā explained
68 Concrete instance of unnahanā. Samunnahanā explained
69 Concrete instance of ukkācanā. Samukkācanā explained
70 Anuppiya-bhāṇitā, cāṭukamyatā, muggasupyatā, pāribhaṭyatā explained

71–73 Cm on the quotation (3. of §59) explaining nemittikatā
71 Explanation of nimitta, nimitta-kamma, obhāsa
Concrete instance of obhāsa-kamma: milch cows and buttermilk-cows
Roundabout talk, sāmanta-jappā: a concrete instance of it:
72 Interjected story of the monk that got the better of the stingy housewife
73 Rest of the explanation of sāmanta-jappā. Explanation of parikathā

74 Cm on the quotation (4. of §59) explaining nippesikatā
Akkosanā, vambhanā, . . . parapiṭṭhimaŋsikatā explained

75 Cm on the quotation (5. of §59) explaining lābhena-labhaŋ-nijigiŋsanatā
Eṭṭhi etc. and esanā etc. explained

76 Cm, taken up again from §58, on the remaining twelve of Buddhaghosa's sixteen words at §43:
yā [pana] . . . vītikkamassa . . . evamādīnaŋ ca pāpadhammānaŋ vasena pavattā micchājīvā virati,— idaŋ ājīvapārisuddhi-sīlaŋ

77–88 Discussion of morality viewed as
a using of the everyday necessaries of life in the right spirit:
Kind 4, tetrad 4, paccaya-sannissita-sīla
Cm on paṭisaŋkhā yoniso etc., quotation 4, §43, from M. 1. 10^5–10^{20}
77 Cm on paṭisaŋkhā yoniso cīvaraŋ paṭisevati
yāvad eva sītassa paṭighātāya, uṇhassa [paṭighātaya]
78 Cm on ḍaŋsa-makasa-vātātapa-siriŋsapa-samphassānaŋ [paṭighātāya]
79 Cm on yāvad eva hirikopīna-paṭicchādanatthaŋ
80 Cm on [paṭisaŋkhā yoniso] piṇḍapātaŋ [paṭisevati]
81 Cm on neva davāya na madāya na maṇḍanāya na vibhūsanāya
82 Cm on yāvad eva imassa kāyassa ṭhitiyā yāpanāya
83 Cm on vihiŋsūparatiyā brahmacariyānuggahāya
84 Cm on iti purāṇañ ca vedanaŋ paṭihaṅkhāmi
navañ ca vedanaŋ na uppādessāmi
85 Cm on yātrā ca me bhavissati anavajjatā ca phāsuvihāro ca.
86 Cm on [paṭisaŋkhā yoniso] senāsanaŋ [paṭisevati]...
utuparissayavinodana-paṭisallānārāmatthaŋ
utu-parissaya explained 1. as 'weather-dangers' (karmadhāraya), and
2. as 'the weather and the dangers' (dvandva)
87 Cm on [paṭisaŋkhā yoniso] gilānapaccayabhesajjaparikkhāraŋ [paṭisevati]
88 Cm on [yāvad eva] uppannānaŋ veyyābādhikānaŋ vedanānaŋ paṭighātāya
abyābajjhaparamatāya

89–112 Four means of attaining the four kinds of morality of tetrad 4 (§43),
Being a practical appendix to sections 44–88:

89–91 By faith (1. saddhā) is attained pātimokkhasaŋvara-sīla
92–100 By mindfulness (2. sati) is attained indriyasaŋvara-sīla
101–104 By strenuousness (3. viriya) is attained ājīvapārisuddhi-sīla
105–112 By wisdom (4. paññā) is attained paccayasannissita-sīla

89–91 Morality dependent on the restraints of the Pātimokkha rules requires
Faith in the moral necessity of the precepts
89 Full faith therein demands that one keep them even at the cost of life
90 Stories of two Elders who sacrificed life in order not to break the precepts
91 Buddhaghosa exhorts to give up life rather than break the precepts (stanza)

92–100 Morality dependent on the restraint of the senses requires mindfulness
Therefore this restraint of the senses should be well cultivated
93 Corollary 1: if it be not so cultivated,
then not even the morality dependent on the restraints enjoined by the rules of the Pātimokkha can be maintained
Two stanzas in negative form as scriptural proofs therefor
Corollary 2: but if the restraint of the senses be so cultivated
that not so much as a thought of evil enters the mind,
then the aforesaid morality may be maintained
The same two stanzas altered to positive form

94–100 Practical advice with concrete exemplary instances
94 The above is the rigorously strict teaching
The mind however is flighty. Therefore, as did Vañgīsa,
one should repel the invasion of evil thoughts by suitable meditation
95 Story of Vañgīsa and how he overcame his hot sexual passion
96 A monk bent on restraining his senses should follow Cittagutta's example
97 Story of Cittagutta, who lived over sixty years in his cave without ever even noticing a lovely wall-painting of the Renunciation of the Seven Buddhas
98 Buddhaghosa's exhortation as to restraint of the sense of sight — two stanzas
99 Story of Mahāmitta, who had never, since entering the Order, looked with lustful thought upon a woman
100 Buddhaghosa's advice as to following this example — one stanza

101–104 Morality as ensuring a blameless livelihood requires strenuousness
101 A monk should strenuously avoid any blameworthy way of getting his daily needs
and be equally strenuous in getting them in a blameless way
The blameless way described
102 For a monk striving after this kind of morality,
four things are in certain cases allowable:
Vague hinting (nimitta), hinting (obhāsa)
Roundabout talk (parikathā), plain talk (viññatti).
In the case of clothing and food, none are allowable
In the case of lodgings, the first three are commonly allowable
In the case of medicine, all four are allowable
But one who avoids them all in every case is called
'one who aims at the ideally simple life' — such as was Sāriputta
103 Story of Sāriputta who refused the milk-rice that would have cured him, because of his plain talk with Moggallāna;
albeit the givers learned of that talk only through a deity
Sāriputta's three stanzas
104 Buddhaghosa urges that this example be generally followed — one stanza

105–112 Morality as a using of the everyday necessaries of life in the right spirit requires wisdom
105 For the exercise of this spirit there are two occasions:
1. When you are getting these daily necessaries (paṭilābha-kāle)
2. When you are using them (paribhoga-kāle)
106 Four ways of using these everyday necessaries of life:
1. As if they were stolen property (theyya)
2. As if they were borrowed property (iṇa)
3. As if they were an inheritance (dāyajja)
4. As if you were the master (sāmi) of them

The first way [§106 continued] is that of the downright immoral person
The second is that of the moral man when he fails to exercise the right spirit

107–108 Excursus.
107: How to avoid the offense of using as borrowed property
1. Clothing; 2. food; 3. lodgings; and 4. things for the sick
In case 4, this spirit should be exercised both on accepting and on using
If you fail to do so on accepting, you may atone by doing so on using
108: For there are four procedures for rehabilitating morality
corresponding each respectively to the four kinds in §43:
1. Confession, for a breach of pātimokkhasaŋvara-sīla
2. Restraint, for a breach of indriyasaŋvara-sīla
3. Right seeking, for a breach of ājīvapārisuddhi-sīla
4. Exercise of the right spirit, for a breach of paccayasannisita-sīla
From this follows the last statement of §107

109 The third way (see §106, above) is that of those who have reached any of the first seven Attainments, i. e. the way of the Sekhas
For these are sons of The Exalted One
and it is an inheritance from him of which they are partakers
The fourth way is that of those who have reached the eighth Attainment
For these, the Arahants, having passed beyond the slavery of lusts,
use the things of life as if masters of them, not slaves
110 The fourth way and the third way are meet for every one
Not so the second — to say nothing of the first
Since the fourth is the best, a monk who would become a master should exercise this right spirit as explained in sections 77–88
Scriptural authorities for this injunction — four stanzas
111 Story of the novice, Saŋgharakkhita the Nephew, who, by thorough exercise of this spirit, attained to Arahantship
112 Therefore, exhorts Buddhaghosa, any who aim at Nibbāna
should cultivate this spirit
113 Thus then morality is of four kinds if viewed
as depending on the restraints enjoined by the rules of the Pātimokkha
and so forth (as in §43)

[Referring to the last line of the Analysis of §25, above]
114–126 Morality is of five kinds in each of two aspects: if viewed —
114–124 Pentad 1.— as a conformity with the precepts on the part of
Five classes: 1. the unordained; 2. the ordained;
3. the Puthujjanas; 4. the Sekhas; 5. the Arahants
125–126 Pentad 2.— as consisting 1. in a getting rid of killing etc.,
2. in abstention, 3. in volition,
4. in restraint, 5. in non-transgression

114 Morality is of five kinds if viewed —
Pentad 1.— as conformity, on the part of the unordained,
(kind 1) with a limited body of precepts (pariyanta-pārisuddhi-sīla: §115);
or as conformity, on the part of the ordained,
(kind 2) with an unlimited body of precepts (apariyanta-p.-sīla: §115);
or as a conformity, on the part of Puthujjanas,
(kind 3) which is complete (paripuṇṇa-p.-sīla: §117);
or as a conformity, on the part of the Sekhas,
(kind 4) which is disinterested (aparāmaṭṭhapārisuddhi-sīla: §120);
or as a conformity, on the part of the Arahants,
(kind 5) which is concomitant with calm (paṭipassaddhi-p.-sīla: §123).
Buddhaghosa gives the above five in the words of the Paṭisambhidā

115 For the unordained the precepts are limited in number
For the ordained, altho the precepts have a limit
(namely 170 billion, 5 million, and 36, or 170,005,000,036),
yet they are virtually unlimited; — like those kept by Mahā-Tissa
116 Story of Mahā-Tissa, in allusive outline, with three stanzas

117 For Puthujjanas, the conformity is so complete,
that it serves as the immediate basis of Arahantship:
as in the cases of the Saŋgharakkhitas, uncle and nephew
118 Story of Mahā-Saŋgharakkhita, who, altho living for more than sixty years from his ordination without even attaining the First Path, nevertheless kept his morality in such complete conformity with the precepts that on his death-bed he attained Arahantship at a bound
119 Excursus: Relative value of learning and morality
Scripture stanzas covering four combinations:
1. Small learning and bad morals; 2. Small learning and good morals;
3. Great learning and bad morals; 4. Great learning and good morals.
Combination 2 is better than 3

120 For the Sekhas, the conformity is "disinterested," as being unmotivated by any false view; and for the Puthujjanas, it is "disinterested,"
as being unmotivated by any passion: case of Tissa, Kuṭumbiya's son
121 Stanzas of Tissa, who, to assure his murderers that he would not run away, broke his two thigh-bones, and, gaining thus a night's time, attained Arahantship
122 Story of an Elder, who, altho in helpless torment from illness, and certain of attaining heaven by death, refrained from putting an end to his misery and attained to Arahantship. His stanzas

123 For the Arahants, the conformity, as bringing an abiding calmness, is called "calmness-conformity"
124 Thus morality is of five kinds from the above five points of view

125 Morality is of five kinds if viewed as consisting —
Pentad 2.— in a getting rid of killing, stealing, adultery, lying, etc.
(kind 1) pāṇātipātassa...sabbakilesānaŋ pahānaŋ sīlaŋ;
or in an abstention from the same
(kind 2) pāṇātipātassa...sabbakilesānaŋ veramaṇi sīlaŋ;
or in a volition
(kind 3) pāṇātipātassa...sabbakilesānaŋ cetanā sīlaŋ;
or in a restraint
(kind 4) pāṇātipātassa...sabbakilesānaŋ saŋvaro sīlaŋ;
or in a non-transgression
(kind 5) pāṇātipātassa...sabbakilesānaŋ avītikkamo sīlaŋ
Buddhaghosa gives the above five also in the words of the Paṭisambhidā
Corollary, also in the words of the Paṭisambhidā:
These five conduce to freedom from regret, to satisfaction...to nibbāna
126 'Getting rid of' is defined as nothing else than
a not allowing the thought of killing etc. to arise
In like manner, abstention, volition, restraint, and non-transgression
are primarily and essentially positive mental activities

127 By the foregoing sections (20–126) the first five of the seven questions
propounded in §19 have been answered

128–134 Discussion of the two remaining questions:
Question 6: How is morality defiled? [§128–§129] and
Question 7: How is morality kept undefiled? [§130]
Practical appendix to the discussion [§131–§134]

128 The defiling of morality may be occasioned
1. by motives which have to do with gain or fame etc., or
2. by motives which have to do with the sexual feelings
The defiled morality may be of four kinds (as suggested by A. 4.54[19]): it
may be
1. not-intact (khaṇḍa), like a garment frayed at the edges;
2. riddled (chidda), like a garment with holes in the middle;
3. streaked (sabala), like a brindled cow; or
4. spotted (kammāsa) like a dappled cow
The defilements from motives of gain etc. were treated at §32; now come
The defilements from motives having to do with the sexual feelings:
129 Seven ways in which these motives manifest themselves:
A monk does not actually have intercourse with a woman, but
1. he likes to have her rub, chafe, bathe, or massage him.

A monk is free from the two aforesaid faults, but
2. he likes to laugh and play with her.

A monk is free from the three aforesaid faults, but
3. he likes to look her in the eye.

A monk is free from the four aforesaid faults, but
4. he likes to hear her laugh, talk, sing, or cry.

A monk is free from the five aforesaid faults, but
5. he likes to think about his former laughs, talks, etc. with her.

A monk is free from the six aforesaid faults, but
6. he likes to see a householder enjoying the pleasures of sense.

A monk is free from the seven aforesaid faults, but
7. he likes to keep his morality in the hope of becoming a deity (and thus enjoying a celestial nymph).

These seven faults are given in the words of A. 4. 54–56 and in diminuendo order.

130 Morality is kept intact, unriddled, unstreaked, and unspotted
1. by avoiding altogether the breach of any precept;
2. by atoning for one that is broken;
3. by freedom from the seven faults just mentioned;
4. by not allowing even so much as a thought of anger etc. to arise; and
5. by moderation of desires, contentment, the simple life, etc.

131 This keeping morality undefiled is brought about in two ways:
1. by realizing the curses of the loss of moral character; and
2. by realizing the blessednesses of lofty morality

Five curses are set forth by a passage cited from A. 3. 252
Buddhaghosa rehearses other grave evils
132 The Exalted One has set forth these curses in the Fire-text (A. 4. 128):
Whether, O monks, is better, to embrace a mighty blazing fire?
or to embrace a tender maid?— Answer: the fire.
133 With continued use of the same text (A. 4. 129),
The accepting or using, on the part of a man of lost character,
of any one of six things, to wit,
salutation, obeisance, clothing, food, furniture, monastery,—
is likened respectively to each one of six tortures, to wit
hair-rope, sharp spear, redhot wrap, redhot ball, redhot bed, boiling pot
134 Buddhaghosa, in twelve āryā stanzas, summarizes §133,
and sets forth the miseries of the man of lost character

135 Buddhaghosa, in ten çlokas, sets forth the blessings of a lofty morality
136 Therefore, he exhorts, one should with all heed keep his morality unblemished

137 The introductory stanza, §1, shows the way of salvation under three heads:
Morality and Concentration and the Higher Wisdom
Morality has now been elucidated
Colophon: Book 1, Exposition of Morality

Book 2.— The Pure Practices.

Book 2 (see §107) is an appendix to book 1, which treats of Morality
1 Preface.— The monk whose morality is kept undefiled
by fewness of wishes, contentment, and so on,
Is thereby established in the "three ancient orders of nobility";
But to be able to attain the fourth, that of delight in meditation,
He must observe the Pure Practices

2 The Exalted One has sanctioned 13 such practices.
Practices 1–2 concern clothing (see §102)
1. Refuse-rag-practice or paŋsukūlikañga
2. Three-robe-practice or tecīvarikañga
Practices 3–7 concern food
3. Alms-practice or piṇḍapātikañga
4. House-to-house-practice or sapadānacārikañga
5. One-sitting-practice or ekāsanikañga
6. One-bowl-practice or pattapiṇḍikañga
7. No-more-practice or khalupacchābhattikañga
Practices 8–12 concern dwelling-place
8. Forest-practice or āraññikañga
9. Tree-root-practice or rukkhamūlikañga
10. Open-sky-practice or abbhokāsikañga
11. Burning-ground-practice or sosānikañga
12 Lodge-anywhere-practice or yathāsanthatikañga
Practice 13 concerns bodily posture
13. Never-lie-down-practice or nesajjikañga

3 Buddhaghosa's outline of this book: eleven topics (in expanded form)
Topic 1: etymological import (attha) of the name of each Practice
[All 13 practices (each by itself in order) are treated together, §4 to §8]
Topic 2: their common characteristic (lakkhaṇa), function (rasa), etc.
[All 13 practices (in a lump — no single one specified) are treated, §9]
Topic 3: the manner of adopting them (samādāna)
Topic 4: the manner of observing them (vidhāna)
Topic 5: the grades [of strictness] (pabheda) in observing them
Topic 6: what constitutes a breach (bheda) of their observance
Topic 7: the advantages (ānisaŋsa) of observing them
[All these 5 topics (each by itself in order) are treated together
for each separate one of the 13 practices, §10 to §92]
Topic 8: of the 3 categories, good [and bad and indifferent] (kusalattika),
the Pure Practices fall under only the first and third [§94]
Topic 9: discussion of "dhuta etc." [§95]. The etceteral term includes
dhutavādo, dhutadhammā, dhutañgāni, and "Who profit by them?"
Topic 10: the Practices as reckoned so as to make the smallest number etc.
Topic 11: the Practices as reckoned so as to make the largest number

4–8 Topic 1: etymological import of the Pāli names for the Pure Practices
4 Pāli names of the Pure Practices concerning clothing
  1. Paŋsukūlikañga (literally, Dust-heaper's practice)
  2. Tecīvarikañga (Three-rober's practice)

5 Pāli names of the Pure Practices concerning food
  3. Piṇḍapātikañga (Pellet-faller's practice)
  4. Sapadānacārikañga (Unbroken-rounds-goer's practice)
  5. Ekāsanikañga (One-sessioner's practice)
  6. Pattapiṇḍikañga (Bowl-fooder's practice)
  7. Khalupacchābhattikañga (Thank-you-No-more-man's practice)
6 Pāli names of the Pure Practices concerning dwelling-place
  8. Āraññikañga (Forester's practice)
  9. Rukkhamūlikañga (Treerooter's practice)
  10. Abbhokāsikañga (Openspacer's practice)
  11. Sosānikañga (Burning-grounder's practice)
  12. Yathāsanthatikañga (Any-bedder's practice)
7 Pāli name of the Pure Practice concerning bodily posture
  13. Nesajjikañga (Sittinger's practice)
8 The Pāli name dhutañgāni, as designation of all 13, may mean:
  1. Practices of the purged (as a tatpurusha compound)
  2. Having "purging-knowledge" as [essential] constituent (a bahuvrīhi)
  3. Purging practices (a karmadhāraya)

9 Topic 2: Characteristic etc. of all the Pure Practices in the lump
  Their common characteristic (lakkhaṇa) is the will to adopt them (§104)
  Their function (rasa) is to extirpate all worldly lust
  Their manifestation (paccupaṭṭhāna) is in freedom from such lust
  Their immediate basis (padaṭṭhāna) is fewness-of-wishes etc.

10–92 Topics 3–7: their adoption, observance, grades, breach, advantages
10 General prefatory remark as to the adoption of any of the Pure Practices:
  The persons in presence of whom they should be formally adopted
11–92 Discussion of all these 5 topics together (but each in its order),
  With reference to each separate one of the 13 practices in order

[From here to §92, one number in parenthesis refers to a practice, and two numbers in parenthesis refer to practice and topic]

11 Refuse-rag-practice (1): Formulas of adoption (1^3)
12 Manner of observance (1^4): 23 kinds of rags permissible for robes
13 Commentary on the names of the 23 kinds
14 What may or may not be taken constructively as refuse-rags
15 Grades of strictness (1^5) in its observance: severe, middling, easy
16 What constitutes a breach (1^6) of its observance
17 The 12 advantages (1^7) of observing it: among them
  Establishment in the "first order of nobility" (ariyavaŋsa: cf. §29)
18 Buddhaghosa praises in 3 stanzas the Refuse-rag-practice

19 Three-robe-practice (2): Formulas of adoption (2^3)
20 Manner of observance (2^4)
21 Grades of strictness (2^5) in its observance: severe, middling, easy
22 What constitutes a breach (2^6) of its observance
23 The 8 and more advantages (2^7) of observing it
24 Buddhaghosa praises in 2 stanzas the Three-robe-practice

25 Alms-practice (3): Formulas of adoption (3^3)
26 Manner of observance (3^4): 14 kinds of food which are not permissible
27 Grades of strictness (3^5) in its observance: severe, middling, easy
Anecdote illustrating the disadvantages of the 2 lower grades
28 What constitutes a breach (3^6) of its observance
29 The 15 advantages (3^7) of observing it: among them
Establishment in the "second order of nobility" (ariyavaŋsa: cf. §17)
30 Buddhaghosa praises in 3 stanzas the Alms-practice

31 House-to-house-practice (4): Formulas of adoption (4^3)
32 Manner of observance (4^4)
33 Grades of strictness (4^5) in its observance: severe, middling, easy
The Elder most distinguished for the practice was Kassapa the Great
34 What constitutes a breach (4^6) of its observance
35 The 8 advantages (4^7) of observing it
36 Buddhaghosa praises in 2 stanzas the House-to-house-practice

37 The One-sitting-practice (5): Formulas of adoption (5^3)
38 Manner of observance (5^4)
39 Grades of strictness (5^5) in its observance: severe, middling, easy
40 What constitutes a breach (5^6) of its observance
41 The 8 advantages (5^7) of observing it
42 Buddhaghosa praises in 2 stanzas the One-sitting-practice

43 The One-bowl-practice (6): Formulas of adoption (6^3)
44 Manner of observance (6^4)
45 Grades of strictness (6^5) in its observance: severe, middling, easy
46 What constitutes a breach (6^6) of its observance
47 The 6 advantages (6^7) of observing it
48 Buddhaghosa praises in 2 stanzas the One-bowl-practice

49 The No-more-practice (7): Formulas of adoption (7^3)
50 Manner of observance (7^4)
51 Grades of strictness (7^5) in its observance: severe, middling, easy
52 What constitutes a breach (7^6) of its observance
53 The 5 advantages (7^7) of observing it
54 Buddhaghosa praises in 2 stanzas the No-more-practice

55 Forest-practice (8): Formulas of adoption (8^3)
56 Manner of observance (8^4)
57 Definitions: village, neighborhood of village, forest; defined also according to texts of Vinaya, Abhidhamma, Suttanta, and Commentaries
58 Permissible modifications of the Forest-practice
59 Grades of strictness (8^5) in its observance: severe, middling, easy
60 What constitutes a breach (8^6) of its observance
61 The 7 advantages (8^7) of observing it
62 Buddhaghosa praises in 4 stanzas the Forest-practice

63 Tree-root-practice (9): Formulas of adoption (9^3)
64 Manner of observance (9^4): trees to avoid
65 Grades of strictness (9^5) in its observance: severe, middling, easy
66 What constitutes a breach (9^6) of its observance
67 The 6 advantages (9^7) of observing it
68 Buddhaghosa praises in 4 stanzas the Tree-root-practice

69 Open-sky-practice (10): Formulas of adoption (10^3)
70 Manner of observance (10^4): permissible occasions of being under cover
71 Grades of strictness (10^5) in its observance: severe, middling, easy
72 What constitutes a breach (10^6) of its observance
73 The 6 advantages (10^7) of observing it
74 Buddhaghosa praises in 3 stanzas the Open-sky-practice

75 Burning-ground-practice (11): Formulas of adoption (11^3)
76 Manner of observance (11^4)
Burning-ground defined; serious business; due precautions
77 Grades of strictness (11^5) in its observance: severe, middling, easy
78 What constitutes a breach (11^6) of its observance
79 The 10 advantages (11^7) of observing it
80 Buddhaghosa praises in 2 stanzas the Burning-ground-practice

81 Lodge-anywhere-practice (12): Formulas of adoption (12^3)
82 Manner of observance (12^4)
83 Grades of strictness (12^5) in its observance: severe, middling, easy
84 What constitutes a breach (12^6) of its observance
85 The 6 advantages (12^7) of observing it
86 Buddhaghosa praises in 3 stanzas the Lodge-anywhere-practice

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87 Never-lie-down-practice (13): Formulas of adoption (13^3)
88 Manner of observance (13^4)
89 Grades of strictness (13^5) in its observance: severe, middling, easy
90 What constitutes a breach (13^6) of its observance
91 The 5 advantages (13^7) of observing it
92 Buddhaghosa praises in 3 stanzas the Never-lie-down-practice

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93 Epanalepsis of stanza 2 of the outline (§3) summarizing topics 8–11

94 Topic 8: of the 3 categories, good [and bad and indifferent] (kusala-ttika) the Pure Practices fall under only the first and third

95–100 Topic 9: discussion of the 5 matters, "1. dhuta etc."
95 The etceteral term includes 2. dhutavādo, 3. dhutadhammā, 4. dhutañgāni, and 5. the question, Who are benefited by the Pure Practices?
96 The word 1. dhuta may mean 'purified,' as applied to a person, or 'purifying,' as applied to a thing
97 The word 2. dhutavādo means 'preacher of the Pure Practices':
Four combinations:
One practices, but does not preach: example, Bakkula
Another does not practice, but preaches: example, Upananda
A third neither practices nor preaches: example, Lāludāyin
A fourth both practices and preaches: example, Sāriputta
98 The word 3. dhutadhammā, 'pure things,' includes 5 such:
Fewness of wishes (appicchatā)
Contentment (santuṭṭhitā)
Love for the simple life (sallekhatā)
Love for solitude (pavivekatā)
Intelligent recognition of his higher needs (idamaṭṭhitā)
99 The word 4. dhutañgāni, 'Pure Practices,' means the 13 already stated,
of which the list is given above at §2,
of which the names are explained at §4–§8, and
of which the general characteristic etc. is stated at §9
100 The question (5): Who are benefited by the Pure Practices? Answer:
Those in whose temperament lust or ignorance or ill-will is dominant

101–104 Topic 10: the Practices as reckoned so as to make the lowest number or a few or the fewest groups (samāsato)
101 The 13 Pure Practices, reduced to lowest number, are 8: namely
Three inclusive Practices, each of which includes one or two others
The House-to-house-practice includes the Alms-practice
The One-sitting-practice includes the One-bowl and the No-more
The Open-sky-practice includes the Tree-root and the Lodge-anywhere
Five non-inclusive Practices (literally, un-confused with others, distinct)
the Forest-practice, the Refuse-rag-practice, the Three-robe-practice
the Never-lie-down-practice, the Burning-ground-practice
[Thus 3 inclusive Practices, with the 5 others which they include,
and with the 5 non-inclusive Practices, make 13]

102 The 13 Practices may be put into 4 groups, according as they concern
1. Clothing; [compare §2, above]
2. Food;
3. Dwelling-place;
4. Strenuousness (or bodily posture)

103 They make 2 groups, according as they concern
1. The everyday needs of life, or
2. Strenuousness

They make 2 groups also according as is is advisable
1. To keep them, or
2. Not to keep them

104 All 13 make a unitary group in so far as
They are all essentially an act of the will: see §9

105–106 Topic 11: the Practices, as reckoned so as to make the highest number (vyāsato), amount to 42: thus

105 Reckoning all 13 of them as for the monks,
8 of them again as for the nuns,
12 of them again as for the male novices,
7 of them again as for the female novices, and
2 of them again as for the lay disciples,— they
Count as 42

106 For each of these 5 classes, the Practices are specifically,— for
Monks, the Practices numbered 1–13 (= 13)
Nuns, numbers 1–6 [7–8 forbidden; 9–10–11 too hard] and 12–13 (= 8)
Male novices, same as for monks, excepting 2,— i. e. 1, and 3–13 (= 12)
Female novices, same as for nuns, ex. 2,— i. e., 1, and 3–6, and 12–13 (= 7)
Lay disciples, without regard to sex, numbers 5 and 6 (= 2)

107 Conclusion: Book 2 sets forth the Pure Practices
As a means of developing the qualities by which
The Morality set forth in Book 1 may be kept undefiled
Colophon: Book 2, Exposition of the Pure Practices

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AN IMPROVED METHOD FOR DETERMINING SPECIFIC HEATS OF LIQUIDS, WITH DATA CONCERNING DILUTE HYDROCHLORIC, HYDROBROMIC, HYDRIODIC, NITRIC AND PERCHLORIC ACIDS AND LITHIUM, SODIUM AND POTASSIUM HYDROXIDES.

By T. W. Richards and A. W. Rowe.

Investigations on Light and Heat published with Aid from the Rumford Fund.

AN IMPROVED METHOD FOR DETERMINING SPECIFIC HEATS OF LIQUIDS, WITH DATA CONCERNING DILUTE HYDROCHLORIC, HYDROBROMIC, HYDRIODIC, NITRIC AND PERCHLORIC ACIDS AND LITHIUM, SODIUM AND POTASSIUM HYDROXIDES.

By Theodore W. Richards and A. W. Rowe.

Received, May 20, 1913.

In a previous communication [1] the authors described a new and accurate method for the determination of the specific heats of liquids, based upon the adiabatic procedure originally outlined by one of us.[2] Any method for this purpose demands an exact quantitative source of heat; and in the present case the device employed was the liberation in parallel experiments of known amounts of heat by the neutralization of an acid with a base. The neutralization was carried out in the center of a calorimetric system consisting primarily either of water or of the liquid under observation. By observing the relative temperature rise in the system when first water and subsequently the other liquid filled the calorimeter, (or better, by so arranging the parallel experiments as to secure uniform temperature changes in the two cases, comparing the relative weight of liquid used with that of the original amount of water) a direct determination of the relative heat capacities of the two systems was rendered possible. The chief advantage of such methods involving the comparison of one set of data with another is that possible errors of direct determination appear in all experiments and are thus eliminated by cancellation.

As originally outlined, the method had one distinct objection, namely, the alkali was at first contained in a reservoir outside the calorimetric system and at a temperature usually somewhat different from the latter. Although the difference was carefully determined and a suitable correction applied for the heat thus abstracted from or brought into the system, this feature introduced uncertainty, especially as the liquid might undergo slight but undetermined changes in temperature during the act of transference. This point was recognized by the authors at the time of the initial publication and at that time a method for its elimination was suggested. A description of the improved apparatus and some of the results yielded by it are embodied in the present communication.

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[1] Richards and Rowe, Proc. Am. Acad., **43**, 475 (1908); Zeit. phys. Chem. 64, 187 (1908).

[2] Richards, Henderson and Forbes, Proc. Am. Acad., **41**, 3 (1905).

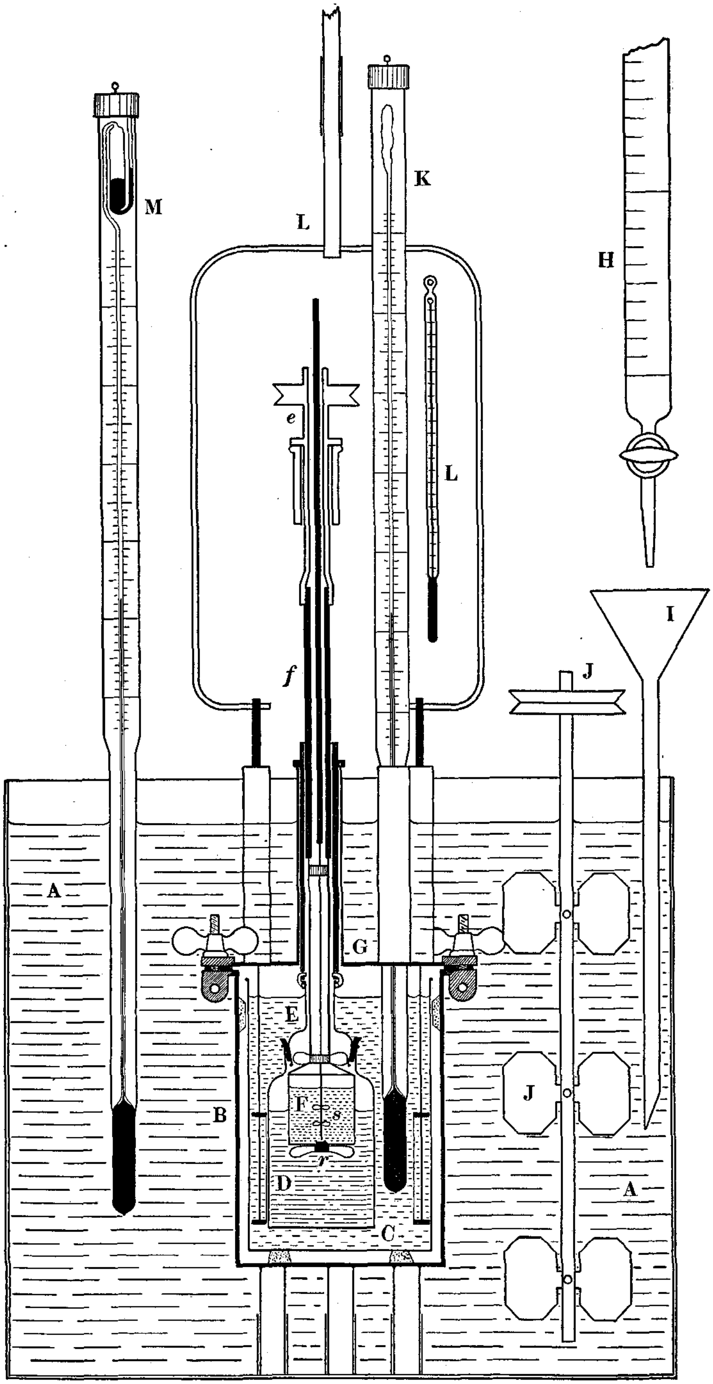
Method and Apparatus.

As in the earlier form, the neutralization of sulphuric acid with sodium hydroxide was selected as the source of heat. The stability of solutions made from this acid renders it especially suitable for such purposes. In carrying out the neutralization a concentrated solution of caustic soda was placed in a small stoppered platinum can, which was suspended within a platinum bottle containing the acid to be neutralized. At the appointed moment, the two liquids were allowed to mix. This bottle, closed with a platinum stopper, had previously been submerged in the liquid to be studied, which was itself held in a large platinum calorimeter. The whole was provided with an environment so arranged as to vary in temperature at the same rate as the calorimeter, and this eliminated the troublesome and inexact cooling correction.

It is perhaps worth while to describe the affair in detail because much time was spent in working out the most advantageous arrangement. The accompanying diagram will assist in the explanation. (Figure 1.)

The platinum bottle (D) used to contain the weighed amount of acid was provided with a very wide mouth, in order that the receptacle for the alkali could be introduced within it. The mouth was narrowed by a hollow platinum stopper or cap (E) of peculiar shape, the joint being made tight by a thin smooth rubber band or collar between the stopper and bottle. The upper end of the open stopper was fastened by stout platinum wires to a hollow tube of hard rubber which passed through an opening in the jacket above, and held the bottle in suspension.

The receptacle for the alkali also was unusual in its form, consisting of a very small platinum can (F) (of a diameter slightly less than that of the mouth of the bottle) with a hole in the bottom closed by a small rubber stopper (r). This can was open at the top and was sus-


M
L
K
H
e
L
I
f
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A
G
E
F
s
B
r
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D
A
C

FIGURE 1.

pended from the periphery by four stout platinum wires symmetrically placed, which in turn were made fast to another hard rubber tube (f). A hard rubber pulley and suitable bearing (e) were fastened to the tube (f) and permitted the mechanical rotation in the horizontal plane of this part of the system. Small platinum wings attached to the outside of the receptacle below caused the rotation to stir thoroughly the surrounding acid. The alkali itself within the very small can likewise needed agitation, which was effected by a tiny platinum stirrer (s) suspended by a hard rubber rod within the tube (f). Upon the effectiveness of the stirring within these receptacles depended the speed with which thermal homogeneity could be established, and the arrangements were found to be very effective for this purpose.

The platinum bottle was immersed, as has already been stated, within the liquid to be studied in a large platinum calorimeter capable of holding about 0.7 liter.

The jacket arrangement for obviating the cooling correction should also be described in detail, as it differs from that used in the last investigation of this kind. As in other recent calorimetric work done in this laboratory, the whole calorimeter and its appurtenances were enclosed in a water-tight vessel, with tubes above for the necessary connections. This vessel was immersed in a pail of dilute alkali. As the reaction within the calorimeter progressed, acid was added to this alkali, so that the change of temperature in the surrounding pail should parallel that in the calorimeter itself. The pail (A) was made of copper and furnished with a powerful propeller-stirrer (J). The water-tight "submarine" vessel containing the calorimeter was furnished with a cover clamped in place and made water-tight by a smooth rubber washer. The submarine vessel was nickelplated and polished inside, and was large enough so as to allow a narrow air space between it and the calorimeter, from which it was kept apart by small wedges of cork. Through suitable tubes at the top of the "submarine" were introduced not only the thermometer and the hard rubber tubes already mentioned, but also the rods bearing the two-stage platinum stirrer which stirred the calorimeter itself, thus equalizing the temperature in the liquid immediately surrounding the wide mouthed platinum bottle. The hard rubber stirrer rods were as small in diameter as possible; for large rods cause a pumping effect and thus introduce outside air into the calorimeter. Great pains must be taken to prevent the circulation of air through these tubes, for the adjustment is so delicate that if the temperature of the room varies considerably from that within the submarine, a distinct effect is produced upon the temperature of the calorimeter by air thus introduced. Accordingly all the tubes were loosely packed with cotton wool which served to prevent undue circulation.[3] A further safeguard against error from this cause was found in increasing the depth of submergence of the "submarine."

Two similar accurately standardized thermometers were used to register the temperature in the calorimeter on the one hand and in the outside bath on the other. For the sake of accuracy these thermometers should be alike in scale and in thermometric lag. Those employed were especially made, and were very well adapted for the purpose. They covered a range from 15.4 to 20.8 degrees, and each degree was 74 millimeters long. A small auxiliary thermometer (L) suspended wholly in the air beside the stems of the larger instruments gave the mean temperature of the exposed portions of these stems. The thermometers were standardized with the greatest care by reference to standard instruments which had been carefully studied at the Bureau Internationale des Poids et Mésures. Their standardization will be described in another place. As the final results usually varied no more from the mean value than 0.05 precent, it is clear that the thermometers were usually to be depended upon to within 0.001°.

In the outer pail of course the concentration of the alkali might vary within wide limits without harm, and the sulphuric acid introduced into it was always the most concentrated grade of pure acid of commerce,[4] which was delivered through a burette graduated in tenths of a degree Centigrade, as already explained in previous papers.

Within the platinum bottle, however, the acid needed very precise determination in order that the heat yielded to the calorimetric system by its neutralization by a slight excess of alkali might be definitely apportioned in each experiment.

The steps taken to prepare these solutions of definite strength are described below. Other experimental details will be discussed later when the individual experiments are described.

With this apparatus, then, were made many series of experiments in which first pure water was contained in the calorimeter-can, and afterwards the various dilute solutions. In each case a definite amount of heat was evolved in the inner vessel by means of known amounts of acid and alkali, and in each case the rise in temperature caused by this added heat was duly read off on the thermometers. Thus with the appropriate calculation, to be described later under the individual experiments, the specific heats of the various solutions were determined.

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[3] The sketch is not strictly drawn to scale and the size of the rods is exaggerated for the sake of clearness.

[4] Crude acid would have answered, but the pure acid was found to be more constant in composition than cheaper cruder grades. As the burettes were graduated to deliver a constant volume of acid for a definite temperature rise, this point was of importance.

Preparation of Materials.

The preparation of the materials needed in the experiment naturally divides itself into two categories; first, the preparation of the acid and alkali needed to evolve measured quantities of heat, and secondly, the preparation of the pure solutions whose specific heats were to be determined. The former of these two categories was a very simple matter and may be described in a few words.

The standard sulphuric acid to be weighed into the bottle within the calorimeter and there neutralized, was prepared twelve liters at a time from the purest obtainable acid and distilled water — the resulting solution being thoroughly shaken to insure complete homogeneity. The concentration was determined by titration against a carefully standardized solution of sodium hydroxide, and finally the liquid was sealed by means of paraffin in two-liter Jena flasks. The results of the experiments made with this acid solution give abundant testimony as to its permanence and constant content of ionized hydrogen. That the acid should undergo no change during use, the device described in the earlier article (l. c.) was utilized. The determinations of the strength of Lot A gave as a mean of the individual titrations the value 6.274 percent. This was subsequently verified by a series of gravimetric analyses, the mean of which was 6.276 percent.

The much more concentrated alkaline solution for the innermost vessel was made up in the manner previously employed, from Kahlbaum's sodium hydroxide, purified by alcohol. This solution was protected from further contamination by carbonate through the use of a double attachment; all entering air must pass first through a tower of soda lime, and further through a wash bottle which contained some of the solution from the container, to prevent evaporation of the standard solution into the tower. (Figure 2) An excess of the alkali was always taken in each thermal experiment, sufficient to render inoperative the carbonate originally present. A series of titrations with a carefully standardized hydrochloric acid solution indicated a total alkalinity corresponding to the concentration 27.825 percent of

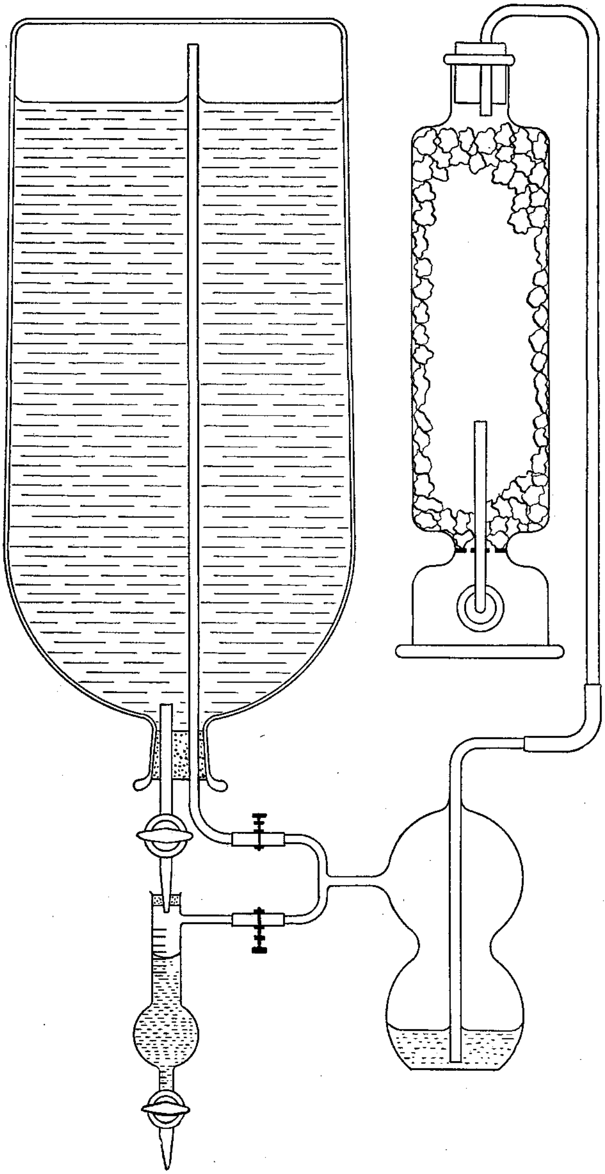

FIGURE 2.

sodium hydroxide — or, expressed in *mols,* NaOH . 5.760 H_2O. The sodium present as carbonate — determined by successive titrations using methyl orange and phenol phthaleïn as indicators,— was 1.2 percent of the sodium present. The standardization of the acid and alkaline solutions used for the titrations was based upon a series of gravimetric analyses of silver chloride.

The substances studied in dilute solution were hydrochloric, hydrobromic, hydriodic, nitric, and perchloric acids, and lithium, sodium, and potassium hydroxides, as indicated by the title. The preparation of these substances will be described in order in the pages immediately following.

Hydrochloric acid.

The acid for this solution was obtained by fractional distillation and condensing in quartz the constant boiling mixture of the purest obtainable acid. The middle third of the middle third was selected from two successive distillations and this sample, one-ninth of the material at first employed, was diluted with freshly distilled water to the desired concentration. Two samples were prepared, the first, sample A, being standardized gravimetrically by the precipitation of silver chloride, taking precautions such as one would use in determining atomic weights, and the second, sample B, being estimated volumetrically by titration with very exact standard alkali, standardized by means of the first. The results of these analyses were as follows: 68.3590 and 69.4105 grams yielded respectively 5.3234 and 5.4048 grams of silver chloride. Using the usual atomic weight Ag = 107.88, this acid is thus shown to contain 100.15 gram-molecules of water to one of acid, the two experiments deviating from the mean by only 0.004 percent. The other acid B was found volumetrically to be slightly more concentrated, having 99.97 gram molecules of water. These are so nearly alike that careful determination of their specific heats showed no essential difference between them.

Hydrobromic acid.

The very pure acid used for these experiments was prepared some time ago for determinations of atomic weights. For the present purpose it was redistilled, using a quartz condenser; the first and last portions of the distillate were rejected. The middle fraction used was perfectly colorless. The two portions (over 62 and 70 grams respectively) were carefully weighed in a weighing burette and titrated with standard alkali contained in another weighing burette, the two analyses giving exactly the same result, a percentage of 4.293 grams of hydrobromic acid, which corresponds to a molecular concentration of 100.5 molecules of water for each molecule of hydrobromic acid. This solution, like all the acid solutions, was immediately placed in a large bottle of resistant glass ("nonsol") and the specific heat was determined as rapidly as possible. It was found that neither this glass nor Jena reagent glass was essentially attacked by the dilute acids during the times in which they were kept therein.

Hydriodic acid.

The preparation of this acid in a state of purity offered considerable difficulty. Three different samples of the purest commercial product which was guaranteed to conform to the usual analytical standard were purchased, but each contained appreciable amounts of free iodine from which it could not easily be purified.

It should be noted, however, that a small amount of iodine would not have a very important effect on the specific heat, although it was exceedingly important that this amount should not increase during the experiment, because such an increase would superpose the heat of oxidation of the acid upon the phenomena concerned with specific heat. Therefore careful analyses with sodium thiosulphate were made before and afterwards in order to make sure that, during the determination of the specific heat, no significant oxidation had occurred. This was the case when the dilution of the concentrated acid was effected by adding water which had been freshly distilled and cooled in an atmosphere of hydrogen, the resulting solution being kept under such an atmosphere protected from oxygen. During the actual measurement of the specific heat it was not convenient to exclude air, but analysis showed that the decomposition which took place during that brief time was wholly negligible.

Each of the three samples of dilute acid was analyzed as before. The first was found to contain 6.635 percent of hydrogen iodide and 0.032 percent of iodine. The percentages in the second were 6.625 and 0.031 respectively. After four experiments on the specific heat another portion of this sample was analyzed and found to contain 6.632 percent of hydrogen iodide and 0.034 percent of iodine, having been essentially unchanged. The third specimen by acidimetric analysis was found to contain 6.626 percent of acid and 0.040 percent of iodine. With these three specimens of acid six determinations of the specific heat were made. They were all essentially of the same strength and the differences between the specific heats of the different samples did not exceed the error of experiment, the determination of sample A giving a value of 0.9210, three of sample B giving 0.9215 and two of sample C yielding 0.9211 as the respective values of the specific heat.

Nitric Acid.

This solution was prepared from the best obtainable nitric acid through fractional distillation, using a quartz condenser. To avoid the action of sunlight in forming oxides of nitrogen the operation was performed in a carefully darkened hood. The middle third of the middle third was reserved. This was diluted to the proper strength with freshly distilled water. The concentrated acid was not absolutely colorless, but the exceedingly pale tinge of yellow could not have indicated an amount of lower oxides large enough to affect the determinations of the specific heat of its dilute solution. The solution was analyzed by titration and properly diluted.

Perchloric Acid.

A solution of this substance was made from a sample of acid of pure material prepared for the determination of the atomic weight of silver and lithium by Dr. H. H. Willard. The authors take pleasure in expressing their thanks for this courtesy. The preparation is described elsewhere.[5] The concentrated acid was diluted to the proper strength with freshly distilled water. Two analyses each showed the presence of 5.788 percent of perchloric acid which corresponds to a concentration of $HClO_4.99.99\,H_2O$.

Sodium Hydroxide.

The preparation and maintenance of the alkaline solutions in a condition free from carbonate offers well known difficulties. The purest sodium hydroxide of commerce contains appreciable amounts of this impurity; and the contamination is not diminished by the usual methods of handling. The frequency with which this fact is overlooked will be discussed in another place. The available data concerning the heat capacities of carbonate solutions suggest however that the amounts of carbonate to be found in hydroxides of the highest grade would have a scarcely appreciable effect upon the specific heat. For example, the specific heat of a solution of sodium hydroxide with 100 mols of water as determined by Thomsen is 0.968 while that of sodium carbonate of an equivalent concentration is given severally by Thomsen as 0.958 and by Marignac [6] as 0.9695. Taking Thomsen's two values as the more strictly comparable, it is evident that each percent of carbonate may be supposed to lower the specific heat by one unit in the fourth decimal place. The highest grade of sodium hydroxide to be found in commerce, as employed in our experiments, contained from 1 percent to 2 percent of carbonate; hence solutions made directly from this material would show a heat capacity 0.01 percent to 0.02 percent lower than a solution of pure hydroxide. Because, however, such assumptions introduce uncertainty; and furthermore, because 0.02 percent is a perceptible quantity, it was obviously best to secure specimens of really pure substances.

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[5] Richards and Willard, Journ. Am. Chem. Soc., **32**, 4 (1910).

Our material was prepared by first adding to a rather concentrated solution of the purest material of commerce an excess of pure barium hydroxide in hot aqueous solution. After the settling of the precipitated barium carbonate, the clear, supernatant liquid was siphoned off in an atmosphere free from carbon dioxide. A series of gravimetric determinations of the excess of barium present in the solution were then made, and the theoretical amount of very pure sulphuric acid was added. After the settling of the precipitate, the supernatant liquid was again siphoned off with the usual precautions, and made up to the proper strength by the addition of freshly distilled water, which had been cooled in an atmosphere free from carbon dioxide. Such alkaline solutions were always kept in paraffin-lined bottles — a procedure which sufficiently preserved their initial purity, as is evidenced in the accompanying analyses.

For example, 101.20 grams of solution were found by titration to be equivalent to 9.8976 grams of a certain solution of hydrochloric acid. On the other hand, 12.9746 grams of this latter solution were found to yield 10.3197 grams of silver chloride. Hence, 100.000 grams of the alkaline solution must be equivalent to 7.8723 grams of silver chloride or must contain 2.171 grams of dry sodium hydroxide. The solution contained therefore 2.171 percent of alkali. Other analyses are tabulated below.

The end point was essentially the same with methyl orange as with phenol phthalein, thus showing that the carbonate had been satisfactorily eliminated. No barium could be found by careful qualitative testing in a solution, but a minute trace of sulphate was found there. This impurity was quite too small in amount to have any effect on the specific heat.

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[6] Marignac, Arch. Sci. Nat., **55**, 113 (1876).

THE CONCENTRATIONS OF THE PUREST SODIUM HYDROXIDE SOLUTIONS

No.	Date	NaOH(%)	Mols.H_2O
1	April, 1909	2.1710	100.06
2	" "	2.1703	100.10
3	May "	2.1709	100.07
			100.08
4	October, 1909	2.1685	100.18

A solution containing enough carbonic acid to combine with 1.3 percent of the alkali present was also made and standardized in the same way, the amount of carbonate present being calculated from the difference between the end points found with the two indicators in the usual way. The specific heat of this solution also was taken in a few experiments. They did not, however, deviate more from those with the pure solution than the possible error of the experimentation, and need not therefore be recorded. Their evidence at any rate was enough to verify the prediction made above that the presence of a small amount of carbonate would have had an almost if not quite negligible effect.

Potassium Hydroxide.

In the case of potassium hydroxide a similar method was used. Two samples were prepared, one made up directly from the purest obtainable commercial product and containing 1.8 percent of its alkali combined with carbonate, and the other freed from this carbonate in precisely the manner described in the case of sodium. The analyses of these solutions are recorded below. They were essentially identical in total alkalinity and no difference whatever could be detected in their specific heat. This again supports the conclusion reached in the case of contamination with sodium carbonate. The object of preparing the two solutions in the case of potassium as well as that of sodium lay not merely in showing the small effect of carbonate, but also in verifying the analytical portion of the work. The agreement of the two solutions showed that no numerical mistake had been made in preparing the solutions, and therefore gave much greater reason for confidence in the accuracy of the final results.

Of the two solutions of potassium hydroxide, the purer was found in two experiments to contain 3.016 and 3.019 percent of the solid substance, and the second less pure solution was found in two other experiments to be equivalent to one containing 3.015 and 3.016 percent of pure substance. The average corresponds to the molecular concentration $KOH.100.1H_2O$. As usual, weights of solution varying from 93 to 132 grams were taken from the weighing burettes in each analysis, hence great accuracy was easily attained.

Lithium hydroxide.

This substance was less easy to prepare, because the material to be had in commerce is very far from satisfactory, as a rule. The substance used in our experiments was of a very high degree of purity, having been made from lithium chloride almost pure enough for the determination of atomic weights. The chloride had been prepared in the first place by Dr. H. H. Willard as one of the preliminary stages of an investigation carried out by him in collaboration with one of us upon the atomic weight of this metal, and its purification has been described elsewhere. We are greatly obliged to him for this help. The salt contained entirely negligible amounts of other bases. After two recrystallizations carried out in order to be sure that no other anions were present, the chloride was converted into the sulphate by heating in a platinum dish with a small excess of very pure sulphuric acid. The pure white salt thus obtained was dissolved in water and the fairly concentrated solution was treated with an excess of hot aqueous barium hydroxide. After settling, the solution was filtered and the excess of barium, determined gravimetrically, was precipitated by precisely the proper quantity of lithium sulphate. The clear solution obtained after settling was decanted in an atmosphere free from carbon dioxide; and after its concentration had been determined, the solution was diluted with enough water, also free from carbon dioxide, to attain the desired concentration. Thus the latter part of the procedure was essentially like that employed in the case of the other alkalies, and the product was strictly comparable with those used in the other cases. In this solution no trace of barium or carbon dioxide were found upon careful qualitative analysis, but here as in the other cases a trace of sulphuric acid was detected. Two analyses gave identical results, showing that the solution contained 1.307 percent of solid hydroxide, thus possessing the concentration LiOH, $100.4H_2O$.

It will be noted that in some cases, particularly in the case of lithium, the solution was not exactly of the concentration MX 100 H_2O. A moment's consideration will show, however, that the specific heat of such a solution is different by only a negligible amount from one of the exact strength. For example, the specific heat of NaOH 100 H_2O is 0.9664 and that of NaOH 200 H_2O is 0.9827. Evidently then the addition of a single H_2O to the 100 H_2O already present will make a difference of less than 2 in the fourth decimal place, and 0.4 H_2O will cause a difference of less than 1 in that place,— an amount beyond the range of accuracy of the experiments.

In making the calculations the following international atomic weights have been assumed O = 16.000, H = 1.008, Cl = 35.458, Br = 79.916, I = 126.99, N = 14.008, K = 39.09, Na = 23.00, Ag = 107.88.

THE EXECUTION OF THE DETERMINATIONS.

Having thus discussed briefly the details of the apparatus and of the purification of the substances employed we may turn to the execution of an experiment. In order to follow the description reference to the figure on page 175 is necessary.

The "submarine" vessel was first secured in position with its lid off, the outer bath was filled almost to the rim of the vessel, and the temperature of its crude dilute alkali was brought to the point at which the determination was to start. The liquid whose specific heat was to be determined was then placed in the platinum calorimeter, brought to the desired temperature, rapidly weighed to the nearest centigram, and placed in position within the "submarine."

The cover of the "submarine," temporarily suspended from a stand in the neighborhood of the calorimeter, had been already fitted with its various attachments, ready to be clamped into place. First, the open cap (E) of the platinum bottle had been arranged in such a way that the rest of the apparatus could be attached below it. Then the receptacle for the alkali, (F) made water-tight by means of a rubber stopper (r) placed in the hole in the lower part, was fixed in place. After the calorimeter had been prepared as described above, the required portion of caustic soda solution was measured into its receptacle (F) through a burette, and the piece was suspended from its proper place in the lid of the submarine vessel below the cap (E). Extremely accurate determination of the amount of alkali was not necessary, for its concentration was such that its own heat of dilution was approximately that required to raise itself through the temperature range of an experiment. Any excess therefore was self-heating and made no appreciable effect on the rise of temperature caused by neutralization; but nevertheless careful measurement with a burette insured the use of very nearly the same quantity in each case. There was always the same slight excess over and above the amount needed to neutralize the acid.

The standard acid, the one reagent whose quantity needed accurate determination, was next weighed into the wide mouthed platinum bottle (D). It was then rapidly brought to the desired temperature and made fast by a water tight rubber collar to its open cap (E), which had been previously suspended beneath from the cover of the submarine vessel above the receptacle for the alkali, as already stated. The calorimeter-stirrer was then added through its proper holes in the lid, and the whole system attached to this lid (or cover) was transferred to the submarine vessel, the bottle being immersed in the calorimeter. After the screws had secured the lid in place and made it water-tight with the help of the large rubber washer, the level of the outer bath was raised by adding dilute caustic soda, and the various stirrers were set in motion. A very few minutes sufficed to establish thermal homogeneity. The addition of a little ice-water or acid permitted the ready regulation of the bath temperature. When the temperature of the system had become uniform, the inner thermometer was carefully read and the result was recorded as the initial temperature of the experiment. The calorimeter-stirrer, operating at a rate of 45 strokes per minute was found to produce practically no mechanical heat in the time of the experiment, and the effect of outside air which might be pumped in through the openings was also found to be negligible during this time, unless the temperature of the room was widely divergent. Hence no corrections had to be applied for these otherwise disturbing side-effects.

All being ready, the rubber stopper was forced out of the receptacle containing the alkali, and the concentrated caustic soda solution was allowed to flow into the standard acid. At the expiration of three minutes the receptacle, still rotating, was lowered almost to the bottom of the bottle, thus permitting a free mixing of the acid and alkali, which was hastened by raising and lowering the receptacle at appropriate intervals. The complete neutralization within the platinum bottle was abundantly proved by the constancy of the readings of the thermometer after six or eight minutes.

This reaction within the platinum bottle furnished heat which raised the outer liquid under investigation about four degrees in temperature. Because the source of heat was completely surrounded by the calorimetric liquid, which in turn was adequately stirred, irregularity of heating effects during the mixing of the acid and alkali could not be attended by any thermal loss. From the curve showing the rise of temperature with time (Figure 3), it is clear that 95 percent of the rise occurs during the first five minutes. This somewhat rapid rise was easily paralleled in the outer vessel by adding acid so that there was no difficulty in making the calorimetric operation fully adiabatic.

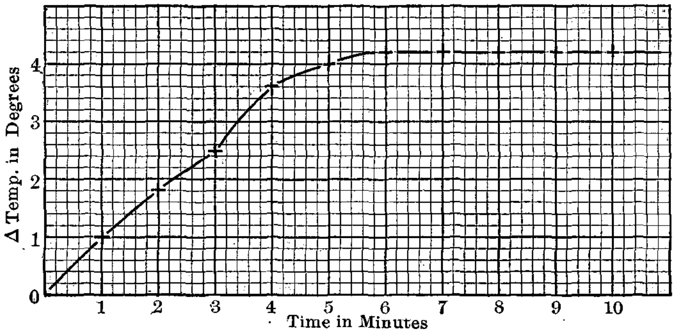

Figure 3. Temperatures in degrees and time in minutes are plotted as ordinates and abscissae respectively. At the expiration of three minutes the inner stirring was greatly increased by the raising and lowering of the innermost vessel.

The total heat capacity of the apparatus was equivalent at first to 12.24 grams of water, made up as follows: 252.22 grams of platinum constituting the calorimeter, stirrer, bottle, cap and suspension, together with the small alkali container within the bottle, was assumed to have the specific heat 0.0324, a value taken as the most probable after comparing a large number of recent determinations. This platinum contained a little iridium, but not enough to affect seriously its specific heat. The 3.3 grams of rubber had a water equivalent equal to 1.59.[7] Later a heavier rubber stopper was used, increasing the water equivalent of the rubber to 2.02. The thermometer had a bulb occupying 3.90 cubic centimeters. Taking its heat capacity per cubic centimeter according to Ostwald-Luther's well known experience, this corresponds to a water equivalent of 1.83. The immersed stem above the bulb was estimated to have a heat capacity of 0.65 on the same basis, by careful weighing of bits of glass tube of the same diameter and thickness. The sum of all these water equivalents was, as has already been stated, 12.24. About 114.8 grams of dilute sulphuric acid (requiring 21.37 grams of the concentrated caustic soda) were used for neutralization. The specific heats of these two solutions were taken respectively as 0.950[8] and 0.8345.[9] The uniformly employed excess of caustic soda (about 3 percent) above the amount needed to neutralize exactly the sulphuric acid could be neglected because its heating effect was enough to raise it through the same range of temperature as the main experiment in the way already explained. Hence the water equivalent of the sulphuric acid was 109.1 and of the caustic alkali 17.83. It will readily be seen that the specific heat of neither sulphuric acid nor caustic soda needed to be known with very great accuracy because they were essentially constant in all the experiments. A slight error therefore would have a wholly negligible effect on the results, which are relative in their character. For example an error of 2 percent in the value assumed for the specific heat of the acid could only affect the final results by a few thousandths of one percent, and there is no likelihood that the assumed specific heat was anything like so much as even one percent in error.

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[7] Haldane, Gee and Terry, Report, Brit. Ass., **59**, 516 (1889).

Data of a Specimen Experiment upon pure Water. Ser. IA, No. 1.

Temperature Data.

Initial Thermometer reading = 16.524°
Total Calibration Correction = − 0.033°
Initial Temperature (corrected) (T_1) = 16.491°
Final Thermometer reading = 20.752°
Total Calibration correction = − 0.038°
Final Temperature (corrected) (T_2) = 20.714°
Temperature Interval (T_2–T_1) = 4.223°
Stem Exposure Correction = + 0.001
T_2–T_1 corrected 4.224°

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[8] Marignac, Arch. sci. nat., **2**, 39, 217 (1870); Cattaneo, Cim., **3**, 26, 50 (1889). Thomsen, Ther. Untersuch.

[9] Determined by the Authors. The experimental data will be given in another paper.

Heat Capacity Data (in terms of water equivalent).
Water in calorimeter = 450.19 grams
Calorimeter and fittings = 12.24 "
21.37 gms. NaOH (sp. ht. = 0.8345) = 17.83 "
114.80 gms. H_2SO_4 (sp. ht. = 0.95) = 109.06 "

Total Heat Capacity = 589.32 "
Total evolution of heat (Q) 589.32×4.224° 2489.3 calories
Heat per 100 grams of acid (Q_{100}) 2168.4 "
Heat per $\frac{1}{2}$ Mol Acid (U) 16949 "

For the sake of convenience, the heat evolved per gram equivalent of sulphuric acid is called U.

All the other experiments upon water were essentially similar in character to this. They were, however, carried out at several different temperatures in order that the temperature coefficient of the reaction might be measured under precisely the conditions of the experiment, because the heat of neutralization varies decidedly with the temperature. The final temperature of the reaction is taken as the temperature corresponding to isothermal evolution of heat, because the heat capacities of the factors were used in computing the total heat capacity.[10] In this and in all the subsequent computations the specific heat of water over the same range is taken as the unit. As the matter is entirely one of comparison between similar experiments, this is entirely permissible and reduces all to the same basis. After these introductory remarks the two series of experiments with water alone in the calorimeter at about 20.7° and 19.8° *C.* are given below in order that the range of accuracy of the experiments may be seen. In this table, as before, Q_{100} stands for the heat evolved during the neutralization of 100.0 grams of dilute acid, and U signifies this same result calculated in terms of the gram equivalent of sulphuric acid.

Inspection of the results shows on the whole a very satisfactory constancy. The three experiments of Series I deviate from the mean by less than one-fortieth of one percent, all having been taken at about the same temperature. The five experiments of the next series are less satisfactory, but the last three are completely concordant with one another and with the mean.

From the two averages 16,946 and 16,993, taken at 20.72° and 19.77° respectively, it is easy to compute that the temperature co-

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[10] Richards. Journ. Am. Chem. Soc., **25,** 209 (1903).

TABLE II.

WATER STANDARDIZATION. SERIES I AND II.

No.	Acid Sample.	Water.	Acid.	T_2.	T_2-T_1.	Q_{100}.	U
		gms.	gms.			calories.	calories.
1	Aα	450.19	114.80	20.72°	4.224	2168.4	16949
2	"	450.18	114.86	20.70	4.224	2167.4	16942
3	"	450.16	114.88	20.74	4.226	2168.1	16947
Total av'ge		450.18	114.85	20.72	4.2247°	2168.0	16946
1	Aα	450.18	114.86	19.72°	4.235°	2173.0	16986
2	Aα	450.17	114.85	19.82	4.237	2175.8	17008 *
3	Aβ	450.15	114.85	19.75	4.233	2173.7	16991 *
4	Aβ	450.17	114.86	19.79	4.233	2173.5	16990 *
5	Aβ	450.18	114.86	19.77	4.233	2173.5	16990 *
Tot'l av'ge		450.17	114.856	19.77	4.2342°	2173.9	16993

* The new and heavier rubber stopper increased the water equivalent by 0.43 grams.

efficient of the reaction $\frac{\Delta U}{\Delta T}$ under these circumstances is –49.5 calories.

From time to time, in order to show that the acid was remaining constant in composition and that each of the two-liter flasks used for the storage of the acid contained material of the same concentration, the experiments were repeated at other temperatures. With the help of the temperature coefficient just calculated, one finds that at 20°.00 the reaction should evolve 16,982 calories, at 20.47°, 16,957 calories, and so on. It was thus possible to compare the observed evolution of heat at different temperatures with that calculated in this way, and thus use these determinations as a check on the concentration of the sulphuric acid,— or, viewed from the other standpoint, as a means of testing the accuracy of the temperature coefficient. The experiments thus made from time to time are given in the following table.

Incidentally it may be noted that Greek letters are used to designate the several two-liter flasks containing different portions of the same acid.

TABLE III.

CHECK DETERMINATIONS WITH WATER.

No.	Acid Sample.	Water.	Acid.	T_2.	T_2-T_1.	U (obs.).	U (calc.).	At Diff. (calories).
		gms.	gms.					
1	Aβ	450.18	114.87	20.47°	4.225°	16957	16958	−1
2	Aγ	450.17	114.85	19.95	4.232	16988+	16985	+3
3	Aϵ	450.19	114.87	20.68	4.222	16946	16947	−1
4	Aζ	450.19	114.87	20.47	4.227	16963+	16958	+5
Total av'ge		450.18	114.865	20.39	4.227			

The agreement of these results is highly satisfactory, two of them giving essentially the calculated values and two others deviating by an amount corresponding to only about 0.001°, the probable error of experimentation. The last experiment was made with the last portion of the original 12 liters of acid, nearly four months having intervened since the first experiment.

The results recorded in these tables warrant the assumption that the acid furnishes a definite quantitative source of heat, which may be depended upon; and also that the method of evolving and measuring this heat has now been brought to within a range of accuracy as great as is needful under the circumstances.

Having thus prepared the way, we may now proceed to describe the work with the various solutions under consideration.

From the above determinations, it was possible to compute the exact amount of heat which would be liberated by the neutralization of a given portion of acid within the temperature limits of the standardization. By replacing the water of these earlier experiments by a definite amount of some other liquid, a direct determination of the specific heat of the latter could be made. As has already been stated, the experiments were so conducted — by regulating the amounts of liquid used — that the temperature rise in all the determinations was substantially the same. The case of $HCl\ 100\ H_2O$ may be taken as typical. After it has been described in detail, the other cases will be given in tabular form. The purification of the acid and the preparation of the solution of the proper strength was already been described.

Although the method was similar to that employed in the case of the water, it will perhaps be best to give the details of a single experiment, before all the data are given in tabular form.

In the case of the hydrochloric acid and in all the subsequent cases the total heat capacity of the calorimetric system was 12.60 grams instead of 12.24 grams as given before, on account of the use of other rubber stoppers weighing 4.05 grams (instead of 3.30 grams). Because of the smaller specific heat of hydrochloric acid as well as its greater density, about 466.5 grams of this acid were used in the calorimeter, instead of 450.2 grams as in the case of water. A preliminary series of eight experiments gave an average rise of temperature under these conditions (the average final temperature being 20.34) of 4.231°. This corresponds to a specific heat possessed by the hydrochloric acid of 0.9635, with a probable error (computed according to the method of least squares) of 0.0002. The individual results need not be given, as they were only approximate, but none deviated more than 0.1 percent from the mean. A single experiment taken at random from the much more satisfactory final series will be recorded in detail before this series is summarized.

DATA OF A SPECIMEN DETERMINATION OF THE SPECIFIC HEAT OF $HCl.100H_2O$.

Series 2B, No. 1. Jan. 17, 1910.

Temperature Data.
Initial thermometer reading = 16.120°
Total Calibration correction = − 0.038
Initial temperature (corrected) (T_1) 16.082°
Final temperature reading = 20.344°
Total Calibration correction = − 0.030°
Final temperature (corrected) (T_2) 20.314°
(T_2–T_1) = 4.232°
Stem exposure corr. = + 0.001°
Temperature interval (T_2–T_1) corrected = 4.233°

Heat evolved by neutralization of acid:—
Weight of acid = 114.87 gms.
Q for 100.00 grams at 20.31° = 2170.7 cals.[11]
Q for 114.87 grams at 20.31° = 2493.4 "

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[11] This value is calculated from the preceding Table II.

Heat capacity data. (in terms of water equivalent)

Heat capacity of acid = x
Calorimeter system = 12.60 gms.
NaOH = 17.83 "
H_2SO_4 (114.87 gms.) = 109.13 "

Total heat capacity = = x + 139.56
Then heat evolved equals $(x + 139.56) \times 4.233 = 2493.4$ cal.

$$\text{Hence, } x = 449.47 \, \frac{\text{calories}}{\text{degrees}}$$

But the weight of $HCl.100\,H_2O$ actually taken in the calorimeter was 466.54 grams. Hence its specific heat

$$= \frac{\text{heat capacity}}{\text{heat capacity of equal weight } H_2O} = \frac{449.47}{466.54} = 0.9634$$

Immediately below is tabulated a series of determinations of this kind, giving all the necessary data.

TABLE IV.

SERIES CONCERNING THE SPECIFIC HEAT OF $HCl.\,100\,H_2O$.

No.	Sample HCl.	$HCl \cdot 100\,H_2O$.	H_2SO_4.	T_2.	T_2-T_1.	Specific $HCl \cdot 100\,H_2O$.	
			Series 2A.				
		gms.					
1	A	466.52	114.86	20.00	4.237	0.9634	
2	A	466.52	144.87	19.99	4.236	0.9637	
3	A	466.51	114.86	20.21	4.236	0.9628	Average. 0.9633
			Series 2B.				
1	B	466.54	114.87	20.31	4.233	0.9634	
2	B	466.55	114.86	19.75	4.237	0.9641	
3	B	466.50	114.86	20.08	4.239	0.9626	
4	B	466.51	114.86	20.50	4.230	0.9636	Average. 0.9634
Total av'ge		466.52	114.863	20.12	4.235	0.9634	0.9634

The mean value, then, for the specific heat of $HCl.100H_2O$ over the range between 15.9° and 20.1° is 0.9634 compared with water over the same range of temperature. This is equal to the results found in the preliminary series, within the range of the probable error (0.0001). It will be noted that the maximum difference from the mean is considerably less than 0.1 percent. The probable error of the seven determinations is only about 1 in the fourth decimal place. These determinations are typical of the experiments made with the other solutions. Because of the great similarity of the experiments to one another, it will not be necessary to record every experiment, but the average values similar to the total averages given on the last line of the table above will be given for each of the essential data, together with the number of experiments.

In the course of the experiments first performed, namely, all those on hydrochloric acid and sodium hydroxide, the original preparation of sulphuric acid was exhausted. This solution (designated A) contained, as has been already stated, 6.275 percent of sulphuric acid. For the remaining experiments it was necessary to prepare a new solution (designated B below) which was found to contain 6.283 percent of acid.[12] In order to make assurance doubly sure, this new solution was used in ten experiments with pure water in the calorimeter which are recorded at the foot of the table below. The slightly greater strength of the acid caused the average rise of temperature to be slightly greater in this last series than in the first. At 20.19° the original acid would have caused a rise of 4.229°, and with this new acid, 0.13 percent more concentrated, the rise was 4.234° — an exact proportionality. No more satisfactory evidence of the consistency of the work could have been desired.

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[12] In this connection a somewhat interesting demonstration of the accuracy of the method was obtained. The sulphuric acid was originally analyzed by titration with a sodium hydroxide solution which had been carefully standardized but which had stood for some months in a paraffin-lined bottle. The mean of the analyses thus conducted, gave the strength as 6.272 percent. When the heat evolved during the neutralization of $\frac{1}{2}$ molecule of acid was determined and compared with that shown by the previous lot of acid, the observed and calculated results showed a serious discrepancy. The strength of the new acid was calculated *from the thermal data* and found to be 6.282 percent. Investigation of the bottle containing the standard alkali showed a break in the paraffin lining which had permitted contact between the glass and the solution; hence the alkali had become slightly more alkaline; a fresh alkali solution was prepared and carefully standardized and by its use the acid was again analyzed. The result of these analyses is the figure given above, namely, 6.283 percent or within 0.02 percent of that calculation from the thermal data. It is clear, therefore, that the calorimetric process had been so effectively arranged as to constitute a very accurate process of quantitative acidimetric analysis.

Of course, in each case where acid *B* was used, the higher value must be used in the computation, whereas in each case where acid *A* was used, the lower value is the proper one.

In the following table each series of experiments used the average quantity 114.863 grams of sulphuric acid per experiment — at least the deviation from this average amount was smaller than the probable experimental error, so that the amount may be taken as a constant. In none of the individual experiments except in the first series with water (Table II), did the amount of acid exceed 114.87 grams or sink below 114.85 grams. The averages for the several series were much closer to 114.863 than this. The heat capacity of the solid apparatus also was constant, being always 12.60 grams except in the first four experiments with water already tabulated; and a constant amount of caustic alkali was used as always. The other quantities found in the experiments are given in the following table, the figures there being (as has been said) total averages similar to those given at the foot of the table just above. Because of the great similarity of the experiments, it does not seem worth while to give each one in detail. Anyone can calculate for himself the average results for the specific heats from the data given below. The only essential figures not immediately indicated by the table are the deviations among the individual experiments, but these are sufficiently shown by the "probable errors" given in the last column, calculated according to the theory of least squares. From this it will be seen that two of the series were somewhat less concordant than that given above, but most of them were fully as good as the sample series, or indeed even better. In another respect, also, perhaps, this table of averages does not quite do justice to the work:— it does not give a vivid representation of the very large expenditure of labor, extending over many months, which was needed to obtain the numerous results so concisely tabulated.

The main object in determining these specific heats was to provide a basis for computing the heats of neutralization of the various substances in question, but the values are nevertheless not without interest in themselves.

The significance of results of this sort are of course most easily perceived when the results are computed in terms of gram-molecules, and after this has been done, two sets of comparisons are possible: — in the first place, the effect of dilution upon the molecular heat of a given substance, and in the next place the effect of varying the ions concerned.

TABLE V.

DATA AND RESULTS OF DETERMINATION OF SPECIFIC HEATS.

	No. of Exp.	Sample of H_2SO_4.	Average Weight of Solution.	Average Final Temp.	Av'ge Rise of Temp. °C.	Sp. Heat (Range about 16-20.2°.)	Probable Error.
H_2O at 19.77°	5[13]	A	450.18	19.77°	4.234	Standard	
H_2O at 20.39°	4[14]	A	450.18	20.39	4.227	Ch'k exp.	
H_2O at 20.72°	3[15]	A	450.18	20.72°	4.225	Standard	
$HCl\ 200H_2O$	8	A	458.99	20.24	4.226	0.9814	±0.0002
$HCl\ 100H_2O$	7	A	466.52	20.12	4.235	0.9634	±0.0001
$HBr\ 100H_2O$	4	B	471.18	20.29	4.276	0.9433	±0.0000
$HI\ 100H_2O$	6	B	480.20	20.41	4.2885	0.9213	±0.0001
$HNO_3\ 100H_2O$	7	B	467.45	20.31	4.249	0.9584	±0.0002
$HClO_4\ 100H_2O$	4	B	471.17	20.34	4.283	0.9466	±0.0001
$KOH\ 100H_2O$	4	B	471.18	20.04	4.232	0.9568	±0.0001
$NaOH\ 100H_2O$	6	A	465.13	20.29	4.233	0.9664	±0.0001
$NaOH\ 200H_2O$	7	A	457.97	20.08	4.231	0.9827	±0.0001
$LiOH\ 100H_2O$	5	B	461.19	20.34	4.2154	0.9813	±0.0001
H_2O at 20.19°	10	B	450.18	20.19	4.234	Standard	

Let us consider first the effect of diluting the hydrochloric acid. The solution $HCl.100\,H_2O$ weighs per gram molecule of hydrochloric acid 36.46 + 1801.5 grams. With a specific heat 0.9634 this gives a molecular heat 1770.6. When 100 gram molecules of water have been added to this solution, it then has a total molecular weight 3639.5 and a molecular heat capacity of 3571.8 calculated from its specific heat 0.9814. The difference between the total molecular heat capacities of these two solutions is 1801.2 or almost the same as the heat capacity 1801.5 of the added water. The slight decrease is no larger than the possible experimental error; but it is probable that the dilute solution really does suffer a small diminution of about this order.

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[13] See Table II.
[14] See Table III.
[15] See Table II.

Carrying out the same calculation for sodium hydroxide we find the molecular heat capacity for NaOH.100 H_2O, 1780, and that for NaOH.200 H_2O to be 3580. The difference here again is slightly less than the heat capacity of the added water, being 1800, instead of 1801.5 — a difference of almost 0.1 percent.

Evidently solutions of strong bases and acids containing as much as 100 molecules of water (about half normal in equivalent concentration) are already so dilute that most of the added water is unaffected,— or at least, such small changes as the water suffers must be balanced by contrary effects. Thus the specific heat of any more dilute solution may be calculated with considerable precision from the data contained in this paper.

Turning now to the comparison of the different substances, all in solutions containing 100 molecules of water for each molecule of dissolved substance, we find other interesting and significant relations. The following table presents the molecular heats of all the substances for comparison:—

TABLE XIV.

The Molecular Heats of the Acid and Alkali Solutions of the Type MX. 100 H_2O

Solution.	Molecular Weight.	Specific Heat.	Molecular Heat.
HCl	1801.5 + 36.46	0.9634	1771
HBr	" + 80.92	.9433	1776
HI	" + 128.00	.9213	1778
HNO^3	" + 63.02	.9584	1787
$HClO^4$	" + 100.46	.9466	1801
LiOH	" + 23.95	.9813	1791
NaOH	" + 40.00	.9664	1780
KOH	" + 56.10	.9568	1777

From this table it is clear that the molecular heat increases with the molecular weight in the case of the halogen acids. The further increase in the case of nitric and perchloric acid is only to be expected from the increasing size and complexity of these molecules. On the other hand it is noticeable that in the case of the alkalies the molecular heats diminish as the atomic weight of the metal increases.

Of course many circumstances lie at the basis of these phenomena; the degree of ionization, the degree of hydration of both the ions and of the undissociated substance; and indeed all those other as yet undetermined factors which together produce the specific volumes and all the other properties of solutions come into play here also. Whatever may be the ultimate interpretation of these matters, it is clear that the data just given must receive consideration in the final estimate of the situation.

It is a pleasure to express our indebtedness for the generous subsidy of the Carnegie Institution of Washington, upon which we have depended for the purchase of the apparatus and chemicals needed in this investigation.

SUMMARY.

In this communication an improved apparatus for the determination of the specific heats of solutions or other liquids has been described. The method depends upon the liberation of definite quantities of heat by the neutralization of definite quantities of pure sulphuric acid.

The apparatus is capable of giving results of great accuracy. The individual deviations are such that with five or six experiments the "probable error" of the average specific heat is usually less than 0.0001. Because of this high degree of accuracy, it is evident that the mercury thermometers used are an adequate means of measuring temperature for this purpose.

With this apparatus the specific heats of dilute solutions of hydrochloric, hydrobromic, hydriodic, nitric, and perchloric acids, as well as of lithium, sodium, and potassium hydroxides have been determined.

From these results the molecular heats of the solutions have been calculated, and relations between them pointed out. It is shown that the molecular heats vary in a systematic manner.

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PETROLOGY OF THE ALKALI-GRANITES AND PORPHYRIES OF QUINCY AND THE BLUE HILLS, MASS., U. S. A.

By Charles H. Warren.

With Two Plates.

PETROLOGY OF THE ALKALI-GRANITES AND PORPHYRIES OF QUINCY AND THE BLUE HILLS, MASS., U. S. A.

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Received May 22, 1913.

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Part I.

The most prominent topographic feature in Eastern Massachusetts is the range of hills, known as the Blue Hills, lying a few miles south and southeast of Boston, chiefly in the towns of Quincy and Milton. The range, which forms the southern rim of what is known as the Boston Basin, begins on the west in a beautifully rounded hill, The

Great Blue Hill, with an altitude of about 600 feet (200 meters) and extends with gradually diminishing elevations, in an easterly direction, with a gentle bow to the south, as a series of rounded hills which die out as the shores of Quincy and Hingham bays are approached.

The Blue Hills proper, or that portion which lies between the Great Blue Hill and the last prominent hills near West Quincy, now form a public park, the Blue Hills Reservation, which ranks as one of the most beautiful parks in the neighborhood of any large city. Although traversed by convenient roads and foot-paths, the natural wild and wooded character of the hills has been perfectly preserved. From the higher points pleasing and extensive panoramas of the surrounding country may be seen on the north south and west, while to the east and northeast, the ocean and its numerous bays form a distant background in a view of great beauty.

With the exception of relatively small areas of Cambrian slate and a few diabase dikes, the abundant rock exposures over this area consist entirely of an alkali-hornblende-aegirite-granite or closely related porphyries, and it is with the petrology of these rocks that the present paper is concerned.

Previous work on the geology of the region.— In an extended memoir entitled "The Blue Hill Complex," Professor W. O. Crosby [1] has furnished us with a very detailed and valuable discussion of the geology of this area. His description of the rocks was essentially only a megascopic one, inasmuch as he had at his command comparatively little microscopic or chemical data, and that which he had appears to have been imperfect and to some extent misleading. Up to the present time the only petrographic work which has appeared describing the rocks are:—brief notes descriptive of the granite by G. N. Hawes [2], M. E. Wadsworth,[3] G. P. Merrill;[4] an imperfect description of the granite and porphyries, by Dr. T. G. White [5]; a brief description of the Quincy granite accompanied by an excellent analysis by H. S. Washington [6]; and a rather detailed description of the granite from the more important quarries of the Quincy district by T. Nelson Dale.[7] The last mentioned paper contains, besides the descriptions of the granite from various quarries, quantitative estimates of the mineral composition as well as many interesting and valuable statements regarding the grain, joint structures, and other features relating to the economic aspects of the granite and quarries.

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[1] Occasional Papers, Boston Soc. Nat. Hist., **4**, 19, (1895).

[2] Tenth Census U. S., **10**, p. 18.

[3] Descriptive Cat. of American & Foreign Rocks, Boston, No. 71 (1883).

[4] Building and Ornamental Stones. Report U. S. Nat. Mus., p. 409 (1886).

[5] Notes on the petrography of the Boston Basin. Boston Soc. Nat. Hist. **28**, No. 6 (1897).

[6] American Journal of Science, **156**, p. 181 (1898).

[7] Bull. No. 354, U. S. G. S. (1908).

A recent paper by G. F. Loughlin [8] discusses the geology of the area particularly with reference to the probable relations of the intrusive rocks to the associated sedimentary formations.

The pegmatites occurring in two of the quarries on North Common Hill, Quincy, have been described together with their minerals by Charles Palache and the writer.[9]

During the earlier part of the writer's work on these rocks he had the assistance of Dr. J. D. Trueman who was at that time pursuing studies leading to the degree of Master of Science at the Massachusetts Institute of Technology, and whose recent lamentable death by drowning while working for the Canadian Geological Survey has deprived geology of one of its most promising and enthusiastic workers. The preliminary results obtained by Dr. Trueman were embodied in his Master's thesis and the writer wishes to acknowledge his indebtedness to Dr. Trueman for much careful and discerning field and laboratory work.

Almost the entire field has been gone over with great care by the writer in person, and in this task he has been greatly helped by the exceedingly full descriptions contained in Professor Crosby's memoir, and has had the further advantage of the latter's company on several field excursions and of his keen interest in the work throughout.

The author wishes here to express his thanks to the Metropolitan Park Commission for permission to collect specimens within the Blue Hill Reservation.

Summary of the geology of the area.— Those wishing to inform themselves regarding the general geologic features of the area are referred to the paper by G. F. Loughlin, previously mentioned, or in case more detailed information is desired, the memoir of Professor Crosby should be consulted. It will be sufficient here to give only a very brief summary of the geology.

The alkaline rock series, as it may be called, occupies a roughly elliptical area having a nearly east-west major axis of about 9 miles (15 km.) in length, and a minor axis of from two to three miles (3 to 5 km.). Its eastern end lies near the Weymouth-Fore river.

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[8] American Journal of Science, **32**, (July, 1911).
[9] These Proceedings, **47**, No. 4 (July, 1911).

Thence it extends to its abrupt termination at the Neponset Valley along the western base of the Great Blue Hill. The Fore River as well as the Neponset Valley probably mark great north and south faults. On the north, along a line crossing northern Quincy and southern Milton, the alkaline rocks are bounded by an east-west fault contact with the carboniferous sediments of the Boston Basin. At the eastern end on the southern side, in northern Weymouth, the alkaline rocks are probably in fault contact (not actually exposed) with an essentially sub-alkaline granite, then for a short distance in fault contact with Cambrian slates or granite through northern Braintree. Further west they are in conformable contact with the coarse (basal) carboniferous conglomerate of the Norfolk Basin. On all sides, then, the boundaries are practically great major faults, and the alkaline rocks comprise essentially a great fault block, or more correctly two, an eastern and western member, and these appear to have been elevated, particularly the western and larger block, which was also tilted up at the north more sharply than the eastern member.[10] The eastern block is crossed by numerous minor, chiefly north and south, faults. These as well as the major boundary faults, and their dissection of the area, have been most fully and carefully worked out and described by Crosby.

The alkaline rocks are intrusive into Cambrian (certainly as late as middle Cambrian) sediments, consisting essentially of slate with quartzitic and limy bands. They are certainly earlier than the adjoining sediments in the Norfolk Basin, an arm of the Narragansett Basin, which are carboniferous, but at present their exact age cannot be stated. In the immediate neighborhood of the granite, the slates have been indurated and somewhat metamorphosed, but extreme metamorphism is not, it is important to note, a characteristic of such of the sediments as now remain.

Although all of the rocks of the series were undoubtedly intruded during a single great period of intrusion, one member, the aporhyolite, has been held by Crosby to be the youngest of the alkaline-rocks. As will be shown beyond, there is reason to believe that it was earlier than the other rocks of the series.

With the exception of the large number of small dikelets of microgranite that are found cutting the slate at the contacts, and of which Professor Crosby has given us a very detailed description, of the dikes of granite cutting the slate in the Pine Tree Brook Reservation and the porphyry in the region about Chickatawbut Hill, of one or two narrow, fine grained granitic stringers cutting the granite elsewhere near its contact or passage into the contact porphyry, and of the small dikes of pegmatite found at one or two points in the area (also near the porphyry cover), dikes, genetically connected with the alkaline magma, are conspicuous by their absence. The complementary dikes formed by differentiation, which are so prominent a characteristic of many intrusions of alkaline rocks, are here entirely absent. This peculiarity is believed to be due in part, as will be pointed out later, to the chemical composition of the magma, and in part to the consolidation of the magma relatively near the surface.

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[10] See Crosby, loc. cit., p. 534 et seq.

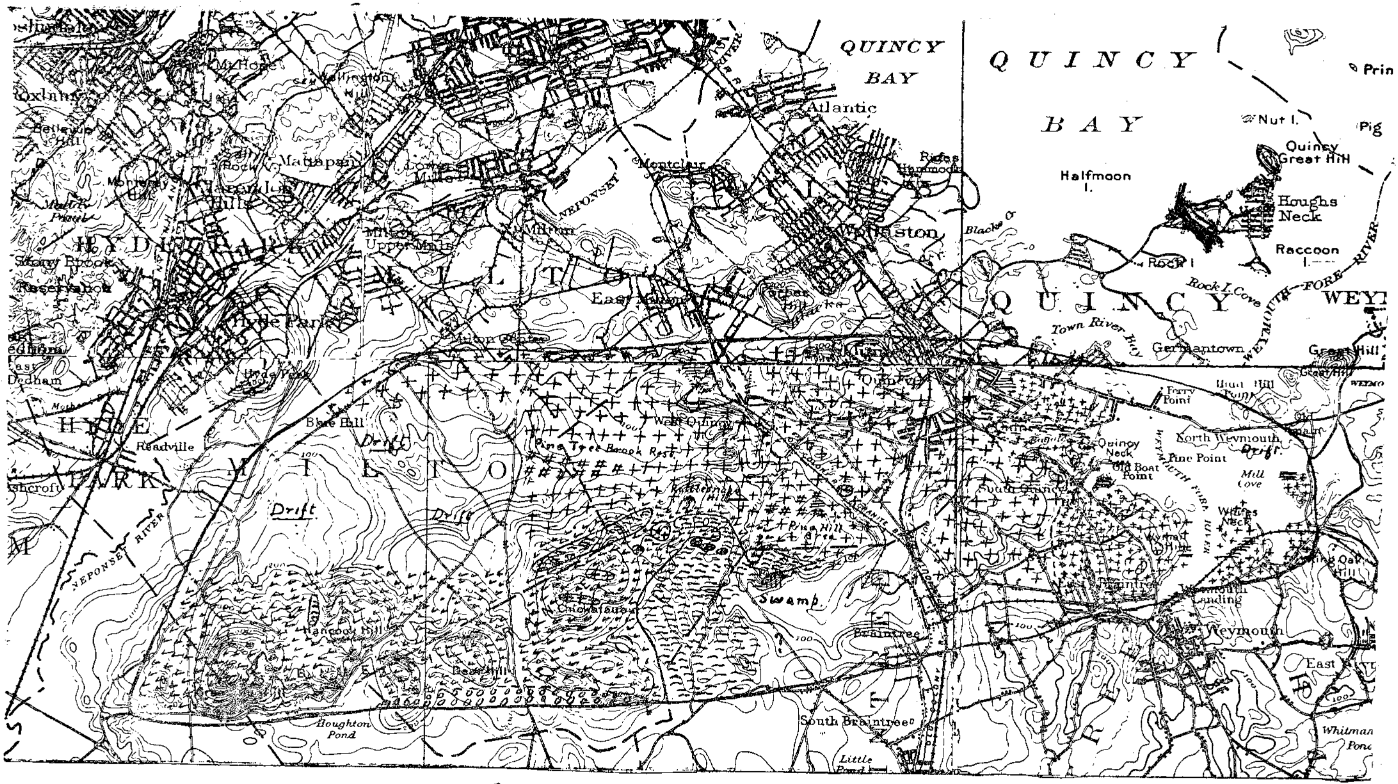
The alkaline rocks as well as the slates have been cut by a later series of basic dikes. These are for the most part heavily altered but appear to have been all essentially diabasic in character and certainly bear no near relation to the alkaline rocks. Neither these nor an older, pre-granitic series of trap dikes cutting the slates in the northern part of the area will be described here.

The entire area, like all of Eastern Massachusetts, has been exposed to erosion since the close of the Appalachian revolution. This erosion has removed all of the carboniferous strata and nearly all of the invaded cambrian or older sediments, together with a great thickness of the upper portions of the alkaline rocks, particularly over the northern and once more elevated part. Much of the original porphyry cover over the southern and western part of the area remains and these rocks now make up practically the whole of the Blue Hills proper.

Besides its alkaline character, somewhat peculiar chemical and mineralogical characters, a leading characteristic of this intrusion is that it consolidated under conditions which resulted in the formation of a thick protecting cover of porphyritic or even glassy rocks, which differ from those beneath chiefly in texture: differentiation did not take place to any great extent and erosion has left the original igneous cover to a considerable extent unimpared for observation and study.

Rock Types. — The rocks of this series are all characterized by the presence of soda-potash feldspars either in the form of a homogeneous mixture (truemixed crystal?), cryptoperthite or microperthite: by the presence of either alkali-hornblends or pyroxenes or both, and, with the exception of one member, by the presence of abundant quartz. They may be divided into the following types: —

I — (a) Medium to coarse grained, riebeckite-aegirite-microperthite-granite (Quincy type): (b) the same, but with an inconspicuous porphyritic habit. (Rattlesnake Hill type.)


QUINCY
BAY
QUINCY
BAY
Prin
Nut I.
Pig
Quincy Great Hill
Halfmoon I.
Houghs Neck
Raccoon I.
Rock I
Rock I. Cove
FORE RIVER
WEYMOUTH
WEY
QUINCY
Town River Bay
Germantown
Great Hill
Atlantic
Riggs
Squantum
Wollaston
Montclair
Black's Cr
NEPONSET
Milton
Milton Upper Mills
Lower Mills
Mattapan
Mt Hope
Wellington Hill
Roxbury
Bellevue Hill
Mother Brook
Stony Brook Reservation
HYDE PARK
Hyde Park
Readville
Dedham
East Dedham
Ashcroft
NEPONSET RIVER
MILTON
Blue Hill
Drift
Drift
Drift
Hancock Hill
Houghton Pond
Chickatawbut
Pine Tree Brook Rest
West Quincy
Quincy
Rattlesnake Hill
Pine Hill
Swamp
Braintree
South Braintree
Little Pond
Ferry Point
Hunt Hill
Quincy Neck
Old Boat Point
Pine Point
North Weymouth
Drift
Mill Cove
WEYMOUTH FORE RIVER
Wessagusset
Weymouth Landing
Weymouth
East Weymouth
Whitman Pond
Kings Oak Hill
100
200

Scale about 1 inch = 1.5 miles.

II — Fine grained granite similar in mineral composition to I, but predominately riebeckitic and of a little more basic composition.

III — The Blue Hill Porphyries: riebeckite, aegrite bearing, quartz-feldspar or granite porphyries.

IV — Dark, alkali-feldspar- or rhombenporphyry.

V — Cognate Xenolithes occurring in I and III. These are for the most part fine grained varieties usually porphyritic and more basic than the granite.

VI — Aporhyolite.

To these may be added, fine-granite and pegmatites, both of rare occurrence.

I.— THE COARSE-GRANITE.

Distribution.— Reference to the map (No. I) will show that the granite, so far as its surface exposure is concerned, occupies somewhat less than one-half of the exposed area and if we assume that the extensive areas in the western section now covered with drift are underlain by granite, which is probably the case, then the porphyries

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GENERAL MAP OF THE QUINCY-BLUE HILL AREA. NO. I.

+++ Coarse Granite.
++++ Fine Granite.
▤ Quartz-Feldspar-& Granite-Porphyry.
Rhombenporphyry - Slate Areas.
≈ Aporhyolite.
≡ Slate (Cambrian).
o o o Conglomerate (Carboniferous.

This map is based on the regular Topographic map of the U. S. Geological Survey, the only available map of the area as a whole. Although not as good as could be desired it will serve to show the general location of the area in question and the approximate distribution of the principal rock types. To show their exact occurrence on a suitable scale would demand an expenditure of time and money that are prohibitive and would serve no very useful purpose.

and fine-granite would hardly cover over one-half of the batholith as at present exposed. The granite occupies a belt stretching across southern Milton on the north of the Blue Hills, comprises most of the rocks in Quincy and West Quincy, but is replaced by the fine-granite facies in eastern and southern Quincy and part of northern Weymouth. In fact the coarse-granite can hardly be said to occur in the Blue Hills proper except at one point (near Rattlesnake Hills), and in the form of dikes, cutting the porphyry cover, in the region just east of Chickatawbut Hill. As a matter of fact, its southern projections at Rattlesnake Hill and on Pine Hill, in both of which places it comes in contact with the porphyry, show a distinct tendency toward a porphyritic texture thus grading toward the porphyry.

Crosby states [11] that normal granite is transitional into "quartz porphyry and fine-granite" in the vicinity of Slide Notch. The writer has examined this section with extreme care and while unable to find any strictly normal granite having certainly the relations ascribed to it by Crosby, has found several well marked, granite dikes essentially of the Quincy type. One of these was first encountered about 75 ft. south of the extreme top of Chickatawbut Hill. It is here about 20 ft. wide, apparently nearly vertical in dip, and strikes about S. 60 E. and reaches a width of certainly 30 ft. It has been traced across the eastern member of Chickatawbut as far as the steep slopes of Slide Notch near its lower end. A second dike, nearly 50 ft. wide at one point, having about the same dip and strike as the first, outcrops at the entrance of the Notch. It has been traced for at least 400 ft. On the western side of the Notch, it seems to flatten in dip and there is also a smaller dike, probably an offshoot of the main one, a few feet to the south. It is to be noted that these dikes show in places along their contact with the granite-porphyry a slight development of very fine graphic-granite, and that they also contain numerous inclusions or segregations of dark porphyry and fine-granite exactly similar to those found in the main granite. Again near the head of Scamaug Notch, relatively coarse granite outcrops in the form of two dikes. While it is clear that at least part of the granite exposed at this point is a dike intrusion, another part of the granite here does not exhibit clearly marked contacts and is probably, as believed by Professor Crosby, an exposure of the underlying granite. The character of the granite-porphyry which seems to grade into the granite in one of its outcrops supports this view.

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[11] Op. cit., p. 366.

The contact relations of the granite and the associated rocks will be discussed later after these have been described.

The appearance and general characteristics of this granite are probably as well known as those of any rock in the country, owing to its wide use as a building and ornamental stone and to the descriptions already made of it by Dale and the other authors previously mentioned. In fact the present description is undertaken partly for the sake of bringing into one place the descriptions of all the alkaline rocks of the area, and partly for the purpose of adding certain mineralogical and chemical data of interest. No attempt will be made here to describe several minor variations in the granite due to local alteration. For these the paper by Dale should be consulted, as well as for many interesting details, relating to the rift, grain and joint structure of the granite.

Petrographic characters — Megascopic.— The normal type is a holocrystalline, coarse, equigranular rock (4 to 8 mm.); prevailingly gray— light to dark gray, bluish or greenish gray; where altered pinkish, reddish or purplish and greenish. The minerals are: — alkali feldspar, colors as above; quartz, clear glassy to dark smoky, less commonly bluish and opalescent; black, lustrous and beautifully cleavable hornblende in prominent, irregular spots, usually intergrown, particularly about the margins, with light to dark green aegirite; occasional separate grains of dark green aegirite; very rarely small purplish spots of fluorite and small brown zircon crystals. Occasionally the hornblende-aegirite spots which have suffered alteration are replaced by a soft brownish or yellowish clay-like material. The rock is remarkable for the beauty of its polished surfaces, particularly its darker varieties.

In places where the granite approaches the granite-porphyry, its marginal phase, the rock is not quite so coarse in grain and the equigranular texture gives place to a distinct but not prominent, granophyric one. (Rattlesnake Hill type.)

In the medium gray type of granite, which is by far the most abundant type, occur irregular and ill-defined darker streaks and cloud-like masses which grade more or less insensibly into the other. The darkest phase of the granite is known under the name of "black granite" and is the most highly prized variety from the commercial point of view. As will be pointed out later, these dark portions of the granite appear to owe their darker color to a greater abundance of the minute crystallizations of riebeckite and magnetite in the feldspars. While this difference may be due in small measure to slight differences in original composition, it is believed to result from a deep-seated differential alteration of the granite mass. In the same way there are streaks and patches of much lighter color than the average. This phase shows many aegirite microliths in the feldspar but little riebeckite. In some of the quarries (Faulkner's, North Common Hill) a rather poorly defined zone of very light granite was observed on either side of a small quartz vein, and there appeared to be some connection between the vein and the whitening of the adjacent rock.

The pink and reddish granites (see later) are the result, for the most part, of surface oxidation of the iron content of the hornblende, etc. Along shear zones and in the neighborhood of trap dikes the granite has often a dark, greenish color due to the presence of streaks, disseminated scales and masses of some dull green, secondary mineral, probably chloritic, but whose composition has not been more closely investigated.

Microscopic.— The minerals observed in thin sections, are: an albite-microcline microperthite, quartz, soda-iron amphiboles, in part riebeckitic, in part cataphoritic, aegirite and sometimes a little of some closely related pyroxene; accessory minerals — aenigmatite, astrophyllite, zircon, titanite, iron oxide minerals, fluorite and very minor amounts of various alteration products.

The feldspar content of the rock is normally almost entirely a microperthite. Its grains are roughly equidimensional in cross-section; their elongation is in the direction of the edge 001/010. The outlines are always xenomorphic, except that occasionally in contact with the other minerals, particularly quartz, they show some development of crystal planes. It, nevertheless, clearly dominates the texture of the rock. The two members of the microperthite are very finely intergrown following the well known law for such intergrowths. The relative amounts of the two feldspars present in different grains is probably pretty uniform, although considerable apparent variations may be noted in the sections of different crystals and even in the same one. From point to point in a crystal there is undoubtedly considerable variation. The plagioclase member, as judged by its optical properties, is very near albite and this is borne out by the chemical evidence derived from the rock analyses. Its most probable composition appears to be $Ab_{98}\,An_2$ to $Ab_{95}\,An_5$. It is commonly, though not always, very finely twinned after the albite law. Locally in crystal sections where it predominates over the microcline, the twinning lamellae are broader and more uniform. A commonly noticed feature of the albite is its predominance about the ends and margin of the crystal. It often extends out with the same orientation in the form of rather sharply bounded projections into the adjoining microperthite grains. When developed along the sides, it not infrequently sends out little prongs and hooks into the adjoining crystal much in the manner described by Prisson [12] for the albite in the perthite of the Red Hill, New Hampshire nephelite-syenite. The microcline member is twinned almost exclusively after the albite law only, and therefore, lacks the characteristic "gitter" structure usually associated with that mineral. The twinning lamellae are relatively shorter and broader than those of the albite member and their boundaries lack sharpness in most cases. The twinning in the microcline is not uniform, even in a single individual, and considerable portions may show little or no twinning. Its optical properties indicate that it is a nearly pure microcline and not a soda-rich variety. The intergrowth as it stands is apparently a mixture of nearly pure albite and potash feldspar.

A small amount of albite occurs in the rock in the form of separately crystallized grains occurring along the sides of the larger crystals, or in the interstices between them. There is often a suggestion, however, that many of these were originally continuous growths with the albite of the adjoining microperthite crystals, and that they have subsequently been deorientated by a slight movement in the mass.

The relations of the two feldspars detailed above is the normal relation seen without substantial variation in a great number of thin sections from the granites of the quarries. In the granite from various parts of the area and particularly from certain quarries, considerable variations from this texture, however, may be seen. The microperthite frequently contains random crystals of albite, often in considerable abundance. These are usually very small and have a slightly elongate, prismoid habit but do not possess sharp terminations. Again the microperthite may contain much larger, curiously irregular grains of albite, often including perthitically intergrown microcline. A number of these albites, much interdented along their contacts, may replace a good portion of an original feldspar crystal. Still again the microperthite may be replaced in part or almost entirely by aggregates of elongated, parallelly arranged, mutually interdented crystals of albite. Some potash feldspar may usually be noted intergrown with the albite. These crystals are often obviously arranged with their longer axes parallel to the original direction of perthitic intergrowth, but not always. They appear sometimes in the center of a microperthite crystal and again encroach on it from the margin. In many cases where the parallel groupings of albite cut across the original perthite strips, their direction of elongation and encroachment has been clearly determined by the structure of an adjoining microperthite grain. This may be in fact always the case, even when not apparent. This replacing feldspar may show simple albite twinning but much of it is untwinned. It is also practically free from the cavities and minute inclusions characteristic of the original perthite; secondary riebeckite and aegirite microliths are, however, quite common in it. In sections from certain portions of the granite in several localities, and particularly from the granite of Cashman's and the Gold Leaf Quarries, not only may all of the above described replacements of the microperthite be seen, but much of the original feldspar is seen to be replaced entirely by a very fine mosaic of feldspar grains among which albite seems to clearly predominate in amount. These mosaics are in part, at least, quite clearly the result of a granulation of the feldspar replacing the original microperthite, although probably the replacement and granulation were nearly or quite contemporaneous processes. All gradations from the normal microperthite crystal up to nearly or quite complete replacement and granulation may be easily traced. The process seems to be clearly one of recrystallization and albitization of the original feldspar which took place, either during the last stages of the solidification of the magma while the last liquids or gases were still very active, and before all movement in the crystallizing mass had ceased — a protoclastic structure —, or during some later recurrence of mineralizing activity and movement incident to a period of dynamic or igneous disturbance through which the region has passed. In either case there seems to have been a considerable increase in the relative amount of albite, at least locally. The first alternative as to origin appeals most strongly to the writer, especially when taken in connection with a closely similar change to be described later, as occurring in the feldspars of the associated porphyries.

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[12] American Journal of Science, **23**, p. 272.

With low magnifying powers the microperthite possesses a more or less dusty appearance and in the darker varieties of the granite, in rather thick sections, it is rendered almost opaque by reason of the very abundant, minute particles which are scattered through it. These included particles are: — 1st.— microliths of soda-hornblende, and aegirite; 2nd — exceedingly minute black specks and crystals (iron oxides); 3rd — minute indeterminate particles and cavities, the latter often more or less filled with brownish or black material.

The total amount of these particles varies widely in different specimens, in different feldspars in the same rock and even in the same feldspar crystal. It may be said, however, that they are most abundant in the granite having a darker color and are undoubtedly largely responsible for that color. The hornblende and aegirite microliths occur together in varying proportions, or one may occur almost to the exclusion of the other. No regularity can be discovered in this variation. The microliths of both show a tendency to an arrangement parallel to the direction of perthite intergrowth, to the direction of the albite twinning plane, and to the cleavage directions. They also occur commonly quite at random. The hornblendes are chiefly of a deep blue color — riebeckitic — although in sections, showing the green, cataphoric variety of hornblende in larger crystals, the microliths have also a corresponding deep-green color. Their habit is either that of short prismoid grains or more commonly of much elongated prisms or even hair-like growths. They sometimes form radiating clusters. The great majority are very minute and the largest rarely exceed 0.02 mm. in breadth or length except the more hair-like forms which attain a greater length. The aegirite microliths vary from the most minute particles up to ones 0.01 or 0.02 mm. in their greatest dimension, which is ordinarily the direction of the vertical axis. They commonly show a curious tendency to arrange themselves end to end, with minute swellings and irregularities along their sides. In color they are pale yellowish green or yellow to almost colorless. a is always nearly parallel to the vertical elongation as it should be in aegirite, a characteristic which obviously excludes their being epidote, a mineral they resemble in general appearance and which they have apparently been mistaken for by Dale. The minute black specks and grains, presumably iron oxides, the indeterminate particles, probably sericitic material or kaolin, and the cavities, show a great preference for the microcline member of the microperthite in which they are astonishingly abundant. It has been frequently noted in feldspars in which the black particles, etc., are more than usually plentiful, that they are concentrated a short distance on either side of cleavage cracks now healed with fresh albite material, also about the later albite crystallizations mentioned above, but never in them. The cavities seldom exceed a few thousandths of a millimeter in their greatest dimension. They are round, ellipsoidal or irregular in form, and with the exception of a little brownish black or reddish material appear to have no filling. Although one doubtless gets a somewhat exaggerated idea of their total bulk from microscopic study, it is difficult to escape the thought that they must affect somewhat the density of the rock, and they must certainly render it more susceptible to the action of chemical alteration.

The quartz is throughout highly xenomorphic in its outlines. The undulatory and broken extinction in most of its grains show that all of it has been subjected to strain. Much of it, particularly in sections which show granulation and secondary recrystallization of the microperthite, is broken and in extreme cases (granite from the Gold-Leaf quarry) it has been reduced to a mosaic of small grains. All of the quartz contains abundant cavities, the majority of the large ones containing liquid with a movable bubble. They are usually arranged in gently curving lines across the quartz and in many cases obviously mark the direction of an old resealed fracture.[13] Minute particles of iron oxides are closely associated with the cavities. The quartz also includes rather rarely small riebeckites, but this is generally, if not always, along fractured or broken zones. The quartz commonly includes, and is intergrown with, aegirite; the same is also true of zircon, although the latter is of course much less abundant than the aegirite.

The hornblende, always with more or less aegirite, forms irregular patches of approximately the same area as the feldspar grains. Two or more crystals are often grouped together. Its crystals are usually broad with a tendency toward prismatic elongation. Toward the quartz it often develops its prism zone, but these are marked by many projecting points and irregularities. Although it often penetrates the borders of feldspar it is also found wrapped about the end of the microperthite grains, and seems generally to have been controlled as to its external form by the more abundant and dominant mineral. Along its prism zone where seen in contact with feldspar and quartz it shows a highly ragged contact surface. Even when low magnifying powers show a fairly well marked line, higher powers resolve the contact surface into a series of irregular projections and indentations. The hornblende is often separated to a greater or less extent from the other minerals by a growth of aegirite. Although the aegirite seems to have attached itself occasionally to an original crystal surface of the hornblende, it is usually found interpenetrated with the latter, the vertical axes of the two being parallel. Outwardly the aegirite develops to the exclusion of the hornblende, particularly about the ends where it is usually seen in greatest abundance. Although the marginal aegirite may form a continuous mass about the hornblende, it more often consists of a number or many individual crystals which may be parallel, slightly divergent, or subradiate in arrangement, and these project out into the feldspar or quartz most irregularly and are often accompanied by numerous semi- or wholly detached particles. It is also not at all uncommon to find inclusions of aegirite lying unorientated in the hornblende. While these latter favor the margins they often lie well within the hornblende substance. Many of the original hornblende grains have evidently been broken apart before crystallization had ceased in the magma, for the separated parts of what was once obviously a continuous crystal may be seen with its broken ends margined with aegirite exactly in the same manner as the natural ends. It is not unusual to find a hornblende grain accompanied by a development of hornblende prisms, sometimes with a divergent arrangement, which project out unto the adjoining minerals: again the hornblende area may consist of a mass of prismoid crystals more or less grouped and of various orientations accompanied by other materials — grains of iron oxide (chiefly magnetite) titanite, calcite, granular feldspar and even fluorite. Such groups appear to be in large part at least recrystallizations. In such cases the aegirite originally present seems to have suffered little or no change. This hornblende is always of the deep blue type (riebeckitic). The minerals mentioned as accompanying the secondary hornblende can be seen to follow in the arrangement of their grains, to some extent at least, the structure of the original mineral. Perhaps the titanite is the only one of these calling for any special notice. Where present its amount may vary from very little to an amount that, in extreme cases, may constitute perhaps one fifth of the whole area. It consists of grains or aggregates of grains whose outward form seems determined by the structure of the original hornblende, or of those minerals which developed simultaneously with it. It thus lacks entirely the habit usually associated with titanite. Its optical properties serve, however, to prove its identity. It seems to be secondary and doubtless derived its titanium from the titantium content of the original hornblende, or in cases where the amount of titanite appears to have been much too large for the amount of titanium present in the hornblende (probably not in excess of 1.5%; see anal. of hornblende from pegmatite — Warren and Palache, loc. cit., p. 124), from included aenigmatite or ilmenite. The lime cannot have come from the hornblende and the presence of calcite with the titanite is suggestive that this constituent was introduced from outside. In fact these areas seem to represent a phase in the alteration of the original hornblende which gave rise to secondary riebeckite accompanied by magnetite and other minerals, in part introduced from the surrounding rock, among these titanite, in cases where the amount of titanium was adequate. In this connection it is interesting to note that titanite having somewhat similar characteristics and accompanied by chlorite and calcite, has been observed by Pirsson and Washington as a secondary mineral replacing hornblende (basaltic) in the fine grained camptonites from the Belknap Mountains of New Hampshire.[14]

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[13] Compare with description of the pegmatite quartz where more details are given. Warren and Palache, These Proceedings, **47**, No. 4, (July, 1911).

These altered hornblendes have doubtless given rise to the small pits and yellowish decayed spots seen megascopically in the finished surfaces of the quarry granites.

The inclusions in the hornblende in addition to the aegirite are: fluorite, in minute, usually anhedral grains; black oxide specks and grains, probably magnetite, scattered for the most part, but also arranged in curiously curved strings or bands, usually traversing only parts of the crystal and then commonly on one side or about a portion of the margin; larger, black grains of iron oxide; and a dark red mineral, probably aenigmatite (see beyond).

It is probably safe to say that the predominating hornblende is a riebeckite. This is certainly true of the granite in the eastern part of the quarry district (North Common Hill and eastward in Quincy). In many slides from the granite of this district it is the only variety of hornblende present. In the West Quincy district and westward in Milton the riebeckite is in considerable part replaced by a hornblende which is apparently a cataphorite or a closely related variety. In many sections this variety is seemingly the only original hornblende present, although it is difficult or impossible in random sections to distinguish always surely between the two. What is here called riebeckite has substantially the same optical properties as those given for the hornblende of the pegmatites previously described by the writer and C. Palache.[15] These are: — $\alpha \wedge c'$ 4° to 5°; Axial plane perpendicular to b, (010); Opt.—, Bisectrix acute $= \alpha$; Axial angle medium to large. Dispersion strong, giving rise to colored axial bars (red and blue). Double refraction very low: pleochrism with low or medium magnifying powers: α, deep-blue to smoky-blue or green; γ, very dark, smoky-green to almost black; β, pale yellow or slightly brownish often with a greenish shade. Absorption; $\gamma < \alpha > \beta$. Sections over 0.03 mm. in thickness are practically opaque for the deeper rays. Sections intermediate in position between the pinacoids often show peculiar dull, bluish-gray or drab tones difficult to describe. To the writer the most characteristic thing about the appearance of this hornblende is the color for the ray very near the vertical axis and the ray vibrating across the cleavage direction in (010) sections. With high powers it can generally be seen that the distribution of color for the α ray is not uniform. The strongest and purest blue appears in the outer parts of the crystal, along cleavage or other cracks, or in thin lamellae lying parallel to the c' axis. The remainder of the crystal is green but assumes a bluish shade as the purer blue parts are approached. But very slight, if any, non-homogeneity can be detected for the other two rays.

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[14] American Journal of Science, **22**, p. 503 (Nov.–Dec., 1906).
[15] loc. cit., pp. 152–3.

The other variety of hornblende present, particularly in the granite from the western part of the granite area, where it appears to be nearly always relatively abundant sometimes almost to the exclusion of the blue type, seems to be closely related to the cataphorites of Brögger and particularly to the sodic hornblende described by Pirsson [16] as occurring in the nephelite syenite of Red Hill, New Hampshire. The pleochrism is strong and somewhat variable: α, light yellow-brown often with a greenish tint; β, dull green to almost black; γ, deep olive-green or less commonly olive-brown. The absorption is very strong, $\beta > \gamma > \alpha$. The angle, $\gamma \wedge c'$ is large and is variable even in crystallographically continuous crystals. This angle has been observed to vary in a single grain from 20° to 32°, and these figures represent variations commonly observed in the run of sections, although extinctions as high as 37° have been noted. The variation in the extinction angle is accompanied by a more or less marked variation in the color of the ray. While in some sections the change is marked by a fairly well defined zonal structure, the variations are often distributed over ill defined areas and are often distinctly gradational. Distinct crystals of the blue and green hornblendes seem to occur in the same rock, but the two are commonly grown together in parallel position (except in cases where the riebeckite is clearly secondary) the riebeckitic or blue type being developed marginally. The general mode of occurrence of the riebeckitic type in the granite and its presence alone in the pegmatitic facies of the granite, seem to point to the conclusion that it is the variety which develops when pneumatolytic agents are active. That such agents, particularly water or water vapor, are of prime importance in its formation is also borne out by its occurrence as a secondary product from the alteration of the original hornblende as noted above. Indeed, the development of the very closely related, or perhaps almost identical, crocidolitic amphibole found in the cavities in the pegmatite pipes and along joint surfaces in the granite, indicate that its solution and recrystallization may be carried out under conditions considerably removed from those prevailing during magmatic period. In this connection it is perhaps worth suggesting that relatively small changes in the hydration of the molecules making up the hornblendes, and particularly in the amounts of ferro-ferri-silicate molecules brought about by the oxidizing or reducing effects of the pneumatolitic agents may cause disproportionally large changes in the colors of the various rays. The chemical composition of the hornblendes will be taken up when the chemical analyses of the rocks are considered.

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[16] American Journal of Science, **23**, p. 268 (1907).

The mode of occurrence of the aegirite has been in part described when speaking of its intergrowth with the hornblende. Its occurrence in the form of microliths in the feldspar has also been noted.

The growths of aegirite are often so considerable that it predominates in amount over the hornblende. It is also found in the form of distinct grains which are most closely associated with the quartz with which there are occasional intergrowths. There was perhaps some tendency on the part of a portion of the aegirite to develop its prismatic zone toward quartz. Aegirite is, however, almost wholly anhedral and the contacts with other minerals are in general characterized by very irregular surfaces, projecting points, nubs, hooks, and by the presence of isolated or but slightly attached small particles, sometimes in considerable abundance. There is usually an elongation in the direction of the vertical axis. In many instances larger grains seem to have been broken apart and separated; again, a greatly elongated crystal or series of crystals end to end, may be seen extending along between the feldspar and quartz, or between two feldspars, or along fracture lines. Many aegirite patches, consist either of a relatively large crystal which contains elongated, prismatic crystals irregularly orientated and often with a subradial arrangement, or they consist wholly of an aggregate of variously placed prismatic crystals. It is also frequently noted that a number of small aegirites though not attached, form nevertheless what might be termed a community of grains. Zonal growths are not uncommon and indicate an earlier stage of development which is evidently connected in many cases, and perhaps always, with an earlier formed pyroxene, remnants of which are occasionally to be seen within the aegirite. As a rule this earlier pyroxene is almost entirely altered or replaced and what remains is filled with inclusions of fluorite and ferruginous matter. It is almost invariably true that in grains which show this pyroxene, the remnants indicate that it was rounded in form, small in size, and that they are enclosed in or indent the feldspar. In sections of the somewhat less acid phase of the granite from near the porphyry contact on Rattlesnake Hill, grains of a nearly colorless to pale brown pyroxene having the general appearance of augite have been noted, surrounded by a well defined rim of aegirite with a very rapid transition between them. The exact nature of this pyroxene has not been made out. Its extinction γ on c' is 35° at least; its double-refraction seems to be lower than common augite. It is probably a calcium-iron-rich pyroxene similar to the pyroxene in the rhombenporphyry. Whatever its original character, it has in general suffered almost complete replacement by aegirite, probably during the later stages of the consolidation.

Occasionally a pyroxene of a deep-green color and otherwise showing substantially the properties ascribed to aegirite-augite occur. These appear to have preceded the purer aegirite but to have followed the other variety just referred to. They are thought from chemical considerations to belong to the aegirite-hedenbergite line of mixtures, probably near the aegirite end, although they may be aegirine-augite.

The aegirite or aegirine-hedenbergite of earlier formation, that is to say the better formed grains, which bear about their margins evidences of later growths of the aegirite (analogous to the growths on the hornblende) commonly contain quite abundant, often very abundant, inclusions. The later formed aegirite rarely contains them. These are, fluorite, in sharply bounded octahedra and rounded grains, sometimes of good size but usually minute, grading down to mere specks: opaque, black grains probably ilmenite; and rarely roundish minute grains of a deep red color, undoubtedly the same as the red mineral forming larger intergrowths with the aegirite and hornblende, and thought to be aenigmatite. (See later.)

The color of the aegirite in thin-section varies considerably and this variation is often seen in grains that are otherwise optically homogeneous crystals. The colors commonly observed are as follows: — α, pale to deep green, sometimes with a slight bluish tone: less commonly almost colorless (usually confined to one part of a grain). β, pale yellowish-green to almost colorless. γ, pale yellow to pale yellowish-green or almost colorless. In many crystals the whole or part possesses a brownish-yellow or even a reddish-yellow color. This is often most pronounced about black oxide inclusions and is thought to be due to a pigment stain of ferruginous character. There at least appears to be no regularity about the distribution of these discolorations. The optical characters are otherwise the usual ones for aegirite.

In thin-sections from the quarries from the western half of the area a dark red mineral has been noted associated with the hornblende and aegirite. Its presence appears to have been first noted by Murgoci [17] who thought it to be a new mineral and tentatively suggested the name Quincyite for it. It appears to be of rather rare and somewhat irregular occurrence. Its crystals are for the most part small and its depth of color render them unfavorable for satisfactory study. So far as determined its properties are as follows: — cleavage-good, apparently prismatic, resembling that of hornblende; pleochrism-deep red or brownish to almost black for the ray making an angle of 30° to 40° with the cleavage; ray perpendicular to this, a bright mahogany-red. Double-refraction weak. The extinction angle is too large for any of the alkali amphiboles and the only mineral that seems to agree with these characteristics is aenigmatite. It is commonly, though not always, in parallel position with aegirite or hornblende and its contacts are usually rounded. It is also seen in the form of small, usually rounded grains sometimes enclosed in aegirite and also closely associated with masses of granular zircon. It has been noted also in close association with astrophyllite — apparently secondary after it — and on one instance it was noted largely replaced by this mineral. V. Hackman has described a secondary mineral having apparently properties very similar to, if not identical with astrophyllite, surrounding aenigmatite in the nephelite syenite from Umptek.[18]

Fluorite, as has been noted, occurs as included grains in the earlier aegirite and is often quite abundant there; it is also found, though less commonly, in the hornblende. Single grains or more often clusters or compact granular aggregates are sometimes seen associated with the quartz, and in such cases zircon and hematite are often present also. It has been noted that fluorite is apt to be more abundant in the granite where it is cut by small quartz veins. Zircon almost always is found in close association with quartz and rarely forms well shaped crystals. The grains are rounded and are to a greater or less extent filled with a dusty, brownish material. It is sometimes intergrown with the quartz much after the manner described for the zircon of the pegmatites.[19] This occurrence of zircon as described appears to be a characteristic of riebeckite rocks (see Murgoci, op. cit.). Titanite has been already described occurring as a probable replacement of the hornblende. It also occurs to a small extent in the form of isolated grains scattered through the rock and commonly associated with ilmenite or magnetite. Ilmenite or magnetite, occur as included grains and more or less fine dust in all the minerals of the rock. While a little of it may be primary, the greater part of it is thought to be secondary even in the relatively fresh granite. Magnetite is more abundant as the alteration of the hornblende has preceded further, and is then noted in the form of well defined octahedra lying in or about the position of the original iron-bearing silicate. Hematite appears to a limited extent in the fresh granite as minute flakes or grains in the feldspar and with the aegirite. In some cases its presence is apparently connected with pneumatolytic processes, but in general it appears only in connection with more superficial alteration. It is abundant in red surface granites.

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[17] Private correspondence.
[18] Mikroskopische Physiographie, Rosenbusch, p. 384.

An interesting and more unusual accessory is the mineral astrophyllite. Attention has been directed to the occurrence of this rare mineral in the Quincy granite by Pirsson.[20] It is only very sparingly present and appears to be of irregular occurrence. It has already been described as occurring about and replacing the aenigmatite and the aegirite in which the latter was intergrown. It generally appears in, or attached to, aegirite or the hornblende, particularly when this is intergrown with aegirite. Pirsson also noted it intergrown with its cleavage direction parallel to the vertical axis of the riebeckite. The writer has also seen it attached to zircon and to grains of a not fully identified mineral, but one which suggested parasite in appearance. Its habit and properties according to Pirsson are: minute elongated laths grouped in bunches; cleavage, micaceous excellent; elongation parallel to c; A = b; B = c; C = a; strongly pleochroic; A, red orange, C, lemon-yellow; absorption, A>C; mean refractive index about, 1.7; extinction parallel to the cleavage cracks; birefringence high 0.04. In convergent light a single biaxial optic axis was obtained on the edge of the field. According to the writer's observations the elongation is more likely parallel to ă than c′. It appears from its mode of occurrence to be a mineral formed by pneumatolytic processes.

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[19] Op. cit., p. 131.

[20] American Journal of Science, **29**, p. 215 (March, 1910).

The writer is indebted to Professor Pirsson for an opportunity to examine the thin-section on which his observations were made. The mineral was more favorably developed for study in this than in any other which the writer has seen.

Calcite is occasionally observed, sometimes filling small interspaces between the other mineral grains, again as small patches within the feldspar, and associated with titanite etc. in the altered hornblende groups. Associated with the calcite and titanite, also alone, grains of a mineral which seems to be siderite has been observed. This has been identified in some of the porphyries. Its presence is not surprising in a rock where iron is so abundant and lime almost lacking.

Special Variations of the Granite.— Four variations from the normal, gray granite may be specially noted. The first is that found at the Gold Leaf quarry already alluded to in the description of the recrystallization of the feldspar, p. (214), and the granulation of the quartz. Macroscopically the striking feature of this variation is the finely granular character of the quartz which is often stained reddish or yellowish with iron oxides. Besides these stains there are numerous red spots that appear in part to be due to an impregnation of small feldspar grains with iron oxide, and in part to the occurrence of distinct grains of some red mineral. Although very difficult to obtain satisfactory data regarding it, it seems to correspond closely to the aenigmatite and is so regarded.

The second variation is that found occurring as a rather sharply defined streak crossing the Ballou Quarry on North Common Hill. Its chief characteristic is its delicate purple shade of color. This is due to the very general distribution of minute scales and specks of hematite through the feldspar. The hornblende groups are nearly all heavily altered, being changed to a mass of riebeckite shreds, magnetite and hematite accompanied by a considerable amount of fluorite and calcite. The aegirite originally with the hornblende has been much less effected by the alteration. This streak appears to have been one in which pneumatolitic action was especially active. More or less of the same changes may be noted in the regular granite of the Ballou Quarry which on this account has been described by Dale[21] as a dark, slightly purplish granite.

The third variation is that known as the *pink* or *red type*. Its distribution is quite general. As clearly pointed out by Crosby[22] it is a superficially altered and oxidized portion of the gray granite, and always passes gradually downward at no very great depths (estimated at 20 ft. in some places where particularly well exposed) into the normal gray type. The original dark silicates have been entirely destroyed, their places being occupied by abundant magnetite crystals, quartz and feldspar and calcite. The pink or red color is due to the presence of exceedingly minute hematite specks or scales resulting from the oxidation of the hornblende microliths originally contained in the feldspars, and in part also to a general distribution of ferruginous products through the rock.

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[21] loc. cit., p. 100.
[22] loc. cit., pp. 334–8.

The fourth, and perhaps the most interesting and important one, is that in which a rather indistinctly marked porphyritic texture is developed. This variation is found wherever the granite approaches the granite-porphyry of the contact zone. Its texture, although not strongly developed, is characteristic, and is due largely to the fact that a part of the feldspars are grown somewhat larger than the rest. This feature is well seen in the granite from the northern slopes of Rattlesnake Hill, on the Great Dome, and particularly over a large part of the Pine Hill area. In the latter location the passage of this phase of the granite into the granite-porphyry of the contact zone is perfect and gradual though always comparatively rapid. On the top of Rattlesnake Hill near the southern edge of the hill, this granite is found in a sharp but perfectly sealed contact with the porphyry. The same phenomena may also be observed elsewhere, and while we cannot doubt that the porphyry is in all cases but a more rapidly cooled phase of the magma, we are forced to conclude that the magma moved to some extent underneath its own cover forming sharp contacts with it.[23] It is to be noted that at such contacts, so far as observed by the writer, there is a more or less marked development, along the immediate contact line, of long, slender riebeckite prisms.

Under the microscope the minerals are seen to be essentially the same as in the normal granite. The texture is also much the same except that a part of the feldspar has attained a larger size and, that in portions of almost every section examined, areas will be found that show the hornblende enclosing small feldspars and suggesting at once a close relation to the groundmass structures of the porphyry above. In the feldspar crystals, particularly the larger ones, distinct outlines, sometimes marked by the inclusion of small crystals of aegirite, of an inner zone of growth may be seen. This boundary marks the slight halt in the growth of the earlier formed crystals which is so clearly shown in the granite-porphyry as will be noted later.

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[23] It is not unlikely that this particular contact, which is steeply inclined, marks a lateral contact of a great dike which broke through or at least pushed up the porphyry cover at this point. "A few hundred feet to the west and a little to the north of the line of contact, rises a prominent knob of granite known as the "Rattle Rock." This rises to about the same elevation as the porphyry on Rattlesnake Hill and probably represents the exposed stump of a great dike or a cupola of granite which domed up or cut through the porphyry.

Chemical Characters.— For chemical analysis a number of good sized fragments were broken from carefully selected samples of freshly quarried rock from the three localities listed below, one from well toward the eastern end of the Quarry section, one from $\frac{1}{4}$ of a mile west of the first and the third $\frac{1}{2}$ of a mile still further west, in the west Quincy district. A sample of the porphyritic phase of the granite from Rattlesnake Hill very near the granite-porphyry cover was also analyzed. Great care was exercised in avoiding xenoliths or parts that showed any discernible variation in grain. The percentages given are the average of closely agreeing duplicates, except that the values for ferrous-iron and alkalies are the average of three determinations each. The methods of analysis advocated by Hillebrand were strictly adhered to. The results are given on page 227.

For the convenience of those who have adopted the so-called "Quantitative Classification of Igneous rocks"[24] the "norm" has been calculated from the average of 1–2–3 given under 4.

	Norm.	
Quartz	32.10	$\frac{\text{Sal}}{\text{Fem}} = 16 > \frac{7}{1}$. Class I 93.70 Salic Minerals, $\frac{Q}{F} = .52 < \frac{3}{5} > \frac{1}{7}$ order 4; quardofelic
Zircon	.30	
Orthoclase	27.24	
Albite	34.06	
Acmite	1.85	$\frac{K_2O + Na_2O}{CaO} > \frac{7}{1}$ = Rang 1; Peralkalic. 5.84 Femic Minerals $\frac{K_2O}{Na_2O} = .75 < \frac{5}{3} > \frac{3}{5}$ = Subrang 3; Sodipotassic; Liparose.
Diopside	1.21	
Magnetite	2.32	
Ilmenite	.46	
	99.54	

The rock may, therefore, be termed a *grano-liparose* or more exactly an alkali-hornblende-aegirite grano-liparose. The calculation of the mineral composition of the granite can only be made approximately, since an accurate estimate of the amount of each mineral present

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[24] Quantitative Classification of Igneous Rocks, by Cross, Iddings, Pirsson, Washington, Univ. of Chicago Press, Chicago, Ill., 1903.

	1.	2.	3.	4.		5.	6.	7.	8.	9.
				%	Molec. Ratio.					
SiO_2	75.08	75.58	73.93	74.86	1.247	72.97	71.65	68.54	78.49	71.34
Z_2O_2	.20	.20	(.20)[25]	.20		.20				
Al_2O_3	11.57	11.17	12.09	11.61	.114	12.13	13.04	15.47	9.99	13.97
Fe_2O_3	2.25	1.71	2.91	2.29	.014	2.77	2.79	2.03	1.94	1.95
FeO	.93	1.26	1.55	1.25	.017	1.09	1.80	2.09	1.18	1.00
MgO	.03	.04	.08	.05	.001	.20	tr	.21	.09	.62
MnO	tr	.05	tr	.02		tr	—	—	tr	tr
CaO[26]	.44	.49	.31	.41	.007	.74	tr	.30	.30	1.63
Na_2O	4.21	4.03	4.66	4.30	.069	4.61	6.30	5.68	3.74	4.84
K_2O	4.62	4.68	4.63	4.64	.049	4.79	3.98	5.75	3.84	3.89
H_2O-	.04	.10		.04		.10	1.11	.59	.72	.89
H_2O+	.19	.34	.41	.31		.35				
TiO_2	.20	.22	.18	.20	.003	.30	—	.14		.43
P_2O_5	tr	tr	tr	tr		.20	—	.10		
Total	99.76	99.87	100.95	100.18		100.25	100.67	100.90	100.29	100.56

Aver. Sp. G. 1, 2 & 3 = 2.661

1. Medium gray granite, Hitchcock Quarry, N. Common Hill, Quincy, Mass. Analyst, C. H. Warren.

2. Very dark granite, Reinhalter Quarry, West Quincy, from about 300 ft. below surface. Analyst, C. H. Warren.

3. Medium dark granite, Hardwick Quarry, N. Common Hill, Quincy, Mass. Analyst, H. S. Washington.

4. Average of 1, 2, and 3.

5. Slightly porphyritic phase from near contact with granite-porphyry. Quarry N. side of Rattlesnake Hill, Blue Hills, Reservation, Analyst, C. H. Warren.

6, **7** and **8** are taken from Rosenbusch's Elemente d. Gesteinslehre, 1910 Ed., p. 86, and are as follows:—

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[25] Assumed to be the same as in 1 and 2.

Fluorine though present in small amount was not estimated.

[26] From 0.1 to 0.28 or an average of 0.19% of this is present as $CaCO_3$, see Dale, loc. cit., p. 94.

6. Riebeckite granite. S. W. Houghnatten, Eftelöt, Sanelsvär, W. from Lougental, Southern Norway.
7. Riebeckite granite, Ekona — Sungale — Krater, Kamerum.
8. Riebeckite, Acmite Granite, Dahamis, Insel, Sokotra.
9. Arfvedsonite-Biotite granite,[27] Stony Brook Reservation, West Roxbury, Mass. Wm. F. Hall, Analyst.

cannot be made, nor if it could be done, could we apportion the various radicles of the soda-iron pyroxene and hornblende accurately as we do not know their exact composition. We may, however, proceed as follows and arrive at a mineral composition which will give us a close approximation to the relative amounts of quartz, the feldspars, the sodic-iron silicates, and the oxide minerals. The albite has been determined at least as sodic as Ab_{95} An_5 and possibly more so. Using this composition for the albite and making the assumption that magnetite is not present in an amount over one percent, which is certainly not far from the truth, further noting the fact that at least one third of the CaO is present as $CaCO_3$ and disregarding the slight excess of Al_2O_3 as being present in kaolin, we arrive at the following composition calculated to 100 percent. The water and fluorine have been also disregarded though they certainly enter into the hornblende present. The omission has the effect of making the hornblende figure somewhat low.

	Quartz	33.3
Microperthite	Albite $Ab_{95}An_5$	28.1
	Microcline	27.5
	Hornblende and Pyroxene	9.6
	Magnetite and Ilmenite Zircon	1.5
		100.00

Ratio, $\frac{\text{Feldspars}}{\text{Quartz}} = 1.67$

Ratio, $\frac{\text{Albite}}{\text{Microcline}} = 1.02$

Albite	50.5
Microcline	49.5
Microperthite	100.00

These percentages agree well with approximate measurements of the relative amounts of the constituent minerals made on thin-sections and with Dale's Rosival measurements[28] made on polished surfaces, the average of which are, quartz 30.6, feldspar 60.6, dark minerals 9.4.

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[27] F. Bascom, Journal Phil. Acad. of Nat. Sciences, **15**, 2d series, p. 135. (March, 1912).

[28] loc. cit., p.

The albite and microcline in the microperthite are almost exactly equal, which is also in agreement with crude microscopic estimates. The $Na_2 Fe_2 Si_4O_{12}$ (aegirite and corresponding hornblende) molecule predominates but the ferrous or ferro-ferri compounds are prominent, a fact to be expected from the known composition of the Riebeckite in the pegmatitic facies where these molecules exceed the $Na_2 Fe_2 Si_4O_{12}$.

Comparing the analyses of other Riebeckite granites given in columns 6, 7, and 8 with each other and with that of the Quincy granite, we note that all are characterized by exceedingly low MgO and CaO, by high iron content in both ferrous and ferric iron, and by high alkalies. The soda predominates, molecularly. With the exception of No. 6, in which the soda is greatly in excess, they show very nearly equal percentages of the two alkalies, though in view of the considerable variation in alumina this perhaps has little significance. An approximate calculation of the mineral composition of each is given below and shows a great variation in the relative percentages of the minerals which are in each case of essentially the same type.

	4a	6a	7a	8a
Quartz	33.3	20.6	12.0	41.4
Albite	28.1	47.2	47.1	30.4
Orthoclase	27.5	22.8	33.8	22.2
Pyroxene Hornblende Magnetite Ilmenite, etc.	11.1	9.4	7.1	6.0
	100.0	100.0	100.0	100.0

Comparison of the three analyses of the Quincy granite with each other shows that there is some variation in the amounts of different oxides, and therefore, in the minerals. These variations were expected from facts revealed by the study of the thin-sections. Thus the granite of the Hitchcock quarry (1) was richer in aegirite and showed less of the riebeckite in the form of microclites scattered through the feldspar. The ferric-iron should therefore be relatively higher than in the dark granite from the Reinhalter quarry, in which is very abundant riebeckite (a hornblende rich in ferrous iron) both in large crystals and distributed through the feldspars. The samples came from localities separated by some distances and different elevations and are representative of the granite as a whole. Taken jointly with the microscopic evidence the chemical analyses show conclusively that there is a small, but noteworthy variation in composition in the rock from different points, though such variations seem to be vicarious in character.

The granite from Rattlesnake Hill is a little lower in silica and higher in alkalies, thus leaning toward the granite-porphyry and fine-granite, (see beyond Nos. 10 and 13) but in the ferrous and ferric iron it is like the normal granite. It thus stands in an intermediate position chemically between the normal granite and its peripheral phases as it does texturally and in structural position.

The granite, No. 9, from the Stony Brook Reservation, West Roxbury, Mass., only a few miles distant (N. W.) from the area under discussion and described by Professor Bascom, differs from the Quincy granite chemically in higher lime, also in having lower silica; soda predominates over potash and there is much less total iron. Mineralogically the two are strongly contrasted, the Quincy is a microperthite granite, the Stony Brook granite, like an enormous preponderance of the granite of Eastern New England, is a two feldspar granite with a strong leaning toward monzonitic types. The former is characterized by soda-iron hornblendes and pyroxenes and never shows epidotic alteration; the latter is characteristically biotitic, perhaps carries an arfvedsonitic amphibole, and is epidotic. Though the Stony Brook granite is perhaps a nearer relative to the Quincy than some of the biotite granite of the Atlantic seaboard, they are still, mineralogically and chemically, sharply contrasted types.

Fine Granite.

It should be noted at the outset in describing this rock that the term "fine-granite" is here used in a more restricted sense than that in which it has been used by Professor Crosby. He used it to include a part of what is here termed "granite-porphyry" and he has also not distinguished between the fine-granite of the alkaline type and that associated in Weymouth with the biotite, subalkaline granite. Professor Crosby held that the fine-granite of the Blue Hills was an intermediate textural phase between the quartz or granite-porphyry and the coarser granite. With the possible exceptions of one or two points, there is no rock that can be properly termed fine-granite, having such relations. The phase of the granite-porphyry as the granite is approached is not readily recognized as a porphyry megascopically, being quite granitic in appearance; nor did Dr. White's microscopic studies inform Professor Crosby as to the true nature of these rocks.

Distribution.— The exceedingly detailed and careful study of the relations of this rock to the coarse-granite and to the slates made by Professor Crosby has proved beyond doubt that it is throughout a contact phase of the granite magma. In the Quincy-Weymouth area (see general map, no. 1), viz. that part of the field lying roughly east of the line of the West Quincy Branch of the Railroad as far north as the West Quincy Station, in the area lying north and northwest of the Station in what has been termed the "Furnace Brook" area and also along the northern edge of the alkaline-rocks wherever the fine-granite is exposed as far west, at least, as Canton Avenue in Milton, this rock appears to be the only contact phase of the magma which was developed and this will be termed for convenience the Ruggles Creek type, since the most satisfactory exposures are found in the region about Ruggles Creek and from here the type specimens were chosen for analysis. The other exposures of the fine-granite, viz. in the Pine Hill area, Pine Tree Brook Reservation and in the area lying between Wampatuck and Fox Hill, all of which are comparatively small in contrast to the development of the rock in the Quincy-Weymouth area of the field, are closely associated with the other contact phases of the magma. The field exposures in these areas, particularly the first two, indicate that in places the fine-granite, often quite porphyritic, replaces the granite-porphyry etc. as the contact phase of the magma against the slate just as it does in the S. Quincy-Weymouth area and along the northern edge of the complex: at other points its relations are more obscure but suggest that there may be fine-granite zone intermediate between the rhombenporphyry and granite-porphyry and the granite. This is perhaps supported by the fact that in such occurrences the rock shows a much stronger porphyritic habit. Actual transitions to the porphyry and coarse granite have not, however, been observed with certainty although numerous sharp contacts have, and it is invaded by dikes of coarser granite just as the porphyry is. No certain contacts can be found for the mass lying between Wampatuck and Fox Hills but there is a strong indication that the fine-granite becomes more porphyritic in the neighborhood of the granite-porphyry and there may be a rapid transition into it.

Although actual contacts of the fine-granite with the slate in the Quincy-Weymouth area are apt to be concealed, enough are satisfactorily exposed to show that the rock is somewhat finer in grain at the immediate contact. Against those slate masses which represent relatively deep projections into the invading magma, the width of the fine-granite zone is measured by a few feet. Where it marks the more elevated main contacts, its thickness is considerable. While it is not possible to estimate accurately what this thickness was in any case it was doubtless measured by tens of feet. Its contacts with the coarse-granite are exposed at several points and are usually, if not always, perfectly sealed but sharp in character. Professor Crosby believed[29] that these sharp contacts were due to differential movement of the still unconsolidated magma against its solidified margin, which was generally fractured and invaded by the granite magma as an incident to the process of intrusion. The facts in the field appear to support this reasonable hypothesis for there are numerous dikes of the coarse in the fine-granite and also inclusions of the fine in the coarse (see Crosby, op. cit., p. 352 et seq.). Professor Crosby has noted the occurrence of at least one contact where there appeared to be some gradation in texture, but in the writer's experience it can be said that the gradation is at most confined to a few inches (compare statements regarding the porphyry-granite contacts beyond).

It is to be noted especially that there are *no segregations* in the normal fine-granite, a point that will be taken up later in discussing the relations of the various phases.

The general distribution of the fine-granite is shown on the map. For a more detailed mapping of this rock and its relations to the slate and coarse-granite the special maps of Professor Crosby should be consulted.[30]

Megascopic.— This rock is a fine grained (one to two millimeters) slightly porphyritic one, of a prevailing light gray color though often light brown or pinkish from alteration. The phenocrysts are alkali-feldspar, are rather sparsely distributed and of characteristically elongated, rectangular outlines (1 by 3 mm. to 2 by 7 mm.). They are more abundant and conspicuous in some localities and become somewhat larger and more prominent in varieties that mark a probable gradation toward the granite-porphyry and probably also toward the slate contacts. Rarely rounded quartz phenocrysts occur, but these are wanting in the normal rock. Scattered quite plentifully through the rock are minute black hornblendes averaging not far from a millimeter in cross section.

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[29] op. cit., p. 355.

[30] It should be noted in this connection that a part of Crosby's fine-granite in northern Weymouth and Hingham is an entirely distinct rock from the fine-granite here considered. It belongs to the biotite, microcline-plagioclase granite which occupies extensive areas to the south and southeast of the alkaline rocks.

Microscopic.— The microscope shows that in the fine-granite of the Ruggles Creek type the minerals present are: — albite-microcline microperthite, quartz and riebeckite with accessory zircon, magnetite and ilmenite, fluorite, a little titanite, calcite, biotite, chlorite and limonite. The last three and a part anyway of the magnetite are secondary, and are present to only a small extent except in the heavily altered surface layer. In the fine-granite of the Pine Tree Brook reservation, we find in addition to the minerals above mentioned, aegirite, a green alkali pyroxene, occasionally small amounts of aenigmatite and rather more abundant fluorite.

The porphyritic texture noted megascopically is inconspicuous in thin section, for the reason that the feldspars are somewhat gradational in size and the larger grains are relatively so few that they are largely lost sight of. While the limits of size of the mineral grains lie between rather narrow limits, the texture of the rock may probably be best described as holocrystalline granular seriate. The feldspars are essentially identical in character and habit with those of the coarse-granite. The quartz is slightly less abundant and shows a tendency to micrographic intergrowth with the feldspar. In fact from some outcrops, probably near to the original contact with the slate, the graphic intergrowths are a prominent feature of the rock. The riebeckite is also essentially the same in habit and optical characters as in the coarse granite. In general it appears that in the fresh rock a green tone predominates over blue (except about the margins) for the ray (α) which is nearest the cleavage. The zircon is usually present in well formed crystals, which is in sharp contrast to the habit of this mineral in the coarse-granite. Aegirite, as has been noted, is present in the fine-granite from the relatively small outcrops on Pine Hill and in the Pine Tree Brook Reservation. Whether it was originally present in the mass northeast of Fox Hill cannot be told, on account of the extreme alteration of the dark minerals, but the general impression is that it was absent. This aegirite has the same relation as those described for the granite. In addition to the aegirite a deep green pyroxene closely resembling aegirite in general appearance but somewhat less pleochroic and of much lower double-refraction and with a larger extinction angle is present. This occurs either in separate, sub- to anhedral grains or is enclosed in the hornblende. It is often strongly altered to ferruginous material. It seems to be essentially the same pyroxene which appears sparingly in the coarse-granite and in the granite-porphyry, and is thought to belong to the aegirite-hedenbergite line of pyroxenes. Magnetite or ilmenite is quite abundant in the form of inclusions in the hornblende. Its appearance suggests that it is secondary after some highly ferruginous mineral other than the pyroxene. Several grains of the dark red mineral, believed to be aenigmatite, have been noted enclosed in the hornblende, and the magnetite may be a replacement of this mineral. Fluorite in the form of included grains is often very abundant in the pyroxene, and it is also present, but somewhat less abundantly, in the hornblende.

Porphyritic Phase of the Fine-Granite.— The tendency of the fine-granite to become porphyritic where it probably grades into the typical granite-porphyry, and also probably for a very short distance where it is in original igneous contact with the slate, have been referred to. (The latter contacts are not satisfactorily exposed and the writer is not altogether certain on this point.) This type is best exposed for study in several outcrops in the Pine-Tree Brook Reservation and at one point just north of the entrance to Scaumaug Notch. The rock while not so profusely porphyritic as the granite-porphyry is still strongly porphyritic, showing abundant rectangular feldspars and rounded quartz grains. The groundmass is finer than the average grain of the typical fine-granite but is distinctly coarser than the groundmass of the granite-porphyry described beyond. The groundmass contains rather abundant "specky" hornblende. Microscopically the groundmass is essentially of the same texture as the fine-granite although it often shows, in the inclusion of the groundmass feldspar and quartz by the hornblende, an approach to the characteristic structures found in the typical granite-porphyry of the Blue Hills.

Chemical characters.— For chemical analysis a specimen of the fine-granite from an old quarry just south of Ruggles Creek, Quincy, was selected. Though stained slightly brown, the rock in thin-section seemed to be the freshest of any that could be obtained. The values are the average of duplicate analyses except that ferrous-iron and alkalies are the average of three and four determinations each.

	10.		4.	11.	12.
	Per cent.	Molec. Ratios.			
SiO_2	71.41	1.190	74.86	76.52	71.63
Z_2O_2	.10	.001	.20		
Al_2O_3	12.74	.125	11.61	12.30	13.71
Fe_2O_3	1.75	.011	2.29	.70	2.09
FeO	2.33	.032	1.25	.56	1.76
MnO	.10	.001	.02	tr	tr
MgO	.06	.001	.05	.16	.19
CaO[1]	.85	.014	.41	.31	1.31
Na_2O	4.59	.074	4.30	5.19	3.24
K_2O	5.00	.053	4.64	4.58	4.49
H_2O+	.56	(.028)	.31	.41	.51
H_2O-	.10		.04	.11	8
TiO_2	.38	.005	.20	.12	.34
P_2O_3	.22	.001	tr		tr
CO_2	.40		—		.41
	100.59		100.96	100.96	99.76

Sp. G. of No. 10 at 20° C. = 2.66

10. Fine-granite, South of Ruggles Creek, Quincy.
4. Average of three analyses of coarse Quincy Granite.
11. Fine-grained or Micro-granite — Neponset Valley, Mass., F. Bascom, Op. cit., p. 137, Intermediate in the field between the granite (No. 9) and the Aporhyolite (No. 12).
12. Aporhyolite. F. Bascom, Op. cit., p. 138, Peripheral phase of granite batholith, Neponset Valley.

The "norm" calculated from the above molecular ratios is as follows: —

	Norm.
Quartz	24.06
Zircon	.10
Orthoclase	29.47
Albite	37.73

$\frac{\text{Sal}}{\text{Fem.}} = 1.14 > \frac{7}{1}$; Class I.

91.36 Salic Minerals.

$\frac{Q}{F} = .35 < \frac{3}{5} > \frac{1}{7}$; Order 4; quardofelic.

Norm.	
Acmite	.92
Diopside	2.71
Hypersthene	1.19
Magnetite	2.09
Ilmenite	.76
Apatite	.34
	99.37

8.01 Femic Minerals.

$\frac{K_2O + Na_2O}{CaO} = > \frac{1}{7} =$ Range 1; peralkalic.

$\frac{K_2O}{Na_2O} = .73 < \frac{5}{3} > \frac{3}{5}$; Subrange 3; sodipotassic.

Liparose.

The rock is therefore a riebeckite-grano-liparose.

In calculating the mineral composition the albite was taken as $Ab_{95}An_5$, which is as near as it could be determined, and the total amount of feldspar was assumed to be that found by a careful Rosival measurement made on two thin-sections. The small amount of calcite and limonite were disregarded, but a part of the water above 110° was used in the proportion in which it was known to be present in the Riebeckite hornblende of the pegmatites [31] which optically appears to be identical with that in the fine-granite.

The results are as follows: —

	Calculated.		Rosival Estimate.
Quartz	23.3	23.3	22.9
Albite $Ab_{95}An_5$	36.93	66.4	66.4
Microcline	29.47		
Hornblende	8.41	10.3	10.7
Magnetite Ilmenite	1.46		
Zircon	.10		
Apatite	.34		
		100.00	100.00

Ratios.

$\frac{\text{Feldspars}}{\text{Quartz}} = 2.84$

$\frac{\text{Albite}}{\text{Microcline}} = 1.25$

Albite = 55.6
Microcline = 44.4
Microperthite = 100.00

The percent of the molecules $Na_2Fe_2Si_4O_{12}$ and $(R'_2R'')_4Si_4O_{12}$ in the hornblende were found to be 44 and 56% respectively. In the pegmatite Riebeckite these were, 42 and 58%, which are in good agreement with those from the granite, and indicate the close similarity of the two hornblendes which was inferred from their optical characters.

Compared with the coarse-granite (4), we note 3.4% less silica; 1.1% more alumina; 0.5% higher total iron oxides, but with ferrous iron in excess over ferric as it should be with aegirite absent. The relative proportion of the alkalies is not notably different but their total is higher by 0.65%. Mineralogically these differences correspond to much less quartz and to a greater preponderence of albite over microcline than in the coarse-granite, the ratios being — Feldspar to Quartz, 2.84 (fine-granite) and 1.67 coarse-granite, and Albite to Microcline, 1.25 and 1.02 resp.

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[31] Warren & Palache, loc. cit., p. 154.

In columns 11 and 12 the peripheral rocks of the Neponset Valley granite intrusion, described by Professor Bascom and already alluded to are given. It can be seen at a glance from these analyses, and the same is fully borne out by the microscopic characters given by Professor Bascom, that the peripheral phases of the Neponset granite are, like the granites themselves, quite sharply contrasted chemically and mineralogically to those of the Quincy, Mass., granite.

Rosival measurements have also been made on thin-sections of the fine-granite from other localities to determine their approximate mineral composition. These are given below: —

MINERAL COMPOSITION OF FINE-GRANITES COMPUTED FROM ROSIVAL MEASUREMENTS.

	Ruggles Creek, Quincy. (type analyzed, No. 10.) (a.)	Wyman's Hill, Weymouth. (b.)	Pine Tree Brook Reservation. (c.)	North of Great Dome (Reservation). (Altered.) (d.)
Quartz	22.9	24.3	24.50	25.8
Microperthite	66.4	67.4	66.	67.1
Riebeckite,	10.7	8.3		
Soda-iron rich pyroxene and Hornblende			9.5	7.1
	100.0	100.0	100.0	100.0

These show a fairly close agreement, but some variation, just as has been shown to exist in the coarse-granite, appears to exist in the fine-granite from different localities. In (a) and (b) the dark silicate is riebeckite; in (c) and (d) some pyroxene is present, and they are also more altered, while the habits of the hornblende and pyroxenes are not quite so favorable to accurate measurement.

THE BLUE HILL PORPHYRIES.

Granite-porphyry: quartz-feldspar-porphyry.

Distribution; Megascopic Characteristics.— The porphyry with its variations into quartz-feldspar-porphyry is the contact phase of the granite magma over that part of the field enclosed within the limits of the Blue Hill Reservation, and occupies, therefore, all of the more elevated part of the field. It is found outside of the Reservation only in the relatively small but, nevertheless, important tract known as the Pine Hill area, which lies immediately east of, and is continuous with the Reservation. Here the porphyry is associated with fine-granite and with a more basic phase of the contact zone, the darker colored, rhombenporphyry. The same is true of the Pine Tree Brook area and in both, it is worthy of note, that patches of the original cover of Cambrian slates remain. The porphyry cover does not reach quite to the northern edge of the Reservation but is cut off along a line that stretches across the top of Rattlesnake Hill and thence runs a westerly and southwesterly direction as far as the alkaline rocks extend. The areal continuity of the porphyry is broken in three places by masses of aporhyolite, or felsite, as it has been called for convenience in the field. The general outlines of these masses are indicated on the map. Considered as later in age than the porphyries and granite by Crosby, this rock is by the writer believed to be of earlier consolidation and it is against the felsite that the magma consolidated with some of its most interesting textural variations.

It is quite impossible to define the extent or exact relative amounts of the different phases of the porphyry, partly on account of their transitions into one another, partly because of their very irregular distribution and partly because of the difficulties of distinguishing minor variations in heavily weathered and lichened outcrops. The relative importance of the different varieties and their relation to each other can, nevertheless, be made out with certainty. What is here termed granite-porphyry is in areal distribution and in volume the most important rock. It is, as would naturally be expected, the phase that immediately overlies the granite and while in some parts of the area (Pine Hill, for example) it passes gradually into the granite, in others it changes suddenly into the porphyritic phase of the granite. The thickness of the porphyry cover varies greatly from a few feet (on Pine Hill) to about 200 feet (estimated) on Rattlesnake Hill.

Though this last thickness may be exceeded in the western part of the Blue Hills, it is thought to represent very roughly the present thickness of the porphyry.

The least altered material obtainable from the old quarries on the east end of Rattlesnake Hill shows the granite-porphyry phase to be a holocrystalline rock having much the appearance of a rather fine grained granite (for which it is usually mistaken on first inspection) of a prevailing light, greenish-grey color. Closer inspection shows clearly that it consists of abundant phenocrysts of alkaline feldspar and quartz embedded in a finely granular groundmass in which appear numerous irregular specks or grains of black mineral. The feldspars are the more abundant phenocrysts. They are elongated parallel to the edge 001–010, have a somewhat tabular habit on 010 and although usually rounded or broken on the ends, show nevertheless, a tendency to form rather acute terminations. They may measure as much as 8 mm. long, 5 to 6 mm. in breadth by 2 to 3 mm. in thickness, but the average is smaller. A chatoyancy, characteristic of the cryptoperthitic feldspars, may sometimes be seen (this is well shown on polished surfaces). The quartz forms rather inconspicuous rounded grains usually smaller than the larger feldspars. The fracture of the rock is much like that of the fine-granite and the jointing is likewise finer than in the coarse granite, with a strong tendency to the formation of sharply prismatic blocks.

While reddish stains may be seen in a few of the feldspars of almost any specimen, there are streaks and irregular patches in which red spots are very abundant and characteristic. The grey variety of the granite-porphyry is interlaminated, as it were, with streaks of considerable regularity and persistence of a darker phase. This is dark grey to almost black, and recalls the association of the dark and light grey coarse-granite with which it is doubtless strictly analogous.

Alteration of the porphyry produces a whitening, generally accompanied by a slight brownish or reddish discoloration of the feldspar phenocrysts, while the groundmass becomes grey or grayish blue owing to the breaking up and dissemination of the dark constituents. More advanced alteration produces a general breaking up of the groundmass.

Over large areas in the hills, the granite-porphyry appears in a form that may more properly be termed in the field a quartz-feldspar-porphyry than a granite-porphyry, for although, as shown by the microscope, the grain of the groundmass is but little finer than that of the variety above described, the distribution of the dark mineral through the groundmass from one cause or another, renders the latter quite dense in appearance and of a peculiar grey or bluish-grey color. The phenocrysts are more conspicuous and all the more so when the feldspar is whitened or slightly stained (pink or brown). This type of rock is abundantly exposed for observation on the broad, gently rounded, smooth ledges characteristic of the northerly slopes of the hills and is a very characteristic feature of the area.

This last rock is scarcely to be distinguished from the phase with a truly dense groundmass into which it grades, and which also has a wide but irregular areal distribution. If anything the phenocrysts are slightly less abundant than in the former, the quartz forms rounder and more prominent grains and the groundmass is more varied in color and often darker. The color may be dark-grey, bluish-grey, pale-green or even dark-red to brown and less commonly of a purplish-red. The last mentioned colors belonging to exposures which have suffered a more complete oxidation of the iron-bearing minerals. Many ledges exhibit a heterogeneous structure. The grey or bluish-grey porphyry contains angular fragments, usually of the yellowish-green color, varying in size from ones measuring a few millimeters across to ones upwards of several inches in their largest dimension; also numerous streaks and blotches of the similar material, of varying width and length. These streaks and fragments by their arrangement often show a pronounced flow structure; again the fragments are so numerous as to constitute a breccia. Alteration further accentuates the heterogeneity. At one point on Heminway Hill the two varieties are so mingled together as to produce the appearance of a tuff. The brecciated character of the porphyry is clearly connected with the immediate contact of the magma with older rocks and is sufficient evidence that the contacts were at most only a short distance from the present surfaces of the rocks as now exposed, and that erosion has removed very little of the intrusive rock, a point that seems to be entirely borne out by the notable scarcity of rocks of the alkaline type in the conglomerates and other sediments of later age. The streaked porphyry referred to by Professor Crosby[32] as occurring on the south side of the Blue Hills both in situ and in the basal beds of the Norfolk-Basin conglomerate is undoubtedly of this type and its heterogenous structure is not, as was supposed by him, due entirely to differential weathering.

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[32] op. cit. p. 359.

In several localities in the Hills, notably on the north and northeast slopes of the Great Blue Hill and on the southern extension of Heminway Hill, there are large outcropings of a porphyry in which the feldspar phenocrysts are megascopically quite inconspicuous. The quartz phenocrysts are fairly numerous but small. The groundmass is aphanitic and usually of a dark brownish red or purplish color. In some places it appears to grade into the more crystalline porphyry about it; in others the transition is sudden, almost sharp, although always perfectly sealed. The sharpness of the transition here as with the other types — granite, fine-granite and granite-porphyry — is a characteristic relation.

In the region about Wampatuck Hill, the porphyry at the contact with the aporhyolite is of the very dense quartz-porphyry variety. The quartz phenocrysts are small and numerous, the feldspar is megascopically subordinate while the groundmass (where not strongly weathered) possesses a peculiar yellowish-green color which the microscope shows is due to the presence of many minute aegirite microliths. Going away from the contact the rock changes within a short distance — varying perhaps from 6 to 18 inches — into the variety with more abundant phenocrysts and this in turn changes rapidly but gradually into the granite-porphyry of the Rattlesnake Hill type. In the finer grained rock of the contact are many fragments, clearly of the contact phase of the porphyry itself and others seemingly of the aporhyolite.

Along the easternmost portions of the contact with the felsite in the Pine Hill Area, the porphyry at the contact presents a somewhat different charcter. At the immediate contact it is of the dense quartz-porphyry type and this shows breccia and flow-structures. Within a few inches this is succeeded quite suddenly by a coarsely porphyritic type which is confined to this mode of occurrence. Here the feldspars are larger and, when not broken or rounded, show acute terminations approaching the "rhomben" type. They are often visibly broken and resealed. In size they frequently measure a centimeter in length by 5 to 7 mm. in breadth by 3 to 4 mm. in thickness. The average is somewhat under these figures. The quartz grains are fewer in number and also larger than in the run of the porphyry, sometimes rivalling the feldspars in size. The groundmass is dense and usually of a grey or bluish-grey color though in streaks and patches it is greyish or yellowish-green. In this coarse porphyry near the contact are streaks and fragments of the finer material. These are in part portions of the coarse porphyry reduced to a fine-grained rock by movements along the contact, in part portions of the finer grained contact quartz-porphyry. The zone of coarse porphyry near the extreme eastern end of the felsite mass is from three to four feet wide. Further west the band broadens out somewhat. Going toward the granite the groundmass becomes gradually coarser in grain, the dark mineral appears in distinct spots resembling more the hornblende of the granite and the rock passes gradually into the granite which, however, retains, for an indeterminate distance, a porphyritic texture and this, as has been already noted, is always an indication of a near approach to the granite-porphyry.

Small cognate Xenoliths of a dark colored rhomben porphyry similar in all respects to those described later under the heading of "Xenoliths" (p. 275) are found, sometimes quite abundantly in the coarsely porphyritic zone, almost up to the actual contact. These are also found, but more sparingly, in the granite-porphyry and granite immediately succeeding.

At the eastern end of the felsite, the breadth of the entire porphyry zone is, so far as it can be determined, only about 15 ft. Going west it broadens rapidly (30 ft.). This width holds nearly to the top of Pine Hill and then still further broadens (50–100 ft.) while at the same time the coarse-porphyry type disappears, and porphyry of the Rattlesnake Hill type replaces it.

In many parts of the Blue Hills the phenocrysts of the porphyry are quite inconspicuous, the rock having a relatively dense and non-porphyritic appearance. The microscope shows, however, that this texture has been produced by a breaking of the phenocrysts through movements in the mass before crystallization had stopped and by the very general recrystallization of the constituents, particularly the feldspar.

Those varieties of the porphyry in which the phenocrysts are less numerous and in which the groundmass is relatively dense might be classed as Paisanites[33] but the greater part of the porphyry appears to the writer to be too abundantly phenocrystalline and its groundmass too coarsely crystalline to be properly classed under this name, although in mineral and chemical composition they are closely related to the original paisanite.

In closing the description of the field characters of the porphyry one more important feature must be noted. Throughout large portions of the higher elevations of the hills, numerous, angular inclusions of a dense to very finely crystalline rock rather sparsely sprinkled with minute black specks of hornblende are found. These are usually small rarely measuring more than a few inches across. When altered they are of a light-grey or light, bluish-grey color and usually show a few small phenocrysts of feldspar. They belong to the alkaline series, and seem to be essentially identical with the porphyry in composition, but appear to differ in texture from the inclusions in the brecciated portions of the porphyry before alluded to. These correspond closely in many respects to paisanite.

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[33] Osann, Tschermaks Min. u. Pet. Mitt. XV Band.

Microscopic characters of the Porphyry: — The granite-porphyry from Rattlesnake Hill in the eastern part of the Reservation will first be described, for it is here that it is perhaps best exposed and where fresh material for sections and chemical analysis can be most easily obtained.

In thin-section the light greenish-grey porphyry, which is considered the normal type, is seen to consist of abundant phenocrysts of feldspar (ca. 40%), quartz (ca. 12%) (see accompanying table, column I), areas of hornblende and pyroxene (mostly aegirite), in part as distinct grains but largely in the form of poikolitic intergrowths with the groundmass, all embedded in a fine groundmass of microperthite, quartz, hornblende, and aegirite. Accessory aenigmatite, magnetite, hematite, zircon, fluorite and occasional calcite and astrophyllite are also present.

TABLE OF ROSIVAL ESTIMATES OF QUARTZ AND FELDSPAR PHENOCRYSTS IN PORPHYRY.

	I	II	III	IV
Feldspar	40.5	32.2	19.0	39.1
Quartz	12.4	16.3	14.5	6.4
Groundmass	47.1	51.5	66.5	54.5
	100.0	100.0	100.0	100.0

I Unaltered Granite Porphyry, Rattlesnake Hill, average measurements of two sections.

II Quartz-feldspar-porphyry, ledge S. of Administration Road, S. of Wampatuck Hill,— 1½ feet from aporhyolite contact. Made on one large section.

III Same, 3″ from same contact.

IV Coarsely porphyritic phase of contact porphyry. Contact east end of aporhyolite Pine Hill, West Quincy. Average of two extra large thin-sections.

Among the phenocrysts, the feldspar greatly predominates. Before modification by breaking and later recrystallization, the feldspar crystals seem to have attained a fair degree of perfection in their crystal form. Cut parallel to the macropinacoid the crystals are often nearly square. There is usually an elongation parallel to the edge 001-010. Again the elongation is parallel to both this direction and the prismatic axis yielding tabular forms. The prism faces are often strongly developed as is also the y, $(\bar{2}01)$ face, the latter tending to give acutely terminated crystals. As noted in the megascopic description, the feldspars occasionally reach a length of 8 mm. or more but the largest dimension usually seen in section will not exceed 3.5 mm. and the average is not much over 1 mm. Twinning is common after the Carlsbad law, less common after the Manebach and still less so after the Baveno law. Leaving aside later enlargements, one may observe many straight edges, but the outlines are very generally curved and the terminations are commonly rounded or irregular. A great many of the phenocrysts have been broken by movements in the crystallizing mass, giving rise to irregular fragments of varying sizes and shapes, and rendering it difficult to tell just how far gradation in size as a result of normal growth extended. It appears, however, certain that there was a continuous gradation in size down nearly to that of the groundmass individuals. Upon the margins of the phenocrysts is a narrow, but sharply marked zone of orientated feldspar substance of later growth (see Figures II and III, Plate 1). Outwardly this zone fades into the groundmass and while often of uniform width entirely around the phenocrysts, it more often varies considerably especially where the contours of the older crystal are irregular, broken or indented. This zone contains many minute, rounded quartz grains, tiny microliths of aegirite and sometimes riebeckite, and is clearly of groundmass age. As it is often seen developed on the broken surfaces of the phenocrysts, its deposition clearly took place subsequent to the fracturing of the latter. This enlargement on many of the smaller, more irregular feldspars is proportionally much broader than on the larger phenocrysts, its area often exceeding that of the enclosed grain. These smaller feldspars grade downward in size practically to groundmass dimensions and form in effect a part of the latter. The original feldspar material includes a few early pyroxene crystals (or such replaced and mantled with aegirite), but is free from other inclusions. Where recrystallized or invaded by albite, the phenocrysts contain abundant shreds and fibers of riebeckite and aegirite grains.

With low powers, the phenocrysts appear fresh and homogeneous, with higher powers considerable portions of the crystals and, rarely, almost an entire crystal, appears also homogeneous. A great part of the feldspar, however, generally shows a very fine lamination, (cryptoperthitic) which is parallel to the usual direction of the perthite intergrowth. As the margin of the original crystal outline is approached, the lamination becomes more distinct and two sets of lamellae can be made out whose optical properties are those of albite and microcline. The perthitic structure of the marginal zone of later feldspar is slightly coarser (see Figure II, Plate 1).

In irregular streaks and patches through the body of the crystals the microperthitic structure is more distinct, and this coarsening is invariably accompanied by the occurrence of fine dust-like particles characteristic of the potash member of the granite microperthite. Sections of the homogeneous and cryptoperthitic parts of the phenocrysts parallel to M, (010) have an extinction measured on the 001 cleavage of from 12 to 13 degrees. Such sections show the immergence of an obtuse bisectrix. Basal sections give a nearly or quite parallel extinction. An acute positive bisectrix is obtained from sections near 100 with an axial angle similar in size to that of orthoclase. These properties are those of the cryptoperthite (anorthoclase) as described by Brogger, and of the feldspar phenocrysts described by Osann [34] in the Paisanite from West Texas.

The smaller, and usually more irregularly shaped crystals lying in the groundmass are for the most part microperthite (see Figure II, Plate 1), this, and the fact that the enlargements of the phenocrysts are also microperthite, as well as the feldspar of the groundmass proper, is important in connection with the relations of the cryptoperthitic and microperthitic structure, as will be pointed out later.

A striking feature of the phenocrysts is the extent to which they are replaced by albite. Many crystals, particularly the longer ones, are crossed transversely by narrow streaks of a more strongly polarizing, colorless material (see Figure III, Plate 1). In it are often minute crystals of aegirite. This substance is evidently albite. It is continuous with, and of the same orientation as the albite member of the microperthite in the later marginal zone. These streaks represent transverse fracture lines in the phenocryst along which albite set free from the unmixing (see later Part II) of the original feldspar substance or from the crystallizing groundmass, or both, have entered. Many instances are also seen where the albite has developed as a line of minute crystals extending across the phenocryst, following the line of cracking, and again the fine material of the groundmass as a whole has frequently been forced in along cracks. Occurring sometimes irregularly in the body of the phenocryst or orientated parallel to the trace of the albite twinning, are often small, irregular to tabular grains of albite. In a great many other cases the replacement of the phenocryst by albite takes the form of curiously irregular masses,[35] which project into the original feldspar substance often replacing as much as a quarter or even two-thirds of the crystal. This albite is sometimes finely twinned, though usually the twinning is confined to a few stripes and is often lacking altogether.

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[34] Tschermaks Mtt. XV, p. 436.

The quartz phenocrysts are less abundant, forming about 12% of the rock. The largest grain measured was 3.5 mm. in diameter but the average is much smaller being in the neighborhood of 1 mm. The size most commonly seen is, however, somewhat larger than this. While occasionally showing a fairly well marked crystal outline, the quartz is usually much rounded and is often embayed by the groundmass. Its grains are sometimes granulated or, like the feldspar, broken apart and almost without exception show abundant evidence of crushing movements in the broken and strongly undulatory extinctions. The quartz contains the same inclusions of bubbles and minute black grains as the quartz of the granite, but is practically free from other inclusions except where fractured and then blue hornblende needles or small aegirites occur in the quartz. A very narrow rim of later growth may frequently be noted about the quartz, but it is much narrower and less conspicuous than the corresponding rim about the feldspar and seems often to be wanting.

The hornblende is relatively abundant and occurs in elongate masses of irregular or rudely elliptical outline which are in great part poikilitic intergrowths with the feldspar and quartz of the groundmass. In size, most of these areas are comparable with that of the smaller phenocrysts of quartz and feldspar and rarely exceed 2 mm. in their longest dimension. In the interior of many of these groups is massive or nearly massive hornblende, frequently enclosing pyroxene (see Figure I, Plate 1). The massive material passes gradually into the poikilitic intergrowth and, while about the edges there are slightly connected or unconnected grains, more distant from these masses there is little hornblende present other than a few shreds of secondary origin.

The poikilitic hornblendes show a strong tendency to form along the margins of the enlarged feldspar phenocrysts which served as points of attachment and sometimes entirely surround them. The smaller microperthitic feldspars of phenocrystalline age with their enlargements are even enclosed by a single group along with groundmass microperthite and quartz (as shown in Figure II, Plate 1). The hornblende, with the possible exception of the massive centers, obviously belongs to the groundmass period of crystallization.

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[35] Compare albitization of feldspar in coarse-granite noted earlier.

In relatively thick sections or with crushed material, the hornblende possesses a prevailingly dark-blue or greenish-blue color. In good thin-sections, however, the characteristic color is a deep green or a bluish-green and the purer blue tones appear about the periphery or in streaks, and only rarely makes up any considerable portion of the more massive parts. The other characteristic ray (across the cleavage in 010 sections) is colored light yellow, light yellowish-green or light brownish-yellow. The extinction, in part, is that of riebeckite, but as in the granite, the extinction of the greener hornblende is much larger (ca 35°), indicating an alkali-hornblende, probably a cataphorite. The optical elongation is very difficult to determine owing to the deep colors and low double refraction but is negative for the blue type and probably positive for the catophorite.

The texture of the hornblende is well described as "spongiform," or more accurately, as domoikic with relatively fine to coarse xenocrysts. The feldspar and quartz xenocrysts show about the same range of sizes. They are irregular to tabular in habit while the quartz xenocrysts are commonly round.

Aegirite is the most abundant pyroxene but some other variety is present, of augitic appearance. The augitic pyroxene occurs in certain of the feldspar phenocrysts and also in the form of larger crystals in the groundmass, often surrounded by the hornblende (see Figure 1, Plate 1). It has sometimes a pale brown color but is usually a pale greenish-yellow. Rarely the augite material extends to a sharply marked line about which is a rim of deep-green aegirite. More often the augite appears as such, only at the center, and is succeeded outwardly by an indefinitely bounded green, aegiritic looking material. The depth of color increases toward the margin. The extreme edge appears to be aegirite. Again no augite can be seen, but the green to deep-green material with, however, the habit of the augite, forms a core which is indefinitely bordered by aegirite. Twinning on *a*, 100 occurs as do also zonal structures, but the latter lack sharpness. The aegiritic material is often finely granular, and may include fluorite grains, and much indeterminate dust. Occasionally the pyroxene core is replaced by many subradiate to diverse, aegirite prismoids. The optical properties of the augite are apparently normal, but with the development of the green material these properties become rather indefinite, the double-refraction is too low and the extinction too large for aegirite. The most characteristic thing about this pyroxene material is the rich yellowish-green to deep-green shade belonging to the alkali-pyroxenes. Scattered through the groundmass are fairly good sized crystals of rich green pyroxene which show no trace of augite and would be taken for aegirite or aegirite-augite, and yet, their double-refraction is also low except about their margins, and the optical properties are poorly defined. The characters just outlined may indicate that augite in small quantity was crystallized at an early period (enclosure in the feldspars) and that subsequently as the concentration of the aegirite molecule increased, the augite was not only encrusted by the aegirite but became in large measure replaced by it. It should be noted that the feldspars in which the augite occurs resemble strongly the feldspars to be described as occurring in the more basic phases, which are marginal differentiates of the magma, and the suspicion is strong that the augite (and its enclosing feldspar) represent crystals which have formed, early in the period of crystallization. The analyses indicate that, as in the rhombenporphyry, the augitic looking pyroxene is rich in the CaF″-molecule. That true aegirine-augite is present the writer can find no very positive proof. The bulk of the aegirite is found as minute and usually irregular grains scattered through the groundmass.

Grains of a red to dark-red mineral, believed to be aenigmatite, occur embedded in the green pyroxene and in the hornblende. The same mineral has also a very characteristic and fairly abundant distribution through the groundmass. Its mode of occurrence may perhaps be described as "clustered." That is to say, it is made up of from perhaps ten to one hundred or more minute grains, for the most part unconnected, lying close together amidst the quartz, feldspar and aegirite grains of the groundmass (see Figure I, Plate 1). The grains are irregular in form with perhaps a slight tendency to an elongation parallel to a poorly developed cleavage. The larger ones will hardly exceed a few hundredths of a millimeter, the rest ranging down to tiny round particles 0.005 mm. in diameter. In a single cluster the majority of the grains have nearly or quite the same orientation. Their color, and such other of their optical properties as can be made out, seem to be the same as those given earlier for the aenigmatite of the granite. The larger grains are commonly slightly attached to small adjacent grains and it may be that the clusters represent loosely connected poikilitic growths, the individual members of which, have been for the most part separated, but only slightly or not at all de-orientated by slight movements in the groundmass. Upon alteration they become opaque, apparently due to the formation of magnetite or ilmenite, or else they alter to limonite (?) blurs. In some instances they are seen accompanied by a development of astrophyllite fibers as is also the case with the aenigmatite of the granite.

The groundmass, aside from the later growths upon the feldspar phenocrysts and the poikilitic portions of the hornblendes and the aenigmatite, all of which are clearly of contemporaneous age with the groundmass and therefore properly belong to it, consists of a microcrystalline, inequigranular mixture of quartz, microperthite and aegirite with some accessory hornblende, mostly of secondary origin, magnetite, hematite, dusty particles, and calcite. The average grain of the quartz and feldspar will probably lie a little under 0.02 mm., the range from about 0.06 to 0.005 mm. The microperthite grains though apt to be sub-rectangular, particularly when enclosed in the hornblende, are usually quite irregular in shape, the quartz often shows a tendency to round outlines while the aegirite is scattered abundantly among the other minerals in the form of very irregular, small grains whose longest dimension rarely reaches 0.02 mm. and is usually measured in thousandths of a mm.

Texture.— Following the descriptive terms as proposed by Cross, Iddings, Pirsson and Washington,[36] the rock is sempatic and skedogranophyric, and in addition the groundmass is in part poikilitic.

Variations from the normal type on Rattlesnake Hill.— A portion of the greenish-grey porphyry is characterized by the presence of a reddish discoloration in the feldspar and a slightly darker color. In such the microscope shows the presence of abundant deposits of hematite along the cracks in the feldspar, of more abundant blue hornblende shreds in the groundmass, while the groups of poikilitic hornblende are bluer in color and often show secondary blue fibers and shreds developed about and upon them.

The strongly marked streaks of dark-grey porphyry are in the nature of indefinite interlaminations in the grey variety and blend rapidly into it. They differ from the lighter type in the presence of very abundant, fine, black dust in the groundmass and in the feldspar phenocrysts, particularly about the margins and along cracks; in the presence of abundant, minute prisms, scales, fibers and irregularly shaped pieces of the blue-hornblende scattered through the groundmass, as well as fibers distributed along the cleavages and the direction of perthitic intergrowth in the feldspar; and in the nearly or complete disappearance of the aenigmatite. It is not probable that the dark porphyry originally differed from the grey in chemical composition and the present differences are undoubtedly due to alteration acting along zones or streaks in the porphyry mass as a whole. The alteration is different from that produced by purely superficial alteration, and is believed to be a result of deep-seated alteration (connected probably with late magmatic conditions) in the same way that the corresponding streaks of dark-grey granite are.

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[36] Texture of Igneous Rocks, Journ. Geol., **14**, No. 8 (Nov.–Dec., 1906).

Characters of the Granite-Porphyry elsewhere in the Area.— Both the light and dark varieties have a very extensive development throughout the Blue Hills, but microscopic study of a large number of specimens from many points shows that there have been considerable changes effected in the various minerals beyond those which are thought to have developed during the late magmatic stage or in one immediately following it. These changes may perhaps have been effected during periods of profound geologic disturbance through which the region has passed, but the writer is inclined to think that they are in the main but a continuation of the modifications described as occurring in the Rattlesnake Hill porphyry, developed during the period subsequent to the consolidation of the porphyry, before the granite magma below had completed its crystallization and was still capable of giving off mineralized vapors, and perhaps also before all movements as a result of upward pressure of the mass beneath had ceased. Upon these are often superimposed the effects of superficial decay although these are limited to a thin surface layer. It thus happens that many exposures afford specimens which depart more or less widely in the details of the structure and composition from the normal type, and not a few in which only the remnants of the original structures remain.

Perhaps the most striking modifications thus effected are those shown by the feldspar phenocrysts. The changes in the original feldspar substance, believed to have been largely brought about during the latter part of the crystallization period or immediately following it, have been described for the normal porphyry, and are to be seen more or less strongly developed everywhere in the porphyry, although they are, to a greater or less extent modified by subsequent and more general alterations. So varied are the details of these changes as a whole that it would be impossible to fully describe them and only brief description of some of the more characteristic and common types will be attempted. The accompanying microphotographs (Figures Nos. IV, V, VI, VII, Plates 1 and 2) will serve, perhaps better than the descriptions, to furnish an idea of the appearance of some of the types of modified phenocrysts. One of their most characteristic features is that great numbers of them are crossed by bands or streaks which are optically continuous with the marginal parts of the phenocryst and with it form a sort of mesh enclosing a heterogeneous mixture of feldspar material. Theses "streaks" nearly always show a distinct central division line representing an original crack now sealed, and extend partly or entirely across the crystal in slightly curved and often ramifying lines. Along the central division line there is usually a narrow streak of albite material, and on either side of this for a variable distance the material shows a faint microperthitic structure as do the marginal parts of the crystal with which it is in fact continuous. Further away from these bands, mingled with remnants of the original feldspar, may be more coarsely developed microperthite, or separately crystallized albite and microcline. Frequently these separate crystals are orientated parallel to the original feldspar or stand normal to the edges of the "streak"; again they are situated quite at random. They may take the form of rather short laths or are entirely irregular in outline. As a rule albite appears to be more abundant than would be expected if the changes were concerned wholly with the rearrangement of the albite and microcline of the original feldspar and there may have been a later introduction of albite. The microperthite can be easily recognized as a rule by the presence of minute specks, cavities, etc. in the potash member just as in the granite microperthite. Wherever this has been broken or replaced, the specks, etc. are absent, but needles of blue amphibole occur, sometimes with aegirite. Both aegirite and amphibole are commonly developed along the original division line in the "streak" and abundantly on either side of the streak as a whole. It appears that the cracks served as channels along which solutions acted, introducing probably some material from without and effecting changes for a short distance on either side in the original feldspar — a more distinct development of the albite and potash members — thus rendering such portions of the feldspar, like the outer margin, relatively more stable and permitting them to persist more or less intact while the less stable interior of the phenocrysts underwent a considerable and often a nearly or complete recrystallization and replacement. The question of the instability of phenocrysts as originally formed will be discussed more fully in a later paragraph when the general question of the crystallization of the rocks will be considered.

In extreme cases, particularly where the rock shows otherwise evidences of extreme modifications, the phenocrysts have been reduced to a fine mixture of two feldspars in size of grain very near to that of the groundmass itself, and, if it were not for the preservation of a part of the "streaks" and marginal portions of the phenocrysts, one would be sometimes in doubt whether the rock contained feldspar phenocrysts or not. The introduction of the granular material of the groundmass along breaks in the phenocrysts is also a very common phenomenon.

Another conspicuous change is seen in the modification of the hornblende groups. The beginning of the alteration shows first in the loss of the highly spongiform appearance of the groups brought about by the development of minute rods or fibers of deep blue amphibole about the edges or replacing the original mineral. Further changes result in a general breaking up of the groups, the inner and more massive portions, where these occur, being also involved. Small clusters of prismatic to almost fibrous, deep blue amphibole of the riebeckite type, associated with abundant magnetite crystals, and often with recrystallized or secondary feldspar and quartz, take the place of the original group. At the same time more or less elongated prisms and more irregular grains develop in the immediate neighborhood, and to a greater or less extent, also make their appearance throughout the rock generally. The aegirite associated with the hornblende seems to be less easily affected but is nevertheless finally involved in the alteration. Still further changes, in which both long continued deep-seated alteration and more superficial decay doubtless play a part, more or less completely destroy the original hornblende and pyroxene and the resulting products, blue riebeckite shreds, magnetite with hematite and limonite or other ferruginous products, become generally distributed through the rock and only clusters of magnetite grains etc. remain to mark the position of the original dark silicates. Of course the microliths of the groundmass also suffer a corresponding alteration.

The quartz phenocrysts are often impregnated with the secondary amphibole, and in the more advanced stages, are broken up and the parts scattered. The quartz of the groundmass in these altered phases is perhaps rather more distinct than in the normal rock. This is in part because it is little affected by the changes, but also in part because at times its grains appear to have suffered some enlargement. It thus happens, as a result of the changes enumerated, that the granite-porphyry is not infrequently reduced to a fine grained aggregate which shows only a faint indication of its originally richly porphyritic character. In such, where superficial weathering has developed a large amount of hematite, etc., the rock has the appearance of a feebly porphyritic, reddish or purplish felsite and may not at first sight be easily distinguished from the aporhyolite of this area.

Microscopic Characters of the Porphyry nearer the Contact.— As noted under the megascopic description, the porphyry undergoes a marked change in texture whenever its contact with the aporhyolite is approached, and although with the exception of the two special areas, Pine Hill and the Pine Tree Brook Reservation, no actual contacts with any other rock occur, there are considerable areas over which the porphyry shows, both in the hand-specimen and under the microscope, unmistakable evidences of being a contact phase.

Thin sections cut from several series of specimens, taken every few inches from the contact going toward the aporhyolite, show that the first change that becomes apparent is in the hornblende groups. They are smaller, relatively more numerous and tend toward an imperfect, short prismatic habit. While still intergrown with the groundmass grains they commonly show a tendency to grow about one or more of the groundmass grains as a center. The massive crystals are less numerous. The aegirite of the groundmass assumes a distinctly more prismoid habit, the crystals being short, stout and better formed, and they often show a tendency to form clusters and to a parallel arrangement. The larger aegirites have much the same habit as elsewhere in the porphyry.

As the contact is approached more closely there is a rather sudden change in the groundmass. It becomes much finer, the aegirite forms very abundant minute prismoids which are commonly arranged with a distinct flow structure about the phenocrysts. The riebeckite also forms minute rods and flakes with a somewhat elongated habit. Minute magnetite octahedra and hematite grains are abundant. The phenocrysts of feldspar and quartz are somewhat less numerous and the quartz has increased relatively to the feldspar (as shown by the Rosival measurements in the table, p. 243, Column II). Though frequently imperfect, particularly on one or two sides, and often broken, the feldspar crystals appear to have been well developed crystallographically as do also the quartz phenocrysts, the latter often showing a quite perfect di-hexahedral habit. The feldspars are now almost wholly recrystallized to an albite-microcline microperthite which occasionally takes on a very curious habit (see below), and although there appears to have been some of the same albitization etc. as has been described for the granite-porphyry, the changes in the contact phases appear to have been more in the nature of a simple recrystallization. The later rims of groundmass age are either lacking or are developed to a slight extent. Many of the quartz crystals, locally, show a well developed rim of later growth. Occasional crystals or groups of hornblende seem to have been present as judged by alteration products. There is often a strong clustering of the aegirite microliths as if in an attempt to form a larger crystal, and there are quite numerous small prismatic crystals of aegirite. Many aggregates of aegirite crystals occur whose outlines and close packing suggest that they were originally formed from a homogenous crystal of pyroxene.

The curious mode of alteration of the feldspar phenocrysts which was referred to immediately above occurs rarely, and is not strongly developed in the contact porphyry as a whole. The finest examples were observed in slides from specimens taken a few feet east of the aporhyolite contact of Hemingway Hill. It is so unusual, so far as the writer's experience goes, that its peculiarities will be noted and illustrated by microphotographs. For the most part, the phenocrysts have been completely recrystallized into curious irregular areas of slightly radiate intergrowths of albite and microcline, giving the impression of a delicate tracery (see Figure VIIa and b, Plate 2). Occasionally the borders of the phenocrysts have been replaced, wholly or in part, by a band of short albite laths alternating with microcline (see Figure VIIa).

At the immediate contact and for a few inches away, the groundmass becomes extremely fine so that it appears almost isotropic with low powers, and is only imperfectly resolved even with very high magnifications. It consists of exceedingly minute prismoids of aegirite mingled with a feebly polarizing aggregate of quartz and feldspar; also with much fine dust, magnetite octrahedra and hematitic material. These latter may be due to alteration which has affected to a greater or less extent all of the specimens which it was possible to collect. Replacements of what appear to have been small pyroxene phenocrysts may be occasionally seen. The feldspar and quartz phenocrysts are less numerous and smaller, and the proportion of quartz relative to feldspar has increased (see Rosival measurements, table p. 243, Column III). The feldspar in all the slides examined is changed into microperthite or a fine mixture of the two feldspars. Both the quartz and feldspar, particularly the former, originally possessed rather sharp outlines and the quartz shows less resorption than in many parts of the porphyry. Flow structures in the groundmass are very pronounced but there are no indications that the rock was ever, even in part, glassy.

As already noted, a characteristic of the porphyry near the contact is the presence of what look like inclusions, or of streaks and spots of different color and texture from the matrix. These are in large part unquestionably parts of the porphyry itself and the microscope indicates that they are in large part fragments of the immediate contact rock. Some of the streaks show evidences of much recrystallization, and seem to be drawn-out and recrystallized fragments. At the contact with the aporhyolite are found many small rounded or subangular fragments which megascopically seem to be certainly pieces of the aporhyolite; microscopically however, they do not show precisely the same structures, nor are they like the other inclusions of the porphyry itself. They are believed to be parts of the aporhyolite subsequently recrystallized and otherwise changed during its inclusion in the hot porphyry mass. At a number of points, where at present no contact with other rocks is to be found but which from their structure show conclusively that a contact was originally only a short distance away (vertically), the brecciated character of the porphyry is most striking. In such, for example, a short distance from the porphyry-aporhyolite contacts on the small knolls south of Wampatuck Hill and again in the extensive ledges northwest of the aporhyolite contacts on Hemingway Hill, the finest example of the brecciated porphyry are to be observed. In these cases the matrix is a quartz-feldspar porphyry with marked flow structure and of the type found two or three feet distance from the actual contacts. The texture of these inclusions is usually quite irregular, and it is doubtful if any of them represent original textures. Many of them appear to have been drawn out and moulded, as it were, by the enclosing matrix. In such there has been much recrystallization of the original constituents. Growths of aegirite and the two feldspars are common, normal to the surfaces of the phenocrysts, and to the margins of the inclusion itself. The aegirite and microperthite form curious radiating intergrowths and in some, a poikilitic intergrowth of aegirite and albite resembling the "diabase" structure, may be seen. Many of the inclusions contain what appear to be vein structures and some are, in common with the enclosing porphyry, crossed by minute quartz veins. Fluorite is present and is often so abundant as to be megascopically visible. These structures seem to the writer to be explained by the hypothesis that they were produced by the breaking and tearing off of the first consolidated portions of the porphyry magma, which had formed against the cooler rocks of the contact zone, by movements in the still partially fluid, and doubtless highly viscous, mass beneath. With these were doubtless fragments of the aporhyolite not now clearly distinguishable. Once included, the fragments were modified in shape and more or less recrystallized by the hot, enclosing mass, with its mineralizing vapors. The porphyry of the contact zone may, therefore, as a whole, be regarded as possessing a somewhat modified taxitic structure.

Contact Porphyry of the Pine Hill Area.— The contact phase of the porphyry as developed on the eastern slopes of Pine Hill are deserving of special notice (see here special map of Pine Hill area). In the first place the distance from the granite to the felsite aporhyolite as has been noted earlier, is much narrower than elsewhere, ranging from perhaps 15 ft. at the extreme eastern end to perhaps 30 ft. a little east of the summit of the hill. The narrower contact zone indicates that here, as held by Professor Crosby, a deep part of the contact zone is exposed and the characters of the rocks fully bear this out. At the immediate contact with the aporhyolite we find a very dense, not conspicuously porphyritic rock. In fact, it is often impossible without microscopic preparations to tell when one is dealing with the aporhyolite and when with the porphyry. The similarity is increased by the fact that the porphyry at the contact, here as elsewhere, is much brecciated, giving rise not only to the appearance of flow structure but also to that of small apophyses of dense material running into the porphyry, and these might easily be mistaken for apophyses of the aporhyolite cutting the porphyry. Such they were supposed to be by Crosby and they naturally constituted the strongest argument for the intrusive nature of the aporhyolite.[37]

A few inches back from the contact, the rock quite suddenly assumes a strongly and coarsely porphyritic habit. The quartz and feldspar phenocrysts here attain the largest size met anywhere in the field and the feldspar is relatively more abundant than elsewhere. (See table, p. 243, Column IV, of Rosival measurements on the porphyry). The feldspars show much the same characteristics as those in the Rattlesnake type, except that there is a stronger development of the y, $\bar{2}01$ face, with a corresponding approach to the "rhomben" type of habit, and that the later additions of groundmass

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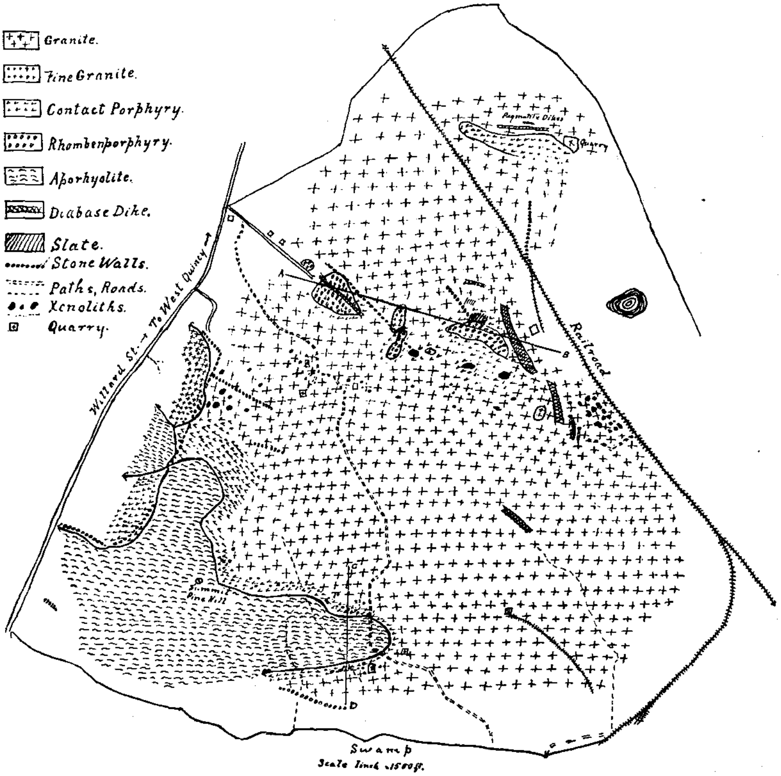
[37] To illustrate how natural was this mistake regarding the nature of this contact the writer may say that he collected several suites of specimens illustrating as he supposed the succession of types across the contact, only to find, when they were sectioned, that they were all of the porphyry and that the aporhyolite had not been reached. The true contact was found two or three inches beyond.


Granite.
Fine Granite.
Contact Porphyry.
Rhombenporphyry.
Aporhyolite.
Diabase Dike.
Slate.
Stone Walls.
Paths, Roads.
Xenoliths.
Quarry.
Willard St. → To West Quincy
Pegmatite Dike
Quarry
Railroad
A
B
D
Summit Pine Hill
Swamp
Scale 1inch =1500 ft.

SPECIAL MAP OF THE PINE HILL AREA, WEST QUINCY, MASS.

This map is based on an outcrop map made for the writer under his direction by Mr. J. D. Mackenzie, to whom the writer is indebted for a painstaking piece of work. The patches of the rhombenporphyry and slate have been purposely exaggerated in size, as have also the xenoliths, in order to have them show on a map of this scale. Many more small xenoliths are scattered through the granite elsewhere in the field but have been omitted from the map. The southern portion of the area has a general elevation of about fifty feet above that of the northern half. While the surface is extremely rough, the actual differences in elevation are inconsiderable.

age are small or fail. The hornblende and aegirite also show much the same characters as in the Rattlesnake type although they have been disturbed more by movements in the rock. Fluorite is particularly abundant in the larger aegirites in the form of small included grains. The groundmass is less regular in texture and grain than in the Rattlesnake Hill type and is on the average a little finer. In fact the groundmass varies much from slide to slide and sometimes in the same slide. In some there is an inequigranular mixture of quartz and feldspar with abundant small prismoids of aegirite, with flakes, shreds, fibers and grains of riebeckite; in others the riebeckite predominates to the exclusion of the aegirite. There is often a very fine poikilitic intergrowth of the feldspar and quartz generally attached to the feldspar phenocrysts (see Figure VI, Plate 1). Flow structures are strongly developed in many parts of the groundmass and are less conspicuous or almost wanting in others. In all of the specimens examined, magnetite grains and octahedra were abundant, and many of the larger hornblendes showed signs of considerable alteration so that it is certain that part, at least, of the fibers and shreds of riebeckite seen in the groundmass are secondary in origin. It may be noted that the irregularities in the structure of this phase of the porphyry are quite distinctly visible to the eye on well glaciated exposures in the field. They appear as rather faintly marked streaks and patches of slightly differing color and texture, and it is easily discerned that the general direction of the flowage in the rock was parallel to the contact. The aporhyolite on the other hand shows, so far as the writer has observed, no megascopic flow structure near the contact.

The band of coarsely porphyritic rock is apparently somewhat variable in width but does not in any case exceed a few feet. It passes rapidly into a rock with a distinctly granular groundmass. The quartz phenocrysts show well rounded outlines and the "rhomben"-like habit of the feldspars persists. Their size is on the average about the same as in the preceding phase and is very close to that of the larger feldspar grains of the granite itself. The hornblende becomes distinctly more granitic in habit, though it still includes the groundmass grains; the groundmass generally begins to assume a finely granular texture.

As one recedes still further from the contact the demarkation between phenocrysts and groundmass becomes less and less distinguishable and the minerals assume gradually the relations to each other found in the porphyritic type of the granite and this, as noted earlier, passes eventually into the normal Quincy granite. The width of the zone measured normally, from the contact to the point, or rather interval, in which the rock can be fairly termed a granite, in some places is certainly not over 10–15 feet while in others, less easily determined, it is probably in the neighborhood of 30–40 ft. How far it is to where the normal Quincy granite is typically developed can not be estimated exactly, but it is safe to say that it is measured by not over a few tens of feet in this part of the area.

After passing the summit of Pine Hill the character of the contact porphyry changes (fault?) and it is less favorably exposed for study. It appears to pass into the Rattlesnake Hill type but as the extreme western end of the area is reached there is again a change, fine granite and more basic phase of the porphyry coming in, which are indicative of a near approach to a slate contact, such as in fact is actually exposed in the northern part of the area as shown on the special map.

On the southern and sharply rounded face a small hill to the east of the railroad track running east of the Pine Hill area, is a thin cover of granite-porphyry. This lies on the granite and forms a sharp contact, marked by the development of abundant long hornblende crystals, which dips a rather low angle to the south. Immediately beneath this porphyry numerous riebeckite pegmatitic dikes and stringers occur in the granite.[38] This porphyry is unusual in its texture. Over small irregular areas it is strongly and coarsely porphyritic (feldspar with subordinate quartz) otherwise it is rather feebly and unevenly porphyritic. The microscope shows, that the feldspar, phenocrysts and groundmass, is a microperthite like that of the granite but contains a greater proportion of albite, and the latter is very strongly developed marginally and as distinct crystals. The quartz tends toward micrographic intergrowths and its total amount is relatively high. Aegirite is abundant alone and intergrown with the riebeckite, which here has often an unusually great elongation; accessories as in the granite. It has possibly been affected by pneumatolitic action connected with the pegmatitic intrusion, but in any case it shows a variation from the other types of granite-porphyry that is of interest. Its feebly porphyritic phase resembles closely the more acid type of xenoliths.

So far the characteristics of the porphyry with relation to its contact with the aporhyolite have been considered. What its contact phases were with relation to the higher slate and intermediate contacts were we can only conjecture. But there appears to be no good reason for supposing them to have been in any essential particulars different from those obtaining for the aporhyolite at the same levels. As for the porphyry contacts with the slates at *deeper levels,* such as in the northern part of the *Pine Hill area* and in the *Pine Tree Brook Reservation,* we find them characterized by the presence of a peculiar and characteristic basic phase which will next be described.

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[38] Warren & Palache, op. cit., p.

Chemical characters.— A specimen of the light grey porphyry was obtained from well inside a large quarried block from the eastern side of Rattlesnake Hill. Thin sections showed the specimen to be almost perfectly fresh, except for a little calcite. The results are the averages of closely agreeing duplicates.

	13.		10.	14.
	Per cent.	Molec. Ratios.		
SiO_2	72.88	1.214	71.41	73.35
ZrO_2	.10	.004	.10	
Al_2O_3	12.30	.122	12.74	14.38
Fe_2O_3	1.67	.011	1.75	1.96
FeO	2.10	.029	2.33	.34
MnO	.10	.001	.10	
MgO	.09	.002	.06	.09
CaO	.87	.014	.85	.26
Na_2O	4.43	.071	4.59	4.33
K_2O	4.90	.052	5.00	5.66
H_2O-	.15		.10	
H_2O+	.31	(.017)	.56	
CO_2	.30	.007	.40	
$T{:}O_2$	.35	.004	.38	
P_2O_5	tr		.22	
Total	100.55		100.59	100.37

Spec. G. of No. 13 at 20° C., 2.667.

13. Granite Porphyry, Quarry, from east side of Rattlesnake Hill, Blue Hills Reservation. Analyst, C. H. Warren.
10. Fine Granite, Ruggles Creek, Quincy.
14. Paisanite, Mosques Canyon, Apache Mountains, Transpecos, Texas; A. Usann. T. M. P. M., XV, p. 439, 1895.

The norm is as follows:

Quartz	26.64	92.23 Salic Minerals.	$\frac{\text{Sal}}{\text{Fem}} = 12.1 > \frac{7}{1}$; Class 1. $\frac{Q}{F} = .40 < \frac{5}{3} > \frac{1}{7}$; order 4, quarofelic.
Orthoclase	28.91		
Albite	36.68		
Acmite	.46	7.60 Femic Minerals.	$\frac{K_2O + Na_2O}{CaO} > \frac{7}{1}$; Rang 1, peralkalic. $\frac{K_2O}{Na_2O} = .7 < \frac{5}{3} > \frac{3}{5}$; Subrang 3; sodipotassic. *Liparose.*
Diopside	3.45		
Hypersthene	.76		
Magnetite	2.32		
Ilmenite	.61		

The rock would, therefore, be termed a sodic-hornblende, aegirite-granophyro-liparose. The phase with a dense groundmass found at or near the contacts would be a graniphyro-liparose.

The mineral composition for the porphyry has been calculated approximately as follows (calculated to 100%).

Granite Porphyry			Fine-Granite
Quartz	26.7	26.7	23.3
Albite	34.8	63.8	66.4
Microcline	29.0		
Pyroxene Hornblende	8.1	9.5	
Magnetite	.76		
Ilmenite	.61		
Zircon	.10		10.3
	100.0		100.0

Ratio: Feldspar to Quartz		
Porphyry		Fine-Granite
2.35		2.84
Albite to Microcline		
1.20		1.25
54.5	Albite	55.6
45.5	Microcline	44.4
100.0	Microperthite	100.0

In chemical composition the rock resembles closely the fine-granite. It appears to be less feldspathic and more quartzose and slightly lower in dark silicates. The mineral composition of the two in these particulars is shown above in parallel columns. The microperthite is closely similar in the two. The soda-iron silicates are, in the case of porphyry, aegirite with some aegirine-augite, cataphorite and riebeckite and a little aenigmatite, while in the fine-granite only riebeckite is present. This illustrates well how slight chemical difference together with different conditions during consolidation (as evidenced by the marked differences in texture of the two rocks) may determine quite surprising differences in the minor essential minerals.

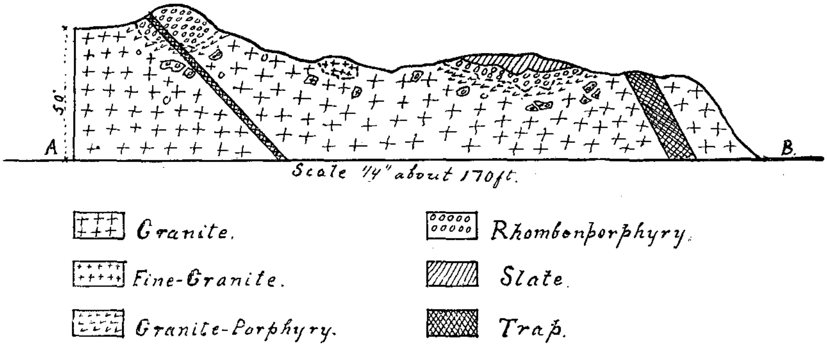
In order to see if there are chemical differences between the granite-porphyry and its contact phases two partial analyses were made: one (No. 11), on as fresh a specimen as could be obtained from the porphyry-aporhyolite contact just south of Rattlesnake Hill. This specimen came from the immediate contact and under the microscope showed small phenocrysts of quartz and feldspar embedded in a very fine groundmass rich in aegirite microlites. The second specimen (No. 12) came from about a foot from the porphyry-aporhyolite contact near its eastern end on Pine Hill, West Quincy. The specimen is of the coarsely and profusely porphyritic type with feldspar phenocrysts of large size, and is the type characteristic of a deeper level of the contact, with these analyses is given in part that of the normal granite-porphyry of Rattlesnake Hill (No. 10).

	10 Granite-Porphyry Rattlesnake Hill	11 Contact Porphyry S. of Rattlesnake Hill	12 Contact Porphyry from E. of Pine Hill
SiO_2	72.88	73.15	72.88
Al_2O_3	12.30	13.07	12.03
Fe_2O_3	1.67 } 3.77	2.18 } 3.89	2.67 } 4.19
FeO	2.10	1.71	1.52
CaO	.87	.68	.40

While the specimens were somewhat altered the analyses indicate that there has been little, if any, real differentiation in the porphyry. The almost exact reversal in the relations of ferrous to ferric iron of the extreme contact phases, in comparison with the rock of the main mass of porphyry, with but a slight gain in the total iron oxides, is striking and in keeping with the strong development of aegirite in the contact rock, and may point to stronger oxidizing conditions near the contact, though in view of the variation in these oxides in the granite itself, its significance is doubtful.

Dark, Alkali-Feldspar- or Rhombenporphyry.

This rock is the one to which Professor Crosby applied the name "basic-porphyry." While the rock is distinctly darker in color and suggests a basic rock in its general appearance, and in fact, is truly more basic than the other rocks of the area, it is not a particularly basic rock containing in any case not much under 60% of silica. The term "rhombenporphyry," which will be used here in describing this rock, seemed an appropriate one on account of the "rhomben" habit of the feldspar phenocrysts, and also, on account of the strong resemblance of the rock, particularly in its microscopic characters, to certain rather fine-grained and somewhat altered rhombenporphyries which the writer had examined from the Laurvik region in Norway. The typical rhombenporphyry, it is true, is characterized by much larger phenocrysts and differs chemically from the present rock in some respects.



Section through the Northern Part of the Pine Hill Area.

This section is intended to illustrate the relations of the intrusive rocks against the intruded slate where the latter formed relatively deep projections into the igneous mass (deep contact levels). The rhombenporphyry and slate masses have been somewhat exaggerated as to size in order to show them to better advantage. The section is however based on the outcrops as shown on the special map of this area.

This porphyry, as held by Crosby, [39] is a marginal differentiate of the magma developed along relatively deep slate contacts of the batholith. If we may include the dark colored, porphyritic knots which are, as will be shown, very closely related chemically and mineralogically and which are believed to be of identical origin with the rhombenporphyry we may say that, a relatively basic feldspar-porphyry phase was developed not only against the deeper projections of slate but also to a small extent against the deeper contacts of the aporhyolite, and in locations such as those just underneath the granite-porphyry in the higher levels of the contact zone, where the magma remained fluid for a sufficient length of time under the cover of its own porphyritic phases to permit differentiation to take place.

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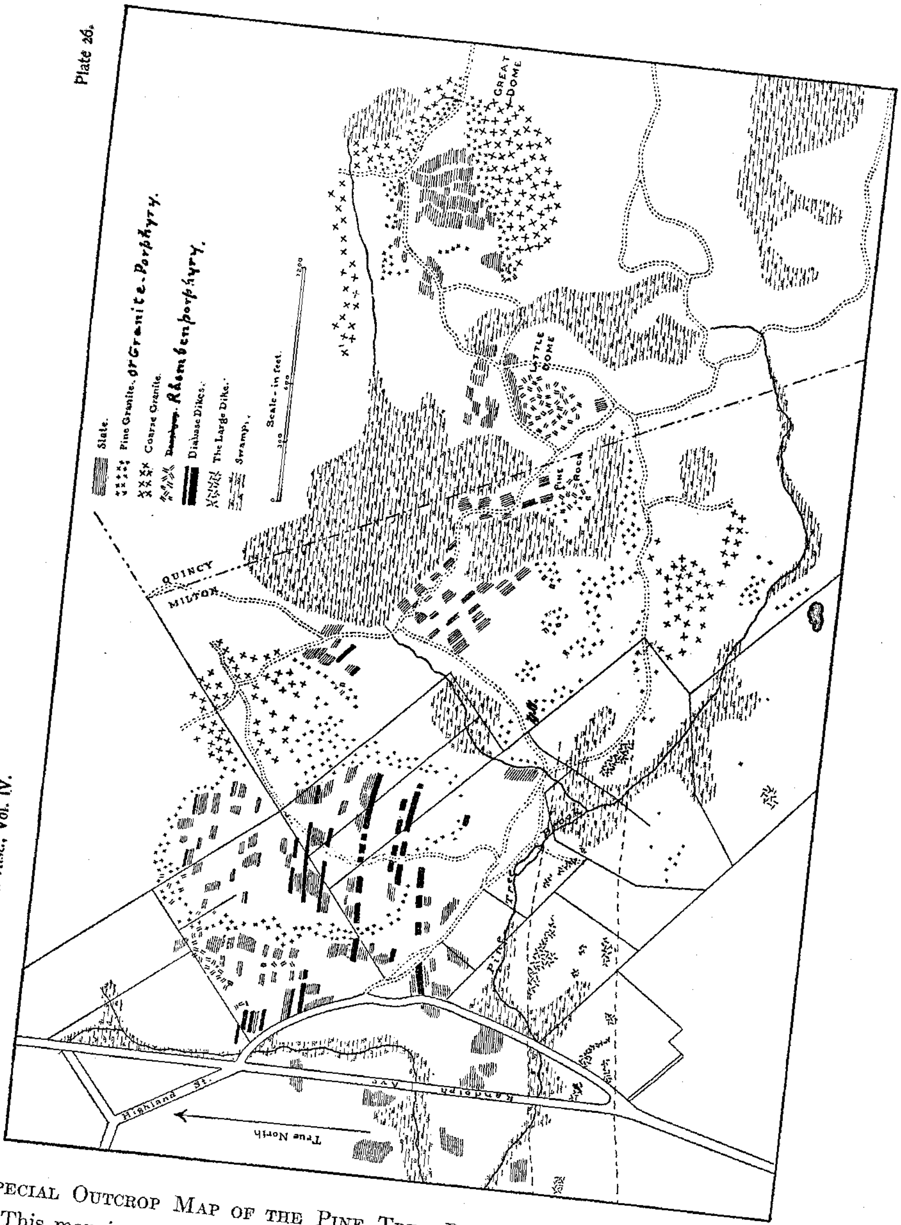
[39] op. cit., pp. 370–371.

Distribution.— The areal distribution of this rock is relatively very small. It is limited in its occurrence to the Pine Hill and the Pine Tree Brook areas, in both of which it occurs at or near the slate contacts, and is associated with developments of the fine-granite and with abundant xenoliths in the adjoining granite-porphyry and porphyritic granite. None of its outcrops are continuous over any considerable area and it is doubtful if any single mass of it has a continuous area of over 10,000 square feet, or a thickness measured by more than a few tens of feet. The majority of its exposures are considerably smaller than the figure just mentioned and the total volume of the rock is relatively small. The best exposure for study and collection of material is found in the northern part of the Pine Hill area (see special map), at the end of a short street which runs east from Willard Street in West Quincy. At this point is a prominent ledge which in the past has been worked for road metal. The rock has been quarried in an open cut which gives excellent exposures for a distance of some 50 feet in length and to a depth of about 15 ft. Going east from this ledge toward the railroad track several other smaller masses of it occur. These are situated very close to two small outcrops of Cambrian slate and are perhaps in contact with it. They are, as shown by abundant contacts, enclosed in a granite-porphyry. At a point a few hundred yards southwest of the first mentioned outcrop a number of smaller masses occur enclosed in the porphyritic granite of the contact type. The largest of these masses will not measure more than a few feet in their greatest dimension and from this they grade down to fragments which are comparable in size to the larger xenoliths found in the granite generally throughout this area, as will be noted later. Small dark patches are abundant in the granite of this type from this location along an east-west line as far as the railroad where, in a large and finely glaciated ledge, the inclusions of the rhombenporphyry, together with inclusions of a somewhat different type, are found in the granite on a scale seen no where else in the field. Judging from the character of the granite along this line and its known relation to the contact porphyry, these inclusions cannot lie more than a few yards, and probably feet, below an original slate contact.

Another prominent exposure of the porphyry is found just north of the Great Dome, particularly beside the Sawcut Notch road which passes by this hill on the north. Small dikelets of the porphyry are to be seen cutting the slate and a considerable exposure of the main mass of the rock at this point is also seen in chilled contact with the slate. A little farther west within the Pine Tree Brook Reservation (see special map) proper, on the Little Dome and about Pine Rock are numerous and excellent exposures although the rock is very deeply weathered in all of them. Several small inclusions of the slate are to be seen in the rhomben-porphyry and a thin veneer of slate may be found on several ledges, thus confirming the intimate relation of the rock with the slate contacts as shown elsewhere. A little further west and north, in the lots just outside of the reservation at near the point where the Sawcut Notch road enters it, abundant patches of the dark colored porphyry are found included in the large granite dike which here cuts across the slates (see special map).

Megascopic Characters.— As regards the relative proportions of phenocrysts and groundmass, this porphyry varies widely. From a rock consisting of a black to dark greenish-black, finely granular groundmass, holding a few white or greyish white to nearly transparent phenocrysts of unstriated feldspar, it varies through more profusely porphyritic phases to those in which the groundmass is practically indistinguishable to the eye, and indeed even under the microscope is so small that the rock is practically a syenite. This later extreme phase, it may be noted, has been found in only one or two good sized ledges in the Pine Tree Brook area. The general run of the porphyry may be described as dopatic to sempatic, viz. groundmass dominant or equal to the phenocrysts in amount.

The feldspar phenocrysts are as a rule somewhat longer than broad and in their longest dimension will range from perhaps 2 mm. to nearly a centimeter. The most characteristic thing about them is the gently curved sides and the acute terminations of a large proportion of them. The habit is quite strongly "rhomben." A considerable number of them are not simple crystals but consist of two or more parts. It may often be noticed that these parts terminate in two or more


Slate.
Fine Granite, or Granite-Porphyry.
Coarse Granite.
Rhombenporphyry.
Diabase Dikes.
The Large Dike.
Swamp.
Scale - in feet.
0
300
600
1200
GREAT DOME
LITTLE DOME
PINE TREE ROCK
QUINCY
MILTON
Pine Tree Brook
Randolph Ave.
Highland St.
True North

SPECIAL OUTCROP MAP OF THE PINE TREE BROOK AREA AFTER CROSBY.

This map is taken from Professor Crosby's Blue Hill Report and serves to show the general relations of the slate, rhombenporphyry fine-granite, granite-porphyry and coarse-granite. Some coarse-granite cuts the fine-granite etc. which cannot be shown on this scale map. There is also coarser, porphyritic granite in the central portion of the westernmost dike, which has been described in some detail in the text as being remarkable for the abundant xenoliths of rhombenporphyry enclosed in the coarser grained part. The original scale of this map has been reduced in reproducing it for this paper so that 1 inch equals approximately 1000 ft.

distinct points on one, or even both ends of the group. Minute dark grains of pyroxene and specks of secondary minerals may be seen in the feldspars, particularly about the margins. (See Figs. VIII and IX, Plate 2).

With a pocket lense the groundmass has a slightly oily lustre in fresh specimens and is of a yellowish-black or to greenish-black color. The grain is rendered somewhat indistinct by the alteration products which are always present. Occasional grains of augite may be seen together with tiny yellowish crystals of epidote, black magnetites or ilmenites and sometimes patches of compact greenish black secondary material. Rarely a grain of pyrite occurs. Superficial weathering causes the groundmass to rust (brown) and retreat leaving the whitened and roughened feldspar in relief.

Microscopic Characters.— The original minerals present in the type analyzed are, soda-orthoclase,[40] crytoperthite or microperthite and augite with accessory quartz, apatite and magnetite or ilmenite. With these are secondary green hornblende, epidote, biotite, titanite, magnetite, calcite, sericite, pyrite and limonite.

The habit of the feldspar phenocrysts at the close of the porphyritic stage of growth seems to have been that of very acutely terminated crystals with rounded contours, the crystal being frequently, in fact, of almost lensiform outlines in cross-section. The tendency to form composite groups is marked, the individual members being united by quite irregular surfaces and often arranged in a slightly divergent manner, and with separate terminations all pointing as a rule in the same direction. The original outlines, doubtless those which give so often the impression in the hand specimen of sharp boundaries, are more or less obscured in thin section by the later growths of groundmass age. The central parts of the feldspar crystals are in part of homogeneous structure, in part very finely striated and in part distinctly microperthitic. Albite or pericline twinning were not noticed. Measurements made on cleavage fragments give extinctions nearly or quite 0° on 001 sections and up to 13° or 14° on 010. The outer part of the original phenocrysts is usually distinctly more perthitic and gives the impression of having a stronger double-refraction, the effect doubtless of the distinctly crystallized albite. The close of the porphyritic stage of growth is not so sharply marked as a rule as in the granite-porphyry, but is nevertheless perfectly clear and extends around the individuals of a single group, showing clearly that the clustering took place during the porphyritic stage of growth. The later border zone of the phenocrysts is microperthite, similarly orientated to the interior and including small augite grains of the groundmass, often abundantly, and these are commonly orientated parallel to the perthite structure. There is little augite that can be truly said to be included in the central and earlier parts of the phenocrysts. The feldspars have suffered more or less from alteration, and besides sericite and calcite contain, often abundantly, minute crystals of hornblende, epidote and magnetite. In the syenitic phase of this rock exposed in the Pine Tree Brook area, the feldspar, while retaining traces of the characteristics above noted, is much more granitic in habit and consists of a finely developed microperthite.

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[40] The term anorthoclase will not be used here for these feldspars as they appear to be monoclinic and the term anorthoclase (used by Brögger for closely similar feldspars in the rhombenporphyries of the Laurvikite area Norway) implies triclinic symmetry. These feldspars may of course be triclinic with a very small extinction angle, in fact an apparent angle of extinction of 1° to 2° was observed on two or three basal cleavage fragments.

The augite appears to have developed only to a small extent during the phenocrystalline stage. A few crystals with, at most, only a feeble attempt at definite crystalline form are found lying for the most part entirely in the groundmass although some of them lie in the marginal parts of the phenocrysts and penetrate perhaps for a short distance into the older portions. These augites include, and are indented by, the feldspar of the groundmass. They rarely measure 3 mm. in length and are usually not over 1 mm. long by perhaps $\frac{1}{2}$ or $\frac{2}{3}$ as broad. Their margins are irregular and have attached to them, particularly on the ends, smaller grains like those of the groundmass generally. They include apatite grains and magnetite or ilmenite. In common with the augite of the groundmass they are of a pale purplish color and appear to be ordinary augite although the chemical analysis of the rock indicates that they are rich in the "CaFe-" molecule. They are occasionally polysynthetically twinned. Their alteration is exactly the same as that of the rest of the augite.

In the more highly porphyritic types it is only rarely that a phenocryst of quartz can be found. When such occurs, the crystal is rounded in outline and is always bordered by a strong development of augite grains.

The groundmass consists essentially of microperthite (rarely a little separate albite) augite and ilmenite or magnitite. The feldspar is in excess and now contains more or less abundant secondary hornblende, epidote, etc. The feldspar grains are xenomorphic and roughly equidimensional and of somewhat variable size. The most commonly observed dimensions are from 00.1 to 00.2 mm., the range from perhaps 0.05 to 0.3 mm. The augite lies between the feldspar, indents it, and is often included in the larger grains as well as in the groundmass additions to the feldspar phenocrysts. They have the habit either of rounded grains, short irregular prisms, or of considerably elongated (parallel to c′) prismoid forms with irregularly developed edges. A strong tendency is shown for the smaller prisms to grow end to end forming a small train, and many of the prismoid crystals are little more than loosely joined shorter crystals grown end to end. This habit is doubtless due in part to the growth of the augite along the direction of the perthite intergrowth in the feldspar, and it thus also happens that the augite crystals have a parallel orientation over small areas. Ilmenite or magnetite grains are commonly present in or about the augite. The augite throughout shows a strong tendency to alter into a green or bluish-green hornblende often accompanied by a lighter green, micaceous mineral apparently a more or less altered biotite. The alteration appears to be a complicated process which involves not only the augite but the magnetite or ilmenite grains usually found with it, and the adjoining feldspar. The resulting products gradually replaced the augite and spread out into the feldspar particularly along cracks and crystal boundaries. The hornblende is mostly of the finely prismatic, aggregated type, though some appears in the form of more massive crystals; the other principle product consists of fibers or plates or is closely felted with a radiate structure. It is strongly pleochroic in light yellow to pale green tones, shows a parallel extinction and a strong double refraction. It appears to be some form of biotite with a chloritic alteration. It often forms patches occupying the position of original augite and ilmenite. In or about these areas are more or less hornblende, epidote prisms and sometimes well formed crystals of titanite. Occasionally these patches may be observed 3 or 4 mm. across, and these doubtless represent not only the replacement of augite, etc., but point to an accumulation of the secondary products about centers of alteration and replacement. In the more syenitic types of this rock (Pine Tree Brook area) the hornblende has been recrystallized into good sized crystals and aggregates of hornblende which replace the already small amount of groundmass and enhance the highly granitoid appearance of this phase of the rock.

As noted, the magnetite, or more probably ilmenite, is almost always closely associated with the augite. Apatite is quite abundant and is found in or about the augite and also in the feldspar. The quartz is mostly confined to the groundmass where it forms extremely irregular masses moulded in between the other minerals. Its amount is usually very small but in some of the less highly porphyritic types it is more abundant and, as will be noted later, it begins to be more plentiful both in the form of phenocrysts and in the groundmass, in xenoliths of the porphyry further removed from the contacts, and particularly in the xenoliths, such as are abundantly developed in the granite of the Pine Hill area.

In some portions of the less porphyritic and more siliceous types a green to greenish-blue, alkali hornblende makes its appearance, poikilitically enclosing the groundmass feldspar. In these, also, the pyroxene in part shows by its green color the presence of the aegirite molecule, and there is an obvious passage toward the more acid phases now represented largely by the cognate xenoliths to be considered later.

A study of the porphyry-slate contact exposed on the Sawcut Notch road shows that the porphyry forms a chilled contact against the slate. The feldspar phenocrysts become somewhat smaller in size and less numerous as the contact is approached, while at the same time the groundmass becomes very fine. The actual contact seen in thin section shows the two rocks in sharp contact. The slate shows a slightly coarsened grain in some of its minerals and there is a patchy development of biotite plates immediately about the contact within the slate. On the whole, while the slate is hard and very dense, the contact metamorphism appears to have been relatively slight.

Chemical Characters.— The great variation in the texture of this rock rendered the selection of a material for analysis difficult. Several specimens, taken from the old road-metal quarry in the northern part of the Pine Hill tract, furnished the best material, although the alteration of the rock even here, is greater than desirable for chemical study. The sample used was assembled from good sized fragments broken from specimens which represented the principle variations in texture noted, varying from profusely and coarsely porphyritic ones to those finer in grain and only moderately porphyritic. The analysis, therefore, is believed to represent very fairly the average composition of this differentiate of the Quincy-Blue Hill magma. The average of duplicate analyses is given under column 13.

	13 Percent		
SiO_2	58.77	.969	58.82
Al_2O_3	15.78	.155	21.06
Fe_2O_3	2.33	.014	3.26
FeO	6.03	.083	.70
MnO	.10	.001	1.38
MgO	.24	.006	3.03
CaO	3.55	.063	6.83
Na_2O	4.47	.073	3.70
K_2O	5.29	.056	1.26
H_2O-	.29		
H_2O+	1.22		
$T:O_2$	.94	.011	
P_2O_5	1.45	.010	
	100.46		100.04

Sp. G. = 2.72

No. 13. Dark, alkali-feldspar- or rhombenporphyry, Pine Hill Area, West Quincy, Mass., Analyst, C. H. Warren.

In the parallel column is an analysis, by G. Forsberg, of a rhombenporphyry from Slotsberg n. Tonsberg, Norway (W. C. Brogger, Z. K., XVI, p. 35, 1890).

The norm of the "Quantitative" Classification is as follows:

Quartz	4.44	$\frac{\text{Sal}}{\text{Fem}}=3 < \frac{7}{1} > \frac{5}{3}$. Class 11.
Orthoclase	31.14	81.06 Salic Minerals.
Albite	38.25	
Anorthite	7.23	$\frac{\text{T}}{\text{Q}}=.05 < \frac{1}{7}$; Order 5: perfelic.
Diopside	.99	
Hypersthene	7.86	$\frac{K_2O+Na_2O}{CaO}=4.9$; Range 2; domalkalic.
Magnetite	3.25	17.13 Femic Minerals.
Ilmenite	1.67	$\frac{K_2O}{Na_2O}=.76$. Subrange 3; sodipotassic; Monzonose.
Apatite	3.36	
	96.19	

It is impossible to calculate, other than approximately, the mineral composition both on account of the altered character of the rock, and the uncertainties as to the exact composition of the various minerals, nor did it appear, in order to help out the situation, advisable to attempt to separate the feldspar and analyze it on account of the included pyroxene and secondary minerals. The approximate mineral composition (to 100%) of the rock has been calculated as follows: —

Quartz	1.00	1.0
Albite $Ab_{95}An_5$	40.2	71.3
Microcline	31.1	
Pyroxene, etc.	19.4	19.4
Magnetite	3.2	8.3
Ilmenite	1.7	
Apatite	3.4	
		100.0

Ratio, Albite to Microcline = 1.30

Albite	56.3
Mic.	43.7
	100.0

The low magnesia indicates that the pyroxene is largely a lime and ferrous iron — rich member of that family (augite-hedenbergite). It is interesting to note that the proportion of Or: Ab + An as deduced by Vogt[41] for the cryptoperthites, (anorthoclase) of the rhombenporphyries associated with the Laurvikite of South Norway is 42: 58, which is very near that found here viz. 43.7: 56.3. With the exception of the silica percentage, high total alkalies, and the relative proportions of the alkalies present in the feldspar, there appears to be little resemblance chemically between this rock and the rhombenporphyry whose analysis appears above. Nevertheless the rock, texturally and mineralogically, belongs to this type and its occurrence with the alkali-granites of this area is interesting and significant.

Cognate Xenoliths.

Patches differing in texture and usually of darker color, are characteristic of the coarse-granite and its porphyritic phase and of the porphyry along the deeper contacts. They are without exception derived from the magma by some process of differentiation and are believed to represent for the most part, if not entirely, fragments, perhaps somewhat modified, derived from marginal facies, immersed and frozen in the consolidating magma. Following Harker[42] we shall call these masses cognate xenoliths. As has been already noted, they are especially abundant in those parts of the field where the contact porphyry is thin, and where also the more basic, feldspar-porphyry is developed — that is, in portions of the field which represent relatively deeper parts of the contact zone, where the magma was less extensively chilled, and where differentiation could take place. They are likewise strongly developed in the porphyritic phase of the granite immediately underneath the heavy cover of granite porphyry (as on the north side of Rattlesnake Hill) where apparently again the magma had an opportunity to differentiate. They also appear in the granite dikes which cut the porphyry cover from the underlying granite, as shown near Slide Notch and in the Pine Tree Brook areas.

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[41] T. H. L. Vogt, T. M. P. M., **24**, No. 6, p. 524 (1906).
[42] National Hist. of Igneous Rocks. A. Harker, p. 347 (1904).

It is noteworthy that in the thick mass of granite-porphyry as exposed at Rattlesnake Hill, and generally in the same rock which is characteristic of the higher levels of the contact, that cognate xenoliths are of very rare occurrence. The same is true of the fine-granite. Inclusions of the fine grained, little or undifferentiated contact phases, as has been noted earlier, are abundant at the contacts and many angular masses, undoubtedly of the same origin, are to be found in many of the porphyry ledges throughout the Blue Hills proper.

Although there is a strong localization of the xenoliths in the regions indicated, they are also common throughout the granite mass as a whole. The contrast in texture and mineral composition is, to be sure, not as marked in the latter class of occurrences, but it is the belief of the writer, founded on extensive observation about the quarries, that there is not a cubic yard of granite in which some more or less marked variation in texture cannot be found on careful examination. Single surfaces of one or more square yards may be obtained which are practically free from noticeable variations in grain, and many such are to be seen in the finished blocks at the quarries, but a careful inspection of the sides and back of the blocks will show one or more patches differing from the normal more or less sharply. Quarrying has penetrated into the granite to a depth of over 300 ft. and the xenoliths are still in evidence, although the writer feels pretty certain that in depth they are either somewhat less numerous than higher up, or at least they are less sharply distinguished in texture.

The Pine Hill area in West Quincy offers the best opportunity to study the xenoliths which are associated with the deeper contact zones. The darker colored, fine grained and usually porphyritic type is common in the granite-porphyry almost up to the aporhyolite contacts and is also found in the porphyritic-granite immediately underlying it. In this phase of the granite generally, and particularly, in the northern part of the area in the immediate neighborhood of the heavy developments of the rhombenporphyry, this type of xenolith is very abundant, but it is associated with types that are lighter in color and with a more granitoid groundmass as well as with some which are feebly or nonporphyritic. In the more normal granite which is exposed over the eastern and southeastern parts of this particular area but which is always prone to pass into the porphyritic granite, and doubtless underlies it everywhere by only a few feet or yards, the xenoliths are relatively abundant, so much so in fact, that all attempts to use the stone commercially have been unsuccessful.

At a point a few hundred feet west of Willard Street, northwest of Pine Hill (see special map), several good sized masses of the rhombenporphyry occur included in the granite, and with them are smaller masses grading down to those only a few inches across. These megascopically and microscopically differ in no essential particular from the less highly porphyritic phase of the type rhombenporphyry, previously described, except that they show a small development of an alkali-hornblende poikilitically enclosing the groundmass feldspar, and in this respect show a gradation toward the granite-porphyry. Again on the eastern side of the area, beside the railroad track, is a beautifully glaciated ledge where is to be seen the finest exposure of xenoliths anywhere in the entire field. The granite is here of the porphyritic type and is literally packed with xenoliths, chiefly of the dark, fine grained porphyritic type, and it is here that they can be studied to the best advantage. In size they vary from tiny patches consisting sometimes of a single feldspar phenocryst, surrounded by a few millimeters of dark groundmass, to masses two to four feet in length by usually $\frac{1}{3}$ to $\frac{3}{4}$ as much in width. Their shape varies greatly: — subangular, round, lenticular, illiptical, greatly elongated with rounded ends and straight or gently curved sides, one side curved and the other irregular or deeply embayed by prongs of granite, or entirely irregular and invaded by tongues of granite. With the darker, are many lighter colored xenoliths, porphyritic, but with a more finely granitoid groundmass, in some cases conspicuously sprinkled with minute hornblende grains. These form separate masses and also sharply separated zones surrounding the darker xenoliths, or attached to them on one or two sides. An occasional mass of fine granite, of the same type as that found as larger masses only a short distance away, may also be found. The contacts of all of the xenoliths with the granite are sharp though perfectly sealed. There is no evidence of textural blending nor are there any reaction rims. The contacts of the lighter colored and more granular xenoliths, while likewise sharp, are naturally not so strongly marked to the eye as in the case of their darker companions. Examination with a lense shows that the contact is not, however, a simple line, but is indented by grains of the surrounding material and sometimes by small apophyses.

The darker xenoliths are here again strikingly similar in appearance to certain phases of the rhombenporphyry, and are composed of a very fine dark green to almost black groundmass, in which is enclosed acutely terminated and often composite phenocrysts of feldspar. In some, the phenocrysts are relatively few, in others they are very numerous. In some of the xenoliths, quartz crystals are sparingly developed, and these are always characterized by a halo of dark mineral grains (usually hornblende) about them. Rarely small patches of hornblende or pyroxene may be seen. In very fresh specimens the feldspar phenocrysts are almost colorless and often show a fine chatoyancy. On weathering the feldspars whiten and the groundmass becomes dull black and less clearly crystalline.

Microscopic characters of the Xenoliths of the Contact Zones.— The close resemblance to the rhombenporphyry is quite as obvious microscopically as megascopically. The feldspar has precisely the same characteristics, even to the sharply marked rim of groundmass age, including prismoids and grains of pyroxene arranged parallel to the direction of perthitic intergrowth. The feldspar of the groundmass is also the same. The same phenocrysts of pale brown augite occur but these show a strong tendency to pass into a green variety, particularly about the margins. The pyroxene of the groundmass has about the same habit as that in the rhombenporphyry but is of a light to rather strong green color often, with a weak pleochrism. Its double-refraction is low, not exceeding that of ordinary augite, and its optical properties otherwise seem to be those of augite, but it probably contains some admixture of the aegirite molecule. Considerable hornblende is also frequently present in the groundmass. It occurs in part as a later growth on the augite, either in the form of minute prisms or needles, or in a more massive form, and in part with a poikilitic habit enclosing the groundmass feldspar. Strongly pleochroic; α, pale greenish-yellow or brown; β, very dark green; γ, deep olive-green; γ makes an angle of as high as 33 degrees on c', it appears to be an alkali hornblende near catoforite. Biotite occurs as a finely foliated alteration product of the augite and ilmenite or magnetite. The latter is abundant in, and associated with, the augite and also scattered through the rock. Magnetite forms grains and sharply bounded octahedra and may be in part secondary, since it has been noticed that in the rocks of this area the magnetite of secondary formation is apt to form sharply bounded crystals. Apatite is present. Most of the xenoliths show considerable alteration resulting in the presence of calcite, kaolin and ferruginous matter.

The darker type of xenoliths pass on the one hand through all gradations into a type lighter in color — greyish green — and with a more distinctly crystalline groundmass and, on the other, there are gradations into fine grained types with few or no feldspar phenocrysts. So varied, in fact, are these xenoliths that detailed description is out of the question and only certain significant features can be noted. Where feldspar phenocrysts occur they are always characterized by a central core of more or less acute habit about which is a later rim usually, if not always, showing inclusions of the groundmass grains. In the more basic xenoliths the core is in part, if not entirely, homogeneous, or it is finely cryptoperthitic; in the more acid types the core is more distinctly perthitic, often wholly so. The quartz phenocrysts, which occur sparingly in the darker types, are always marked by a more or less strongly developed rim of pyroxene or hornblende; in the lighter colored and more siliceous types the quartz is more abundant and the margin of dark silicates is less marked or nearly wanting. In the more acid types, while some augite is present, it shows a tendency to pass into a green variety, and most of the pyroxene is distinctly green, slightly pleochroic, and seems to be an aegirine-bearing augite probably near augite, since the double-refraction is always much too low for true aegirine-augite. It is of earlier age than the hornblende, which in this type becomes rather abundant and grows about the pyroxene in part, and in part, occurs separately, enclosing poikolitically the feldspar of the groundmass. Most of the hornblende appears to be related to the catoforites although it is in part riebeckitic, particularly about the margins. The latter is also disseminated as shreds and fibers through the rock. Magnetite is an abundant alteration product, accompanied by some biotite. It may be noted here that some xenoliths occur in which little or no pyroxene is present, its place being taken by hornblende, apparently original, although it is true that all the xenoliths of this type that have come under the writer's observation are quite heavily altered, and it is possible that a part of the hornblende may be secondary after pyroxene. Apatite is common in the more basic types, less common in the others. Titanite is present in all, forming irregular masses sometimes associated with the hornblende, but it often lies between or wrapped about the groundmass feldspar and is believed to be wholly of secondary origin.

In the acid types fluorite is present in small grains with the dark minerals and also, like the titanite, wrapped about the feldspar. A little zircon is also present. In most of the xenoliths examined, alteration has produced considerable calcite, kaolin and other products.

The xenoliths of the fine-granite type show little evidence of gradation into the other types, in this respect resembling the fine-granite masses in the immediate vicinity from which they are thought to have been derived.

The xenoliths found in the Pine Tree Brook Reservation, and in the immediate vicinity, are essentially of the same character as those already described. When we come to consider the xenoliths found in the granite of Rattlesnake Hill just underneath the thick mass of granite-porphyry which there covers it, we find much the same characters but with certain differences. In the first place, the acute habit of the feldspar phenocrysts is not noted to the same extent as in the Pine Hill tract, and while there is a just as sharply marked cessation of the phenocrystalline period of growth, the inner core of feldspar is usually almost, if not quite, as distinctly microperthitic as the feldspar of the groundmass and often almost as coarse as that of the surrounding granite. The hornblende seems more highly poikilitic resembling more nearly some of the hornblende in certain phases of the overlying porphyry. Occasional grains of aenigmatite are also present with the dark minerals, and this mineral is also found in the porphyry above and in the granite. In the more siliceous xenoliths, round quartz grains make their appearance in the groundmass rims of the feldspar phenocrysts, and the quartz of the groundmass has also a distinctly rounded habit or is even poikilitically enclosed in the feldspar, a relation that is also observed in the granite-porphyry above. The many dark greenish-black or greenish-grey, fine-grained and non-porphyritic xenoliths which occur here contain abundant aegirine-augite and aegirite, together with hornblende, and although of much finer grain, are texturally much the same as the surrounding granite. Occasionally these show a feeble banding as if they had been drawn out during inclusion in the granite.

In general it may be said of the xenoliths which occur near the contacts, that they partake to a striking degree of the mineralogical and textural characteristics of the contact facies of the magma which are developed *en masse* in their immediate neighborhood. Though their margins are moulded, and more or less invaded by the enclosing rock, the actual contacts are sharp and there is no evidence of reaction between them, nor of any notable transfer of material from the enclosing magma to the xenolith. Their probable mode of origin as well as that of the xenoliths of the normal granite will be considered later, when the general process of crystallization, etc., of the magma is taken up.

Microscopic characters of the Xenoliths of the Normal Granite.— The prevalence of xenoliths in the coarse granite of the quarries has been noted above. Mr. Dale[43] states that their sizes range from one half an inch to 2 feet by 1 foot, 6 inches; 2 feet, 6 inches by 2 feet, 6 inches; 3 feet by 4 inches, and 6 feet by 2 feet, but that they are usually small and roundish or elliptical in outline. The present writer is quite in accord with these statements. Mr. Dale also divides the "segregations" into three classes, which so far as their megascopic characters are concerned are substantially the divisions given below.

Although there appears to be more or less gradation and no sharp line can be drawn, three types may be made as follows: —

(1) Essentially fine-grained to almost dense xenoliths of a *dark* to *medium bluish* or *greenish-grey color;* usually irregularly, though not abundantly porphyritic, the phenocrysts being chiefly feldspar and quartz with occasionally irregular patches of pyroxene or hornblende. The phenocrysts are sometimes rudely clustered.

(2) Essentially fine to medium grained xenoliths of *light* to *medium grey color:*— about the same shade as the enclosing granite; usually contain a few, sometimes a good many phenocrysts of feldspar and quartz and occasionally irregular crystals of pyroxene or hornblende. The phenocrysts are not as a rule very evenly distributed and tend to form clusters.

(3) Fine-grained *greenish* or *yellowish-green* xenoliths often having a feeble banding; not usually porphyritic. These are substantially like many of type (1), and appear to differ from them chiefly in being more altered and in having been sometimes sheared, thus developing a banded structure.

The contacts of these xenoliths is essentially a sharp one, though naturally the grains of the surrounding granite project into the xenoliths about the margin and occasionally tongues of the granite penetrate them. There is never any sign of chemical interreaction between the two.

Thin sections of the darker colored xenoliths of type (1), show that they are essentially a pretty even-grained mixture of microcline-microperthite and some quartz, aegirite-augite, aegirite and green or blue alkali-hornblende. The latter and a part of the pyroxene often encloses the feldspar in poikilitic fashion. With these are the usual accessory minerals found in the granite. The lighter colored xenoliths of this type (1) are more siliceous and contain proportionally more aegirite. The phenocrysts of feldspar when present — which is usually the case — consist of a fine microperthite, and show, as in all of the xenoliths previously described, an inner core about which is a later margin enclosing few to many grains of the dark minerals or quartz. Here, however, the texture of the core is about the same as that of the rim. The feldspar and quartz of the groundmass are sometimes entirely xenomorphic, in other instances the feldspar is quite rectangular in outline, in others the quartz is characterized by a distinctly round habit and then is apt to be poikilitically enclosed in the feldspar. The pyroxene forms irregular elongated prismoids lying between the feldspar and quartz, also commonly enclosing the feldspar. The hornblende is wrapped about the feldspars, often forming poikilitic groups of some size. Slender shreds and fibers of blue, secondary hornblende may be abundantly distributed through the rock. Accessory minerals are as usual, although, as in the case of type (2), fluorite, zircon and titanite may be locally abundant closely associated with the quartz and often wrapped about the feldspar in very irregular masses. This suggests a later introduction of these constituents. In certain of these xenoliths the pyroxene shows a peculiar alteration and replacement. The centers of the majority of the pyroxene crystals are replaced, sometimes by a material which appears to be siderite, sometimes by a fibrous or foliated material of medium, mean refraction and high double-refraction, probably muscovite, but usually by both materials present in varying amount. Minute grains of magnetite and sometimes fluorite are associated with these. Decomposition of the siderite develops a yellow stain, and this can be seen in the hand specimen and is believed to account largely for the yellowish color of so many of the fine-grained xenoliths particularly of type (3). The replacement of the center of the pyroxene seems to have been accompanied by more or less recrystallization of the marginal parts which as a rule are granular and very irregular. Material from these has spread out into the surrounding microperthite, particularly along the lamellae of the potash member. The remaining pyroxene is in part aegirine-augite, though outwardly it appears to be aegirite. As augite is abundantly present in so many of the xenoliths of the contact types it is not improbable that the central parts of the pyroxene in these cases was augite, and that it has been decomposed and replaced by processes perhaps connected with pneumatolytic activities in the enclosing granite.

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[43] loc. cit., p. 96.

Microscopic study of the xenoliths of type (2) show that they are uniformly more quartzose and richer in aegirite than the others. In the finer grained xenoliths the grain is quite uniform and will average under a millimeter; in the coarser, the grain may average from one to three millimeters or about the same as that of the fine-granite so abundantly developed in the eastern part of the area. The general texture of this type is granitic, with a feeble and usually irregular porphyritic tendency. The phenocrysts of feldspar, sometimes with quartz, may be clustered in patches of coarser granitic habit. Some trace of an inner core of distinct form may be seen in at least some of the feldspar phenocrysts, even in the coarsest grained types, but the phenomenon is more distinct in the finer grained ones. The feldspar in many of the xenoliths is precisely similar to that of the granite outside, except in size. In others there is a more distinct separation of the microcline and albite. Sometimes separate crystals of almost pure microcline and albite may be seen; but generally the two are intergrown, the microcline forming relatively good sized patches surrounded by finely twinned albite which determines the outlines of the crystal. The quartz is highly xenomorphic in some, in others it is rounded in outline and may be inclosed in the feldspar.

The pyroxene is aegirite. It is in part massive, of the same character as that of the granite, and is to some extent intergrown with riebeckite. The larger massive grains lie about the feldspar, sometimes enclosing them. Much of the aegirite is found lying between the feldspar crystals and penetrates quite deeply into their margins. In such instances it is of very irregular habit often being little more than a skeleton of loosely joined grains and minute prismoids tending to form a single elongated crystal. Sections cut through these irregular aegirites where they penetrate into the feldspar give the impression of a "spatter" of aegirite grains enclosed in the middle of a feldspar crystal. Aegirite in the form of small microlites together with shreds and fibers of blue hornblende are scattered through the feldspar generally. In some of the xenoliths, fluorite is not only present in the form of minute grains in the aegirite (as in the granite) but it also forms masses of considerable extent replacing the quartz. Occurring with it and of the same habit, is zircon. These minerals are both probably of pneumatolytic origin. Titanite is also present in formless grains and is perhaps of similar origin with the zircon.

Other inclusions in the Granite-Porphyry. The rarity of cognate xenoliths in the granite-porphyry of the Rattlesnake Hill type has been noted; also the occurrence in some parts of the porphyry, of very fine grained, angular fragments. These are perhaps best exposed for study in the vicinity of Scamaug Notch, and in many of the smooth glaciated ledges which form Kitchimakin Hill. They vary in size from small fragments an inch or two across to those measuring upwards of a foot on a side. In color, when not oxidized, they are of a light to medium gray or bluish-gray. They are feebly porphyritic containing a few small phenocrysts of feldspar (rarely quartz) up to two millimeters in length. Microscopically they are found to consist of the same feldspar as the enclosing porphyry, quartz and riebeckite, magnetite and alteration products. The riebeckite is partly in the form of abundant small prismatic or flaky aggregates enclosing the groundmass minerals and in part in the form of tiny shreds and fibers. The quartz and feldspar are xenomorphic and somewhat variable in grain. The feldspar phenocrysts are often broken, as are the few quartz phenocrysts that have been noted in these inclusions. These inclusions resemble most nearly some of the fine-grained contact phases of the porphyry and doubtless are derived from such a source, having been broken off and included in the still fluid mass beneath the contacts, and then sunk, or were carried away, from their original place of formation.

Chemical characters.— Analyses of a number of the various types of xenoliths described would doubtless yield interesting results. The writer has, however, confined himself, to two types, partly on account of the labor involved and partly because it is believed that the microscopic evidence is sufficient to show the connection chemically and texturally between the various types. The two types chosen are extreme ones. One is a rhombenporphyry type resembling megascopically very closely the moderately porphyritic type of the rhombenporphyry, and came from a recently quarried block of granite from the northern part of the Pine Hill Tract not far south of the type locality for the rhombenporphyry. The microscope showed that the feldspar was the same as in the type rhombenporphyry; quartz is lacking almost entirely; augite is present but is much less abundant than a green, feebly pleochroic soda-iron type; soda hornblende of the green and blue types is present, and some black oxide minerals; some secondary biotite and other alteration materials are also present. The second chosen was a fine granite type of xenolith, the most quartzose and aegiritic found, and was taken from the granite of the Hardwick Quarry, Quincy. The results of duplicate analyses follow on p. 282.

In comparison with the chemical composition of the type rhombenporphyry the xenolith is slightly higher in silica, also in total iron oxides, and the ferric iron is proportionally higher as might be expected

	16		15	17	3
	Per cent	Ratios			
SiO_2	60.02	1.000	58.77	71.84	73.93
Al_2O_3	14.86	.145	15.78	13.55	12.09
Fe_2O_3	2.80	.018	2.33	2.50	2.91
FeO	6.57	.090	6.03	.39	1.55
MnO	.20	.002	.10	—	tr
MgO	.38	.009	.24	tr	.08
CaO	3.33	.059	3.55	.85	.31
Na_2O	5.64	.090	4.47	10.00 estimated	4.66
K_2O	4.26	.045	5.29		4.63
H_2O+	.78		1.22		.41
H_2O-	.20		.29		
TiO_2	.90	.001	.94		.18
P_2O_5	.63	.004	1.45		
	100.57		100.46		100.75

Sp. G. of No. 16 at 20° C. = 2.80.

16. Rhombenporphyry xenolith, from granite, northern part of Pine Hill, West Quincy, Mass. Analyst, C. H. Warren.
15. Rhombenporphyry,— Pine Hill, Mass. Analyst C. H. Warren.
17. Fine-granite type of xenolith, most quartoze type, Hardwick Quarry, Quincy, Mass. Analyst, C. H. Warren.
3. Granite from Hardwick Quarry. Analyst, H. S. Washington.

from the greater amount of pyroxene and hornblende and the presence of the $Na_2Fe_2'' Si_4O_{12}$ molecule; Al_2O_3 is lower. Soda is here considerably in excess of the potash for the same reason. Lime is lower, though still relatively high for the Quincy rocks owing to the presence of the lime and ferrous-iron rich, augite. If we attempt to calculate the mineral composition for the groups,— feldspars, pyroxenes + hornblende, and accessories,— on the assumption that the proportions of potash feldspar to Ab + An is the same as in the rhombenporphyry (No. 15) it becomes at once apparent that much too great an amount of alumina will then fall to the pyroxene and hornblende to agree with the general alumina-poor character of these minerals in the Quincy magma; also considerable free quartz would result, whereas little is really present. It, therefore, appears that albite is considerably more abundant, in the groundmass, than in the rhombenporphyry (and this agrees with microscopic observations so far as it is possible to judge in so fine-grained a rock). Working on the assumption, that with the exception of some soda combined with the ferric iron, this oxide goes into albite, we may venture a very rough approximation of the mineral composition as follows: —

Feldspars	65.0
Pyroxene, Hornblendes, etc.	30.0
Accessories	5.0
	100.00

Although less feldspathic and richer in pyroxene, etc. than the average of the rhombenporphyry, the xenolith is more siliceous owing to the greater amount of silica called for by the albite and by the soda-pyroxenes and hornblendes.

It is easy to trace with the microscope the gradual change from the moderately porphyritic type of rhombenporphyry, which is developed in the larger masses near the slate, with only the colorless or pale brown augite, through types, occurring as smaller masses down to those comparable in size with the average xenolith, in which pyroxene of the green, sodic type becomes more and more abundant, in which sodic-hornblende becomes also gradually more abundant and in which quartz may appear sporadically, to a type like the one analyzed. From this we pass with increase of silica, decrease of iron and lime, into types which resemble closely some phases of the granite porphyry of the lower and thinner contact zones.

Turning now to the quartzose and aegirite rich xenoliths we may note that in contrast with its enclosing granite (No. 3) that it is lower in silica, and that the iron is lower and nearly all ferric, as would be expected from the presence of aegirite alone without the ferrous-iron bearing hornblendes found in the granite.

The more siliceous, finely granitic and slightly or non-porphyritic xenoliths are often almost the exact counterparts of some of the fine-granite of the contact zones, others seem to depart somewhat from this type. But as has been pointed out, the fine-granite from different localities (different parts of the contact zone) varies somewhat in composition, and it is probably true that there is a close resemblance between the members of these types and the various fine-granites of the contact zone. For example, while the xenolith analyzed (17) differs sharply from the fine-granite of the Ruggles Creek type in having aegirite instead of riebeckite, it does bear a very close resemblance in this respect, as well as others, to the fine-granite and granite-porphyry covering the coarse granite with its pegmatite dikes on the knoll just east of the R. R. track, N. E. of the Pine Hill area. If we disregard minor variations, the resemblances, of the porphyritic xenoliths to the rhombenporphyry and to various phases of the granite- and quartz-feldspar-porphyry characteristic of different parts of the contact zone on the one hand, and that of the finely granitic feebly- or non-porphyritic xenoliths to the fine-granites on the other hand, are striking and have an important bearing on the origin of the xenoliths as a whole.

The Aporhyolite.

Distribution.— This rock has a relatively large development in the area. It occurs in at least three separate masses (see general map), which collectively cover several square miles. The first and largest occupies the southwestern part of the Pine Hill tract, and may be connected with the large area of aporhyolite which occurs within the Blue Hill Reservation lying to the south of Rattlesnake Hill, extending southward to the borders of the Reservation where it is lost beneath the great swamp in northern Braintree. According to Professor Crosby it again appears at one point beyond the southern border of the swamp. The area covered by this occurrence is certainly not less than two square miles. The second mass in size is that found lying to the north and northwest of Fox Hill. The third is a relatively smaller mass occupying a portion of the top of Hemingway Hill.

The Pine Hill aporhyolite (see special map) begins a few hundred feet east of the summit of the hill. It here forms a nearly north-south contact with the porphyry phase of the granite intrusion for about fifty feet. The supposed apophysis of aporhyolite described by Crosby as cutting across the contact-porphyry at this point, appears on microscopic study to be in reality only the very fine (felsitic), extreme contact phase of the porphyry, the contact here being irregular in direction for several feet. From the north-south contact just referred to, the aporhyolite mass extends to the west, and broadening out, one contact runs in a west-southwesterly direction for a few hundred feet when it is entirely lost beneath a heavy mantle of drift, but ledges of the rock extend continuously nearly to Willard Street. The other, and northern contact, is beautifully exposed and runs in a nearly east-west direction to a point a little northeast of the summit. Up to this point, as along the southern contact, aporhyolite is continuously bordered by the coarsely porphyritic and relatively thin zone of quartz-feldspar- and granite-porphyry (with xenoliths of rhomben-porphyry type) earlier described in detail as characteristic of the deeper contact levels. The immediate contact of the two is often very difficult to make out and thin sections are often required to really certainly distinguish the two rocks at the contact. As noted, the porphyry, is at the immediate contact, very dense and feebly porphyritic besides being characterized by breccia and flow structures and gives on megascopic examination alone, the impression that it belongs rather to the aporhyolite than to the porphyry. As can be seen by referring to the special map of this area, the trend of the contact at a point northeast of the summit is irregular, and finally takes a sharp turn to the southwest running along the western base of the hill. The contour of this mass is clearly most irregular. The contact is usually steeply inclined so far as can be told. At one point near the extreme eastern end of the mass it is clearly exposed on a steep cliff marking the western wall of a small valley, and the contact porphyry can here be seen undercutting the aporhyolite with a dip to the south of about 45 degrees.

Whether the aporhyolite on Pine Hill is actually connected with that within the reservation to the west cannot be told, owing to the rather deep, drift filled valley that intervenes. It is probable, as suggested by Crosby that this valley was once occupied, in large part at least, by slate which was eroded away much more easily than the igneous rocks. Contact with the porphyry is found again south of Rattlesnake Hill and can be traced in a westerly direction across the northern slopes of Wampatuck Hill, thence in a gentle curve around the western top of this hill and back along its steep southern face. South of this last line of contact, on the low prominent ledges lying immediately south of the main hill, granite porphyry appears again and extends nearly to the road where it is in contact with the aporhyolite again. The low, rounded knolls that occur directly south and southeast of the road at this point, consist in part of quartz-feldspar porphyry or granite-porphyry, and in part of aporhyolite with characteristic contacts exposed in several places. These are the "islands of quartz-porphyry" referred to by Professor Crosby in his report. Beyond these "islands" to the south, the aporhyolite, so far as can be determined, extends continuously to the borders of the Blue Hill Reservation and probably beyond.

The second, and next most important occurrence begins, as can be seen from the general map, at a point a little north of Fox Hill and runs in a southwesterly direction with a width of at least 500 feet for one-half mile, or as far as Cedar Rock. Beyond this point a heavy mantle of drift makes it impossible to say positively whether it extends further in this direction or not, but it probably does, since as Professor Crosby has pointed out, outcrops of the same rock occur further west beyond Randolph Avenue. The southern and more elevated contact with the porphyry is exposed at several points and it appears to be a nearly straight line with minor irregularities. The northern contact is unfortunately unsatisfactory. At one point Professor Crosby states that it is in igneous contact with slate, but the extremely altered condition of the exposures renders their study of little value. The changes in the porphyry along the southern contact are, as has been noted earlier, more suggestive of the contacts exposed at the eastern end of the Pine Hill mass than that about Wampatuck Hill, although the coarsening of the porphyry is not so marked. As the granite is exposed only a very short distance north of what must be the northern contact of the aporhyolite near its eastern end, the intervening porphyry zone must be a thin one. From these facts, and from the generally massive character of this mass of aporhyolite, it is thought that the southern and more elevated contact represents a deeper zone than that exposed about Wampatuck, but more elevated than that on the eastern side of Pine Hill; and also that if the contact was exposed along the northern side it would be like that east of Pine Hill, just as in the adjoining Pine Tree Brook area the slate contacts are the same in character as those in the northern part of the Pine Hill tract, viz., of the deeper level type.

The third mass of aporhyolite occupies the northern top of Hemingway Hill, as shown on the map. It is an elongated mass of no great width and shows the same contact phenomena with the porphyry as those found about Wampatuck Hill, except that there is perhaps rather more brecciation of the porphyry, particularly near the northern end of the hill. A marked flow structure with taxitic structure characterizes a part of the exposed portions and the rock generally appears like that on Wampatuck and Pine Hill.

Megascopic characters.— The prevailing color of the aporhyolite is a dark reddish-brown or purple. Locally, where it has suffered from strong surface weathering, it becomes whitish with brown rust spots. In places it is an almost perfectly dense, structureless rock though it usually shows a few small rectangular feldspar phenocrysts and less abundant minute quartz blebs. Locally the porphyritic texture becomes more prominent. Flow structures are common and in certain localities, such as on the top and northern slopes of Pine and Wampatuck Hill, and on the top of Hemingway Hill, the flow structure is very strongly and beautifully developed. Taxitic structures are common, especially where the flow structures are most in evidence. Spherulitic textures may also be seen in many places, but the latter is not a striking megascopic characteristic. Under the hammer the rock is tough and breaks with a sub-concoidal fracture. It is finely jointed, breaking up into small, angular, prismatic blocks. Along many joints and fracture lines, quartz or calcite, or both have been deposited. The large mass of aporhyolite lying northwest of Fox Hill is more uniform and massive than the other occurrences and shows little of the flow and taxitic structures.

Microscopic characters.— Microscopically the rock shows no unusual features for this class of rocks and a brief description will suffice. Thin-sections and the chemical composition show conclusively that the aporhyolite belongs to the same series as the granite, etc. Phenocrysts are few and irregular in distribution, feldspar predominates, quartz appearing only in the form of minute grains. The feldspar phenocrysts are as a rule, when not broken mechanically, fairly sharp in outline, and of a square or rectangular form. The large ones may measure as much as 2½ mm. on a side but are usually considerably smaller. They now consist entirely of a fine microperthite verging toward cryptoperthite in places. They are often fractured and broken apart. The quartz phenocrysts, when they occur, are small and are apt to be rounded or irregular. They do not show marked resorption. No dark silicates are developed as phenocrysts, and in fact these are missing from the rock as a whole. The body of the rock shows considerable variation in texture. No part of it is now glassy, but much of it is so fine as to appear isotropic with low magnifications and it is only with very high powers and strong light that it is seen to be entirely crystalline. The greater part of the aporhyolite is a fine, variously textured mixture of alkalic feldspar and quartz. Most of the feldspar appears to be microperthitic although distinct albite laths can often be detected. Through this mass is everywhere sprinkled, more or less irregularly, tiny grains of hematite and magnetite, the latter commonly showing sharp octahedral outlines. The other alteration products are, kaolin, serecite, calcite, siderite and limonite.

Spherulites made up of segments of finely fibrous quartz and feldspar are common. They may be scattered through the mass or they may make up almost the entire field, again they are arranged in bands. The spherulites contain abundant grains of hematite or magnetite, or both, arranged parallel to the radiation, and the same minerals are often thickly clustered about the margins of the spherulite. In plane light one can detect the presence of a spherulite by means of these little rayed clusters of iron-oxide grains. Again the rock may be largely made up of alternating bands of a fine irregular mixture of the quartz and feldspar and of a material similar in structure to the segments of the spherulites. The phenocrysts are often included in these bands and hence follow flow lines through the rock. It seems doubtful if the soda-iron silicates ever developed in the rock as originally solidified nor did they form on devitrification, but iron oxides and silica separated instead. In some sections examined, a few minute grains having the appearance of a hornblende, and a few tiny specks of what is thought to be biotite have been noted, but these are probably secondary.

In some parts of the aporhyolite, micropoikilitic structures are developed. These are especially strong in much of the rock from the large mass lying northwest of Fox Hill, which, as has been noted, is of a more massive character generally than the rest of the aporhyolite. Over considerable areas of this occurrence the rock consists almost entirely of a groundmass of micropoikilitic material enclosing fairly numerous phenocrysts of microperthite. The micropoikilitic material forms small roundish, elliptical or mutually moulded areas, which are of about the same size as the feldspar crystals themselves — that is from a few tenths to two or three millimeters, measured along their greatest dimension. About the boundaries of these, magnetite or hematite is often abundant and the same is sprinkled more or less plentifully throughout the rock. The general effect with low powers and crossed nicols is that of a rather finely granular rock. The micropoikilitic areas appear to consist of an intergrowth of microperthite and quartz with a sufficient uniformity of orientation to give them individuality. Some unorientated crystals of feldspar and quartz are scattered through them.

It is perhaps worthy of note that in some of the altered types of the aporhyolite, siderite in tiny rhombohedra and irregular grains and masses is sometimes quite abundant. The same mineral has been seen, or strongly suspected, in the granite, identified in the porphyry, and has been also found in some of the xenoliths in the granite. The presence of this mineral as an alteration product is not common in igneous rocks so far as the writer is informed. In the present series of rocks its presence is perhaps not remarkable because lime, to form the more usual calcite, is present in very small amount, while iron oxide is relatively abundant.

To determine the chemical composition of the aporhyolite a specimen of rock showing a few small feldspar phenocrysts was secured from some fresh excavations made along the road just south of Wampatuck Hill. The results of an analysis of this specimen yielded the results given below (18). With it are given the results (19) of some determinations made for Professor Crosby by students and also analyses of aporhyolite (11 & 20) from the nearby Neponset Valley intrusion.

	18		19	11	20
	Per cent	Molc. Ratios			
SiO_2	76.37	1.273	74.52	76.52	75.46
Al_2O_3	12.15	.119	13.95	12.30	13.18
Fe_2O_3	1.65	.010	2.72	.70	.91
FeO	1.06	.014		.56	
MnO	.07	.001		tr	
MgO	.10	.002		.16	.10
CaO	.17	.002		.31	.95
Na_2O	3.64	.058		5.19	6.88
K_2O	4.68	.050		4.58	1.09
H_2O+	.08			.41	.93
H_2O-	.13			.11	
TiO_2	.18	.003		.12	
	100.28			100.96	99.50

Sp. G. of No. 18, at 20°C. = 2.645.

18. Aporhyolite, Wampatuck Hill, Blue Hill Reservation, Mass. Analyst, C. H. Warren.

11. Rhyolitic faces of Neponset Valley, Mass., granite. Analyst, W. H. Hall. F. Bascom, op. cit., p. 138.

19. Average of several determinations quoted by Crosby (op. cit., p. 380) of aporhyolite from Pine Hill.

20. Aporhyolite (flow) Neponset Valley, Mass. Analyst, W. H. Walker. F. Bascom, op. cit., p. 144.

The aporhyolite (18) is more acid than the granite or its contact facies, the total iron lower and the potash relatively higher. The averages given by Crosby (No. 19) of the rock from Pine Hill, show lower silica and higher alumina. Variation is probably to be expected in different parts of a rock of this character. Devitrification and the attendant alteration have undoubtedly modified the rock as a whole and the true composition of the rock as originally solidified cannot now be determined. But there is no doubt whatever of its belonging to the Quincy type of magma.

The norm is as follows: —

Quartz	36.84	95.49 Salics.	$\frac{\text{Sal}}{\text{Fem}} = 2.0.$ Class I. Persalane.
Orthoclase	27.80		
Albite	30.39		
Anorthite	.56		$\frac{Q}{F} = 0.6.$ Order 3. Quarfelic.
Corundum	.92	4.56 Femics.	$\frac{K_2O+Na_2O}{CaO} = 54.$ Range 1. Peralkalic.
Hypersthene	.86		
Magnetite	2.32		
Ilmenite	.46		$\frac{K_2O}{Na_2O} = .86.$ Subrange 3. Sodipotassic.

The rock is, therefore, an alaskose and may be termed a granialaskose. The mode is essentially like the norm. The ratio of Ab: Or is 1.09 that of the granite 1.02; or Ab, 52.3; Or, 47.7, the granite Ab, 50.5; Or, 49.5.

In columns 11 and 19 are given the compositions of the rhyolitic rocks associated with the neighboring Neponset Valley granite intrusion. They show the characteristic differences of the two magmas, viz. higher iron and predominance of potash over soda in the Quincy magma, whereas soda dominates, sometimes greatly, over potash in the Neponset rocks. Thus the chemical evidence agrees entirely with the microscopic, in showing that the Blue Hill aporhyolite belongs to the Quincy granite magma and is quite distinct from the abundant volcanics developed in the neighboring areas of granitic rocks.

Slate-Granite Contacts, North Common Hill, Quincy.

The contacts of the coarse-granite with the slate along the northern side of North Common Hill, Quincy, show no intermediate contact phases and therefore call for special mention. As pointed out by Crosby [44] there are numerous patches of slate lying in the coarse granite.

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[44] loc. cit., p. 28 et seq., also special map opp. p. 428.

Some of these have a length of nearly one hundred feet by somewhat less in width and are stated by Crosby to show a very constant strike of N. 80 E. with a dip, S. 80–85, the metamorphism by the granite not having been sufficient to obliterate the true bedding of the slate. This constant orientation of isolated pieces of the slate is held by Professor Crosby to indicate that they are — "roots of a once continuous body of slate."

The granite remains coarse up to within an inch of the slate and then is fine-grained to the contact, nor was the granite necessarily rendered finer because of chilling, but may have developed simply a finer grain induced by direct physical contact with the slate surfaces. The slate is very dense, hard, and somewhat, though not very highly, metamorphosed, and shows no evidences of having received any additions from the magma with which it was in contact. The absence of extreme metamorphism certainly does not argue in favor of any excess of heat nor of volatile products in this part of the magma, a point that is of importance in any consideration relating to the method by which the magma came into its present position.

The absence of the contact phases, which are elsewhere developed between the granite and the older rocks, and the present higher elevation of the slate-granite contacts on North Common Hill relative to other parts of the field where the contact phases are strongly developed — Pine Hill for example — makes it necessary to assume, that either these contacts were originally much deeper seated portions of the contact and since have been elevated, or that the slate patches represent sunken blocks frozen in the granite, and that somewhere above the present plane of erosion there once existed a cover of the contact porphyries. The constant orientation of isolated blocks of slates over so considerable an area, and the presence of a well defined fault contact lying but a little way to the north with a probable upthrow of the igneous rocks relative to the sediments [45], points very strongly to the correctness of Crosby's conclusion that the slate here represents remnants of the original slate contacts.

PEGMATITE "PIPES."

Three pegmatitic masses having the form of elongated, pipe-like bodies occur in the granite of North Common Hill, Quincy. These are remarkable for their structure and crystallizations and have been made the subject of an extended description by Professor Charles

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[45] See G. F. Loughlin, op. cit., p. 29.

Palache and the writer [46] to which reference may be made for details. These pipes lie wholly within the granite, and it is certain that one, and probably two, did not even reach the present eroded surface of the granite. It is thought that they represent relatively siliceous, water-rich segregations formed at greater depths in the magma.

Vein phenomena.

Occasional quartz veins occur in the granite and granite-porphyry. They are generally small affairs measuring from ¼ to 1 inch in width but have usually a considerable length and appear to reach deep into the rock. Several of these have been referred to by Dale as occurring in some of the quarries. They usually contain small amounts of fluorite and some of them sphalerite, galena and chalcopyrite. Similar sulphides were noted in the pegmatite pipes.

Dike Phenomena.

As has been noted earlier in the paper, dike phenomena connected with the intrusion of the alkaline rocks are inconspicuous. With the exception of a few small pegmatitic dikes or streaks and of two narrow and short microgranite dikes cutting the granite, the dike phenomena of the area is confined to the few granite dikes cutting the cover-porphyry in the region about Slide and Scamaug Notches, and to those dikelets or apophyses invading the slates for a short distance from the actual contact. These latter have been described and figured in some detail by Professor Crosby and appear to be merely offshoots from the main mass of granite. They are for the most part small affairs though in a few instances they appear to have attained considerable size. Some of the larger ones are of about the same grain as the granite; the smaller ones are quite fine. None of them with the exception of the dikes in the Pine Tree Brook Reservation appear to possess any noteworthy characteristics not fully covered by Crosby's descriptions.

The last mentioned dikes cut the slate and also the only series of diabase dikes which are known to be older than the alkaline rocks of eastern Massachusetts (see special map Pine Tree Brook Area). The slate cover in this area is very thin, in fact, forms hardly more than a thin skin about and through which the underlying igneous rocks appear constantly. Against the slate, the normal development is first the relatively basic feldspar-porphyry. This is succeeded by a thin zone of granite-porphyry passing into the porphyritic coarse-granite. In places, fine-granite appears to come in as a contact phase as noted earlier. The coarser granite intrudes the fine-granite and both porphyries in many small irregular dikes, and in the extreme western edge of the area, running for about one half of its length (in part outside the northern boundary of the reservation) is a large dike of variable width and quite irregular trend which has a total length of something over a thousand feet. The width varies from a few feet to as much as 100 at the widest part. Just north of the boundary fence this dike is well exposed for study. Its eastern side, westward from the slate for perhaps ten feet (exposed), consists of a fine-granite-porphyry containing fairly abundant phenocrysts of feldspar and quartz and abundant specky hornblende. The center consists of the porphyritic type of coarser granite in which are embedded a great number of inclusions of the basic feldspar-porphyry of the same composition and character as that found in the massive ledges only a little distance away where the slate cover has been worn away. These inclusions are of all sizes from one-half centimeter to those which will measure two feet across, though the average will not probably exceed seven or eight inches. In shape they are sub-angular, rounded, elliptical or irregular, closely similar in fact to the inclusions described as occurring in the granite of the glaciated ledge of the Pine Hill area, West Quincy (p. 274). Some slate inclusions are also present. Though the western side of the dike is not well exposed, the fine-granite-porphyry appears to form the border of the western contact also. The dike is doubtless a very shallow one, hardly more than an upward protuberance of the invading igneous mass into the slate cover. The composition of this dike is satisfactorily explained if we assume that prior to the intrusion of the dike, the magma beneath the slate had differentiated and partly crystallized with the development of the rhomben-porphyry next to the slate. Pressure from below then caused the magma to burst through the cover of slate above, breaking up the layer of basic feldspar-porphyry and carrying its fragments, together with slate, up into the dike channel mingled with the granitic magma. Toward the margin, the dike assumed a finer grained and porphyritic texture, but centrally crystallized as granite.

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[46] These Proceedings, **47**, No. 4 (July, 1911).

Small dikes and patches of pegmatitic character always containing some fine-granite material[47] are found; one on the Rattle Rock;

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[47] See Warren & Palache, loc. cit., p. 127.

several on the small hill south of North Common Hill, Quincy, and just east of the northern extension of the Pine Hill Area; and several small dikes and patches of irregular outline in the coarse-granite near its contact with the fine-granite in the old quarry just off Quincy Ave. in Weymouth. In all of these occurrences it is noteworthy that the pegmatitic dikes occur either at the contact with the granite-porphyry (Rattle Rock and northeast of Pine Hill), or with its fine-granite equivalent (Weymouth). Their origin is probably to be found in the filling of fractures in the upper parts of the granite by injection of fluid material from beneath. The banded texture of the larger veins — coarse margins, fine centers, etc.— have been discussed elsewhere.[48]

PART II.

GENERAL DISCUSSION.

Chemical and Mineral characters.— Although these characters have been more or less fully discussed under each type it may be well to briefly summarize certain important features brought out by the chemical analyses.

The entire series are characterized by relatively high iron and alkalies, by exceedingly low magnesia and also by almost equally low lime, except in the case of the rhombenporphyry and the rhombenporphyry xenoliths, in both of which the lime reaches about three percent owing to the presence of a lime-iron rich pyroxene probably a hedenbergitic augite.

There is a considerable fluctuation (see table No. I) in the total iron oxide content even between the coarse-granites (compare 1–2 with 3), while the rhombenporphyries show a great difference, compared with the rest, in their high iron content. It is to be noted that there is a strong sympathy between the iron oxides and high alkalies; as the amount of feldspar increases the sodic-iron silicates increase also. The relative amounts of K_2O and Na_2O are not far from equal, with the exception of the rhombenporphyry xenoliths, and this abnormality may point to some later addition of soda from an enclosing hot magma. The potash on the whole slightly predominates in amount.

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[48] Palache & Warren, op. cit., p. 127.

TABLE NO. I.

	Coarse Granites				Fine Granite	Granite-Porphyry	Rhomben-Porphyry	Rhomben-Porphyry Xenolith	Aporhyolite	Riebeckite Granite. S. Norway	Riebeckite Granite. Kamerum	Riebeckite Granite. Socotra	Biotite Hornblende Granite. Neponset, Mass.
	1	2	3	5	10	13	15	16	18	6	7	8	9
$Al_2O_3 + Fe_2O_3$ =	.128	.121	.137	.137	.136	.133	.169	.163	.129	.145	.165	.110	.149
$Na_2O + K_2O$ =	.117	.114	.124	.125	.127	.123	.129	.135	.108	.145	.153	.100	.118
Ratio $\frac{R_2O_3}{R_2O}$ =	1.09	1.06	1.10	1.09	1.07	1.08	1.31	1.20	1.19	1.00	1.07	1.10	1.26
Average of 1 to 13 inclusive = 1.08; average of 6–7–8 = 1.05.													
Na_2O =	4.21	4.03	4.66	4.61	4.59	4.43	4.47	5.64	3.64				
K_2O =	4.62	4.68	4.63	4.79	5.00	4.90	5.29	4.26	4.68				
Total	8.83	8.71	9.29	9.40	9.59	9.33	9.76	9.90	8.32				
Ratio $\frac{K_2O}{Na_2O}$ =	1.10	1.16	0.99	1.04	1.09	1.10	1.16	.75	1.01	Contact Phases Granite-Porphyry			
Fe_2O_3 =	2.25	1.71	2.91	2.77	1.75	1.67	2.33	2.80	1.65	2.18	2.67		
Fe O =	.93	1.26	1.55	1.09	2.33	2.10	6.03	6.57	1.06	1.71	1.52		
Total =	3.18	2.97	4.46	3.86	4.08	3.77	8.36	9.37	2.71	3.89	4.19		

A study of the ratios existing between the sum of the molecular proportions of alumina and ferric oxide, to that of the soda and potash reveals a high degree constancy if we except the rhombenporphyry phases (15 & 16 Table No. I). Inasmuch as no allowance has been made for magnetite nor the small amounts of lime and ferrous-iron which, of course, effect this ratio, some variation is to be expected. The departure from constancy in the rhombenporphyries is caused by the larger amounts of lime and ferrous iron, whose relations cannot be very precisely determined, but the direction of the divergence is the one to be expected.

Another point of considerable interest is the proportions of the soda and potash feldspar molecules present in the microperthite resp. cryptoperthite or soda-orthoclase. These proportions are shown in Table No. II as calculated from the analyses: —

TABLE NO. II.

	Quincy Granite	Fine Granite	Granite Porphyry	Rhomben-Porphyry	Aporhyolite	Average	Vogt's[49] Anchi Eutectic Feldspar
	4	10	13	15	18		
Quartz % in rock	33.3	23.3	26.7				
Albite % in rock =	28.1	36.9	34.8	40.2	30.4		
Microcline % in rock =	27.5	29.5	29.0	31.1	27.8		
Microperthite resp. Cryptoperthite, Soda-orthoclase =	55.6	66.4	63.8	71.3	58.2		
Ratio $\frac{Ab}{Mic}$ =	1.02	1.25	1.20	1.29	1.09	1.17	1.27 to 1.50
Albite % =	50.5	55.6	54.5	65.3	52.3	53.8	56 to 60
Microcline % =	49.5	44.4	45.5	43.7	47.7	46.2	44 to 40

There is, in spite of a rather large fluctuation in the total amounts of the two feldspar molecules, a rough approximation toward a constancy in their relative proportions, but the ratio is lower than the lower limit for feldspar intergrowths as estimated by Vogt. In calculating the Ab: Mic. ratio several assumptions had to be made which would obviously affect the ratio so that they are at best only approximations.

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[49] Tschermak's Mineral. u. Petrog. Mitt. Vol. XXIV, No. 6, 1906, p.

However, had it been assumed that the albite was as sodic as $Ab_{98}An_2$ instead of $Ab_{95}An_5$, the ratio would, for the average of the granites (No. 4), been only 1.21 at most, and this would have reduced the aegirite and hornblende percentages considerably below their true value and raised the magnetite above a probable value. A similar change in the albite, would in the case of the fine-granite, affect the ratio very little since this was controlled by a quantitative, microscopic estimate of the minerals present, and this ratio also falls just below Vogt's minimum. The Quincy rhombenporphyry, however, does fall within it. Despite the possible errors (at least these are perhaps no greater than enter into Vogt's admittedly approximate estimates) in such calculations where some assumptions have to be made, and where slight changes in the Na_2O values used cause much greater changes in the resulting minerals, the writer believes that the values given are worthy of some weight and indicate a wider fluctuation in the proportions of soda to potash feldspar in their intergrowths than those estimated by Vogt. It is also to be noted that a small portion of the albite occurs in these rocks outside of the intergrowth. This is included in the above calculations because of the impossibility of estimating its amount, but if allowances for it could be made it would have the effect of still further lowering the proportion of albite that is present in the intergrowth with the microcline.

The relative proportions of quartz to feldspar are also interesting. Table II, p. 296, shows that these are, when calculated to 100%:— 37.4%, quartz to 62.6%, feldspar in the coarse-granite; 25.9% to 74.9% in the fine-granite and 29.5% to 70.5% in the granite-porphyry. Although there is admittedly some chance of error in the calculation of these percentages, the writer believes that the possible error is not sufficient to account for the considerable differences shown by these figures and that they represent real differences of composition. While the fine-granite and granite-porphyry are not so far apart, the coarse-granite shows a wide divergence. Dispite the probability that the considerable amounts of sodic-iron silicates present in the rocks might effect the composition of a possible quartz-feldspar eutectic as compared with purer quartz-feldspar granites by an amount impossible to estimate at present, it is interesting to note that the figures for the fine-granite fall on one side of the ratio for the "granite eutectic" as estimated by Vogt[50] and the granite-porphyry on the other side. Vogt's estimate is quartz, 27.5% feldspar, 72.5%. The coarse-granite departs very widely from this figure, whereas it is the latter that should correspond most nearly to the eutectic composition on the basis of the theory. It is of course by no means necessary, in a magma representing, like the present one, a very complicated chemical system in which there are certainly a considerable number of as yet imperfectly understood solid-solution relations existing between the various components, that the end product of differentiation should be a eutectic mixture. In any case the figures above given serve to illustrate the present uncertainties of the eutectic theory as applied to such systems.

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[50] Tscherm. Min. Pet. Mitth. (2), **25**, pp. 361–2, 383–5 (1906).

Fluorine is present usually in small amount: locally near quartz veins, in the pegmatitic pipes, and in certain contact facies of the porphyry cover it is quite abundant in the form of fluorite, which is its usual mode of combination. Small amounts are present in the riebeckitic hornblende, and in the pegmatite pipes it is also present in the mineral parasite. Zirconia is generally present in small amount as zircon. Titanium is present in ilmenite, aenigmatite, astrophyllite, titanite (in part or wholly secondary) and probably to a small extent in the hornblende. The cerium earths make their appearance in the parasite of the Pegmatites and are doubtless present elsewhere in the granites, etc. Though very small in amount their presence is interesting and is probably, like zirconia, characteristic of this type of rock. Traces of molybdenium (as molybdenite), lead (galena), zinc (sphalerite) are present in the magma since they are found in the quartz veins and pegmatitic phases.

Although it is the writer's opinion that the greater part of the original mineralizers were retained within the granite by the quick chilling of the upper zones, these acting as a protecting cover, it is possible, if not indeed probable, that the more volatile contents of the extreme upper portions of the invading magma were in great part lost by rapid and easy diffusion through the relatively thin cover of sediments. The occasional strong development of fluorite in certain localities where the indications are, that the rock as now exposed was originally not far removed from an original contact, supports this view. The retention of mineralizers in the magma appears to be a characteristic feature of other occurrences of this type of rock, particularly that of Dobrogea on the Danube which seems to resemble the present occurrence in many respects.[51]

The riebeckite of the fine-granite appears to correspond closely to that of the pegmatite pipes which have been analyzed and shown to consist essentially of such molecules as $Na_2Fe_2Si_4O_{12}$, $(R''_1R'_2)\ Fe_2Si_4O_{12}$, where R'' equals chiefly Fe'' and R', soda, but with small amounts of fluorine and hydroxyl water. Doubtless the riebeckite of the coarse-granite is in large part closely similar, but it appears to pass easily into an alkali-iron hornblende closely allied to the catoforites. The aegirite of the pegmatite pipes consists of the almost pure aegirite molecule $Na_2Fe_2Si_4O_{12}$ and much of the aegirite of the granite is doubtless of the same pure variety, but in the earlier formed pyroxene noted, though for the most part rich in aegirite compound, other molecules enter, chiefly one rich in calcium and iron. This molecule appears also as important in the early pyroxene of the rhomben- and granite-porphyry. Again in the granite and granite-porphyry we have aenigmatite or some closely allied species appearing. All this illustrates well the great complexity of the relationships existing between these soda-iron rich minerals. There are doubtless a number of solid-solution relationships, complicated by polymorphism, concerned in their growth. The formation of one or the other is doubtless effected by the presence of mineralizers, principally fluorine, by differences of pressure and rate of cooling, but of all these things we have at present such wholly inadequate information that speculation regarding the thermo-chemical relations of these minerals seems too uncertain to be worth venturing further with at present, and such conclusions as those arrived at by Murgoci [52] regarding the formation of riebeckite and aegirite, interesting and suggestive though they are, must be held open to much question.[53]

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[51] Murgoci, op. cit., pp. 142–144.

The intrusion of the Batholith.— The facts developed by the present investigation are obviously too meagre so far as their bearing on the general question of igneous intrusion is concerned to warrant a lengthy discussion of this problem. As bearing on the problem of the intrusion of this particular mass of rock they are believed to be instructive and fairly complete, and it is hoped that a brief discussion of this particular problem may add a little of value to the more general one.

There is, the writer believes, convincing evidence [54] that before the intrusion, the space now occupied by the alkaline rocks was formerly filled by Cambrian and pre-Cambrian sediments, and probably also in part by an older biotite granite intrusive into at least a part of these. Whatever the exact manner in which these older rocks were replaced by the batholith [55] these certainly had no appreciable effect on the chemical composition of the batholith as now exposed by erosion and quarrying. The Cambrian sediments now extant in contact with the batholith or in its neighborhood, nor the pre-Cambrian rocks existing in nearby areas, could not, by any process of assimilation, produce the present highly specialized chemical character of the batholith unless such assimilation were accompanied by a drastic differentiation which it is quite certain has not taken place in the batholith as exposed. That the magma of the present batholith was originally produced at greater depths by the differentiation of a magma which had during some past epoch assimilated older rocks, is not denied, indeed, the writer believes that such a process did take place. The assimilation of any noteworthy quantity of sediments and subsequent differentiation demands an enormous amount of heat, or a long continued period of sufficient heat to ensure the necessary mobility, which does not appear to have been the case in this magma. This is shown by the lack of extreme metamorphism of the Cambrian slates found at the contacts, even of the coarse granite on North Common Hill, Quincy, which must represent relatively deep portions of the original contact, as has already been pointed out. There is for example much less metamorphism in these sediments than that described by V. M. Goldsmidt [56] for the sediments in contact with, or included in,

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[52] American Journal of Sciences, **20,** No. 116, (Aug., 1905).

[53] See also in this connection Warren & Palache, op. cit., p. 144.

[54] The evidence is not direct, so far as the pre-Cambrian sediments and granites are concerned, since the Quincy rocks are nowhere found in igneous contact with these rocks. Both to the north in Essex Co., and to the south in northern Rhode Island, there are intrusions of alkaline granites so similar to the Quincy that there can be no doubt that they are of the same age. These are known to cut an older, biotite granite. Their intrusion was preceded,

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at least in Essex Co., by acid alkaline intrusive and effusive rocks which either lie upon an eroded surface of the older granite, etc. or cut them. The bulk of the evidence referred to is contained in the unpublished thesis of Dr. C. H. Clapp (Thesis for the degree of Doctor of Philosophy in Geology, C. H. Clapp, Mass. Inst. of Technology, Boston, Mass., 1911.) An abstract of this thesis has been published setting forth the main results and a paper will appear later as a bulletin of the U. S. Geological Survey. The facts regarding the alkaline granite in Rhode Island will be published by the writer and Mr. Sidney Powers in a future paper now in preparation.

[55] Although the mass of alkaline rocks exposed is not large, perhaps not large enough to warrant the use of the word batholith according to the ideas of some geologists, the writer has used it as a convenient term. In all probability the mass is connected with a much greater mass of alkaline rock beneath, which has sent up protuberances as it were, of which the Quincy-Blue Hill mass is one, the Rockport, Mass. Granite another, and those near Diamond Hill, Rhode Island another. There is no evidence whatever of anything in the nature of a laccolith about the Quincy-Blue Hill mass.

[56] Videnskapsselskapeta Schrifter, **1**, Mat. Naturv. Klasse, No. 1 (1911).

the igneous rocks of the Christiania region where no chemical assimilation in place has taken place in the opinion of the workers in that field. The relatively thick cover of porphyry, representing as it does the chilled upper portion of the invading mass, which covered a great part of the batholith, is itself another strong evidence that there was not enough original heat, or that the solidification took place so near the surface that the magmatic heat was too rapidly dissipated, even to keep the temperature sufficiently high for a sufficiently long interval to permit of the assumption of a plutonic texture throughout. No differentiation to amount to anything appears to have occurred in either the fine-granite nor the granite-porphyry, excepting in the thin zone of porphyry found for a short distance along the aporhyolite contact on Pine Hill and at the slate-granite contacts in the Pine Tree Brook area. It is only along the deeper contact levels such as represented the two areas just mentioned, or in the granite directly under its own porphyry cover, where the magma remained hot and permitted differentiation to take place, that we find evidences of differentiation. Furthermore, complementary dike phenomena, either in the granite or in the adjoining country rocks, are wholly wanting and this is thought to be further evidence that extensive differentiation did not take place. It is believed that such as did occur can now be seen in large part, at least, in the batholith as it stands today. If these deductions are correct the production of the magma by the actual melting of the sediments as proposed and set forth at length by Professor Crosby in his report on the region, is out of the question.

That the magma took the place of large volumes of pre-existing rock is certain, and that it did so without appreciably effecting the chemical composition of the magma appears almost equally certain. To accomplish this only two methods of intrusion appear possible to the writer. The first, is that the magma actually pushed up a great block of rock (bysmalithic) thus bringing it up to its present level, which was so near the surface that the magma became chilled so that little differentiation could take place and the heavy mantle of porphyry was developed as a chilled cover. It is certain that the alkaline rocks have suffered some differential movement and that they have been tilted up, relatively at the north. These movements took place along great major fault lines and it is perhaps possible that these may represent lines of weakness along the original lateral contacts of a bysmalithic intrusion. This hypothesis would do away with many grave difficulties but there is no proof whatever. The other possible method of intrusion which offers an explanation of the facts as presented is, that of "stoping" as advanced by R. A. Daly[57] and in a somewhat less highly developed form, independently, by J. Barrell.[58] If the Quincy magma has reached its present position by replacement and not by displacement of the country rock, and has not dissolved the invaded rocks, then the writer can see no escape from the so-called stoping method of replacement.

The first solidified portions of the magma are found in place in several parts of the field. In eastern and southern Quincy and northern Weymouth we find the fine-granite as the contact phase. Considerable portions of this were broken up or invaded by the still fluid magma beneath, as shown by the perfectly sharp igneous contacts of many of the larger masses of fine-granite with the coarse, by the included masses of fine-granite and by the dikes of coarse-granite cutting the fine. This part of the area undoubtedly represents relatively deep levels of the contact (as held by Professor Crosby). In the Pine Hill tract and the Pine Tree Brook reservation the contact-porphyry is a relatively thin zone passing gradually and rapidly into granite and here also are found the basic marginal differentiate of the magma, the rhombenporphyry, and also the other phase of lower contact levels, the fine-granite. In both areas there is positive evidence of the movement of the underlying magma, which often resulted in a breaking up of the earlier consolidated rocks or in invasion of them by dikes, or both. In the Blue Hill Reservation proper, where the cover-porphyry attained a great thickness, the transition from the granite-porphyry to the porphyritic phase of the coarse-granite is either very rapid, or as observed on Rattlesnake Hill, the contact is sharp, indicating that there the magma moved under its own cover. Again in and about Slide Notch we find actual dikes of granite (with xenoliths of the dark feldspar-porphyry and fine-granite types) of considerable dimensions cutting the porphyry cover. In several places among the higher hills we have also noted the occurrence of considerable areas of fine, feldspar-quartz-porphyry essentially identical with the contact phases (against the aporhyolite of Wampatuck Hill for example) of the granite-porphyry, but in many instances, as for example, on the northern and northeastern slopes of the Great Blue Hill, the contacts of these fine porphyries and the associated granite-porphyry are either very rapid transitions or are really sharp though perfectly sealed contacts, again indicating a movement accompanied by breaking off,

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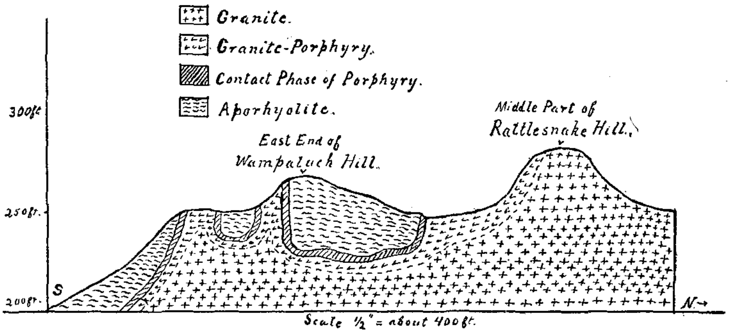
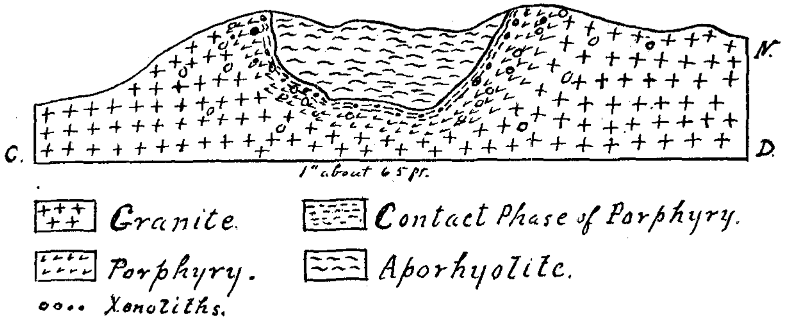
[57] American Journal of Science, No. (1903) **26**, (July, 1908).
[58] U. S. Geol. Surv., Prof. Paper, No. 57; (1907).


Granite.
Granite-Porphyry.
Contact Phase of Porphyry.
Aporhyolite.
300ft
250ft.
200ft.
East End of Wampatuck Hill.
Middle Part of Rattlesnake Hill.
S
N→
Scale ½" = about 400ft.

SECTION THROUGH RATTLESNAKE AND WAMPATUCK HILLS, BLUE HILLS.

This section, drawn approximately to scale, is taken through the eastern and lower end of Wampatuck Hill and the central portion of Rattlesnake Hill and is intended to illustrate the writer's conception of the relations existing between the aporhyolite, contact-porphyry, granite-porphyry etc. The actual width of the chilled contact zone immediately against the aporhyolite is exaggerated in order to show it on this scale diagram.


N
C.
D.
1" about 65ft.
Granite.
Contact Phase of Porphyry.
Porphyry.
Aporhyolite.
Xenoliths.

SECTION THROUGH THE APORHYOLITE-PORPHYRY CONTACT EAST OF PINE HILL, WEST QUINCY.

This section, drawn approximately to scale, illustrates the relation, as worked out by the writer, existing between the aporhyolite, granite-porphyry etc. along those portions of the contact between these rocks found at relatively deeper levels than that shown in the Rattlesnake Hill section. Here the very dense immediate contact rock is succeeded by a narrow zone of coarsely and profusely porphyritic rock which in turn passes gradually, but rapidly, through granite-porphyry into the porphyritic phase of the coarse granite. Flow structures are strongly developed near the contact, and dark colored xenoliths are found in the coarsely porphyritic phase and also in the granite-porphyry and granite somewhat more distant from the contact.

and immersion of, the broken blocks in the yet unconsolidated magma beneath. The writer is strongly inclined to believe that the masses of aporhyolite in like manner represent masses of the first consolidated portions of the invading magma, broken up and partially or perhaps wholly immersed in their own magma. R. A. Daly, as a result of his acquaintance with the region in question, has suggested tentatively that these masses of aporhyolite may be foundered blocks of the original roof rocks.[59] The fuller knowledge of the characters and relationships of the rocks of the region which we now have, supports the view that the aporhyolite represents portions of the invading magma originally consolidated very near the surface, at least. This view of the origin of the aporhyolite brings us into direct opposition to Professor Crosby's conclusions and the question of its relative age must be examined more in detail. The characteristics of the porphyry at the contact with the aporhyolite have been described and shown to be everywhere indicative of a chilled contact and this fact was fully recognized by Crosby. He offered two hypotheses as to the nature of the aporhyolite. First, that it is in the main truly intrusive through successive zones of the batholith and interformational with reference to the slate and contact rocks; and secondly, that it is truly effusive. The first as stated by Crosby is as follows:[60] "Hence we seemed forced to the conclusion that after the development of the contact zone of quartz-porphyry in the usual manner the extrusion of the magma now represented by the felsite" (aporhyolite) "took place in such a way as to form, not great dikes extending up through the Cambrian strata, possibly to the surface, but rather a laccolithic accumulation between the contact zone and the Cambrian cover, with a bending down or falling in of the edges of the contact or porphyry zone sufficiently marked to account for the great width of the dike, for the narrowness of the porphyry zone, for the fact that in spite of very unequal erosion the felsite is nowhere found in contact with the granite, and for the high inclination of the felsite-porphyry contact. . . ." "A sill[61] extending south from a laccolithic trough between the contact zone of the batholith and its Cambrian cover would, perhaps, best express the idea."

The second hypothesis, and the one favored by Crosby, is stated by him as follows:[62] — . . . . "the felsite is truly effusive, post-dating. . . .

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[59] These Proceedings, **47**, No. 3, p. 62 (June, 1911).
[60] op. cit., p. 386.
[61] op. cit., p. 387.
[62] op. cit., p. 387.

the erosion requisite to lay bare the batholith, and occupying in its broader, dike-like development between the east side of Pine Hill and the summit of Wampatuck Hill, a steep-walled valley due to the erosion of a deeply included body of slate. Depressions having this origin exist in the modern topography, the valley of Ruggles Creek being a good example. If now, we conceive such an erosion trough, with its wall of quartz-porphyry essentially intact, as the Ruggles Creek Valley is today bordered by the contact zone of fine-granite, to be traversed longitudinally by a fault fissure with a down-throw to the south, as the valley of Ruggles Creek unquestionably is, the outflow of acid lava filling the depression and connecting fissure, and flowing out away to the south, would seem to account for all the facts as they are now developed in the field."

Under the first view it seems to the writer very difficult to conceive of a magma forcing its way in between the contact porphyry and its slate cover in such a way as to carefully remove all of the slate over a great area of what was certainly a most uneven contact surface without leaving, so far as known, a single fragment of it between the porphyry and the aporhyolite.[63] Under the second view, it seems very unlikely that erosion, accompanied by even the most favorably disposed faulting, could have removed the slate cover *just so* far as the fine-grained contact phase of the porphyry *and no farther*, for the aporhyolite *never* comes in contact with any other phase of the batholithic rocks; and this over a wide area, at various elevations and along a most devious line of contact. Furthermore, it would seem almost inevitable under either hypothesis that the lava would have included within itself fragments of either the slate or the porphyry, or both, and such inclusions are nowhere to be found, at least along the great length of contacts which have been examined by the writer. The porphyry at all of its exposed contacts with the aporhyolite bears all the characteristics of a chilled contact phase against *something*, and it appears most natural to assume that the chilling was done by the rock with which it is now in contact — the *aporhyolite*. It is wholly inconceivable that the aporhyolite itself, a rock, that in large part, at least, solidified as a vitreous rock, should have so altered the granite-porphyry as to reproduce in it the characteristic contact structures which it now possesses.

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[63] It should be noted that the aporphyolite is in contact, according to Crosby, with slate northwest of Fox Hill, but the precise relation of the slate to the nearby granite and porphyries is unknown. The only other slate anywhere near the aporhyolite is a fragment, 2 ft. across, embedded in the granite-porphyry just east of the eastermost, and very narrow end, of the aporhyolite in the Pine Hill area.

An alternative hypothesis to the one proposed above, viz. that the aporhyolite represents engulfed blocks of the original chilled roof, might be that, the aporhyolite was first intruded, at a distinctly earlier period into the relatively cold slates relatively near the surface, in the form of great dikes, or more irregular masses, or even was extruded, and that later followed the greater invasion of the main mass. In such case the magma would be expected to have enveloped much of the rocks first invaded by the rhyolite along with it, which is certainly not the case. It must be frankly admitted that the view that it is an earlier formed rock is not free from serious difficulties and the conclusion finally reached in any case is perhaps determined by a "choice of evils", and the writer has chosen what appears to him to be the lesser one, namely those which make the aporhyolite an earlier consolidated part of the alkaline intrusion.

Consolidation of the Magma.— From the relationships previously set forth we may conceive of the consolidation of the batholith having taken place somewhat as follows: — at the highest levels, near the surface, sudden chilling resulted in the formation of considerable masses of highly vitreous rock; a little lower down, in the denser forms of the feldspar-quartz-porphyry. These earliest consolidated rocks were fractured and broken off by movements of the fluid mass beneath, large blocks of them became immersed in the magma, perhaps sunk, and against them the magma consolidated with sharp but tightly sealed contacts showing a variable amount of local chilling. Further cooling of the magma, still at relatively high levels, resulted in the formation of a thick zone of feldspar-quartz-porphyry and granite-porphyry. This thick cover of still relatively hot rock, being a poor conductor of heat, acted as an effective blanket, protecting the magma underneath and permitting it to solidify with sufficient slowness to develop a truly granitoid texture. In places, perhaps very generally, the magma beneath this porphyry-cover moved relatively, and sometimes broke through, forming the granite dikes like those noted as occurring in and about Slide Notch. As these dikes contain numerous cognate xenoliths, — rhombenporphyry and fine-granite types—and as these are also found in the granite directly underneath the porphyry, as at Rattlesnake Hill, it appears that the magma, wherever it remained hot and sufficiently fluid for some time, differentiated to a small extent yielding more basic phases, which were more or less broken up and scattered, itself becoming in consequence slightly more acidic.[64]

At lower levels of the contact, such as those represented by the area located in eastern and southern Quincy and northern Weymouth, the magma first consolidated as the fine-granite against the slate. Here again the contact phase was fractured and to some extent engulfed in the underlying magma as shown by the included blocks, the dikes in the fine-granite and the sharp contacts generally existing between the two granites.

The fine-granite and the granite-porphyry characteristic of the chilled cover at lower and the other at higher levels are, so far as extent and thickness are concerned, by far the most important marginal phases of the batholith, and although they differ sharply as to texture and to some extent in the character of their dark silicates, they are, as we should expect, almost identical in chemical composition. They are both slightly more basic than the average of the coarse-granite. Their textures and the absence of differentiation products in them suggest that, being more quickly cooled, they did not differentiate but solidified with substantially the composition of the magma as intruded, and it is only beneath them, or at deeper contacts against the slates, that the magma differentiated.

At other relatively deep contacts, such as those represented by the Pine Hill and the Pine Tree Brook areas, we find the first formed secretion of the magma against the slate to be the rhombenporphyry, indicative of more considerable differentiation. The rhombenporphyry, like the first formed rocks generally, was broken through by the underlying magma and portions of it are found abundantly distributed through the associated granite and porphyry. The magma on breaking through this early phase, in part consolidated as a thin zone of coarse porphyry, in part as a porphyritic phase of the fine-granite, or as fine-granite. All of these were again broken through by the magma, as shown by the dikes which are so prominent a feature of the Pine Tree Brook area.

An objection might here be raised, that at the granite-slate contacts found on North Common Hill, which seem to represent the deepest contacts anywhere exposed and which have been brought up to their present relatively high positions by faulting and elevation at the north, no rhombenporphyry is now found at the contact, whereas if it is true, as had been assumed, that it is in places where the magma remained hot for a long period, thus retaining its fluidity and permitting differentiation to take place, it is precisely here that the products of differentiation should be found. In view of the fact that elsewhere in the field the first formed consolidations have generally been broken and torn away from their original position at the contacts, it is to be expected that at deeper contacts where the magma was not chilled and retained its mobility longer, any differentiation products that might form, would sink or be otherwise moved from their original place of formation and dispersed through magma. Indeed, the numerous patches of varying texture and more basic composition, found everywhere throughout the granite, are believed to be evidence that such action has in fact taken place. It does not, however, appear necessary to assume that differentiation would occur at very deep contacts in the same manner that it would at higher elevations, where the temperature gradient would be much greater where other conditions would likewise be different.

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[64] The aporhyolite was originally in large part glassy and has since suffered devitrification and alteration so that its exact original chemical composition is in doubt. As it stands, it appears to be on the average about as siliceous as the granite, perhaps slightly more so. This might be held to favor its being of later origin. On the other hand it may well represent a relatively siliceous and originally more aqueous upper portion of the invading magma doubtless closely similar in this respect to much of the fine-grained feldspar-quartz-porphyries of the higher elevations.

In places, as in the neighborhood of the contacts in Northern Pine Hill, abundant xenoliths of what are quite obviously fragments of the contact phases are very abundant, having been caught in the freezing liquid before they had had time to move, or be moved, far from their original positions. The general distribution of the contact phases through that part of the magma which in time formed the granite is held to account for the presence of the xenoliths generally. In this connection it should be noted that the irregular variation in the composition of the granite, as shown by the microscope and the chemical analyses, is suggestive of movements in a magma not perfectly homogeneous in composition, and this is in keeping with the other evidences above set forth of more or less strong movement in the intruding magma before complete solidification took place. As further bearing on the question of movement, we should note the strong testimony of the broken phenocrysts of the granite-porphyry and of the protoclastic structures in certain portions of the granite (that of the Goldleaf Quarry).

Order of Crystallization in the various phases of the Magma.— The crystallization of the various rock types has been purposely left until they could all be discussed together. The relations of the various minerals to each other in the granite, as revealed by the microscope, does not in itself lead to any very precise conclusion regarding the order of crystallization.[65] It might be inferred[66] that the feldspar was perhaps first to crystallize, that the quartz and aegirite and perhaps a part of the feldspar (albite) was the last to cease crystallization. Furthermore, the great complexity of the chemical system represented by the granite, its many components and the several complex, isomorphous relationships known to exist between some of them, would render any predictions as to the probable order of crystallization of very difficult, even if we had anything like a full knowledge of the thermal and other data bearing on the question. Owing to the differences in the chemical composition of the coarse-granite and the granite-porphyries etc., and the resultant differences in mineral compositions, we cannot look upon the porphyry as the exact quenched-quickly cooled- equivalent of the granite. However, the figures given show that the differences in mineral composition which would be effective in modifying the order of crystallization are those relating largely to the quartz and feldspar, so that with this in mind we shall find that a comparison of the crystallization in the porphyries with that in the granite leads to a better understanding of the crystallizing process as a whole.

The phenocrysts of the quickly cooled porphyry represent the first crystallizations from the magma. These appear to be quartz and the homogeneous soda-potash-feldspar crystals. If we should assume that the proportions of quartz to feldspar in Vogt's quartz-feldspar eutectic are approximately correct, then the rather close approach in the amounts of these minerals to this proportion might lead us to expect a simultaneous crystallization. The greater proportion of the quartz phenocrysts to those of feldspar in the contact phases of the granite-porphyry found at the higher contact levels, as shown by the Rosival estimates given in columns II and III, p. 243, and the greater resorption of the quartz phenocrysts are probably indicative that the quartz preceded the feldspar in the quickly chilled phases. On the other hand, at deeper levels of the contact, the proportion of the quartz to feldspar phenocrysts is reversed and the amount of the former is relatively small (see column IV, p. 243). Further, if we consider the rhombenporphyry and assume that, in general, the marginal differentiates of a magma are richest in those compounds which are the first formed secretions from the magma, and observe that the soda-potash-feldspar is clearly the first mineral to crystallize in this porphyry, we may infer that the feldspar was the first mineral to crystallize, and that the possible appearance of the quartz first, may be due to a reversal of the normal order through supercooling, a condition that did not exist to the same extent at the deeper levels.

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[65] In this connection see an interesting paper by N. L. Bowen, Journal of Geology, **20**, No. 5 (July–Aug., 1912), dealing with the order of crystallization in rocks.

[66] See Warren & Palache, op. cit., p. 143–4; also Murgoci, op. cit., p. 137.

In the extreme contact phases at higher levels we find pseudomorphs of what were pretty certainly pyroxene crystals: we have also noted the occurrence of a few phenocrysts of a calcic-iron pyroxene in the granite-porphyry, and that of a more highly sodic variety of pyroxene in the form of small crystals included in the outer parts of the feldspar phenocrysts, as well as in the form of separate crystals. In the rhombenporphyry a calcic-iron pyroxene began to crystallize before the phenocrystalline feldspar had ceased its growth. From these facts we infer that the less sodic pyroxenes began to crystallize shortly after the feldspar.

After a considerable amount of quartz and feldspar with small amounts of pyroxene (possibly aenigmatite) had grown as phenocrysts, there occurred a sharp and clearly marked pause in the crystallization of these constituents. The cause of such a pause is undoubtedly to be found in the supersaturation of the residual liquor, a phenomenon which appears to the writer to be an inevitable incident in the crystallization of a quickly chilled rock magma, such as that of the granite-porphyry unquestionably was. It is clear that before and perhaps during this pause the magma was in movement, for we find fragments of feldspar phenocrysts as well as the broken and irregular ends of the crystals covered by the later growth, itself unbroken. Furthermore, the sealed cracks crossing so many of the phenocrysts extend only to the edge of the original crystal, but not across the later rim. During this pause it is possible that the resorption of the quartz phenocrysts took place. It is also possible that the pyroxene (of more sodic type) continued its growth, and the massive parts of the hornblende began their growth, although their crystallization appears to belong to the second period of crystallizating activity, for they never come normally in contact with the original phenocrystalline feldspar but always lie outside of the later groundmass addition to the feldspars. As we should expect in a supercooled liquid, the crystallization when it was resumed took place about vastly more numerous centers, and so far as can be told there was a practically similtaneous growth of the various minerals. A portion of the feldspars attached themselves as a parallel growth of microperthite to the already existing feldspar, forming a relatively narrow rim about the larger ones, but a much broader one about the smaller and later phenocrysts. This marginal feldspar includes many quartz and aegirite grains, indicating its contemporaneous age.[67] The first hornblende formed small massive crystals suggesting a slight priority of growth, and as its crystallization progressed, it enclosed poikilitically the quartz and feldspar of the true groundmass, as well as some of the smaller and later feldspar phenocrysts. The greater part of the aegirite, excepting the crystals of distinctly earlier age, is distributed through the groundmass in the form of minute crystals or clusters. The aenigmatite appears to have followed about the same plan of growth as the hornblende.

In the long period during which the granite beneath was solidifying, the porphyry must have remained at a temperature not far below its crystallizing interval, and during this period certain mineral and textural changes took place. The most obvious ones were those connected with the recrystallization of a part of the phenocrystalline feldspar and its albitization, fully described in the earlier part of this paper and to be referred to again. There were doubtless other changes among them, possibly those resulting from some coarsening and modification of the texture by "sammelkrystallization," as it has been termed by F. Rinne.[68]

If we examine the phases of the granite-porphyry which are nearer the granite in the field, we find the same order of progression, but the pause is perhaps less sharply marked, the grain of the groundmass is coarser indicating a longer period of cooling, during which there was time for a more nearly granular type of texture to develop. In the porphyritic phase of the granite, which comes next in order, we find that the pause is almost or quite obliterated and that the groundmass is barely distinguishable from the rest of the rock. Passing now to the coarse-granite, we cannot doubt that the first crystallizations were substantially the same as in the porphyry. The larger amount of quartz relative to the feldspar in the granite suggests that the quartz preceded the feldspar in the beginnings of its crystallization. In any case there can be little doubt but that the quartz and feldspar were the first minerals to crystallize. Owing to the very slow cooling (long time period) there was abundant time for a perfect equilibrium to establish itself, no supersaturation ensued, and the minerals continued to grow chiefly by addition to crystals already formed and the crystallization of all these proceeded, accompanied as we shall see by certain transformations, chiefly in the feldspar, throughout the remainder of the main (magmatic) period of crystallization. Succeeding this, and continuous with it, certain changes were effected by the action of late mineralizing vapors or solutions, later growths of albite, aegirite, quartz and zircon being evidence of this. In fact no line can be drawn marking the close of the main period of crystallization, and there is doubtless in all magmas a slight but continuous mineralizing action going on for a long time after the main crystallization has virtually ceased, as the temperature remains relatively high for a long period, and the water, etc. have not been wholly eliminated.

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[67] In the extreme contact phases the rims about the feldspars are very narrow or impreceptible. This is to be expected, owing to the greater degree of supercooling resulting in a greater impediment to free molecular movement in the highly viscous residual liquors. It should be noted that in some parts of the porphyry a later rim is also found about the quartz phenocrysts, as might be expected.

[68] Fortschrift fur Mineralogie, Krystallographie u. Petrographie, **1**, p. 209, et seq.

Zircon appears to have grown throughout the entire period of crystallization but to have especially favored the last stages. This agrees with the observations of others on the zircon of similar rocks.[69]

The element of time appears here as the great conditioning factor. A long period of time made the establishment of a relatively perfect equilibrium possible and so smoothed out, as it were, the cooling curves of the magma, and left in the textural relations of the various minerals little or no trace of critical points in the progress of the crystallization.[70]

In the fine-granite, found as the contact phases in the eastern and southeastern parts of the area, we find in the texture good evidence that these contacts were at lower levels of the original contact than were those of the granite-porphyry. The rock is somewhat porphyritic, the phenocrysts being microperthite and quartz. These merge into the grain of the rest of the rock, which is like that of the coarse-granite but much finer. We have here probably to deal with a sudden, initial chilling of the magma for a certain distance from the slate contacts, followed, however, by a relatively slow dissipation of the heat when compared with that of the quickly cooled magma at higher levels, which developed the granite-porphyry. We may assume that the sudden, initial chilling caused a sufficient increase in the viscosity to determine more numerous centers of crystallization, while the relatively slow cooling that ensued under the thicker cover of slate permitted the development of an essentially even grained though fine texture. The conditions, like the results, were intermediate between those of the granite-porphyry on the one hand and the coarse-granite on the other. It is also conceivable that the magmatic water and other materials capable of aiding crystallization were retained better in the deeper seated magma than they were in the porphyry-magma, located as it was much nearer the surface.

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[69] See Murgoci, op. cit., p. 137; also Warren & Palache, op. cit., p. 144.

[70] In this connection the writer wishes to direct attention to the discussion of the development of the porphyritic texture given by Professor Crosby in his Blue Hill report (op. cit., pp. 360–1), and in a later paper (On the Origin of Phenocrysts and the Development of the Porphyritic Texture in Igneous Rocks. Amer. Geologist, **25** (May, 1900). Although several statements in this paper are hardly correct in the light of our present knowledge of the laws governing the crystallization of heterogeneous systems, much of his explanation of this texture appears to be substantially in accord with the most recent views on this subject, but does not seem to have been referred to as it deserves. Compare also a paper on the same subject by Professor L. V. Pirsson (Amer. J. Sci., **7** (April, 1889) whose views are discussed by Crosby in the second paper referred to.

Differentiation of the Rhombenporphyry.— We may next consider the crystallization of the rhombenporphyry and its bearing on the differentiation of this rock. Here there is no question but that the rhombenfeldspar phenocrysts — homogeneous mixed crystals as we hold them to be — were the first minerals to crystallize. Before the close of the growth of these feldspar phenocrysts the calcic-iron pyroxene began to crystallize. Again we note a pause, marking the close of the phenocrystalline stage of growth, followed by the consolidation of the groundmass about more numerous centers, with a practically simultaneous growth of the minerals and with the addition of a part of the groundmass feldspar — now cryptoperthite or microperthite — to the feldspar crystals already formed. In such a texture there is, in the writer's opinion, proof that the rock was not formed by any process of fractional crystallization. It indicates that the rock was crystallized from an individualized—differentiated—magma of homogeneous composition, at least through such parts as now possess a uniform texture and composition. It is true that the rhombenporphyry as a whole does not possess a perfectly homogeneous composition, and as sodic-pyroxenes and hornblende enter into its composition in some parts, we have doubtless a gradation toward the granite-porphyry. The composition of a part of the xenoliths, obviously fragments broken off from the main masses of this porphyry, show a somewhat further gradation toward the granite-porphyry. The actual contacts between the rhombenporphyry and the other rocks are always sharp, and if there ever was a complete transition between them it is necessary to assume that those types which were truely gradational in character have now disappeared from sight. It is the writer's opinion, however, that there always existed a sharp or very sudden change from the rhombenporphyry into the other types — a practical discontinuity. A number of similar sudden or sharp contacts between the peripheral differentiates of a magma and the remaining portions have been described from other localities, among the better known examples being those described by Weed & Pirsson in the Shonkin Sag Laccolith, Montana,[71] by Weed & Pirsson in the Square Butte Laccolith, Montana [72] and by Pirsson and Rice at Tripyamid Mountain, N. H.[73] In such cases the evidence all points to a strong diffusion of mineral compounds [74], having an earlier period of formation, toward the cooling surface followed by crystallization of the magma thus formed. Movement of the remaining magma against these differentiated phases does not seem sufficient to fully account for the sudden or even sharp change from one to the other, though as in the present case, it may account for a certain portion of the contacts. Assuming such a diffusion toward the cooling surface to have taken place, may not the process have tended to overrun itself, so to speak, thus bringing about a condition of supersaturation which resulted finally in a more or less sharp pause in the process, a pause analogous to that which has been described as marking the end of the phenocrystalline stage of growth in the case of the feldspar phenocrysts of the porphyries of this area. The result of such a pause is a practically discontinuous contact. It is not the same thing as the formation of immiscible liquid phases in a silicate magma, but the final result is not very different.

The Origin of the Cognate Xenoliths.— The opinion has been expressed that the patches of different texture and more basic composition which occur in the granite and to some extent in the porphyry are in the nature of inclusions of differentiated, peripherial portions of the batholith, and they have been called cognate xenoliths. That some of these xenoliths, as for example those found about the rhombenporphyry on Pine Hill, are nothing but fragments of the rhombenporphyry is perfectly certain, and it may be safely inferred that many others are of the same origin, though they are not so obviously connected in the field with the rhombenporphyry. That all of the xenoliths were formed in this way is by no means so certain.

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[71] Amer. Jour. Sci., **12**, p. 351 (1896).

[72] Bull. Geol. Soc. Amer., **6**, pp. 404–5 (1895).

[73] Amer. Jour. Sci., **31** (April, May, 1911).

[74] In this connection see also Brögger's conclusions to the same effect in a paper on the basic rocks of Gran. Quart. Jour. Geol. Soc., **1**, p. 15 (1894).

So far as the strongly porphyritic xenoliths are concerned it is important to note that they all exhibit the same characteristics as the granite-porphyry and the rhombenporphyry, in that they have phenocrysts of feldspar, usually of more or less distinct "rhomben" habit, which show a pause in the progress of crystallization followed by a simultaneous crystallization of the groundmass minerals. Furthermore the feldspar phenocrysts in the great majority of cases, and always in the xenoliths of the "rhomben" type of phenocrysts, are, or show positive evidence of having been, either a homogeneous soda-potash feldspar or very fine cryptoperthite, viz.— the same feldspar which is characteristic of the quickly cooled (quenched) phases of the magma. It seems improbable that we should find a small mass of rock possessing a texture indicative of relatively rapid cooling and containing phenocrysts of a type of feldspar, which elsewhere in the field formed only under conditions of rapid cooling, developing such peculiarities *in situ* enclosed in a rock which itself is typically granitoid and developed under conditions of slow cooling. It might be contended that these xenoliths have developed about one or more crystals of the feldspar, acting as nuclei, by some process of fractional crystallization, and that the rim of later growth was due to a growth of larger crystals at the expense of smaller ones during the period in which the material was surrounded by the hot, granite magma. The presence of poikilitically developed hornblende perhaps might also be cited to support this contention. It appears, however, to the writer that in such a case the same action of the hot magma would pretty certainly have effected a recrystallization of the feldspar phenocrysts since, as we have shown, these are very easily changed, and such changes have not been observed to be common in the feldspar phenocrysts of this type of xenolith; nor does it seem likely that so perfect a reproduction of the normal porphyritic texture could result. Furthermore, a process of gradual accumulation from the surrounding magma could hardly be expected to result here in sharp contacts, which are a characteristic of all of the xenoliths. The rounded, irregular and indented contacts of the xenoliths with the enclosing granite, so beautifully exposed on the glaciated ledge beside the railroad track east of Pine Hill, and the flow structures sometimes observed in them, suggest simply a softening and moulding of the xenolithic masses by the hot granite magma. It might also be contended that they represent consolidated liquid segregations formed *in situ* and that their finer texture was due to their more basic (than the granite) character, or to a tendency of the earlier formed minerals to form smaller crystals.[75] The same objection raised above, that the feldspar phenocrysts are not recrystallized, would operate strongly against this view, nor is there any evidence that because of a more basic or different chemical composition there should result such a texture as that described: nor is it the earlier formed minerals that are the smaller in size, but exactly the reverse. In this whole group of xenoliths (those with a marked porphyritic texture) there is little, if any, evidence that there has been any addition of material from the enclosing magma, and little, that there has even been any noteworthy recrystallization of the xenolith since its first formation.

In the case of the feebly or non-porphyritic xenoliths, particularly those of finely granitic texture, there is less positive evidence as to their origin. It is true that many of them contain phenocrysts of microperthite which show traces of an earlier formed core. But this is, in character, like the margin. It may be that this type represents spots in the granite which for some unknown reason developed a finer texture. But why they should have sharp contacts or why, in the case of those so near the granite in chemical composition as to possess, at best, but very slight differences effective in modifying the texture, they should show a contrasted texture, is not at all clear. It has been pointed out that, allowing for possible changes in chemical composition effected perhaps by the enclosing granite magma, the xenoliths as a whole correspond rather closely, and in part almost exactly, to the various types of rock found as peripheral phases of the batholith. For reasons above set forth, the writer believes that the porphyritic xenoliths are simply fragments of the peripheral phases of the batholith, broken up and more or less moulded in form by movements in the hot magma, and that these fragments have in part sunk in the magma or been moved from their original positions by the same movements. He is also strongly inclined to believe that all of the patches of contrasted texture are the result of the same process.

The above statements regarding the origin of the xenoliths appear to be in substantial accord with the views of Professor Crosby. Inasmuch as he relied on Dr. White's descriptions of their microscopic characters some of his statements regarding them differ materially from the writer's. We are, however, agreed that the porphyritic texture is a plain indication that the crystallization followed, and did not precede differentiation.[76] Crosby's final statement regarding their origin is as follows [77]: — ". . . . segregation on a large scale and subsequent crystallization developed a continuous zone of basic-porphyry: and that as the energy of the process diminished, or where it was primarily weak, it became more or less localized, developing isolated segregations which were subject to various accidents — distortion, cracking and crowding — during the gradual stiffening of the enclosing magma." The writer is disposed to insist that the evidence, as above given, shows that the segregation referred to by Crosby was confined to the immediate contacts and that it did not form small localized masses (the size of the present xenoliths) in the magma at points distant from the contact.

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[75] See Harker, op. cit., p. 348.

The Relations between the Soda and Potash Feldspars.— In the descriptive part of this paper it has been noted that the feldspar which occurs in the form of phenocrysts in the porphyries, when unaltered, is a soda-orthoclase (?) — or a very fine cryptoperthite. The phenocrysts consist centrally of a homogeneous material or a very fine (almost homogeneous) cryptoperthite, but toward the margins they become more distinctly perthitic though the structure is still very fine. In the groundmass of the rhombenporphyry and in the later rims of groundmass age about the phenocrysts, we have cryptoperthite or coarser microperthite; in the granite-porphyry the rims about the phenocrysts are of microperthite as are also the small crystals in the groundmass. In the granites, on the other hand, the feldspar is throughout a strongly developed and fairly coarse microcline-microperthite, with a very subordinate amount of separately crystallized albite, and this is located about the ends or sides of the microperthite crystals, usually as a continuation of the albite within the body of the crystal.

Recently J. H. L. Vogt [78] in an elaborate paper, has developed the view that the anorthoclase — cryptoperthite — feldspars are eutectic mixtures and proposes to designate them as "eutectic feldspars." As the result of a statistical study of the available chemical analyses of feldspars and the rocks in which they occur, he has computed that the eutectic mixture lies between the limits 40–44 Or: 60–56 Ab + An; that the mixed crystal phases forming the eutectic are about Or, 12%: Ab + An 88%, and 72% Or: 28% Ab + An. That is, he believes that mixtures of potash-feldspar and plagioclase belong to type V of Roosebooms'[79] classification of mixed crystals; viz.— they have a limited miscibility in the crystalline condition with a eutectic point between the two resulting types of mixed crystals. His figures are admittedly only approximations and future investigation will doubtless modify them in some particulars, but nevertheless his views are to the highest degree suggestive and important. At temperatures lower than that at which the anorthoclase or cryptoperthite crystallizes, some diminution of the solubility of the one feldspar in the other in the mixed crystals, would probably result connected, probably, with a transition to another crystalline modification in the case of at least one component, and a transformation, an unmixing (*entmischung*), in the solid condition would then take place. The perthite or microperthite which might in such case result, would consist of a potash feldspar containing less Ab or Ab + An than it would at higher temperatures, and a plagioclase member, likewise poorer in Or. Vogt estimates that the potash member would contain in such cases 10–15% of Ab or Ab + An. That the perthitic intergrowth is due in general to an unmixing of an earlier existing homogeneous mixture, has also been entertained by several other petrographers,[80] although none have put their ideas into such definite form as has Vogt.

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[76] op. cit., p. 372.
[77] op. cit., p. 373.
[78] Tschermak's Mineral. u. Petrog. Mitt., **24**, No. 6 (1906).

It appears to be pretty well established that there are all gradations between microperthite and cryptoperthite; that there is a truly gradual gradation into a perfectly homogeneous soda-potash feldspar is not so certain. It may be entirely correct to look upon the homogeneous anorthoclase as being a crystallized eutectic mixture, but the writer is led to go somewhat further and suggest that the apparently homogeneous mixtures may be a true mixed crystal belonging to Type 1 of Rooseboom, viz.— the two feldspars are miscible in all proportions in the solid crystalline state.[81] It may be noted that anorthoclase presents certain peculiarities that perhaps suggest a mixed crystal. It is a familiar fact that its crystals frequently show a division into fields when examined under crossed nicols, in fact, these may be visible megascopically. This phenomenon is common in mixed crystals such as the garnets and alums. Again the crystalline habit of the anorthoclase commonly differs from ordinary feldspars. The crystals are often curiously distorted, acute terminations being common. These may be due to the physical conditions of the solution under which they grew, but one cannot but recall that such abnormalities are characteristic of certain mixed-crystals among laboratory salts.

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[79] Zeitschr. f. Phys. Chem., **30** (1897).

[80] Rosenbusch, F. Becke and others.

[81] Since this was written a paper by Dr. E. Dittler (T. M. P. M., Nos. IV–V, **31**, pp. 511–22, 1912) has come to the author's attention, in which experimental evidence is given to the effect that the alkalic feldspars form mixtures belonging to type 111 of Rooseboom's classification; viz.— a continuous line of mixed crystals tending toward a minimum freezing point.

There seems, however, to be more direct evidence of the complete isomorphism of the soda and potash feldspars. P. L. Barbier and A. Prost[82] of Lyon, France, have recently described monoclinic feldspars in which the soda is present in a molecularly greater amount than the potash. This has been confirmed by Dr. W. T. Schaller[83] of Washington, who has suggested for the new monoclinic modification of soda feldspar, thus shown to exist, the name "Barbierite" in honor of its discoverer. F. Angel[84] has also described a "Soda bearing, monoclinic sanidine containing 4.95% Na_2O and 6.75% K_2O from Mitrowitza. Although anorthoclase appears to be triclinic at ordinary temperatures it is worthy of note that according to Forstner[85] it becomes monoclinic at higher temperatures.

It appears to the writer that at the temperature of crystallization we may assume that the potash and soda-feldspars form mixed-crystals of Type 1, and that this condition of equilibrium continues to hold for an interval, probably a short one, below this temperature. With lowering temperatures both of the components may pass through an inversion point, going over into other crystalline modifications, albite and microcline. This produces a radical change in the equilibrium of the system, with the result that there is an unmixing of the original mixed-crystal phase, and the formation of two new mixed-crystal phases having a eutectic point between them — that is, they pass from Type 1 of Roozebooms' to Type V. One of these phases is albite, or a highly sodic feldspar of the albite type, holding still in solution some of the other component, and the other is a microcline likewise with some albite dissolved in it. If this be so, it would appear that the alkali feldspars exist in two modifications each, an α and a β form. Vogt[86] has discussed this question for microcline and arrived at the conclusion that orthoclase and microcline stand to each other in the relation of an α and β form, and he compares the polysynthetic twinning structure of the latter with that of the well known case of leucite. It is true, so far as the writer can ascertain, that the potash member of such microperthites, where this has been determined carefully, is microcline either with the usual gitter structure, or with the albite twinning alone, or even without twinning.

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[82] Bull. Soc. Chem., p. 111 (1908).
[83] Bull. No. 509, U. S. Geological Survey (1912).
[84] Neues Jahrb. fur Min., Biel-Band, **30** (1910).
[85] Zeit. f. Kryst., etc., **9**, p. 333 (1884).
[86] op. cit., p. 540.

The theory above put forth does not contradict the conclusion of Vogt that many, perhaps most microperthites and cryptoperthites are essentially eutectic mixtures. It supplements it by way of adding something to the theory of feldspar mixtures so far as the alkalic members are concerned, and it is believed may lead to somewhat better understanding of their relations in igneous rocks generally. The approximation to constancy in the composition of the cryptoperthite and microperthite feldspars as shown by Vogt, is a strong argument in favor of their being eutectic in nature. Their peculiar structure is also favorable to this view. For, if we reason from the analogy of the well known eutectic structures in alloys and laboratory salts in which one of the characteristics is an intimate mechanical or crystallographic mixture of the two phases forming the eutectic, we should expect to find in feldspar mixtures, which are of nearly or exactly eutectic composition, a corresponding tendency to form an intimate and curious intergrowth. In view of the well recognized sluggishness associated with transformations in the solid state, we should expect that the unmixing at a transition point, such as the one suggested above, would in many, perhaps most cases be more or less incomplete, and in mixtures which originally departed more or less widely from eutectic proportions, it is probable that the resulting crypto- or microperthite would, owing to the incomplete separation of the two phases, be only an approximate eutectic ("anchi-eutectic" of Vogt). To the sluggishness of transformations in the solid state must be added in the case of the feldspars a further, and doubtless considerable, impediment to the progress of the readjustment caused by the extraordinarily viscous character of the feldspar substance. This may be inferred from the thermal properties of the feldspars as revealed by the brilliant work of Dr. A. L. Day[87] and his associates at the Geophysical Laboratory in Washington, D. C. On account of their peculiar thermal properties it is to be expected that except under the most favorable conditions, critical points on their cooling curves will be readily passed by through supercooling, and that we should rather speak of critical intervals than points. For the same reasons, while the feldspars doubtless tend to form a eutectic mixture theoretically, it is highly probable that in the actual crystallization this tendency may often fail of complete realization, and that Vogt's term "anchi" — approximate — may be appropriately applied to the intergrowths of the two feldspars as eutectics, and to the two mixed-crystal phases as well.

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[87] The Thermal Properties of the Feldspars, Carnegie Institution, Washington, D. C., (1905), and American Journal of Science, **14** (Feb., 1905).

The phenocrysts of lavas and of the contact phases of an intrusive igneous mass, like the one under consideration in the present paper, grow with relative rapidity and are then frozen in the quickly consolidated groundmass. These are precisely the conditions which are favorable to the preservation of the crystal in the condition in which it first formed, even after the temperature falls below a possible transition point, provided the tendency to readjustment, which would normally ensue, is not very strong, and the actual change itself is hindered sufficiently by the molecular immobility of the now solid crystal as well as by the relatively rigid surroundings. In other words, a phase, stable at higher temperatures, is caught by the quenching of the magma and rendered relatively stable at lower temperatures. If a crystal in a metastable condition be long held at a temperature, at or below, its transition point, or is acted upon at relatively high temperatures by a liquid or vapor, especially if the stable phase is present, it will, as is well known, generally go over to the stable phase. Precisely this thing appears to have occurred in the case of the feldspar phenocrysts of the Blue Hill porphyries. Parts, perhaps almost the entire phenocryst of feldspar, will be, so far as the microscope shows, of entirely homogeneous structure, but through this, streaks and patches are found, of cryptoperthite or very fine microperthite. As we approach the margin of the crystal the same thing is observed, and the crystal for an indefinite, short distance from the margin consists of a perthitic intergrowth. The streaks and patches in the inner parts of the crystal are believed to be due to an *unmixing* under the action of long continued heat, accompanied probably by the action of heated vapors or solutions permeating the rock. The marginal microperthite may be due to the same cause, but it seems more probable that at the time when this grew the temperature had fallen to a point below the transition interval, and the two phases separated more or less completely, forming the crypto- or microperthite; the cryptoperthite here, and in general, representing a structure resulting under conditions less favorable to a *free unmixing* than the microperthite. It is to be noted that the generally widespread and clearly marked recrystallization of the phenocrysts in the porphyries is, it is believed, an evidence of the generally unstable condition of the feldspar phenocrysts as first formed from the magma. As still further bearing on this question we may note that the smaller and later formed phenocrysts are largely microperthite, and that the feldpsar which developed in the groundmass of the porphyries, in part attaching itself to the older phenocrysts, is all microperthite. This part of the feldspar content of the rock obviously solidified at lower temperatures than the phenocrysts, and, if our theory is correct, formed the region lying below the transition interval.

When we come to the coarse-granite we find that all of the feldspar is a relatively coarse microperthite. In that part of the magma which eventually formed the granite a very long cooling period obtained, during which there would be ample opportunity for the establishment of more perfect equilibria in general. The mixed crystals of soda-potash feldspar which first formed would pass through the transition interval, but, in sharp contrast to conditions obtaining in the porphyry magma, there would be no supercooling, and enough time for the readjustments to take place. The new phases did not move far, but rather withdrew from each other, and being still closely similar crystallographically, finally took up the relative positions in which we now find them; viz.— they formed an intimate, crystallographic intergrowth.

Regarding the composition of the soda-potash-feldspar eutectic, there is still, as is to be expected, both on account of the peculiar physical properties of the feldspars alluded to above, as well as on account of the character of the available evidence bearing on the question, much uncertainty. The approximate compositions of the feldspars, as calculated from the analyses of the Quincy-Blue Hill rocks, show some divergence from the figures worked out by Vogt. The limiting values for the concentrations of one feldspar in the other, as given by that author, appear to the writer as being open to some question. In the present rocks the sodic member, so far as can be told, appears to be a very pure albite, likewise the potash member seems to be a highly potassic microcline. It would appear, if the writer's deductions are correct, that the concentrations of the two feldspars in the mixed crystal phases may be lower, at least for the highly alkalic feldspars, than is indicated by the figures of Vogt. That the eutectic composition lies in the general region indicated by him appears probable. It is of course true that Vogt's conclusions include more calcic plagioclases than are concerned in the present case, and it seems not improbable that with the presence of more of the lime feldspar molecule, different conditions of equilibrium come into play, and that for a certain concentration of anorthite, complete crystal miscibility may not obtain, but only partial miscibility, and that such feldspar systems belong to type V only of Roosebooms' classification.

The unmixing of a previously homogeneous mixture is believed to offer also an explanation of the occurrence of albite about the margins of the microperthite grains in the granite, and also in part, of the presence of strongly marked albitization of some of the original feldspar in the porphyries. Assuming that the albite present in the soda-potash mixed-crystal was in excess of the eutectic proportions stable at lower temperatures, we should expect that, in the process of readjustment which would take below the inversion point above postulated, that some of the albite phase would be set free and that it would be forced out upon the final crystallization of the microperthite. Much of it would be expected to attach itself directly to the albite exposed along the margins of the microperthite crystals, while some of it would grow alongside as separate crystals or even migrate to more distant parts of the rock. The same thing would happen in the case of the feldspar growing in the groundmass of the porphyry, but there would be present the metastable, earlier formed phenocrysts, which would be readily effected by the liquors rich in albite, thus producing the albitization described as so commonly observed in the porphyries. This theory relieves us of the necessity of calling upon the underlying magma to supply albite bearing solutions in the considerable amount necessary to furnish all of the later albite in the porphyry. It does not, however, by any means exclude the probability, that the underlying, crystallizing magma has furnished some part of the albite.

Development of Microliths in the Feldspar. — The writer has expressed the opinion in this paper and elsewhere [88], that the microliths of aegirite found so abundantly in the feldspar of the granite were not products of earlier crystallization, but were of later origin. During the readjustments in the feldspar to meet changed conditions of equilibrium have we not a most favorable time for the introduction of those minute crystallizations of a mineral which is known to have been growing during the later period of crystallizing activity? During the readjustments within the feldspar, would we not have the degree of molecular mobility and openness of structure, if we may use such a term in this connection, to favor the introduction of extraneous material, aegirite, etc? Does not also the presence of the innumerable, minute vesicles in part, at least, date from this period of change? It is of course possible that the aegirite, etc. represents material that was originally held in solution in the feldspar material, and that it separated out during the unmixing of the original crystal. No microliths of aegirite nor of any other mineral appear in the homogeneous parts of the phenocrysts in the granite-porphyry, and this alone is believed to be very strong evidence that they made their appearance during the unmixing and probably by introduction from sources outside the feldspar crystals themselves. This has, of course, no connection with those microlithes and larger crystal grains, such as the riebeckite, which are connected with later alterations in the rocks.

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[88] Warren & Palache. Pegmatites of the Quincy Granite, etc., op. cit. p. 145.

In this connection it is perhaps worth noting that the pyroxene and amphiboles may also undergo transformations of which we have as yet no definite information.

Summary.

The alkaline granitic magma of Quincy and the Blue Hills was intruded as a small batholithic mass, later than the middle cambrian, but earlier than the late carboniferous period. The method of intrusion is believed to have been one of "stoping." The magma as a whole is believed to have stoped its way upward to a relatively high elevation in the pre-existing formations, and a considerable portion of it reached a position very near the surface. As a result, the upper portions consolidated to highly vitreous rocks, while somewhat lower portions formed thick masses of porphyritic, crystalline rocks,—quartz-feldspar and granite-porphyries. The extreme upper portions were perhaps more siliceous and richer in volatile constituents, which in large part may have escaped; lower portions consolidated without substantial differentiation or change. As an incident to intrusion, the earlier consolidated portions were much broken, engulfed in the magma beneath, and against these, as against the original contacts, the residual magma consolidated with a varying degree of chilling, according to position. At considerably lower contact levels now exposed in the eastern part of the field, the magma consolidated with a marginal zone of fine-granite as the contact phase. Here also the first formed rock was intruded by, broken up, and to some extent engulfed in the magma beneath. Under the thick cover of porphyries acting as an effective insulator, as well as along the deeper original contacts, the remaining magma cooled with sufficient slowness to permit of the development of a medium coarse, even grained rock, the granite.

Locally, against deep projections of slate, as well as to a small degree under its own porphyry cover, the magma differentiated with the formation of a relatively basic phase, the rhombenporphyry. Some part of this basic differentiate still remains in place against the slates, but considerable portions of it were broken up, and together with fragments of the other contact phases, were scattered by sinking, and movements in the magma. Many of these were eventually frozen in the magma, particularly near their original places of formation, and now form the cognate xenoliths (knots) so abundant in the granite.

Long continued erosion has removed nearly all of the original slates etc. invaded by the magma, together with some portion of the first formed marginal cover of igneous rocks. Faulting and elevation of the mass, particularly along its northern edge, has brought up into the plane of erosion a portion of the original deep seated slate-granite contacts.

Owing probably to the small capacity possessed by the magma for differentiation, combined with the rapid refrigeration of a great part of the magma, no complementary dike phenomena or other intrusions of complementary nature were developed, and are, in striking contrast to many other batholithic intrusions of granitic rocks, absent from this area.

In the case of the rhombenporphyry and the cognate xenoliths derived from it, it is held that their crystallization followed differentiation, and that the process of differentiation was one of diffusion of compounds, which were normally the first to crystallize from the magma, toward the cooling surface. It is held that while a part of the sharp contacts between this basic phase and the granite-porphyry and granite are due to mechanical causes (breaking and movements in the magma) there was originally a practical discontinuity — sharp or very sudden change — between the rhombenporphyry and the main magma.

Chemically the magma as a whole is characterized by high silica, high alkalies, relatively high iron and by very low lime and magnesia. The alkalies are about equal, the potash slightly predominating in amount, but molecularly less important than the soda. Chief accessory constituents are zirconium, titanium and fluorine.

The feldspar is substantially a mixture of soda and potash molecules. In the phenocrysts of the porphyries much of it appears as a homogeneous mixture of the two. Irregularly through parts of the phenocrysts, and toward their margins, cryptoperthite or microperthite appears. A later growth of fine microperthite attached to the phenocrysts, but of groundmass age, occurs. The feldspar of the groundmass is microperthite. In the granites, the feldspar is substantially all microperthite, a little free albite occurring about the ends and sides of the crystals. The microperthite throughout, so far as can be told, is a microcline-albite microperthite. Later changes of a deep-seated character have profoundly altered much of the phenocrystalline feldspar of the porphyries, either recrystallizing it to an aggregate of albite and microcline, or in part replacing it with albite. Similar replacements have occurred to some extent in the granitic feldspar.

The predominant dark silicate is hornblende, of which two varieties appear. One of these is riebeckite or a closely allied type, the other appears to be closely related to the cataphorites. Both are generally present: on the whole the riebeckitic variety clearly predominates and hence the rocks have been characterized as riebeckitic. The predominant pyroxene, by a wide margin, is aegirite. In the most basic member of the rock-series a variety of pyroxene rich in the calcium-ferrous iron molecule appears. This is found to a very small extent in the more siliceous porphyries but gives place to a more highly sodic type, which also occurs in the granites to a small extent. The rare mineral aenigmatite occurs in the granite and in the granite-porphyry.

The predominant molecules in the hornblendes are: — $Na_2Fe_2Si_4O_{12}$, $(R'_2,R'') Fe_2Si_4O_{12}$ — where R′ is soda with small amounts of fluorine and hydroxyl, and R″, ferrous iron chiefly — and $Fe_4Si_4O_{12}$: In the pyroxenes $Na_2Fe_2Si_4O_{12}$ strongly predominates but $Ca\ Fe\ Si_2O_6$ is often present. The conditions determining the formation of these minerals is very obscure, but slight variations in the concentrations of the constituent oxides present, local variations in the amounts of mineralizers, particularly fluorine, and the rate of cooling, are doubtless the most important factors concerned.

The hornblendes are readily effected by alteration, and change first to a deep-blue, riebeckitic or crocidolitic variety and eventually change to magnetite and other iron oxides. The aegirite is less readily altered.

Alteration, deep-seated or superficial has not much effected the granites, except locally, but the porphyritic rocks have suffered more generally, so much so, that their original structures are often profoundly changed.

Quartz and a soda-potash feldspar (mixed crystal) appear as the first crystallizations from the porphyry magma, followed closely by the less sodic-pyroxenes. It is thought, however, that the feldspar was the first crystalline phase to form normally, followed quickly by the quartz and pyroxene. These were followed by the other minerals. It is inferred that in the granite the order of crystallization was substantially the same as that of the granite-porphyry. In the porphyries there was a sharply marked pause closing the phenocrystalline stage of growth, due, it is believed, to supersaturation, after which crystallization was resumed, but about vastly more numerous centers, the various minerals continuing to grow almost or quite simultaneously, in part attaching themselves to the earlier formed crystals, but largely forming a fined grained groundmass, in part with a poikilitic fabric. In the porphyritic phase of the granite, into which the granite-porphyry passes, the pause in the progress of the crystallization was less marked, and in the normal granite it did not occur, owing to the more perfect conditions of equilibrium conditioned by very slow cooling. The later crystallizations added themselves to the crystals already present, and thus obliterated all distinction between phenocrysts and groundmass.

It is held that the granite-porphyry represents substantially the composition of the magma as intruded, and that beneath this protecting cover the remaining magma was able to differentiate to a small extent forming a more basic phase, the rhombenporphyry, itself becoming in consequence more siliceous. Assuming that by this differentiation the granite moves toward a eutectic composition, which is by no means a necessary or certain procedure, it is pointed out that the proportions of quartz to the feldspars departs quite widely from the granite eutectic proportions as estimated by Vogt. In fact the proportions of these minerals in the granite-porphyry is nearer to Vogt's eutectic ratio than are those of the granite, which appears to be a variation in the wrong direction to agree with his theory. The effect of the presence of the sodic-iron silicates on the composition of a possible eutectic is not yet determinable, and in any case, the composition of the granite eutectic appears to be a very open question.

It is suggested that the potash and highly sodic-feldspars may first crystallize as homogeneous mixed-crystals (type I, of Rooseboom) and at somewhat lower temperatures pass though a transition point, or interval, becoming then only partially miscible in the solid state, with a eutectic point between the two resulting types of mixed crystals, albite and microcline (type V of Rooseboom). The homogeneous feldspar phenocrysts, or such parts of them as are still homogeneous, represent the first formed type caught in the rapidly chilled magma, and now exist in an unstable condition. The cryptoperthite and microperthite generally throughout the porphyries, and granites, are regarded as the result of an unmixing below the transition point or interval, and probably represent an approximate eutectic mixture, or a true eutectic, provided that the readjustment had time to complete itself. It is suggested that the prominence of the albite phase about the margins of the granitic microperthite crystals, and elsewhere in the rocks as an apparently late crystallization, is due, in part at least, to the excess over eutectic proportions of that constituent in the original mixed-crystals, and that it was set free during the unmixing and thus became active during the late stages of crystallization. This period of unmixing was probably that during which the aegirite microliths, etc. were introduced into the feldspar of the granite. This and the albite crystallizations were doubtless aided by the last liquids (mineralizers) of the magma. It is thought that, at least for mixtures containing as in the present case, a highly sodic member, the concentrations of one feldspar in the other in the two phases of the eutectic mixture, are probably not as great as those calculated by Vogt, and that the true eutectic proportions may depart somewhat from the value deduced by him, but that this value is not much in error.

It is pointed out that following the transformations which are believed to have taken place during the late magmatic period, there were further and more or less profound alterations of a deep-seated character in the minerals of the porphyries, and to a less extent in the granites. Subsequent surface alteration has, in many localities, resulted in still further changes chiefly affecting the iron-bearing silicates.

The characteristics of these rocks show a close analogy in many respects to those described for other intrusions of riebeckite-aegirite granites, and in this respect are in general agreement with the generalizations of Murgoci regarding this class of rocks.

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PLATE 1.

Fig. I.— Granite-Porphyry, Rattlesnake Hill, Blue Hills. At the bottom on the right, is a pyroxene crystal surrounded by poikolitic hornblende enclosing groundmass grains; above appears a cluster of aenigmatite grains; on the left, below, is a cross-section of a pyroxene crystal with a small slightly darker margin of aegirite; above is a massive hornblende becoming poikolitic outwardly and then showing enclosed groundmass grains. With the exception of a small phenocrysts of feldspar, barely seen just above the center, the rest of the section is all groundmass in which tiny microliths of aegirite can be seen.

Photomicrograph, plane light. Magnification about 15 diams.

Fig. II.— Granite-Porphyry, Sassoman Pass, Blue Hills. On the right is shown the edge of a large phenocryst of nearly homogeneous soda-potash feldspar terminated by a sharply marked line outside of which is a narrow margin of microperthite enclosing tiny groundmass crystals; near the bottom is a small phenocryst of feldspar with a relatively broad, later rim; the rim shows a delicate perthite structure which is also developed for a short distance within the inner and earlier formed part. A similar phenocryst appears at the top of the section. At the extreme lower side is seen a small phenocryst of relatively late age enveloped by a relatively very wide rim of later age. In this rim are minute enclosures of groundmass grains. The dark part of the field is hornblende, poikolitically enclosing feldspar (microperthite) and quartz crystals.

Microphotograph, plane light. Magnification about 25 diams.

Figure III.— Granite-Porphyry, Rattlesnake Hill, Blue Hills. This shows a phenocryst of soda-potash feldspar surrounded by a narrow rim of later microperthite enclosing groundmass grains. There is a faint perthite structure developed in the inside portion of the phenocrysts next the rim but this is very indistinctly shown with the magnification used. The phenocryst is crossed by streaks representing cracks sealed with albite. A part of the body of the crystal has suffered a recrystallization, shown as flower-like areas.

Microphotograph, crossed nicols. Magnification about 25 diams.

Fig. IVa.— Granite-Porphyry, Chicatawbut Hill, Blue Hills. This shows one end of a largely recrystallized feldspar phenocryst. The boundary of the phenocryst is marked by a sharp line outside of which is a rim of later groundmass age. Old cracks, now sealed with albite, and having a deposit of minute aegirite microliths on either side of them cross the phenocryst.

Microphotograph, plane polarized light. Magnification about 25 diams.

Fig. IVb.— Same as fig. IVa, but with crossed nicols. Nearly the whole interior of the phenocryst is seen to be recrystallized to a fine aggregate of feldspar-microcline and albite. A good part of the crystals stand normal to the direction of the albite streaks which are seen to run into the marginal parts of the original feldspar substance, and these have not yet suffered recrystallization, having been of more stable composition than the interior.

Fig. V.— Altered granite-porphyry, between Hemingway and Hancock Hills, Blue Hills. This shows the remnants of two feldspar phenocrysts. All that remains of the original crystals is a part of the margin. The white streaks of albite, which mark the position of cracks in the original crystal, remain (see figs. III and IV). Between these is now a mass of albite and microcline grains which are scarcely distinguishable from the surrounding groundmass. This illustrates an extreme stage in the destruction of the feldspar phenocrysts. The groundmass still shows most of the original aegirite, but the hornblende has suffered much modification.

Microphotograph, crossed nicols. Magnification about 25 diams.

PLATE 2.

Fig. VI.— Granite-porphyry from about 20 ft. back from the contact, east of summit Pine Hill, West Quincy. Shows a feldspar phenocryst crossed by streaks of albite and recrystallized to a curiously mottled microperthite. About the end of the crystal is seen a micrographic intergrowth of quartz and feldspar. The groundmass is here much coarser in grain than in the porphyry of the Rattlesnake Hill type and the grain is fast approaching that of the granite into which this rock passes within about twenty feet.

Microphotograph, crossed nicols. Magnification about 25 diameters.

Fig. VIIa.— Quartz-feldspar-porphyry, from near the top of Hemingway Hill, Blue Hills. This shows the greater part of a feldspar phenocryst which has undergone a partial and unusual alteration. Considerable portions of it are still a perfectly homogeneous, but about its margins, also along the edges of a break which crosses it, it has been replaced by a narrow band of normally orientated lath-like crystals of albite and microcline. A large part of the interior has been changed to small areas of microperthite of curious pattern (see Figure VIIb). The groundmass has been forced in along the break referred to.

Microphotograph, crossed nicols. Magnification about 25 diams.

Fig. VIIb.— Same section as Figure VIIa. Shows a feldspar phenocryst to some extent broken and entirely changed to a curious aggregate of small microperthite areas. The structure of these areas is irregular and slightly divergent giving the effect of a curious and beautiful tracery.

Microphotograph, crossed nicols. Magnification about 25 diams.

Fig. VIIIa.— Rhombenporphyry Xenolith, Pine Hill Area, West Quincy. Shows a characteristic phenocryst of soda-orthoclase (anorthoclase?) The shape of the crystal is characteristic of much of the feldspar in the rhombenporphyry as a whole, likewise the minute rod-like pyroxene microliths, located just outside the sharply marked boundary but lying in a band of feldspar substance attached to the phenocryst. The groundmass consists of microperthite and pyroxene with some hornblende.

Microphotograph, plane light. Magnification about 15 diams.

Fig. VIIIb.— Same crystal as in Figure VIIIa but with crossed nicols. The attached border of microperthite of groundmass age is well shown.

Fig. IX.— Rhombenporphyry, Pine Hill Area, West Quincy. This shows a composite feldspar phenocryst, the acutely terminated points of the crystals pointing the same way. The development of minute pyroxene microliths in the marginal growth of feldspar just outside the boundary of the original phenocryst is well shown.

Microphotograph, plane light. Magnification about 15 diams.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series.—No. XLI.

By Sidney F. Blake.

I. A Redisposition of the Species heretofore referred to *Leptosyne*.

II. A Revision of *Encelia* and some related Genera.

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.— NEW SERIES, No. XLI.

By Sidney F. Blake.

Presented by B. L. Robinson 14 May, 1913. Received 12 June, 1913.

I. A REDISPOSITION OF THE SPECIES HERETOFORE REFERRED TO LEPTOSYNE.

Among the numerous genera of the Helianthoid *Compositae* scarcely any has had a more involved history than the genus *Coreopsis* L., its synonymy embracing more than a score of generic names. On the one hand closely allied to the still larger genus *Bidens*, and perhaps not clearly separable from it, it is related on the other to various smaller and much more distinct genera. One group of about a dozen species, characterized by fertile rays and the presence of an annulus on the tube of the disk-flowers, has by many authors been kept distinct under the name *Leptosyne* DC., but by Bentham and Hooker,[1] Hoffmann,[2] and more recently by Hall,[3] has been reduced to *Coreopsis*, apparently with justice. In habit, involucre, and achenes its members are closely similar to various species of genuine *Coreopsis;* and while most of the species have a thickened hairy annulus at base of throat in the disk-corollas, this is glabrous in some species [4] and entirely absent in others,[5] while the rays although usually fertile are sometimes sterile or neutral in the section *Pugiopappus;* so that in the absence of any quite constant diagnostic character and because of the general very close similarity, it seems advisable to follow the authorities above mentioned in referring the genus definitely to *Coreopsis*.

The genus *Coreocarpus* Bentham, on the other hand, although made a section of *Leptosyne* by Gray and included by Hoffmann in his section *Leptosyne* of *Coreopsis*, departs in its isomorphic involucral scales so strikingly from the distinct dimorphism of the bracts, running with considerable uniformity through the whole *Coreopsis* series, as to deserve generic rank. The bracts are imbricated in two rows, subequal and all similar, subherbaceous, and ovate, which in connection with the cymosely panicled small heads makes the genus easy of recognition.

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[1] Gen. Pl. ii. 385 (1873).

[2] In Engler & Prantl, Nat. Pfl. iv. Ab. 5. 243 (1890).

[3] Univ. Calif. Pub. Bot. iii. 139 (1907).

[4] *L. maritima*, *L. gigantea*.

[5] *L. mexicana*, *L. insularis;* and *Electra* has no annulus.

A scapose Mexican perennial, *Leptosyne pinnata* Robinson, described from material without ripe fruit and referred to this genus with considerable doubt, proves to confirm in achenial characters the distinctness already suggested by habit and is described further on as a new genus.

The relation of the groups in question would seem to be best shown thus:

COREOPSIS L. (*κόρις* bug, and *ὄψις* likeness, from the form of the achene in the original species, *C. lanceolata* L.) Heads radiate or rarely and abnormally discoid, the flowers all yellow; rays neutral or styliferous and fertile or rarely sterile, disk-flowers mostly fertile. *Involucre double, scales of each series slightly connate at base; the inner membranaceous, 1–2-rowed, brown or yellow; the outer narrower* (*except in C. calliopsidea*), *herbaceous, usually shorter than the inner.* Receptacle flat or nearly so; pales flat, membranaceous. Ray-florets ligulate, entire or 2–3-dentate; disk-florets regular, tubular, with slightly enlarged throat and (4–)5-toothed limb, often with a thickened glabrous or pilose ring at base of throat. Anthers entire or barely bidentate at base. Style-tips truncate or with short subulate hispid appendages. Achenes obcompressed, sometimes meniscoid and much thickened on one face, orbicular to oblong, those of the ray when fertile commonly broader than the others, glabrous or pubescent, sometimes villous on the margins, wingless or with a chartaceous wing sometimes pectinately lobed, epappose or with two teeth, short ciliate scales, or glabrous or upwardly hispidulous awns or scales, or with a small cupule in place of pappus.— Herbs or rarely shrubs, glabrous or pubescent. Leaves alternate or usually opposite, undivided and entire or toothed, or ternate, or usually ternately or pinnately dissected. Heads of medium size, solitary or corymbose-panicled.— Gen. 263, no. 670 (1737), and Sp. Pl. ii. 907 (1753), in part.

Subgenus **Leptosyne** (DC.) Blake, n. comb. Rays styliferous, mostly fertile, rarely neutral in the section *Pugiopappus;* disk-flowers usually with a thickened and generally hairy annulus at base of throat. — *Leptosyne* DC. Prod. v. 531 (1836); Gray, Proc. Am. Acad. xvii. 218 (1882), Syn. Fl. i. pt. 2. 299 (1884). *Coreopsis* sect. *Leptosyne*

O. Hoffm. in Engler & Prantl, Nat. Pfl. iv. Ab. 5. 243 (1890) (excluding *Epilepis*, *Coreocarpus*, and *Acoma*).— Twelve species, ranging from California to Guatemala, chiefly Mexican.

Sect. 1. **Electra** (DC.) Blake, n. comb. Suffruticose, with opposite oval to lanceolate coriaceous sometimes ternately parted leaves. Heads solitary or paniculate-corymbose, radiate. Outer involucral scales about 5, oblong; inner about 8, longer, oval-oblong. Rays about 5, 2–3-dentate, oblong to elliptic, fertile, the tube pubescent; disk-flowers with pubescent tube shorter than the cylindric-funnelform throat, and (4–)5-toothed limb; annulus none. Style-branches with subulate hispid appendages. Achenes strongly obcompressed, glabrous, margined, the outer broad, the inner much narrower, all pappusless or the inner rarely with a pair of smooth slender awns.— *Electra* DC. Prod. v. 630 (1836); Gray, Pl. Wright. i. 110, footnote (1852).— Three species of Mexico and Central America.

* Heads numerous in ternate corymbose panicles.

1. C. MEXICANA (DC.) Hemsl. Shrubby, nearly glabrous, 0.6–2 m. high; leaves lanceolate to lance-ovate, acute to acuminate at both ends, sharply serrate, often trifoliately cut nearly to the midrib, glabrate on both sides or retaining a sparse pubescence chiefly along the veins, the blades 4–11 cm. long, on narrowly margined petioles 1–2.5 cm. long; heads 1–1.3 cm. high, 2.5–4 cm. in diameter including rays; achenes 6–9 mm. long.— Biol. Centr.-Am. Bot. ii. 196 (1881). *Electra mexicana* DC. l. c. *Electra Galeottii* Gray, l. c. *Coreopsis Galeottii* Hemsl. l. c. 195.— In an authentic example of *E. mexicana* in the Gray Herbarium, collected by Mendez, the tube of the ray is distinctly hirtellous, and the narrowly lanceolate leaves still show a slight appressed pubescence beneath, while one of the younger heads is also sparingly hairy at the base, so that the characters relied upon by Dr. Gray in separating *E. Galeottii* entirely fail to hold. *Galeotti* 2086, represented by a fragment in the Gray Herbarium, as well as the Baites specimens cited in the original description, is practically glabrous, while *Galeotti* 2087 somewhat approaches the next form.

GUANAJUATO: "circa Villalpando ultra Guanaxuato," *Mendez* (COTYPE in Gray Herb.); Guanajuato, 1895, *Dugès* 472; near Cadereyta, 22 Aug. 1905, *Rose* 9717; HIDALGO: sunny rocky slopes, Pachuca, Sept. 1905, *Purpus* 1550; clay banks, Dublan, alt. 2070 m., 15 Oct. 1902, *Pringle* 9895; Sierra de Pachuca, 2900 m., 14 Sept. 1899, *Pringle* 8218; near Metepic Station, 2530 m., 20 Sept. 1904, *Pringle* 13041; MEXICO: barranca above Santa Fe, 2600 m., 1 Sept.

1905, *Pringle* 13547; OAXACA: Cerro San Antonio, 1650 m., 26 June 1906, *Conzatti* 1431; CHIAPAS: 1864–1870, *Ghiesbreght* 133, 539 (both with panicle and under side of leaves along veins loosely pubescent); Mexico without locality: 1864, *Baites; Galeotti* 2086, 2087 (TYPES of *E. Galeottii* Gray, in Gray Herb.). GUATEMALA: Dept. Alta Verapaz, Dec. 1907, *Türckheim* II 2043 (large-leaved); Dept. Amatitlan, near Amatitlan, 20 July 1860, *Sutton Hayes;* Dept. Jalapa, Laguna de Ayarza, Sept. 1892, *Heyde & Lux* 3792.

1β. C. MEXICANA (DC.) Hemsl. var. **hyperdasya** Blake, n. var., foliis infra ubique dense pubescentibus, supra venis exceptis glabratis vel scabriusculis, caulibus et gemmis et inflorescentia fuscis cum pilis tomentosis lente subglabratis.— OAXACA: ravines of hills near Oaxaca, 1830 m., Sept. 1894, *Pringle* 4896 (TYPE SHEET in Gray Herb.); La Carbonera, 2165 m., 20 Sept. 1895, *L. C. Smith* 808; Cerro de San Felipe, 2000 m., 1 Sept. 1897, *Conzatti & González* 545, 546; San Juan del Estado, 18 June 1894, *L. C. Smith* 25; near Reyes, 1830–2290 m., 17 Oct. 1894, *Nelson* 1718 (approaching the next form); HIDALGO: Mineral del Monte, *C. Ehrenberg* 354.

1γ. C. MEXICANA (DC.) Hemsl. var. HYPERDASYA Blake f. **holotricha** Blake, n. forma, foliis parvis utrimque cineraceis pube densa subscabra.— PUEBLA: vicinity of San Luis Tultitlanapa, July 1908, *Purpus* 3099 (TYPE SHEET in Gray Herb.).

* * Heads long-peduncled, solitary or rarely somewhat panicled.

+ Leaves 2–5 cm. long, cuneately oblanceolate or obovate; disk-corollas 5-toothed.

2. C. CUNEIFOLIA Greenm. Suffruticose, trichotomously branched, the young growth pubescent, later glabrate; leaves pale green especially beneath, with a few loose hairs when young, mucronately 5–11-toothed above the middle, tapering to a sessile margined base; heads 8–10 mm. high, 2 cm. across the rays.— Proc. Am. Acad. xl. 43 (1904). — JALISCO: dry rocky mountains above Etzatlan, 2 Oct. 1903, *Pringle* 8781; Sierra de San Estaban, near Guadalajara, 1830 m., 21 Oct. 1903, *Pringle* 11900; DURANGO: 16 Aug. 1897, *Rose* 2344.

+ + Leaves 1–2 cm. long, oval; disk-corollas 4-toothed.

3. C. **parvifolia** Blake, n. sp., fruticosa trichotome ramosa juventate appresse pubescens denique glabrata cortice cano; foliis parvis 1–2 cm. longis ovalibus supra appresse pubescentibus infra paullum crinitis vel glabratis supra partem inferiorem integram utroque ca. 5-mucronato-dentatis, summis imminutis subintegris; pedunculis solitariis ramos terminantibus 3–6.5 cm. longis subpubescentibus;

capitulis 1–1.5 cm. altis 3 cm. diametro (radiis inclusis); squamis exterioribus subcrinitis oblongo-spatulatis obtusis, interioribus oblongis obtusis apice fimbriatis (8–10 mm. longis 4–5 mm. latis); radiis ca. 5 ovalibus 13 mm. longis 8 mm. latis; corollis disci 7 mm. longis infra hirtellis 4-dentatis.— PUEBLA: dry rocky hillsides, Esperanza, Aug. 1907, *Purpus* 2581 (TYPE in Gray Herb.).

Sect. 2. **Anathysana** Blake, n. sect., herbae perennes caulibus pluribus radice lignea foliis oppositis integris vel pinnatiformibus lobis paucis filiformi-linearibus. Involucrum ut apud §1, squamis interioribus 8–12. Flosculi radii fertiles; ei disci saepius exannulati. Styli rami apice incrassati breviter appendiculati. Achenia ut apud §1, epapposa.— Type species *Leptosyne mexicana* Gray (= *C. cyclocarpa* Blake).— Three species of Mexico and Socorro Island.

* Leaves entire, linear-filiform.

4. C. **cyclocarpa** Blake, n. nom. Stems numerous from a thick woody base, 6–7 dm. high, slightly pubescent below; leaves 2–6 cm. long, entire or very rarely 3-lobed from near the middle, ciliate at base; heads rather few, long-peduncled, 6–8 mm. high, 1.5–2.5 cm. in diameter including the 8–10 rays; outer scales about half as long as inner; disk-florets exannulate.— *Leptosyne mexicana* Gray, in Wats. Proc. Am. Acad. xxii. 429 (1887), not *C. mexicana* (DC.) Hemsl. (1881).— Named from the orbicular indistinctly margined achenes, those of the ray 4.5 by 4.5 mm.— JALISCO: Rio Blanco, Sept. 1886, *Palmer* 568 (TYPE of *L. mexicana* in Gray Herb.); near Guadalajara, 10 Sept. 1890, *Pringle* 3570, 24 Sept. 1891, *Pringle* 3841, 4 Oct. 1903, *Pringle* 11546.

* * Leaves pinnately divided into 3–7 linear lobes, the uppermost sometimes entire.

+– Heads larger, inner involucre 6–8 mm. long; leaves mostly with 3 lateral pairs of lobes; disk-flowers with hairy annulus.

5. C. **pinnatisecta** Blake, n. nom. In habit, pubescence, and involucre very similar to the last; leaves 2–3 cm. long, the lobes mucronate-tipped; achenes obovate, 3.5–4 mm. long, 2.5 mm. broad.— *Leptosyne Pringlei* Rob. & Greenm. Am. Journ. Sci. ser. 3. l. 155 (1895), not *C. Pringlei* Rob. Proc. Am. Acad. xliii. 41(1907).— OAXACA: Sierra de San Felipe, 2135 m., 7 Aug. 1894, *Pringle* 4871 (TYPE in Gray Herb.); PUEBLA: Cerro de Paxtle, near San Luis Tultitlanapa, Sept. 1909, *Purpus* 4098.

+ + Heads smaller, inner scales 4–5 mm. long; leaves mostly with 1 pair of lobes; disk-flowers exannulate.

6. C. **insularis** (Brandeg.) Blake, n. comb. Decumbent (base unknown), nearly glabrous; leaves 1–2.5 cm. long, 3–5-lobed; heads axillary and terminal, about 1.3 cm. in diameter including the small rays; outer bracts $\frac{2}{3}$ as long as inner; achene 4.2 mm. long.— *Leptosyne insularis* Brandeg. Erythea vii. 5 (1899).— SOCORRO ISLAND: March–June 1897, *Anthony* 394 (TYPE COLLECTION): 27 May–3 July 1903, *F. E. Barkelew* 223.

Sect. 3. **Tuckermannia** (Nutt.) Blake, n. comb. Stout perennials, with alternate fleshy 2–3-pinnately dissected leaves and large heads. Outer involucral scales 5–8, lance-oblong, about equaling the inner; the latter about 12, oblong. Rays large, fertile. Disk-flowers with nearly glabrous annulus. Achenes obcompressed, glabrous, narrowly winged, epappose or rarely with margins produced into short teeth or awns.— *Tuckermannia* Nutt. Trans. Am. Philos. Soc. ser. 2. vii. 363 (1841). *Leptosyne* sect. *Tuckermannia* Gray, Bot. Calif. i. 356 (1876), & Syn. Fl. i. pt. 2. 300 (1884).— Two species of California, Lower California, and adjacent islands.

* Heads few, on very long naked peduncles, 6–8 cm. broad including rays.

7. C. MARITIMA (Nutt.) Hook. fil. Stems fleshy-herbaceous, spreading, 3–8 dm. high, from a thick woody base; leaf-lobes linear, 1.5–3 mm. broad; peduncles 2–5 dm. long; rays 14–20; achenes rarely with 2 teeth or awns.— Bot. Mag. t. 6241 (1876). *Tuckermannia maritima* Nutt. l. c. *Leptosyne maritima* Gray, Proc. Am. Acad. vii. 358 (1868).— Coast of San Diego County, California, northern Lower California, and adjacent islands.

* * Heads numerous, cymosely clustered toward tips of branches on peduncles mostly 1–1.5 dm. long, smaller (5–6 cm. broad).

8. C. GIGANTEA (Kellogg) Hall. Stems fleshy-woody, erect, 3–30 dm. high, often 1 dm. thick; leaves mostly clustered toward tips of branches; leaf-lobes finer, 1–1.5 mm. broad; rays 10–16; pappus none.— Univ. Calif. Pub. Bot. iii. 142 (1907). *Leptosyne gigantea* Kell. Proc. Calif. Acad. iv. 198 (1872).— Coast of southern California and islands, from Los Angeles County to San Luis Obispo County; also Guadalupe Island, Lower California.— A form occurs on San Nicolas Island (*Blanche Trask* 76, in part) with discoid heads, the receptacle very chaffy, the disk-florets showing dialysis of corolla with more or less complete abortion of sexual organs.

Sect. 4. **Pugiopappus** (Gray) Blake, n. comb. Annuals, branched from the base, with 2–3-pinnatifid leaves mostly basal, and medium-sized heads solitary on long nearly naked peduncles. Outer involucral scales 5–7; inner about 8. Rays usually styliferous and fertile, sometimes neutral or with short included styles, broad and many-nerved. Disk-flowers with bearded ring. Achenes dimorphous: those of ray epappose, corky-margined and more or less corky-ridged on the faces; those of disk long-villous on margins, bearing a pair of linear-lanceolate denticulate paleae.— *Agarista* DC. Prod. v. 569 (1836), not D. Don 1834 (*Ericaceae*). *Pugiopappus* Gray, Pacif. R. Rep. iv. 104 (1857). *Leptosyne* sect. *Pugiopappus* Gray, Syn. Fl. l. c.— Two species of southern California.

* Outer scales linear-lanceolate.

9. C. Bigelovii (Gray) Hall. Simple or branched below, 1–6 dm. high; leaves 5–10 cm. long, or smaller in starved specimens; outer scales 6–11 mm. long, the inner ovate, 8–12 mm. long; rays 1–2 cm. long; disk-achenes black, 6 mm. long, glabrous on both faces or slightly pubescent on inner, twice as long as the awns.— Univ. Calif. Pub. Bot. iii. 141 (1907). *Pugiopappus Bigelovii* Gray, Pacif. R. Rep. l. c. *Leptosyne Bigelovii* Gray, Syn. Fl. l. c. *P. Breweri* Gray, Proc. Am. Acad. viii. 660 (1873). *L. hamiltonii* Elmer, Bot. Gaz. xli. 323 (1906).— Southern California, not on the coast, from Tulare County to the Colorado Desert.— The annulus of the disk-flowers, in the types and other specimens examined, is very distinctly bearded, not glabrous as originally described and as repeated in the Synoptic Flora and by Hall.

* * Outer scales deltoid-ovate.

10. C. calliopsidea (DC.) Gray. Rather stouter and more leafy-stemmed, with broader leaf-lobes; outer scales united for about half their length, the free deltoid tips 5–6 mm. long; inner scales 11–14 mm. long; rays 1–2.5 cm. long; disk-achenes villous on inner face, nearly equaled by their pappus.— Bot. Mex. Bound. 90 (1859). *Agarista calliopsidea* DC. Prod. v. 569 (1836). *Leptosyne calliopsidea* Gray, Syn. Fl. l. c. *L. calliopsidea* var. *nana* Gray, l. c. (a dwarfed form). *Pugiopappus calliopsidea* Gray, Proc. Am. Acad. viii. 660 (1873). *P. calliopsideus* Gray, Bot. Calif. i. 355 (1876).— Southern California, from Cholame (San Luis Obispo County) to Santa Barbara County and the Mohave Desert; north to middle California according to Hall.

Sect. 5. **Euleptosyne** (Gray) Blake, n. comb. Similar to last section in habit and involucre (outer scales linear); rays glabrous, fertile. Disk-flowers with an annulus, nearly glabrous in one species. Style-branches enlarged at tip, short-appendaged. Achenes corky-winged, sometimes meniscoid, with a cupule in place of pappus.— *Leptosyne* sect. *Euleptosyne* Gray, Syn. Fl. i. pt. 2. 299 (1884).— Two species of Arizona, California, and northern Lower California.

* Achenes with numerous clavellate hairs on both faces; disk-corollas with bearded annulus; leaf-divisions nearly filiform.

11. C. DOUGLASII (DC.) Hall. Scapes solitary or few, 1–3.5 dm. high; leaves chiefly in a dense basal tuft, entire or mostly 1–2-pinnately dissected into linear-filiform lobes, 2–10 cm. long; outer involucral scales linear, 5–8 mm. long; inner yellow, scarious-margined, multinervose, ovate, slightly longer.— Univ. Calif. Pub. Bot. iii. 140 (1907). *Leptosyne Douglasii* DC. Prod. v. 531 (1836). *L. californica* Nutt. Trans. Am. Philos. Soc. ser. 2. vii. 363 (1841). *L. Newberryi* Gray, Proc. Am. Acad. vii. 358 (1868).— Southern California and southern Arizona; also San Quentin, Lower California, 1889, *Palmer* 677.

* * Achenes without clavellate hairs, glabrous on outer face, more or less papillose on inner; annulus nearly or quite glabrous; leaf-divisions about 1–1.5 mm. broad.

12. C. **Stillmanii** (Gray) Blake, n. comb. Somewhat stouter than last, more leafy below; corky margin of achene rugose.— *Leptosyne Stillmanii* Gray, in E. Durand, Journ. Acad. Nat. Sci. Phila. iii. 91 (1855), and in Torr. Bot. Mex. Bound. 92 (1859). *L. Stillmani* Gray, Bot. Calif. i. 356 (1876).— CALIFORNIA: Calaveras Co., *Heermann;* valley of the Sacramento, *Stillman* (TYPE in Gray Herb.); hillsides, Auburn, April 1865, *Bolander* 4520; dry sand hills, Antioch, 16 April 1868–9, *Kellogg & Harford* 439; fields, Middle Tule River, 240–305 m., April–Sept. 1897, *Purpus* 5004.

COREOCARPUS Benth. (*κόρις* bug, and *καρπός* fruit, from the peculiar achenes). Heads heterogamous, radiate, the flowers all yellow; rays styliferous, fertile, disk-flowers mostly fertile. Involucral scales 5–8, 2-rowed, subequal, submembranaceous, dark-lineate, ovate to ovate-oblong, the outer obtusish, the inner acuminate; heads sometimes with a few bractlets at base. Receptacle flat, with narrow membranaceous pales subtending the flowers. Ligules small, 4–5-nerved, entire or emarginate; disk-corollas regular, tubular, with slightly enlarged funnelform throat and 5-toothed limb, with a hairy annulus at base of throat. Style-branches with subulate hispid appendages. Anthers entire at base. Achenes obcompressed, with an entire or pectinately cut crustaceous wing, calvous or with two retrorsely hispidulous slender awns, often granular on one or both faces. — Annuals or suffrutescent perennials, with opposite 1–2-pinnately divided leaves, and small slender-peduncled heads (less than 2 cm. in diameter including the rays) in somewhat ternate cymose clusters at the ends of the branches.— Bot. Voy. Sulph. 28, t. 16 (1844); Gray, Proc. Am. Acad. v. 162 (1861). *Acoma* Benth. l. c. 29, t. 17 (1844). *Leptosyne* sect. *Coreocarpus* Gray, Syn. Fl. i. pt. 2. 301 (1884).— Three species of the Sonoran region.— Well distinguished by inflorescence, and by the involucre of few similar scales, not double as in all members of the genus *Coreopsis*.

* Herbaceous annuals.

1. C. PARTHENIOIDES Benth. Slender, 2–4 dm. high, branched above, nearly glabrous, bearing few heads in somewhat ternate terminal clusters; leaf-blades 2–4.5 cm. long on petioles nearly as long, somewhat thickish, bipinnatifid, the primary lobes deltoid, 1–2 cm. long, nearly as broad, entire or 3–4-lobed with broad divisions; heads 5–6.5 mm. high, about 1 cm. broad including the rays; rays oval, about 5, 5 mm. long, yellow, often drying whitish with purplish veins; achenes oblong, crenate-margined, in the only specimens examined; figured by Bentham as oval, shortly 2-awned, with imperfectly dissected wing.— Bot. Voy. Sulph. 28, t. 16 (1844). *Leptosyne parthenioides* Gray, l. c.— Bentham's type came from Bay of Magdalena, Lower California. The only specimens examined are: SONORA: high in the mountains, Guaymas, Oct. 1887, *Palmer* 299.

1β. C. PARTHENIOIDES Benth. var. **heterocarpus** (Gray) Blake, n. comb. Leaves bi-tripinnatifid with finer divisions, the ultimate ones nearly linear; margin of achenes entire and incurved or dissected into lobes; awns sometimes present.— *Coreocarpus heterocarpus* Gray, Proc. Am. Acad. v. 162 (1861). *Leptosyne heterocarpa* Gray, Syn. Fl. l. c. *L. dissecta* Gray, l. c. (as to plant, not synonym). *L. parthenioides* var. *dissecta* Wats. Proc. Am. Acad. xxiv. 56 (1889), as to plant cited, not as to name-bringing synonym. *Coreocarpus involutus* Greene, Pittonia, i. 290 (1889).— While *C. heterocarpus* Gray appears to be merely a form of *C. parthenioides*, as long ago suggested by Dr. Watson, indistinguishable by achenial characters alone, it nevertheless differs sufficiently in foliar characters to retain varietal rank. *C. involutus* Greene, of which no authentic specimen has been seen, is judging from the description inseparable from this variety. Plants collected by Brandegee on Natividad Island and distributed as this species are slightly stouter and more pubescent than any other specimens examined but show no essential differences.— Lower California: Lagoon Head, Mar. 1889, *Palmer* 795; Natividad Island, 10 April 1897, *Brandegee;* Cape San Lucas, &c., Aug. 1859–Jan. 1860, *Xantus* 62 (type of *C. heterocarpus*); San José del Cabo, 8 Mar. 1892, *Brandegee* 339; La Paz, 20 Jan.–5 Feb. 1890, *Palmer* 19; Santa Agueda, 4–6 Mar. 1890, *Palmer* 248; mountain sides, Los Angeles Bay, Dec. 1887, *Palmer* 660; Lower California, without locality, *Dr. Street.*

* * Suffrutescent perennials.

+– Achene-wing pectinately dissected.

2. C. **arizonicus** (Gray) Blake, n. comb. Much branched from a woody base, nearly or quite glabrous; leaves 7–10.5 cm. long, pinnately divided into 3–5 linear lobes 1–3 mm. wide; heads rather numerous in panicled few-headed cymes; involucral scales 5–6 mm. long; rays 5–6, 7 mm. long; achenes with the wing split into flattened cuneate lobes, the inner achenes narrower; retrorsely spinulose awns sometimes present.— *Leptosyne* (*Coreocarpus*) *arizonica* Gray, Proc. Am. Acad. xvii. 218 (1882). *Coreopsis arizonica* O. Hoffm. in Engler & Prantl, Nat. Pfl. iv. Ab. 5. 243 (1890).— Arizona: along streams, Ft. Lowell, Aug. 1880, *Lemmon* (cotype); by streams of the Santa Catalina Mts., 760–1060 m., April 1881, *Pringle* (cotype); Santa Catalina Mts., May 1881, *Lemmon* 211; mountains, Lowell, 8 May 1884, *W. F. Parish* 112. Sonora: Touibabi, 18 Nov. 1890, *F. E. Lloyd* 407; Alamos, 1890, *Palmer* 384 (approaching the next form); Chihuahua: southwestern part, 1885, *Palmer* 294.

2β. C. arizonicus (Gray) Blake var. **pubescens** (Rob. & Fern.) Blake, n. comb. Whole plant pubescent with short rather soft hairs.— *Leptosyne arizonica* var. *pubescens* Rob. & Fern. Proc. Am. Acad. xxx. 118 (1894).— Sonora: Granados, 905 m., 15 Nov. 1890, *Hartman* 222; Huchueraçhi, 1220 m., 5 Dec. 1890, *Hartman* 296; Agnos Blanco, 9 Dec. 1890, *Lloyd* 406.

2γ. C. arizonicus (Gray) Blake var. **filiformis** (Greenm.) Blake, n. comb. Leaf-lobes linear-filiform, less than 1 mm. wide, the lower 4–6 cm. long.— *Leptosyne arizonica* var. *filiformis* Greenm. Proc. Am. Acad. xl. 44 (1904).— Sinaloa: Sierra de Choix, 80 km. NE. of Choix, 15 Oct. 1898, *Goldman* 258 (cotype in Gray Herb.).

+– +– Achene-wing thick, often rugose, entire or barely crenulate; leaves fleshy, bipinnatifid.

3. C. **dissectus** (Benth.) Blake, n. comb. Suffrutescent, trichotomously branched, glabrous; leaves mostly crowded near the base of

the young branches, 1–2-pinnatifid, 1.5–2.5 cm. long, ternately cut into short fleshy linear lobes, on petioles nearly as long; heads cymosely arranged in nearly naked panicles, as large as those of last species; achenes 4 mm. long, 2.5 mm. broad.— *Acoma dissecta* Benth. Bot. Voy. Sulph. 29, t. 17 (1844). *Leptosyne dissecta* Gray, Syn. Fl. i. pt. 2. 301 (1884), as to synonym. *L. parthenioides* var. *dissecta* Wats. Proc. Am. Acad. xxiv. 56 (1889), as to synonym only.— Bentham's type came from Cape San Lucas. The only specimens examined are three sheets from Magdalena Island, LOWER CALIFORNIA, collected 12 Jan. 1889 by *Brandegee* (see Brandeg. Proc. Calif. Acad. ser. 2. ii. 176 (1889)).

3β. C. DISSECTUS (Benth.) Blake var. **longilobus** Blake, n. var., foliis 5–7.5 cm. longis pinnatiformibus, segmentis (3–5) lineari-filiformibus, lobis inferioribus 2–3 cm. longis 3–5-lobatis; acheniis ut in forma typica sed margine crenulatis.— LOWER CALIFORNIA: Carmen Island, 1–7 Nov. 1890, *Palmer* 877 (TYPE SHEET in Gray Herb.; distributed as *L. dissecta* Gray).

STEPHANOPHOLIS Blake, n. genus Compositarum Coreopsidearum (στέφανος crown, and φολίς scale). Capitula heterogama radiata, radiis fertilibus. Involucrum duplex squamis liberis, exterioribus 5–6 herbaceis obtusis ovato-lanceolatis; interioribus circa 8 submembranaceis ellipticis atroviridibus apice rotundatis margine angusto scario fimbriatulo exteriores subaequantibus. Receptaculum conicum, paleis planis membranaceis flavis apice rotundatis. Radii corollae ca. 12 ligulatae oblongae tridentatae supra albidae infra atroplumbeae glabrae obscure ca. 8-nervatae. Disci corollae flavae glabrae exannulatae 5-dentatae tubulo breve. Antherae basi subintegrae apice appendice deltoideo munitae. Styli rami apice incrassati. Achenia dimorpha: ea radii valde obcompressa ovalia glabra epapposa; disci paullo crassiora oblonga supra appresse pubescentia, pappo coroniformi e squamis brevissimis inaequalibus lacerato-fimbriatis vix junctis ad angulos plus minusve exaggeratis composito.— Herbae perennes scaposae pratincolae radicibus fasciculatis caulibus a foliorum basibus fibrillosis persistentibus lanugine brunnea intermixta vestitis. Folia multa longa integra vel pinnatiformia. Scapi pauci nudi vel 1–2-bracteati capitula solitaria majuscula radiis albidis gerentes. Genus habitu pappo clinio conico bene distinguitur. Type species *Leptosyne pinnata* Rob.— One species with a variety, in mountain meadows of southern Mexico.

1. S. **pinnata** (Rob.) Blake, n. comb. Smooth except base and scapes; leaves 1–3.5 dm. long, with 3–6 pairs of small oblong lobes and a much enlarged slightly glandular-crenulate terminal one 3.5–9.5 cm. long; scapes very rarely branched, densely appressed-pubescent above, exceeding the leaves; head about 1 cm. high, 3 cm. broad including the rays.— *Leptosyne pinnata* Rob. Proc. Am. Acad. xxvii. 176 (1892). — MEXICO: wet meadows, Del Rio, 30 Aug. 1890, *Pringle* 3668 (TYPE in Gray Herb.); wet meadows, Valley of Toluca, 19 Aug. 1892, *Pringle* 4194; wet alpine meadows, Sierra de las Cruces, 2990 m., 28 Aug. 1904, *Pringle* 13067.

1β. S. PINNATA (Rob.) Blake var. **integrifolia** (Greenm.) Blake, n. comb. Leaves entire, narrowly lanceolate, only very slightly crenulate, 1.5–2 dm. long; pappus slightly more developed.— *Leptosyne pinnata* var. *integrifolia* Greenm. Proc. Am. Acad. xl. 44 (1904).— DURANGO: near El Salto, 12 July 1898, *Nelson* 4580 (COTYPE in Gray Herb.).

II. A REVISION OF ENCELIA AND SOME RELATED GENERA.

In the course of a revision of the genus *Encelia*, as at present understood, it has been found necessary for clearness of definition to remold the group by the reference of a number of species to the related genera *Viguiera*, *Flourensia*, and *Verbesina*, and by the recognition of several genera long treated as synonymous; and in view of the changes in generic boundaries involved it seems desirable to consider briefly the history of some of these related genera and to contrast their characters.

Only two genera of this immediate relationship were known to Linnaeus. *Helianthus*, characterized by its thickish achenes with promptly deciduous pappus of paleaceous awns and sometimes also squamellae (short intermediate scales), is today taken in its original interpretation, save that the small and very distinct genus *Heliopsis* was later erected by Persoon on one of the original species (*H. laevis*). The Linnaean genus *Verbesina*, on the other hand, was very composite, its ten original species (reducible to nine or eight) representing seven modern genera, two only of the species being now included in the genus. It is well distinguished by the generally fertile rays and the chartaceo-cartilaginous wings of the flat achene, but these being usually invisible or indistinct in the ovary young material is easily misplaced, and several species of *Verbesina* have been described under *Encelia*.

Encelia Adanson[6] was based on "Cotula marit. Peruana," cultivated in the Jardin de Roi at Paris, which is *Encelia canescens* Lam.,[7] an alternate-leaved perennial with epappose achenes villous on the edges and narrowly white-margined. In 1789 L'Héritier[8] redescribed the plant as *Pallasia halimifolia,* a new genus, quoting as a synonym *Coreopsis limensis* Jacq.,[9] but not referring to Adanson's genus. The species was again described and figured by Cavanilles[10] in 1791, Lamarck being correctly accredited with the authorship of the species, which has nevertheless since been universally attributed to Cavanilles.

In 1807 Persoon [11] described the genus *Simsia,* basing it upon three species published by Cavanilles under *Coreopsis,* of which one, *S. ? heterophylla,* has since become the type of *Iostephane* Benth., while the others, both reducible to the species long known as *Simsia auriculata* DC. or *Encelia mexicana* Mart., have always been retained in *Simsia* — characterized mainly by the biaristate not villous-edged achene — until that genus was merged with *Encelia* in 1873.

Both genera were recognized by De Candolle in his Prodromus in 1836, *Encelia* with four species and *Simsia* with eight, several species being here first published, and the new genus *Armania* Bertero [12] was described, based upon *Hopkirkia fruticulosa* Spreng.,[13] a species not since identified but certainly a *Simsia.*

In a communication by Gray to the American Academy, apparently first published [14] in 1847, two new genera of this group were proposed, *Barrattia* Gray & Engelmann for a species closely allied to *Simsia* but with epappose achenes, and *Geraea* Torr. & Gray for an annual with narrowly cuneate villous achenes having well developed margin and crown and two strong awns. Two years later both genera were reduced to *Simsia* by Dr. Gray.[15] In Bentham's treatment twenty-four years later in his Genera Plantarum [16] they were recognized as sections of *Encelia,* to which *Simsia* was here also for the first time definitely subordinated. Dr. Gray, in a paper of 1883 [17] and in the Synoptical Flora, carried the reduction a step further by including

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[6] Fam. ii. 128 (1763).

[7] Encycl. Meth. ii. 356 (1786).

[8] In Ait. Hort. Kew. iii. 498 (1789).

[9] Coll. ii. 299 (1788), & Icon. iii. t. 594 (1786–1793).

[10] Icon. i. 45. t. 61 (1791).

[11] Syn. ii. 478 (1807).

[12] Prod. v. 576 (1836).

[13] Sys. iii. 444 (1826).

[14] Am Journ. Sci. ser. 2. iii. 274–5 (Mar. 1847).

[15] Pl. Fendl. 85 (1849).

[16] Gen. Pl. ii. 378 (1873).

[17] Proc. Am. Acad. xix. 7–9 (1883).

Geraea in the section *Euencelia,* and *Barrattia* in *Simsia.* Since Dr. Gray's treatment nothing has been done in the way of a revision of the group as a whole, nor has any new species been described under any of the genera *Pallasia, Armania, Geraea, Barrattia,* or *Simsia* since 1859.

In 1871 D. C. Eaton [18] described as *Tithonia argophylla* a remarkable new species from Utah, with large solitary heads, squamellaceous corona between the awns of the achene, and densely silvery-pubescent basal leaves, which two years later was referred to *Encelia* by Dr. Gray,[19] who at the same time added a very similar new species (*E. nudicaulis*). Ten years after Gray [20] transferred them to *Helianthella,* instituting for their reception the new section *Enceliopsis.* In 1909 Aven Nelson [21] elevated the group to generic rank, mainly on the basis of habit, enumerating five species, one of them new, which I have not been able to separate from *E. nudicaulis.*

Hemsley [22] in 1881 listed 17 species of *Encelia* from Mexico, describing one new species and making many new combinations of names which had been first published under *Simsia.*

In recent years the boundaries of the genus *Encelia* have been stretched to include a number of shrubby Mexican species, usually described from material without ripe fruit, which in the light of all their characters require transferral to other genera (*Viguiera, Flourensia, Verbesina*) if generic distinctions in this group are to be preserved. Six species (*E. hypargyrea, maculata, montana, Pringlei, rhombifolia, squarrosa*), with achenes plumpish when mature, so far as known, and a persistent pappus of two aristate or paleaceous awns and several short truncate squamellae, exactly agree with *Viguiera* in essential characters and are further on transferred to that genus. Another fascicle of six species (*E. collodes, glutinosa, microphylla, oblonga, resinosa, suffrutescens*) is not so easily placed owing to lack of ripe fruit in nearly every species, but all differ in more or less essential characters from the true genus *Encelia,* and may by a slight extension of character be included in *Flourensia* DC.[23] This genus, wrongly referred by Bentham [24] to *Helianthus,* was based on four species, two radiate Chilean plants and two discoid Mexican species, the latter taken by Gray [25] as typical of the genus. One of the Chilean species,

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[18] In Wats. Bot. King's Rep. v. 423 (1871).

[19] Proc. Am. Acad. viii. 657 (1873).

[20] Proc. Am. Acad. xix. 9 (1883).

[21] Bot. Gaz. xlvii. 432 (1909).

[22] Biol. Centr.-Am. Bot. ii. 183–5 (1881).

[23] Prod. v. 592 (1836).

[24] Gen. Pl. ii. 376 (1873).

[25] Proc. Am. Acad. xix. 7 (1883).

F. corymbosa DC., is a true *Viguiera* and was transferred to that genus by Gray in 1883 under the new name *V. Poeppigii*, the name *corymbosa* being rejected as inappropriate; but Reiche [26] says: "El estremo de los tallos corimboso-ramoso, rara vez indiviso. Cabezuelas terminales en las ramas hácia arriba desnudas"; and in any case the name *corymbosa*, not being preoccupied, must be retained.[27] The remaining three species, alternate-leaved glutinous shrubs with villous achenes noticeably thicker than in true *Encelia*, and with a pappus of two slender awns disposed to be trifid from near the base, with or without slender acute squamellae, form a rather definite group which has since been increased to about ten species. Of the six Encelias above mentioned four (*E. collodes, microphylla*,[28] *oblonga, suffrutescens*) agree well with these characters, except that *E. oblonga* and *E. suffrutescens* are scarcely glutinous, while the remaining two species, fully mature fruit of which is greatly to be desired, in their general characters are so close to the others as to justify their allocation here.

The genera *Encelia, Geraea*, and *Simsia* are here separated mainly on the strength of characters to which attention has not previously been directed. The fourteen species included in *Encelia* are all perennials with leaves all alternate, achenes very flat, villous at least on margins, narrowly white-bordered and generally pappusless, bluntish short-hairy style-branches, and receptacular chaff softly scarious, bluntish, falling with the achenes. The two species included in *Geraea* are annuals or biennials, with all or nearly all the leaves alternate, pales as in the last, longer and more hairy style-branches, and narrowly cuneate villous achenes with strong white border, awns, and conspicuous crown, the last represented on the ovary in at least one species by a squamellaceous corona. The twenty-two species included in *Simsia* are mostly annuals, with always some at least of the lower leaves opposite, marginless thin-edged not villous achenes, attentuate hispid style-branches, and stiff acuminate pales persistent long after the achenes have fallen. The characters of these and some related genera are contrasted in the following key.

In the course of this revision some 670 sheets have been studied, representing all the material in the Field Columbian Museum, the

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[26] Fl. Chile, iv. 93 (1905).

[27] VIGUIERA **corymbosa** (DC.) Blake, n. comb. *Flourensia corymbosa* DC. Prod. v. 592 (1836); Reiche l. c. (q. v. for vars.). *Helianthus corymbosus* Poeppig in DC. l. c. as syn. *H. revolutus* Meyen "Reise i. 311 (1843)." *Viguiera Poeppigii* Gray, Proc. Am. Acad. xix. 6 (1883).

[28] The close resemblance of *E. microphylla* to *Flourensia* was commented on by Dr. Gray (Proc. Am. Acad. xix. 7, and Syn. Fl.).

National Herbarium, and the Gray Herbarium. I wish to thank Mr. W. R. Maxon and Dr. C. F. Millspaugh for the loan of the material of *Encelia* under their charge, Mr. M. E. Jones for the loan of 11 sheets of *Enceliopsis* and for assistance in other ways, M. Casimir de Candolle for a photograph of the type of *Simsia lagascaeformis* and critical notes, and Dr. Philip Dowell for aid in proof-reading. I am greatly indebted to Miss Mary A. Day of the Gray Herbarium for assistance in proof-reading and for constant help in bibliographical matters, and above all to Dr. B. L. Robinson for his advice and guidance throughout the whole course of my work.

KEY TO ENCELIA AND SOME RELATED GENERA.

Achenes very flat; no squamellae except in *Helianthella* and *Enceliopsis*.
Achenes winged; rays usually fertile.....................VERBESINA L.
Achenes wingless; rays neutral.
Squamellae present.
Usually leafy-stemmed herbs of mountainous regions, with green leaves and frequently foliaceous outer involucral bracts; squamellae mostly narrow, laciniate, and united at base; achenes not villous or white-margined. HELIANTHELLA T. & G.
Scapose desert plants with canescent or silvery broadly oval or rhombic leaves; outer scales never foliaceous; squamellae short and indistinct, mostly united into a low sometimes entire crown; achene villous except in *E. grandiflora*, and strongly white-bordered........ENCELIOPSIS (Gray) A. Nels. (p. 351.)
No squamellae.
Scapose, with broad leaves and large solitary heads..(ENCELIOPSIS)
Leafy-stemmed (except *E. scaposa*, which has linear leaves); heads several (except in two species with linear leaves), small or medium-sized.
Pales soft, bluntish, falling with the achenes; leaves alternate; achenes villous at least on margins.
Perennials; style-branches bluntish, not villous; achenes without crown, usually epappose. ENCELIA Adans. (p. 358.)
Annuals or biennials; style-branches longer, villous; achene narrowly cuneate, with strong white margin, awns, and crown.....................GERAEA T. & G. (p. 355.)
Pales rigid, acute or acuminate, persistent; lower leaves opposite; style-branches attenuate, hispid-villous; achenes not villous-ciliate.....................SIMSIA Pers. (p. 376.)
Achenes thickened; squamellae often present.
Pappus caducous, of paleaceous awns and rarely short squamellae; herbs. HELIANTHUS L.
Pappus more persistent[29]; awns often aristate, squamellae usually present; herbs or shrubs.
Squamellae none, or narrow and acute; achenes usually densely villous; alternate-leaved usually glutinous shrubs......FLOURENSIA DC.
Squamellae present, mostly short, rounded, fimbriate; herbaceous or frutescent, very rarely resiniferous, often opposite-leaved. VIGUIERA HBK.

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[29] Caducous in some Viguieras, e. g. *V. Mandoni* Sch. Bip.

Herbs, with alternate linear leaves and four-angled achenes, or scapose, with linear-lanceolate leaves . . . . (HELIANTHELLA)

ENCELIOPSIS (Gray) A. Nels. (*Encelia*, and ὄψις likeness).— Heads large, many-flowered, radiate or discoid, the rays neutral; flowers all yellow. Involucre hemispherical, the scales 2–3-seriate, subequal or graduated with the outer shorter, lanceolate to lance-ovate, equaling or somewhat exceeding the disk. Receptacle somewhat convex; pales soft and scarious, with abruptly narrowed hairy tip, enfolding the achenes and falling with them. Rays long (1.5–4.5 cm.) and narrow, several-nerved, pubescent on back and tube, entire or tridenticulate, absent in one species; disk-corollas with cylindric tube abruptly widened into the throat, and 5-toothed pubescent limb. Anthers sagittate at base. Style-branches bluntish, pubescent. Achenes of ray triquetrous, sterile, rarely maturing and developing thin corky wings; of disk compressed, very flat, villous particularly on the margins (or glabrate in one species), with blackish body and white cartilaginous border passing above into 2 teeth or awns, these connected by a fringe of short confluent squamellae, sometimes completely united into a thick entire crown.— Scapose xerophytic perennials, with stout root and often much branched caudex, the short branches bearing tufts of thick oval or rhombic 3–5-nerved leaves, and one or several naked or 1–2-bracteate monocephalous scapiform peduncles. Type species *Encelia nudicaulis* Gray.— Four species of very arid regions of the southwestern United States.

Distinguished from *Helianthella* by the generally shorter squamellae, from *Encelia* and *Geraea* by usual presence of squamellae, and from all three by habit. Forming a connecting link between *Geraea* and *Helianthella*, and probably having developed as an adaptation to desert conditions of the mountain loving genus *Helianthella*.

Helianthella § *Enceliopsis* Gray, Proc. Am. Acad. xix. 9 (1883), & Syn. Fl. i. pt. 2. 283 (1884).

Enceliopsis A. Nels. Bot. Gaz. xlvii. 432 (1909).

* Heads discoid; plant hispid-canescent.

1. E. NUTANS (Eastw.) A. Nels. Root tuberiform, becoming very thick (3 cm.) and woody, bearing a short lignescent caudex from which proceed the 1–5 scapes and the tuft of crowded leaves; leaves oval, obtuse to rounded at tip, rounded at base, hispid-canescent with appressed hairs, 2–5 cm. long, on margined petioles 2–6 cm. long; scapes hispid with somewhat reflexed hairs, 1.5–2.5 dm. high, naked or with one or two narrow bracts; heads nodding in fruit, 2–4 cm. wide, 1.5–2 cm. high; scales densely hispid, lanceolate, 2–3-seriate,

11–15 mm. long, the outer shorter; pales about 15 mm. long, faintly nerved, pubescent on the back and subherbaceous narrow tip; disk-corollas 8 mm. long, tube $\frac{2}{3}$ length of throat; achenes 1 cm. long, obovate, very villous, the callous margin emarginate at apex.

Encelia nutans Eastw. Zoe ii. 230 (1891); Jones, Proc. Calif. Acad. ser. 2. v. 701 (1895).

Enceliopsis nutans A. Nels. Bot. Gaz. xlvii. 433 (1909).

Verbesina scaposa Jones, Zoe ii. 248 (1891), & Proc. Calif. Acad. l. c. (1895).

Specimens examined: COLORADO: Grand Junction, May 1892, *Alice Eastwood* (GN [30]); UTAH: Green River, alt. 1340 m., 23 May 1895, *Jones* 11859 (hb. Jones). Type material collected by Miss Eastwood on Orchard Mesa, Grand Junction, 17 May 1891. *Verbesina scaposa* Jones was described from material collected in sandy deserts near Grand River, eastern Utah, at Cisco, 2 May 1890.— This species grows in very poor clay soil containing a little active alkali (sodium carbonate) in open deserts (Jones, in litt.).

* * Heads radiate; plants densely white-pubescent.

+– Heads smaller, the rays [31] 1–2.5 cm. long; pubescence rather dull, not silvery; leaves mostly obtuse or rounded.

2. E. NUDICAULIS (Gray) A. Nels. Caudex more or less branched from a thick woody root, the branches short and stout, woolly and covered with the thick crowded bases of former leaves; leaves tufted, ovate to orbicular, subacute to rounded at both ends, white with a dense fine simple pubescence of several-celled glandular-based hairs, 2.5–6.5 cm. long, 1–6.5 cm. wide, on margined petioles one to three times their length; scapes 1.5–2.5 dm. high, pubescent like the leaves, usually bractless; heads 4–8 cm. in diameter including the rays; involucre densely pubescent like the leaves and stem, 3-seried, the scales slightly unequal, subulate-lanceolate from an ovate base, bluntish, 1–2 cm. long, equaling or barely surpassing the disk; rays about 20, glandular-pubescent on tube and back, tridentate, about 11-nerved, 1–2.5 cm. long, 2–6 mm. wide; disk-corollas 7 mm. long, with short tube and thick-cylindric throat, more or less glandular-pubescent; pales 12–15 mm. long, scarious, pubescent on back and tip and somewhat glandular, often laterally 1-toothed; mature achenes cuneate, 9 mm. long, 3.5 mm. wide, rather shortly silky-

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[30] In citation of specimens F = Field Museum; G = Gray Herbarium; N = National Herbarium.

[31] Ligule measurements are taken exclusive of tube.

villous except for a submarginal naked border, narrowly white-margined, awnless or with two stout subulate teeth or slender upwardly pubescent awns $\frac{1}{6}$–$\frac{1}{3}$ their length, connected by a fimbriate crown of nearly fused squamellae.

Encelia (*Geraea*) *nudicaulis* Gray, Proc. Am. Acad. viii. 656 (1873); Jones, Proc. Calif. Acad. ser. 2. v. 701 (1895).

Helianthella nudicaulis Gray, Proc. Am. Acad. xix. 9 (1883).

Enceliopsis nudicaulis A. Nels. l. c. (1909).

Enceliopsis tuta A. Nels. l. c. (1909).

Specimens examined: IDAHO: rather rare, dry rocky bluffs, Salmon River near Bay Horse, 5 Aug. 1895, *Henderson* 3653 (N); dry sage brush hills, above Salmon River, 6 Aug. 1895, *Henderson* 3653 (N); NEVADA: Candelaria, Esmeralda Co., 1881, *Shockley* (G); Hawthorne, Lepantha Mine, alt. 1677 m., 25 May 1897, *Jones* (N); compact clay slopes, alt. 305 m., Las Vegas, 29 April 1905, *Jones* 11857 (hb. Jones); limestone clays, Las Vegas, 4 May 1905, *Goodding* 2271 (G, type collection of *E. tuta*); clay, Horse Spring, alt. 915 m., 17 April 1894, *Jones* 5069k (hb. Jones); UTAH: St. Thomas or St. George, *Capt. F. M. Bishop* (HOLOTYPE in Gray Herb.); gravel at foot of precipitous slopes in very poor clay soil, Marysvale, alt. 1830 m., 4 June 1894, *Jones* 5376 (hb. Jones); Ferguson Spring, alt. 1920 m., 14 June 1900, *Jones* 6403 (hb. Jones); halfway station W. of Wa Wa, alt. 2135 m., 15 May 1906, *Jones* 11856 (hb. Jones).— Inhabits rocky or hard clay knolls where the soil is very compact (Jones, in litt.).

I am unable to separate satisfactorily *E. tuta* from the older *E. nudicaulis*. The type of the latter has medium-sized orbicular leaves, connected by the Shockley and Jones plants with the small subacute ones of *E. tuta*, while the Henderson plants, largest- and broadest-leaved of all, bear some smaller leaves identical in shape and tip with those of *E. tuta*, indicating that the latter represents only a starved phase of *E. nudicaulis*. The achenes appear to be rather variable in pubescence when young, and at maturity are strongly bidentate or with two awns of varying length, the longest that I have seen being about $\frac{1}{6}$ the length of the mature fruit, although when young they are often longer relative to the ovary. The squamellae, fairly distinct when young, become fused into a barely fimbriate crown on the ripe fruit.

+— Heads larger, rays 2–4.5 cm. long; pubescence silvery-velutinous; leaves rhombic-ovate, acute.

++ Achenes puberulent or glabrate; rays 3.5–4.2 cm. long, 6–12 mm. wide.

3. E. GRANDIFLORA (Jones) A. Nels. "Stems very thick and tufted, branched and very short, woody, densely covered with very

thick leaves;"[32] leaves all basal, broadly rhombic-oval or orbicular, subacute, velvety with appressed hairs especially when young, 4.5–8 cm. long, 3–6 cm. wide, on broadly margined petioles of about the same length; scapose peduncles channeled and finely pubescent, 3.5–4.5 dm. high, with 1–2 linear bracts; heads 11–12 cm. in diameter including rays; involucre densely short-pubescent, triseriate, the bracts little graduated, about 18 mm. long, tapering from an ovate base, exceeding the disk; rays 24–33, oblong, subentire or faintly tridentate, 3.5–4.2 cm. long, 6–12 mm. broad, about 11-nerved, pubescent on back chiefly along the veins; disk-corollas with slender tube and cylindric throat, glabrous except for the pubescent teeth, 6–7 mm. long; pales glandular-pubescent at apex, 11–14 mm. long, about 11-nerved; immature achene 6 mm. long, 2.5 mm. wide, appressed puberulent on body and margin or nearly glabrous, with two short ascending awns and a corona of short confluent squamellae; mature achene broadly obovate, 10 mm. long, 6.5 mm. broad, with blackish sparingly puberulent or glabrate body and broad whitish-yellow margin and fimbriatulate crown, the smoothish awns 1 mm. long.

Encelia grandiflora Jones, Proc. Calif. Acad. ser. 2. v. 702 (1895), not *E. grandiflora* (Benth.) Hemsl. (1881).

Enceliopsis grandiflora A. Nels. l. c. (1909).

Helianthella argophylla Coville, Contr. U. S. Nat. Herb. iv. 132 (1893), not Gray.

H. Covillei A. Nels. Bot. Gaz. xxxvii. 273 (1904).

Specimens examined: CALIFORNIA: Panamint Cañon, alt. 610 m., 3 May 1897, *Jones* 11855 (GN, and hb. Jones); banks of apparently calcareous clay, Hall Cañon, Panamint Mts., alt. 450 m., 18 April 1891, *Coville & Funston* 698 (G).— This species and the next are found only where sodium chloride is abundant, on cliffs adjoining salt deposits, but apparently never directly on salt flats (Jones, in litt.).

++ ++ Achenes villous; rays 2 cm. long, 4 mm. wide.

4. E. ARGOPHYLLA (D. C. Eaton) A. Nels. Base as in the last; leaves in a thick tuft, oblong-obovate or rhombic-obovate, silvery-velutinous, acute at apex, tapering to a broadly margined base, 3-nerved, 3.5–7 cm. long, 1.1–3.2 cm. wide, the petioles shorter; scapose peduncles 1 or 2, 2.5–3.7 dm. high, short-pubescent, bractless; heads 4.5–7.5 cm. in diameter including rays; involucre 2 cm. high, triseriate, the short-silky scales subulate-tipped, with broadly ovate base, exceeding the disk, the outer loose and reflexed; rays about 30,

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[32] Jones, l. c.

linear-oblong, 2 cm. long, 3–4.5 mm. wide; disk-corollas 7 mm. long (tube 3 mm.), hairy on teeth and base of tube; pales 13.5 mm. long, laterally 1-toothed, glandular-hairy on keel and tip; achene oblong, 10 mm. long, 3.5 mm. wide, silky-villous on body and margin, awnless or with two subulate awns 1.8 mm. long, the squamellae almost completely united.

Tithonia argophylla D. C. Eaton in Wats. Bot. King's Rep. v. 423 (1871).

Encelia (*Geraea*) *argophylla* Gray, Proc. Am. Acad. viii. 657 (1873); Jones, l. c. 702 (1895).

Helianthella argophylla Gray, Proc. Am. Acad. xix. 9 (1883); Coville, l. c. (1893), as to name only.

Enceliopsis argophylla A. Nels. l. c. (1909).

Specimens examined: UTAH: St. George, 1870, *Palmer* (fragments of TYPE in Gray Herb.); NEVADA: salty cliffs, salt mine near Stone's Ferry, near the Colorado River, alt. 366 m., 11 April 1894, *Jones* 5032q (hb. Jones).

GERAEA Torr. & Gray (γεραιός old, from the canescent-villous achenes). Heads medium-sized or rather large, many-flowered, radiate or discoid, the rays neutral; flowers all yellow. Involucre hemispheric, the scales 2–3-seriate, linear or broadly oblong, equaling or shorter than the disk. Receptacle flattish; pales softly scarious, conduplicate, falling with the achenes. Rays when present cuneate, the tube hairy; disk-corollas with cylindric tube and broader throat, limb hairy and 5-toothed. Style branches long, hairy. Disk-achenes strongly compressed, villous especially on the edges, narrowly cuneate with narrow whitish margin produced into two strong awns decurrent into the conspicuous crown.— Annuals or biennials (base unknown in *G. viscida*), glandular-pubescent, simple or branched, with alternate dentate leaves and usually few paniculate heads. Type species *G. canescens* Torr. & Gray (*Encelia eriocephala* Gray).— Two species of southwestern United States and adjacent Mexico.— The squamellaceous corona of the ovary, from which the thick crown of the fruit is at least in part developed, is distinct enough in *G. viscida*, although visible in *G. canescens* only as a narrow border connecting the decurrent based awns. It seems quite analogous with the corona of completely fused squamellae in such a species of *Enceliopsis* as *E. nutans*.

Geraea Torr. & Gray, Am. Journ. Sci. ser. 2. iii. 275 (Mar. 1847); Proc. Am. Acad. i. 48 (1848).

Simsia section *Geraea* Gray, Pl. Fendl. 85 (1849).
Encelia section *Geraea* Benth. & Hook. fil. Gen. Pl. ii. 378 (1873).

* Heads radiate; involucral scales linear, densely ciliate; lower leaves narrowed to a petiolar base.

1. G. CANESCENS Torr. & Gray. Erect annual, 1–6 dm. high, simple or branched from the base, hirsute with white hairs intermixed with stalked glands; leaves lance-oblong, ovate, or obovate, narrowed to a margined base, 1–7 cm. long, 0.3–0.4 cm. wide, acute, 3-nerved, bluntly or acutely toothed mostly above the middle, those of inflorescence bracteiform, all alternate or rarely one or two pairs of opposite ones at base of stem; heads few, somewhat panicled, terminating branchlets or long-peduncled from the upper axils, 5 cm. in diameter when well developed; involucre 8–12 mm. high, 2–3-rowed; the scales subequal or the outer a little shorter, linear-lanceolate, glandular on the back, 3-nerved inside, densely ciliate except at tip with long white hairs; rays 10–14, golden yellow, cuneate, subentire or tridentate, 11–21 mm. long, 6–11 mm. wide, the tube hairy; disk-corollas 6 mm. long, with short glandular-hairy tube and cylindric-funnelform throat; pales 9 mm. long, fimbriate-margined, glandular-hairy toward the narrowed apex; achenes narrowly cuneate, 6–7 mm. long, silky-villous, with flattened denticulate awns half their length, the body black, the narrow white margin continuous with the awns and the thick entire yellowish crown.

Geraea canescens Torr. & Gray, l. c. (1847).
Simsia (*Geraea*) *canescens* Gray, Pl. Fendl. l. c. (1849).
Encelia eriocephala Gray, Proc. Am. Acad. viii. 657 (1873).

Specimens examined: UTAH: southeastern part, 1870, *Palmer* (N); NEVADA: El Dorado Cañon, Lincoln Co., Jan.–Apr. 1895, *Lyra Mills* 12 (N); valley of the Virgin River, 6 May 1891, *V. Bailey* (Death Valley Exp. no. 1911: N); CALIFORNIA: Furnace Creek Cañon, Funeral Mts., alt. 100 m., Feb. 1891, *Coville & Funston* 361 (N); 8 miles N. of Salton Sea, Apr. 1910, *Mrs. C. H. Everhart* 2 (F); Palm Springs, alt. 150 m., 19 Apr. 1907, *S. B. Parish* 6079 (F); The Needles, 3 May 1884, *Jones* 3783 (FN); Agua Caliente, Mar. 1881 and Apr. 1882, *Parish Bros.* 228 (FN); Agua Caliente, *W. G. Wright* (G); Colorado Desert, Apr. 1889, *Orcutt* (N); same locality, *Wright* (FN); loose soil, near Calexico, 28 Mar. 1903, *Abrams* 3153 (G); in sandhills, Jan., Ft. Yuma, *Schott* (G); diluvial banks of the Colorado near Ft. Yuma, *Schott* (F); Colorado River, *Newberry* (G); interior of California, *Fremont* 393 of 1844, *Coulter* 304 (COTYPES in Gray Herbarium);

Arizona: Ft. Mohave, 1860–1, *Cooper* (GN); Beaver Dam, 1877, *Palmer* 237 (GN); Santa Rosa to Casagrande, 13 Mar.–23 Apr. 1903, *Griffiths* 4008, 4025 (N); Tule Desert, W. of Monument no. 180, 9 Feb. 1894, *Mearns* 2794 (N); without locality, 1880, *Lemmon* (G). Sonora: near El Capitan, southwest of Sonorita, west of Pinacate, 5 Feb. 1910, *Lumholtz* 27 (G).

1β. G. canescens Torr. & Gray var. **paniculata** (Gray) Blake, n. comb. Greener and less hirsute than the ordinary form, paniculately much branched above, the very numerous heads only 2 cm. in diameter including the rays.

Encelia eriocephala var *paniculata* Gray, Syn. Fl. i. pt. 2. 282 (1884).

Specimens examined: Arizona: mesas near Phoenix, 17 June 1882, *Pringle* 1271 (FGN, type collection).

* * Heads discoid; involucral scales oblong, densely glandular; leaves all sessile by a clasping base.

2. G. **viscida** (Gray) Blake, n. comb. Stout, at least 8 dm. high (the base unknown), hirsutely villous and viscid-glandular throughout; stem channeled, simple or branched above, leafy; leaves thin, oval or broadly ovate-oblong, acutish to obtuse or rounded, sessile by an auriculate or cordate base, irregularly dentate on the wavy margin, the midvein prominent, 3–10 cm. long, 1.5–5 cm. wide; heads rather few, terminating branchlets or long-pedunculate from the upper axils, 1.5–4 cm. broad, hemispherical; involucre shorter than the disk; scales 2–3-seriate, subequal or the outer shorter, oblong, obtuse, 3-nerved, densely glandular, 11–15 mm. long, 2–5 mm. broad; disk-corollas 7–8 mm. long, with rather long slender tube and cylindric throat, somewhat glandular particularly on the tube; pales 14 mm. long, glandular-pubescent toward apex; achenes 7–9 mm. long, with two villous awns more than half their length, narrowly wedge-shaped, silky-villous, with blackish body and white margin and crown.

Encelia (*Geraea*) *viscida* Gray, Proc. Am. Acad. xi. 78 (1876).

Specimens examined: California: near Larkin's Station, 80 miles east of San Diego, 1875, *Palmer* (type in Gray Herb.); Warner's Ranch, and other elevated places in San Diego Co., June 1880, *Parish* 241 (G); Campo, June 1880, *Vasey* 327 (N); mountains near Campo, 24 July 1883, *Orcutt* (F); dry hills near Campo, 26 May 1903, *Abrams* 3633 (FGN); Potrero Valley, San Diego Co., June 1889, *Orcutt* (N). Lower California: without locality, 1883, *Orcutt* (G).

ENCELIA Adans. (to Christopher Encel, who published a work on oak-galls in 1577).— Heads small or medium, radiate or rarely discoid, flowers yellow or purple. Involucral scales 2–3-rowed, subequal or the outer shorter, lanceolate to ovate-lanceolate. Receptacle convex; pales scarious, soft, embracing the achenes and falling with them. Rays entire or 2–3-toothed or -lobed, yellow, rarely absent; disk-corollas with short tube and usually cylindric-funnelform throat, the limb hairy and 5-toothed. Style-branches obtuse, short-pubescent outside. Disk-achenes compressed, very flat, oblong or obovate, villous on margins and glabrous or pubescent on the sides, narrowly white-margined, usually pappusless but in some species with two slender upwardly pubescent awns.— Alternate-leaved generally pubescent perennials, sometimes frutescent, with solitary to paniculate heads of usually yellow flowers. Type species *E. canescens* Lam.— About 14 species of western America, in arid regions and on the sea-coast, from Nevada to Lower California and central Mexico, and again from Peru to central Chili; one species on the Galapagos Islands.

Encelia Adans. Fam. ii. 128 (1763).

Pallasia L'Hér. ["Diss. (1784)"] in Ait. Hort. Kew. iii. 498 (1789), not of Houtt. 1775, nor Scop. 1777, nor L. fil. 1781, nor Klotzsch 1853 (the last a valid genus of *Rubiaceae*, the others all synonyms of various genera).

Eucalia Raeuschel, Nom. ed. 3. 251, 385 (1797), a nomen nudum.

Enchelya Lem. in Orb. Dict. Hist. nat. v. 300 (1844).

Encelya and *Enchelia* Baillon, Dict. Bot. ii. 517 (1886).

Key to the Species of Encelia.

A. Suffrutescent, leaves laciniately lobed.
  B. Leaves linear.................................1. *E. ventorum.*
  B. Leaves ovate.................................2. *E. laciniata.*
A. Leaves linear, unlobed.
  B. Heads panicled; plant resinous..............14. *E. stenophylla.*
  B. Heads solitary; leafy-stemmed; leaves glabrous beneath. 12. *E. angustifolia.*
  B. Heads solitary; scapose; leaves puberulent both sides. 13. *E. scaposa.*
A. Leaves oblong to ovate, unlobed.
  B. Heads paniculate, numerous; branches of inflorescence glabrous. 3. *E. farinosa.*
  B. Heads few or solitary; peduncles pubescent.
    C. Leaves cordate...............................11. *E. Palmeri.*
    C. Leaves rounded or cuneate at base.
      D. Shrubby; disk yellow. 4. *E. frutescens.*
      5. *E. albescens.*

D. Mostly herbaceous; disk purple.
  E. Leaves tomentose or canescent with a rather soft pubescence; South American........................9. *E. canescens.*
  E. Hispid-canescent; plant of the Galapagos Islands. 10. *E. hispida.*
  E. Leaves greener, less pubescent; Mexican and Californian.
    F. Involucre densely tomentose, or scabrous-pubescent in a variety..........................8. *E. californica.*
    F. Scales dorsally glandular, ciliate toward tip; leaves ovate, acute.....................7. *E. halimifolia.*
    F. Scales glandular-ciliate; leaves oval-oblong, obtuse. 6. *E. conspersa.*

SYNOPSIS OF SPECIES.

*LACINIATAE. Suffrutescent, with laciniately lobed leaves; achene pappusless; disk purple.

+– Leaves linear; peduncles a centimeter long.

1. E. VENTORUM Brandegee. Suffrutescent, much branched, 0.9–1.2 m. high, stem 5–7.5 cm. thick; the young branchlets glandular; leaves crowded toward tips of branches, fleshy, linear, with 1–5 linear alternate lobes above the middle, 3–6.5 cm. long, 1–2 mm. wide; heads "fragrant," glutinous, nodding on short peduncles, solitary at tips of branches, hemispheric, 10–12 mm. high; scales about 3-seriate, somewhat unequal, rather loose, lanceolate to lance-ovate, ciliate and glandular-dotted, becoming reflexed and somewhat woody in age; rays about 10, small, 8 mm. long, truncate, with rather long hairy tube; disk-flowers about 50, the corolla 5 mm. long, with short tube and cylindric-funnelform throat; pales greenish and glandular-puberulent on the keel, about 3-nerved on the sides, 7–11 mm. long; achenes 5.5–8 mm. long, oblong, truncate, narrowly margined, with villous margin and apex, glabrous on the sides.

Encelia ventorum Brandeg. Proc. Calif. Acad. ser. 2. ii. 175 (1889).

Specimens examined: LOWER CALIFORNIA: Lagoon Head, 6–15 Mar. 1889, *Palmer* 828 (GN); Playa Maria, July–Oct. 1896, *Anthony* 118 (FGN). Originally collected by Brandegee "on the narrow strip of sand between the lagoons and the ocean near the Boca de Las Animas."

+– +– Leaves broader; peduncles 2.5–6.5 cm. long.

2. E. LACINIATA Vasey & Rose. Suffrutescent, 0.6–0.9 m. high, much branched, more or less glandular-pubescent, and usually hispid with ascending hairs on the younger parts; leaves ovate or obovate in outline, acute or obtuse, unequally and laciniately lobed with the lobes sometimes toothed, narrowed to a margined petiole, 3–5.5 cm. long, 1–2.5 cm. wide, lamina 2.5–6 mm. broad between the lobes; heads

terminal and long-peduncled from the upper axils, nodding in fruit, 10–12 mm. high; scales 2–3-rowed, loose, lanceolate, somewhat glandular, ciliate and tomentose; rays about 12, oval, subentire, 7 mm. long, with hairy tube; disk-corollas as in the last, glandular at base and tip, 5 mm. long; pales few-nerved, glandular-hairy toward the loose subherbaceous tip, 8–10 mm. long; achenes 5–6.5 mm. long, obovate, emarginate at apex, densely spreading-villous on the margin and with a few hairs toward the apex.

Encelia laciniata Vasey & Rose, Proc. U. S. Nat. Mus. xi. 535 (1889).

Specimens examined: LOWER CALIFORNIA: sand plains and hills above the bay, Lagoon Head, 6–15 Mar. 1889, *Palmer* 804 (FGN, TYPE COLLECTION); Ascension Island, Mar.–June 1897, *Anthony* 435 (G). Also reported by Brandegee from San Gregorio.— Anthony's plant differs from Palmer's in its thicker more bluntly lobed leaves, like the stem finely glandular-pubescent, nearly without the rough white hairs of the types.

* * HALIMIFOLIAE. Herbaceous or frutescent; leaves entire or merely repand-toothed, oblong to ovate; disk yellow or purple; achene rarely with 1 or 2 weak awns.

+– Heads numerous, paniculate, the branches of inflorescence smooth; leaves chiefly basal.

3. E. FARINOSA Gray. Much branched from a woody base, sometimes 1.6 m. high, the stems and branches exuding a fragrant resin, white-mealy becoming glabrate; leaves mostly basal, broadly ovate to lanceolate, acute or obtuse, entire or rarely repand-toothed, the margin often undulate, densely white-farinose occasionally becoming subglabrate, the nerves rather prominent beneath, 3–10 cm. long, 2–5 cm. wide, on narrowly margined petioles 1–4 cm. long; panicle nearly naked, the branches whitish-yellow, glabrous or rarely with a few hairs, often glandular-hairy just below the heads; heads terminating the branches, often nodding in fruit, radiate, disk 1–1.5 cm. in diameter; scales imbricated in 3–4 rows, the outer or sometimes all linear, the inner usually successively longer and with broader bases, loosely hairy when young, often glabrate when older, all blunt, the longest 3.5–7 mm. long, shorter than the disk; rays[33] about 12, usually conspicuous, 7–11 mm. long, oval-oblong, 3-lobed; disk-corollas 3.5–4.5 mm. long, glandular on the tube, yellow including the limb; pales 6–7 mm. long, glandular on keel, faintly nerved, entire or laterally 1-toothed; achene 4.5 mm. long, obovate, emarginate, villous all over except for a submarginal naked border, awnless.

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[33] In *Coulter* 327 (hb. Gray) some at least of the rays are styliferous, the only such instance known to me in the genus.

Encelia farinosa Gray in Torr. Bot. Emory Rep. Mil. Recon. 143 (1848).

Specimens examined: NEVADA: Muddy Valley, Lincoln Co., alt. 518 m., 1 May 1906, *Kennedy & Goodding* 8 (N); boulder washes, Las Vegas Mts., 13 May 1905, *Goodding* 2363 (G); CALIFORNIA: Panamint Cañon, alt. 610 m., 3 May 1897, *Jones* (N); Furnace Creek Ranch, Death Valley, 25 Mar. 1891, *Coville & Funston* 476 (FGN); near Bennett Wells, Death Valley, 22 Jan. 1891, *Coville & Funston* 202 (N); Los Angeles, 25 May 1902, *Braunton* 287 (N);[34] San Bernardino Mts., 1880, *Vasey* 286 (N); San Bernardino, 12 Jan. 1880, *S. B. Parish* (G); May 1881, *Parish* (F); 2 May 1896, *C. E. Cummings* (G); dry hills near San Bernardino, 27 Apr. 1891, *S. B. Parish* 2192 (N); vicinity of San Bernardino, alt. 305–457 m., 27 Apr. 1895, *S. B. Parish* 3638 (GN); foothills near San Bernardino, 15 May 1891, *S. B. Parish* (F); San Bernardino Hills, Apr. 1881, *Parish* (F); near Barstow, 12 Apr. 1905, *T. E. Wilcox* (N); The Needles, 8 May 1884, *Jones* 3853 (FN); Elsinore, 4 Apr. 1903, *Baker* 4151 (FG); Temecula Cañon, alt. 800 m., 31 Mar. 1898, *Leiberg* 3209 (N); Box Springs, San Diego Co., 10 May 1882, *Orcutt* (F); Signal Mt., Colorado Desert, 2 Apr. 1903, *Abrams* 3159 (G); Colorado Desert, Mar. 1881, *W. G. Wright* 185 (G); southeastern California, 1876, *Palmer* (G); southern California, 1876, *Parry & Lemmon* 181 (F); California, *Coulter* 327 (G); Colorado River, interior of California, 1854, *Bigelow* (G); Mexican Boundary Survey under Emory, 1846 (TYPE in Nat. Herb., no. 46083); eastern base of Coast Range, edge of Colorado Desert, 7 May 1894, *Mearns* 2969 (N); ARIZONA: Fort Mohave, 1860–1, *Cooper* (G); Beaver Dam, 1877, *Palmer* 239 (G); Grand Cañon of the Colorado, Journey of 1885, *Gray* (G); Verde Mesa, 1867, *E. Smart* 201 (N); buttes, Tempe, 19 Apr. 1892, *Ganong & Blaschka* (G); along the Gila, Mar. 1852, *Parry* 54 (G); Coyote to Santa Rosa, 13 Mar.–23 Apr. 1903, *Griffiths* 3986 (N); Laosa to Lavare via Babuquivari, 1903, *Griffiths* 3615 (N); Santa Catalina Mts., 19 Apr. 1881, *Pringle* (FG); Camp Grant, 2 Apr. 1867, *Palmer* 124 (G); Tucson Mts., Sept. 1907, *Thornber* (N); Tucson Mts., 1903, *Griffiths* 3475 (N); hills near Tucson, 8 May 1883, *Pringle* (F); Tucson, 8 Apr. 1892, *Toumey* (N), and 1 May 1892, *Toumey* 687 (N); Tule Mts., Mexican boundary line, 11 Feb. 1894, *Mearns* 2800 (N); near Monument no. 178, Mex. bound. line, 8 Feb. 1894, *Mearns* 2791 (N); Chimehuevis, alt. 915 m., 21 Apr.

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[34] A MS. note attached to the sheet says in part: "This plant was just 10 ft. from root to fruit." The species is not included in Abrams' Los Angeles Flora.

1903, *Jones* (N). Sonora: common on plains from Huerigo to Granada, Guasabas, alt. 915–1220 m., 15 Nov. 1890, *Hartman* 233 (GN); Hermosillo, 10 June 1897, *F. S. Maltby* 229 (N); Torres, 10 June 1897, *Maltby* 179 (N); El Grupo (?), 13 Nov. 1895, *McGee* (N); hillsides, Guaymas, July 1887, *Palmer* 111 part (GN); Hermasillo, 4 Mar. 1910, *Rose, Standley, & Russell* 12354 (N); Empalme, 11 Mar. 1910, *Rose, Standley, & Russell* 12625 (N); Carral, 12 Mar. 1910, *Rose, Standley, & Russell* 12651 (N); Guaymas, 23 Apr. 1910, *Rose, Standley, & Russell* 15051 (N); Magdalena, 25 Apr. 1910, *Rose, Standley, & Russell* 15098 (N); Sinaloa: thickets along Rio Fuerte, San Blas, 24 Mar. 1910, *Rose, Standley, & Russell* 13365 (N).— The only *Encelia* of any economic importance, and that but slight, the resin being burned as incense in the churches of Lower California, giving the plant the local name of "Incienso" (Brandegee, Zoe i. 83 (1890)).

3β. E. farinosa Gray f. **phenicodonta** Blake, n. forma, disco purpureo. Disk-corollas purple above; otherwise as in the typical form.— Specimens examined: California: Riverside Mt., *Newberry* (G); Arizona: Williams Fork, Mar. 1876, *Palmer* 251 (FN). Lower California: west side of Lake Maquata, Colorado Desert, 27 Jan. 1890, *Orcutt* 2023 (N); cañon near San Quentin, 22 Apr. 1886, *Orcutt* 1341 (cotypes in FGN); old diggings, Calmalli, alt. 366 m., Jan.–Mar. 1898, *Purpus* 33 (FN); La Paz, 20 Jan.–5 Feb. 1890, *Palmer* 50 (N); Santa Rosalia, 24 Feb.–3 Mar. 1889, *Palmer* 186 (GN); Sonora: Papago Tanks, Pinacate Mts., 14 Nov. 1907, *MacDougal* (N); hillsides, Guaymas, July 1887, *Palmer* 111 in part (GN).

3γ. E. farinosa Gray var. **radians** Brandeg. in herb., n. comb.— Leaves glabrate or nearly so; involucre nearly or quite glabrous, its bracts chiefly linear-oblong; disk purple.

Encelia radians Brandeg. Proc. Calif. Acad. ser. 2. ii. 176 (1889).

Specimens examined: Lower California: San José del Cabo, Mar.–June 1897, *Anthony* 433 (GN); same locality, Jan.–Mar. 1901, *Purpus* 398 (GN).

+ + Heads few or solitary; peduncles usually pubescent; stem leafy.

++ Shrubby, even the branches woody; heads solitary at tips of long naked usually scabrous peduncles terminating the branches, often discoid; disk yellow; awns often present.

4. E. frutescens Gray.[35] A low much branched shrub, 1.3–1.6 m. high, white with a dense short very scabrous pubescence at least

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[35] Often confused with *Viguiera Parishii* Greene, which is rather similar in aspect but has mostly opposite cordate-deltoid leaves and a pappus of squamellae as well as awns.

on the younger parts, the branchlets ending in long naked monocephalous peduncles; leaves short-petioled, oblong to ovate, obtuse or acute, cuneate or truncate at base, scabrous with scattered white hairs with persistent tuberculate bases, 1–3 cm. long, 0.6–1.6 cm. wide; involucre 6–10 mm. high, its scales somewhat unequal, 3-rowed, hispid-scabrous and sometimes slightly glandular, varying from linear-lanceolate to ovate-acuminate; heads 1–2.5 cm. broad; rays rarely present, then about 12, 3-lobed, about 9 mm. long; disk-corollas 5–6 mm. long, with glandular-hairy tube and hairy limb; pales glandular-pubescent, 1 cm. long; achenes black with narrow white margin, villous on the edges and somewhat pubescent on the sides, 6.5–8 mm. long, 2.5–3.2 mm. broad, awnless or with 1 or 2 weak villous awns.

Simsia frutescens Gray in Torr. Bot. Mex. Bound. 89 (1859).
Encelia frutescens Gray, Proc. Am. Acad. viii. 657 (1873).
E. frutescens f. *radiata* Hall, Univ. Calif. Pub. Bot. iii. 135 (1907).
E. frutescens f. *ovata* Hall, l. c.

Specimens examined: CALIFORNIA: Mohave Desert, near Bagdad, 12 Apr. 1905, *Wilcox* (N); The Needles, 3 May 1884, *Jones* 3812 (FN); near Cañon Springs, Apr. 1905, *Hall* 5859 (N); Palm Spring, alt. 61 m., 10 May 1903, *Jones* (N); wash near Coyote Wells, Colorado Desert, 3 Nov. 1890, *Orcutt* 2200 (GN); Signal Mt., 2 Apr. 1903, *Abrams* 3156 (FGN, cotype number of f. *ovata* Hall); interior of California, 1849, *Fremont* (G, COTYPE); ARIZONA: ravines in gravel plain, Ft. Mohave, 1860–1, *Cooper*, (GN, mixed with *Viguiera Parishii* Greene); Sierra Prieta near Ft. Yuma, *Schott* (G, COTYPE); Agua Caliente, 1846, *Emory* (G, COTYPE); Yuma, 1881, *Vasey* (N); west bank of Colorado River below Yuma, 7 Apr. 1894, *Mearns* 2853 (N); Yuma, 25 Apr. 1906, *Jones* (N); Santa Catalina Mts., Apr. 1881, *Lemmon* 188 (G); Red Rock, 11 June 1892, *Toumey* 671 (N); Carrizo Creek, *Hayes* 446 (G); mesas near Tucson, 8 May 1884, *Pringle* (FGN, cotype number of f. *ovata* Hall); mesas, without exact locality, 15 May 1881, *Pringle* (FG); foothills, Santa Rita Mts., 1902, *Griffiths & Thornber* (N); high plains, Lowell, 9 May 1884, *W. F. Parish* 110 (G); Wilmot Siding, 8 miles southeast of Tucson, on the mesas, 800 m., 6 June 1903, *Thornber* 83 (N); without locality, 1869, *Palmer* (N).

The form *ovata* described by Hall seems hardly distinct enough for recognition; apparently the lower leaves are always more or less ovate or ovate-oblong, the upper usually oblong but sometimes broader, but without any distinct line of demarcation; and the presence of rays, also, is a character scarcely requiring recognition by name.

The following sheets, with ovate leaves more or less glandular and with fine appressed pubescence, seem intermediate between this and var. *virginensis*: ARIZONA: near Ft. Verde, 20 June 1883, *Rusby*, (FN); along Bright Angel Trail to Grand Cañon, alt. 1000 m., 10 Sept. 1901, *Leiberg* 5926 (N); Hackberry, 24 May 1884, *Jones* (FN).

4β. E. FRUTESCENS Gray var. **resinosa** Jones in litt., n. var., hispido-scabra et plus minusve glandulosa interdum glandulosissima; foliis tenuibus margine sinuatis late ovatis 1–2 cm. longis latisque subacutis obtusisve basi truncatis vel rotundatis rare cuneatis utrinque plus minusve glandulosis scabris pilis albis basi tuberculatis; petiolis glanduloso-scabris 5–7 mm. longis; pedunculis 1–2 dm. longis glandulosis sparse scabris; capitulis 1–2 cm. diametro radiatis; involucri 2–3-seriatis squamis ovato-acuminatis vel lineari-lanceolatis dense glandulosis exterioribus interdum paucis pilis hispidis; paleis dense glandulosis; acheniis exaristatis.

Specimens examined: UTAH: near Great Salt Lake, *Capt. Bishop* (G); ARIZONA: without locality, 1869, *Palmer* (N); Little Colorado near Winslow, 10 June 1890, *Jones* (TYPE COLLECTION, GN); half mile below Tanner's Crossing, Little Colorado, 18 May 1901, *L. F. Ward* (N).

4γ. E. FRUTESCENS Gray var. **virginensis** (A. Nels.) Blake, n. comb. Leaves broadly ovate, cinereous-scabrous with a fine glandular pubescence intermixed with stouter tuberculate-based hairs like those found in the type; outer involucral scales linear-lanceolate, the inner ovate-acuminate; rays apparently always present; otherwise as in the typical form.[36]

Encelia virginensis A. Nels. Bot. Gaz. xxxvii. 272 (1904).

E. frutescens f. *virginensis* Hall, l. c. (1907).

Specimens examined: UTAH: valley of the Virgin River near St. George, 1874, *Parry* 142 (FG); La Verken, alt. 1036 m., May 1894, *Jones* 5195 (FN); southern part, 1875, *Johnson* (N); NEVADA: "The Pockets," valley of the Virgin River, 30 April 1902, *Goodding* 666 (FGN, COTYPES of *E. virginensis*); dry washes, Mesquite Well, 1 May 1905, *Goodding* 2259 (G); ARIZONA: northern part, 1872, *Wm. Thompson* 380 (N).

The following, with white-pubescent but scabrous leaves and more or less ovate-acuminate outer scales, seem intermediate between this variety and the next; UTAH: St. George, 1877, *Palmer* 238 (G);

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[36] According to Nelson's description the lower leaves are opposite, but I have been unable to find evidence of this in the material at hand, and opposite leaves are unknown elsewhere in the genus.

NEVADA: Moapa, Lincoln Co., alt. 518 m., 12 May 1906, *Kennedy* 1112 (N); ARIZONA: 1869, *Palmer* part (N).

4*b*. E. FRUTESCENS Gray var. **actoni** (Elmer) Blake, n. comb. Leaves ovate, cuneate or truncate at base, very rarely toothed, whitened with a rather soft fine pubescence; pubescence of stem and peduncles also softer than in the typical form; involucral scales mostly ovate-acuminate; rays apparently always present; otherwise as in the typical form.

Encelia actoni Elmer, Bot. Gaz. xxxix. 47 (1905).

E. frutescens f. *actoni* Hall, l. c. (1907).

Specimens examined: NEVADA: Esmeralda Co., 31 May 1886, *Shockley* 416 (G); Candelaria, alt. 1677 m., June 1887, *Shockley* 540 (G); without locality, 1872, *Lt. Wheeler* (N); alt. 1050 m., Beattie, Nye Co., 5 June 1912, *Heller* 10422 (N); CALIFORNIA: Pleasant Cañon, Panamint Mts., alt. 1677 m., 6 May 1897, *Jones* (N); near Independence, 29 May 1906, alt. 1160 m., *Hall & Chandler* 7217 (G); hillsides, Argus Mts., alt. 1220–1525 m., 1897, *Purpus* 5383 (GN); Tehachapi foothills, Kern Co., 30 May 1905, *F. Grinnell* (N); east slope of Walker Pass, Kern Co., alt. 1300 m., 21 June 1891, *Coville & Funston* 1020 (N); Rock Creek, desert slopes of San Gabriel Mts., alt. 1158 m., July 1908, *Abrams & McGregor* 548 (N); Liebre Mts., June 1908, *Abrams & McGregor* 402 (GN); Acton, June 1902, *Elmer* 3724 (N, TYPE NUMBER of *E. actoni*[37]); Mohave Desert, June 1887, *S. B. Parish* (F); near Hesperia, Mohave Desert, 30 May 1901, *S. B. Parish* 4873 (N); San Francisquito Creek, 27 May 1905, F. Grinnell (N); San Francisquito Cañon, 26 May 1905 (N); Victor, alt. 792 m., 17 May 1903, *Jones* (N); San Jacinto, June 1901, *Hall* 2007 (N); San Felipe Cañon, 22 June 1888, *Orcutt* 1483 (N); San Jacinto, 14 Aug. 1907, *V. Bailey* (N); San Felipe, June 1852, *Thurber* 634 (G); San Isabel, June 1852, *Thurber* 634 (G); near San Felipe, 10 Oct. 1858, *Sutton Hayes* 443 (N); dry rocky hills, Jacumba, 31 May 1903, *Abrams* 3667 (GN); Smith's Mt., San Diego Co., 1880, *Vasey* 285 (FN); ARIZONA: Colorado River, *Schott* (G)[38]; "Jesup Rapids," 18 Feb. 1858, *Ives Expedition*, in part (N).

5. E. ALBESCENS Gray. Frutescent (?); branches striate, whitened with a roughish appressed pubescence, terminating in naked

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[37] Described by Elmer as having the ray-flowers styliferous, but I do not find them so.

[38] Originally taken by Gray (Bot. Mex. Bound. 88) to be *E. conspersa* Benth. and afterwards considered (Proc. Am. Acad. viii. 656) a small variety of *E. californica*, but best placed here.

similarly pubescent monocephalous peduncles; rameal leaves ovate, obtusish, barely toothed, whitened with an appressed scabrous pubescence particularly beneath, 2–2.5 cm. long, 0.9–1.2 cm. wide, short-petioled; involucre 7 mm. high, the scales subequal, in 2 rows, linear-lanceolate, appressed-pubescent and scabrous without; rays suborbicular, 3-lobed, 12 mm. long, pubescent on the back; disk-corollas 5 mm. long, pubescent on tube and teeth; pales 7 mm. long, scantily glandular-pubescent on back, fimbriate at apex; immature achene 4.7 mm. long, villous on margin, appressed-pubescent on the sides, bearing 2 unequal weak awns, or awnless.

Encelia albescens Gray, Proc. Am. Acad. viii. 658 (1873).

Specimens examined: SONORA: 1869, *Palmer* 21 (GN, COTYPES).— A doubtful species, known from very insufficient specimens, too close to *E. frutescens* var. *actoni*.

++ ++ Mostly herbaceous (except no. 6); heads always radiate, usually somewhat racemose; disk purple; awns absent (rarely present in one variety).

= Frutescent; leaves small, oblong, green; involucral scales glandular-ciliate.

6. E. CONSPERSA Benth. Shrubby, branched, the scabrous bark white; branches leafy below, terminating in nearly naked 1–3-branched peduncles, the branches monocephalous; leaves oblong to ovate, obtusish, cuneate or truncate at base, green, scabrous-pubescent especially beneath, 2–2.5 cm. long, 8–10 mm. wide, on petioles 3.5–7 mm. long; peduncles slightly scabrous; disk 1–1.3 cm. in diameter, 8 mm. high; involucre 5–6 mm. high, its scales unequal, 3-seriate, the outer lanceolate, the inner ovate, glandular on back and white-ciliate nearly to the tip; rays oval, slightly 3-lobed, pubescent on tube and back; disk-corollas 5 mm. long, with short tube, glabrous; pales 6 mm. long, about 9-nerved, glandular on the back; immature achenes 2.5 mm. long, villous on the margin, pubescent on the sides.

Encelia conspersa Benth. Bot. Voy. Sulph. 26 (1844).

Specimens examined: LOWER CALIFORNIA: Magdalena Island, 18 Jan. 1889, *Brandegee* (GN). Bentham's type came from Bay of Magdalena.— Brandegee's specimens well agree with Bentham's description, except that the branches are not "albo-tomentosis" nor the scales "lineari-lanceolatis." The species must remain somewhat in doubt until Bentham's type can be re-examined. As is suggested in Index Kewensis, it seems in the specimens at hand too close to *E. halimifolia* Cav.

= = More herbaceous; leaves larger, generally canescent or white-tomentose; involucral scales generally tomentose or canescent-pubescent.

× Leaves green; scales mostly ciliate.

7. E. HALIMIFOLIA Cav. Suffrutescent, branched, apparently diffuse, the stems, young branches, and in a less degree the peduncles canescent with fine incurved hairs; leaves ovate, acute or obtusish, cuneate or nearly truncate at base, entire or slightly repand-toothed, somewhat pubescent with incurved hairs but not canescent, 2.5–3.5 cm. long, 1.2–2 cm. wide, on petioles a centimeter long; peduncles axillary and terminal, simple or 2–3-branched, monocephalous, nearly naked; heads 8 mm. high, 1.2–1.4 cm. wide excluding rays; involucre 5–6 mm. high, the scales somewhat unequal, 2–3-seriate, linear-lanceolate, glandular-ciliate; rays 10–12, broadly oval, barely 3-lobed, pubescent on tube and back; disk-corollas 4.5 mm. long, with short tube, glabrous; pales 7.5 mm. long, glandular-pubescent on back; immature achene 3.6 mm. long, villous on margin, otherwise nearly glabrous.

Encelia halimifolia Cav. Icon. iii. 6. t. 210 ("1794" = 1795).

Pallasia grandiflora Willd. Sp. Pl. iii. 2261 (1804).

Specimens examined: SONORA: Yaqui River, 1869, *Palmer* 12 (FGN). There is also a sheet in Gray Herb. (ex herb. Klatt) labeled "Ex horto Scholae centralis Monspeliensis. *Mexico?* Peruvia!" but the specimen is doubtless from Mexico.

× × Leaves canescent or tomentose, or involucre densely pubescent.

○ Leaves green; involucre usually densely and softly tomentose; plants of California and Lower California.

8. E. CALIFORNICA Nutt. Much branched, somewhat spreading from a frutescent base, 6–10 dm. high, or on San Clemente Island becoming 3–3.6 m. high, 2.5–10 cm. in diameter; stem and peduncles canescent with fine incurved-spreading rather soft hairs; leaves from lanceolate to ovate, acute, cuneate or rarely slightly rounded at base, entire or somewhat repand-dentate, appressed-pubescent with soft hairs but distinctly green, 3–6 cm. long, 1–3 cm. wide, on petioles 0.5–3 cm. long; peduncles long, terminal and axillary toward ends of branches, nearly naked; disk 1.5–2.5 cm. in diameter; involucre 1–1.3 cm. high, the scales 2–3-seriate, lanceolate, densely tomentose; rays oblong, 14–20, pubescent on tube and exterior, 1.5–3 cm. long; disk-corollas 5 mm. long, glabrous or the teeth pubescent; pales 7.5–10 mm. long, glandular-puberulent above; achenes 5.5–6 mm. long, villous on margin and pubescent down the middle of each side, awnless.

Encelia californica Nutt. Trans. Am. Philos. Soc. ser. 2. vii. 357 (1841).

Specimens examined: CALIFORNIA: Santa Barbara, 1873, *Bolander* (G); also June 1875, *Rothrock* 82 (F), and May 1902, *Elmer* 3899 (FN); ocean bluffs, Santa Barbara, 16 May 1908, *Eastwood* 129 (FGN); near the 35th parallel, Whipple's Exp., 1853–4, *Bigelow* (N); San Fernando Valley, Feb. 1861, *Brewer* 187 ("bad smell", GN); beach above San Buenaventura, 5 Mar. 1866, *S. F. Peckham* (N); Pasadena, May 1903, *Grant* 395 (F); Santa Monica, Oct. 1881, *Parish Bros.* 964 (F); also 24 Jan. 1897, *J. H. Barber* 29 (N); near beach, Long Wharf, near Santa Monica, 13 Aug. 1910, *Blake* 693 (G); hillsides, May 1885, Los Angeles, *Hasse* (N); foothills, Griffith Park, 13 Mar. 1902, *Braunton* 242 (N); hillsides near Inglewood, 8 Mar. 1903, *Abrams* 3107 (G); Redondo, 25 May 1902, *Braunton* 288 (N); hills, Wilmington, 1 Apr. 1882, *Pringle* (FN); near Riverside, alt. 366 m., Apr. 1902, *Hall* 2922 (N); San Diego, *Parry*, *Newberry*, *Nuttall* (G); also 1875, *Palmer* 169 (F), 28 Feb. 1884, *Orcutt* (F), 15 Mar. 1882, *Jones* 3069 (N), 29 Jan. 1894, *H. A. Sheldon* 45 (N), 30 Mar. 1896, *C. E. Cummings* (G), 1 May 1902, *Brandegee* (Baker distr. 1657, FGN), May 1906, *Brandegee* (F); La Jolla, 16 Feb. 1895, *Snyder* (F); seashore, 30 m. from Lower California, 24 Feb. 1883, *G. C. Deane* (G); Catalina Island, 30 Jan. 1874, *Baker & Dall* (GN); Avalon, May 1898, *Trask* (N); San Clemente Island, 25 Aug. 1894, *Mearns* 4067 (N); Chalk Cañon, San Clemente Island, June 1903, *Trask* 204 ("10–12 ft. tall; 1–4 ins. diam.", N); ARIZONA: Ft. Mohave, 1860–1, *Cooper* 324 (N). LOWER CALIFORNIA: Todos Santos, 13 May 1882, *H. E. Fish* (F); San Quentin Bay, Jan. 1889, *Palmer* 661 (N), 662 (GN).— Bailey (Cyc. Am. Hort. ii. 529 (1900)) states that this species is in cultivation.

8β. E. CALIFORNICA Nutt. var. **asperifolia** Blake, n. var., frutescentior capitulis foliisque plerumque minoribus; disco 1.2–1.8 cm. diametro; pube caulium foliorumque asperiore; foliis ovalibus vel ovatis 1.5–3 cm. longis 1–1.5 cm. latis scabris subintegris; involucro breviore, pube densa aspera, squamis subglandulosis ad apicem subglabratis; acheniis rare cum aristis 1–2 tenuibus.

Specimens examined: LOWER CALIFORNIA: near Rosario and San Fernando, 4 May 1886, *Orcutt* 1346 (FGN); Lagoon Head, 40 miles inland, 6–15 Mar. 1889, *Palmer* 822 (GN); Cedros Island, 1875, *Streets* (GN), also 18–20 Mar. 1889, *Palmer* 702 (FGN: 1 or 2 weak awns present), Mar.–June 1897, *Anthony* 292 (COTYPES in FGN); San Benito Island, *Streets* (G); San Bartolome Bay, Mar. 1889, *Lt.*

C. F. Pond (N).— Often confused with *E. frutescens* Gray, from which the purple disk at once distinguishes it.

∘ ∘ Leaves canescent or tomentose; involucre not densely tomentose; plants of South America (except one species of Lower California, with cordate leaves).

+ Leaves cuneate or rounded at base; plants of South America and the Galapagos Islands.

— Pubescence usually soft; heads (except in starved specimens) long-peduncled; South American.

9. E. CANESCENS Lam. Suffrutescent below, suberect, canescent with a rather soft pubescence, the stem about 6 dm. long; leaves broadly ovate, rounded at base, obtuse or rounded at tip, canescent with a soft pubescence, 2.5–5 cm. long, 1.6–3.5 cm. broad, on petioles 0.5–2 cm. long; inflorescence few-headed, terminal, the heads racemose or corymbose-panicled; disk 1–1.5 cm. in diameter; involucre 5–7 mm. high, its rather loose bracts lanceolate to lance-ovate, somewhat triseriate, tomentose; rays about 12, broadly oval, faintly 2–3-lobed, 7 mm. long; disk-corollas 4.5 mm. long, the short tube glandular, the teeth hairy; pales 8 mm. long, glandular-hairy on the back; achenes 6 mm. long, 2.7 mm. wide, blackish with narrow white villous margin, pilose down the middle of the sides, awnless.

Encelia canescens Lam. Encycl. Method. ii. 356 (1786); Cav. Icon. i. 45. t. 61 (1791).

Coreopsis limensis Jacq. Coll. ii. 299 (1788), & Icon. Pl. Rar. iii. t. 594 (1786–1793).

Enselia limensis Jacq. Coll. l. c. 300; *Encelia limensis* Steud. Nom. ed. 2. 420 (1840).

Pallasia halimifolia L'Hér. in Ait. Hort. Kew. iii. 498 (1789).

Encelia alternifolia Raeuschel, Nom. ed. 3. 251 (1797).

Eucalia canescens Raeusch. l. c. (1797).

Specimens examined: PERU: dry seacoast, Payta, *Col. Hall* (G); Lima and San Lorenzo, *Gaudichaud* 112 (G); without definite locality, *Dombey* (G), *McLean* (G); *Wilkes Expl. Exp.* (N), approaching var. *oblongifolia*; CHILI: sandy places, Copiapo, Sept. 1854, *Lechler* 2801 (G).— Passes into the following varieties.

9β. E. CANESCENS Lam. var. PARVIFOLIA (HBK.) J. Ball. Pubescence denser, almost tomentose; leaves rhombic-ovate or ovate-lanceolate, cuneate at base, acute or subacute at tip; achenes rather more hairy.

Encelia parvifolia HBK. Nov. Gen. iv. 206 (1820).

Encelia canescens var. *parvifolia* J. Ball, Journ. Linn. Soc. xxii. 151 (1887).

Pallasia procumbens Spreng. Sys. iii. 610 (1826).

Encelia paucifolia Walp. Linnaea xiv. 505 (1840) [err. cler.].

Encelia pilocarpa Rusby, Bull. N. Y. Bot. Gard. viii. 131 (1912).

Specimens examined: Peru: Arequipa, 8 Aug. 1901, *Williams* 2526 (GN, type number of *E. pilocarpa*); without definite locality, *McLean* (G); Chili: desert of Atacama, Sept.–Oct. 1890, *Morong* 1311 (FGN, distr. as *E. tomentosa* Walp.); sandy places near Caldera, May 1882, *Ball* (G). With no locality: *Wilkes Expl. Exp.* (N).

9γ. E. canescens Lam. var. tomentosa (Walp.) J. Ball. Apparently more frutescent; stem and the small broadly ovate blunt short-petioled leaves densely white-woolly.

Encelia tomentosa Walp. Linnaea xiv. 504 (1840).

Encelia canescens var. *tomentosa* J. Ball, Journ. Linn. Soc. xxii. 160 (1887).

Specimens examined: South America: *Wilkes Expl. Exp.*, without locality (N). Walpers says of his species: "e *Chili* misit *Filter*."

9δ. E. canescens Lam. var. **oblongifolia** (DC.) Blake, n. comb. Leaves oblong-lanceolate to rhombic-ovate, somewhat less pubescent than in the typical form, about the shape of those of var. *parvifolia* but much less pubescent; heads apparently fewer, on much longer peduncles.

Encelia oblongifolia DC. Prod. v. 567 (1836).

Specimens examined: Chili: Coquimbo, *Gaudichaud* 85 & 86 (G, cotype); Coquimbo, July–Aug. 1856, *Harvey* (G); without definite locality, *Gay* (G).

— — Pubescence harsh; heads racemose, on peduncles about 3 cm. long; plant of the Galapagos Islands.

10. E. hispida Anderss. Branching, erect, 0.6–1 m. high, canescent with a dense hispid pubescence, the branches striate; leaves oblong, cuneate at base, subentire or repandly toothed, 4–5 cm. long, 1.4–2.2 cm. broad, on spreading-hirtous petioles 1–1.8 cm. long, appressed-hirsutulous above, more densely so beneath particularly along the veins; heads racemose in inflorescences terminating the branches, the arcuate-spreading peduncles linear-bracted at base, 2–3.5 cm. long, densely villous-hispid; heads small, 8 mm. high, 9–11 mm. wide exclusive of rays; involucre 4–5 mm. high, the scales lanceolate, 2-seriate, somewhat unequal, densely villous-hispid, the outer a little loose; rays oval, tridentate, pubescent on the back, 4.2 mm. long; disk-corollas 3.5–4.5 mm. long, the short tube and teeth pubescent;

pales 4.7 mm. long, upwardly villous; achenes (not quite mature) narrowly obovate, 4.5 mm. long, villous on margin and pubescent medially on the sides.

Encelia hispida Anderss. Om Galap.-Öarnes Veg. 73 (1853).

Specimens examined: GALAPAGOS ISLANDS: dry grassy places, Chatham and Charles Islands, *Andersson* (G, TYPE COLLECTION).

+ + Mature leaves cordate-ovate; plant of Lower California.

11. E. PALMERI Vasey & Rose. Suffrutescent, branched, 1 m. high, the stems whitened with a dense hispid pubescence, the branches rather sharply angled; leaves broadly ovate, cordate or rarely truncate at base, rounded or obtuse at apex or the younger acute, hispid-canescent on both sides or sometimes almost velutinous, entire or with blunt triangular teeth, 1.5–4 cm. long, 1.1–3.5 cm. broad, on petioles 3–10 mm. long, with ampliated base; heads peduncled, several in a glandular-hispid paniculate inflorescence, the bracts minute; disk 1–2 cm. broad; involucre 1 cm. high, the rather loose scales lanceolate to linear-lanceolate, triseriate, somewhat unequal, glandular on the back, densely villous on sides nearly to tip, somewhat pubescent inside; rays about 20, 10 mm. long, broadly oval, slightly trilobed, pilose on tube and back; disk-corollas 5 mm. long, nearly glabrous, the disk brownish purple; pale 8 mm. long, glandular-pubescent on the back above; achenes 4.5 mm. long, villous on the angles and slightly pubescent down the midline of the sides.

Encelia Palmeri Vasey & Rose, Proc. U. S. Nat. Mus. xi. 535 (1889).

Specimens examined: LOWER CALIFORNIA: Lagoon Head, 6–15 Mar. 1889, *Palmer* 805 (FGN, TYPE COLLECTION): San José del Cabo, Mar.–June 1897, *Anthony* 326 (GN); La Paz, 20 Jan.–5 Feb. 1890, *Palmer* 15 (GN).

* * * ANGUSTIFOLIAE. Leaves narrowly linear; awns present; disk yellow.

+ Leafy-stemmed; heads solitary, terminal; leaves glabrous beneath.

12. E. ANGUSTIFOLIA Greenm. An herbaceous perennial, the stems several, erect, 3–4 dm. high, striate, purplish below, slightly pubescent above, from a woody root; leaves scattered, narrowly linear, attenuate, subsessile, with 3 strong somewhat reticulated nerves, glabrous or nearly so below, appressed-hirsutulous above, 2.5–8 cm. long, 1.5–2.5 mm. wide, remotely serrulate; heads pedunculate, solitary, terminating the stems, about 1 cm. high, the peduncle bearing a slender bract just below the head; scales apparently few and rather loose, lance-attenuate, ciliate, 1–2-seriate; rays about 5, oblong, barely tridentate, 15 mm. long, 4 mm. wide; disk-corollas

rather few, 5.5 mm. long, pubescent on tube and teeth; pales 8 mm. long, pubescent; immature achene 2.5 mm. long, villous on the margin and slightly pubescent on the sides, bearing 2 upwardly pubescent awns about its own length.

Encelia angustifolia Greenm. Proc. Am. Acad. xxxix. 110 (1903).

Specimen examined: Territory of Tepic: in the Sierra Madre, 13 Aug. 1897, *Rose* 3453 (G, type collection).

+ + Scapose; head solitary, terminal; leaves puberulent both sides.

13. E. scaposa Gray. Herbaceous perennial, simple and erect, 3.5 dm. high, the leaves all clustered at the base, the monocephalous scape naked except for two linear bracts; the lowest leaves scalelike; leaves linear, 3.5–8.5 cm. long, 1.5–4 mm. wide, attenuate, subsessile, with whitish cartilaginous margin and tip, rough-puberulent both sides; scape puberulent especially above, striate, whitish; head 1 cm. high, 2 cm. wide excluding the rays; scales linear-lanceolate, subequal, about 2-rowed, rather loose, white-hispid on the back; rays pubescent on tube and back, apparently rather numerous, oval, 15 mm. long, 6 mm. wide; disk-corollas 5 mm. long, pubescent on teeth and short tube; pales 10 mm. long, rather narrow, about 9-nerved, pubescent above; immature achenes 4.5 mm. long, pubescent on the sides, villous on margin and top, as are the two awns which are about as long.

Simsia? (Geraea) scaposa Gray, Pl. Wright. ii. 88 (1853).

Encelia scaposa Gray, Proc. Am. Acad. viii. 657 (1873).

Specimen examined: New Mexico: stony hills between the Mimbras and the Rio Grande, Oct. 1851, *C. Wright* (G, type collection).

+ + + Heads numerous, panicled; plant resinous.

14. E. stenophylla Greene. Perennial, the stems several and erect from a somewhat branched woody base bearing the scars of former leaves, more or less glutinous; leaves mostly crowded toward the bases of the stems, linear, 4.5–8 cm. long, 1–2 mm. wide, mucronulate, somewhat fleshy, glutinous, slightly hairy along the margin, the edges more or less revolute; heads corymbose-panicled toward tip of stem, on peduncles 1–3 cm. long; disk 8 mm. high, 7–11 mm. wide; involucre 3.5–4 mm. high, the scales lance-ovate, about 3-seriate, somewhat unequal, resinous and slightly ciliate, rather strongly ribbed; rays about 6, oval, entire, resin-dotted on tube and back, 5.5 mm. long; disk-corollas 4.5 mm. long, resin-dotted, the throat broadly oblong; pales 7.5–8.3 mm. long, rounded at tip, resin-dotted;

achene 5 mm. long, villous all over, the awns 2 mm. long, upwardly pubescent.

Encelia stenophylla Greene, Bull. Torr. Club, x. 41 (1883).

Specimens examined: LOWER CALIFORNIA: Cedros Island: *Veatch* (GF, TYPE COLLECTION); 3 May 1885, *Greene* (G); 18–20 Mar. 1889, *Palmer* 734 (FGN); Mar.–June 1897, *Anthony* 309 (FGN); 7 Apr. 1897, *Brandegee* (F).

TRANSFERRED AND DOUBTFUL SPECIES.

Encelia adenophora Greenm. Proc. Am. Acad. xxxix. 109 (1903) is to be transferred to *Simsia.*

Encelia amplexicaulis Hemsl. Biol. Centr.-Am. Bot. ii. 183 (1881) is a synonym of SIMSIA AMPLEXICAULIS (Cav.) Pers.

Encelia calva Gray, Proc. Am. Acad. viii. 658 (1873), is a synonym of SIMSIA CALVA Gray.

Encelia cedrosensis Rose, Contr. U. S. Nat. Herb. i. 17 (1890) = VERBESINA HASTATA Kell. (*V. venosa* Greene).

Encelia Chaseae Millsp. in Millsp. & Chase, Field Col. Mus. Bot. iii. 125 (1904), is to be transferred to *Simsia.*

Encelia collodes Greenm. Proc. Am. Acad. xxxix. 110 (1903), from Chiapas = FLOURENSIA **collodes** (Greenm.) Blake, n. comb. Habit very much like that of *F. laurifolia* DC., but heads fewer and radiate; immature achenes villous, the two awns united at base to several slender lacerate squamellae.

Encelia Conzattii Greenm. l. c. 111 (1903) = VERBESINA HYPOGLAUCA Sch. Bip. (as noted by Greenman in Gray Herb.).

Encelia cordata Hemsl. l. c. 183 (1881) is a synonym of SIMSIA CORDATA (HBK.) Cass.

Encelia dentata Poir. Encycl. Suppl. v. 665 (1817) = VERBESINA DENTATA (Humb. & Bonpl.) HBK.

Encelia exaristata Gray in Hemsl. l. c. 183 (1881) = SIMSIA EXARISTATA Gray.

Encelia foetida Hemsl. l. c. 183 (1881) is a synonym of *Coreopsis foetida* Cav., which is a *Simsia.*

Encelia fruticulosa Hieron. Bot. Jahrb. xix. 54 (1894) is based as to name on *Hopkirkia fruticulosa* Spreng. Sys. iii. 444 (1826), which is also *Armania fruticulosa* Bert. in DC. Prod. v. 576 (1836), a species not identified but certainly a *Simsia.* Hieronymus has renamed his specimens *Encelia Sodiroi,* q. v.

Encelia Ghiesbreghtiana Hemsl. l. c. iv. 57 (1887) is a clerical error for *E. Ghiesbreghtii* Gray.

Encelia Ghiesbreghtii Gray, Proc. Am. Acad. viii. 658 (1873), is to be transferred to *Simsia.*

Encelia glutinosa Rob. & Greenm. Am. Journ. Sci. ser. 3. l. 155 (1895) from Oaxaca = FLOURENSIA **glutinosa** (Rob. & Greenm.) Blake, n. comb. Habit much as in *F. collodes,* but petioles and young branches tomentose, heads numerous and corymbed, and the decidedly villous young achenes 2-awned but without squamellae.

Encelia grandiflora Hemsl. l. c. ii. 184 (1881) is a synonym of SIMSIA GRANDIFLORA Benth.

Encelia heterophylla Hemsl. l. c. 184 (1881) is a synonym of SIMSIA HETEROPHYLLA (HBK.) DC.

Encelia hirsuta Ktze. Rev. Gen. iii. pt. 2. 145 (1898), including f. *radiata* Ktze. l. c., is to be transferred to *Simsia.*

Encelia hispida Hemsl. l. c. 184 (1881) is a synonym of SIMSIA HISPIDA (HBK.) Cass.

Encelia hypargyrea Rob. & Greenm. Am. Journ. Sci. ser. 3. l. 155 (1895), from Oaxaca and Puebla, = VIGUIERA **argyrophylla** Blake, n. nom. (not *V. hypargyrea* Greenm. Proc. Am. Acad. xxxix. 105 (1903)). Pappus of two awns with several short fimbriate scales between; achene thickened and appressed-pubescent.

Encelia lagascaeformis Gray in Hemsl. l. c. 184 (1881) = SIMSIA LAGASCAEFORMIS DC. (as to name only).

Encelia maculata Brandeg. Zoe v. 259 (1908), from Puebla and Oaxaca = VIGUIERA **maculata** (Brandeg.) Blake, n. comb. Pappus and achene of *Viguiera.* Related to *Viguiera eriophora* Greenm. The specimens examined (*Purpus* 4127) have the ray-flowers styliferous.

Encelia megacephala Sch. Bip., a nomen in Ktze. Rev. Gen. iii. pt. 2. 145 (1898), is a *Simsia.*

Encelia mexicana Mart. in DC. Prod. v. 578 (1836) is a synonym of SIMSIA AURICULATA DC.

Encelia microcephala Gray, Proc. Am. Acad. viii. 657 (1873) = HELIANTHELLA MICROCEPHALA Gray, l. c. xix. 10 (1883).

Encelia microphylla Gray, l. c. xv. 37 (1880), from Coahuila = FLOURENSIA **microphylla** (Gray) Blake, n. comb. Habit of *Flourensia;* achene somewhat thickened (perfectly ripe fruit not seen), densely villous in the manner of *F. laurifolia* DC., with or without a pappus of 2 upwardly pubescent awns. The likeness to *Flourensia* of this species was commented on by Dr. Gray (Proc. Am. Acad. xix. 7, and Syn. Fl.).

Encelia montana Brandeg. Univ. Calif. Pub. Bot. iii. 394 (1909), from Cerro de Paxtle, Puebla (*Purpus* 3103) = VIGUIERA HELIAN-

THOIDES HBK. The ray-flowers are described as neutral by Brandegee, but in the specimens of the type collection examined are styliferous but undoubtedly sterile.

Encelia nivea Benth. Bot. Voy. Sulph. 27 (1844) has been variously identified, but from the opposite leaves and the thickened achenes cannot have been an *Encelia,* and was probably a *Viguiera.*

Encelia oblonga Rob. & Fern. Proc. Am. Acad. xxx. 118 (1894), from Durango and Chihuahua, is a synonym of *Helianthella Pringlei* Gray, Proc. Am. Acad. xxi. 389 (1886). The species, an alternate-leaved frutescent plant with large heads solitary and long-peduncled at tips of stems, and involucre of ovate-based long-attenuate somewhat foliaceous bracts much exceeding the slightly resinous disk, is somewhat anomalous in appearance, but seems best referred to *Flourensia* as F. **Pringlei** (Gray) Blake, n. comb. The achenes are oblong, 11 mm. long, much thickened, striate, densely pubescent, and tend at maturity to lose the 2 awns which compose the pappus.

Encelia pilosa Greenm. l. c. xxxix. 111 (1903) is a synonym of SIMSIA LAGASCAEFORMIS DC.

Encelia pleistocephala J. D. Sm. Bot. Gaz. xiii. 189 (1888) = VERBESINA PLEISTOCEPHALA (J. D. Sm.) Rob. Proc. Am. Acad. xliii. 41 (1907).

Encelia polycephala Hemsl. l. c. 184 (1881) is a synonym of SIMSIA POLYCEPHALA Benth.

Encelia Pringlei Fern. Proc. Am. Acad. xxxv. 573 (1900), from Hidalgo, has the pappus and achene of *Viguiera* and should be referred to that genus as VIGUIERA **trachyphylla** Blake, n. nom. (not *V. Pringlei* Rob. & Greenm. Proc. Am. Acad. xxix. 387 (1894).

Encelia purpurea Rose, Contr. U. S. Nat. Herb. i. 336 (1895) = SIMSIA LAGASCAEFORMIS DC.

Encelia resinosa Brandeg. Zoe v. 240 (1906), from Hidalgo, an alternate-leaved resinous shrub with a few racemose axillary large heads, seems best referred to *Flourensia* as F. **resinosa** (Brandeg.) Blake, n. comb. The young achene is flattish, scantily haired on the sides and thickly at the apex, with two long awns disposed to be trifid from near the base.

Encelia rhombifolia Rob. & Greenm. Am. Journ. Sci. ser. 3. l. 155 (1895), from Oaxaca = VIGUIERA **rhombifolia** (Rob. & Greenm.) Blake, n. comb. Pappus of two awns with intermediate short squamellae; achene thickened and pubescent.

Encelia sanguinea Hemsl. l. c. 185 (1881) = SIMSIA SANGUINEA Gray.

Encelia sericea Hemsl. l. c. 185 (1881) is a *Simsia.*

Encelia Sodiroi Hieron. Bot. Jahrb. xxix. 43 (1900) is a *Simsia.*

Encelia squarrosa Greenm. Proc. Am. Acad. xxxix. 112 (1903), from Guerrero = VIGUIERA **squarrosa** (Greenm.) Blake, n. comb. Achene and pappus quite of this genus. A very distinct species.

Encelia stricta Seaton, Proc. Am. Acad. xxviii. 120 (1893), from Mt. Orizaba and Esperanza, Mexico = VERBESINA **Seatoni** Blake, n. nom. (not *V. stricta* (Hemsl.) Gray, l. c. xix. 13 (1883)). Closely related to *V. hypomalaca* Rob. & Greenm. and to *V. stricta* (Hemsl.) Gray; distinguished from the latter by the not scabrous under leaf-surface, and from the former by the shorter broader crenate-dentate leaves not cordate-clasping at the base.

Encelia subaristata Gray in Hemsl. l. c. 185 (1881) is a synonym of SIMSIA SUBARISTATA Gray.

Encelia suffrutescens R. E. Fries, Nova Act. Soc. Sci. Upsal. ser. 4. i. no. 1. 83. pl. 6. fig. 1–3 (1905), from northern Argentina = FLOURENSIA **suffrutescens** (R. E. Fries) Blake, n. comb. Young achene plumpish, with two upwardly pubescent awns; habit of *Flourensia*, the heads radiate but scarcely resinous.

Encelia tenuis Fernald, Proc. Am. Acad. xxxiii. 94 (1897) is a *Simsia*.

SIMSIA Pers. (dedicated to Jacob Sims, editor of Curtis' Botanical Magazine from 1784 to 1816).— Heads small or medium, radiate or discoid, the flowers yellow or purple. Involucral scales in 3 or 4 rows, subequal or distinctly seriate, lance-ovate to lance-linear. Receptacle slightly convex; pales scarious, stiff, acuminate, conduplicate about the achenes, persisting after the fall of the latter. Rays slightly bidentate, yellow or rarely purple, sometimes wanting; disk-corollas with short usually pubescent tube and cylindric throat, 5-toothed, yellow or purple, sometimes changing color with age. Style branches attenuate, hispid-villous. Disk-achenes strongly compressed, very flat, obovate or oblong, glabrous or more often appressed-pubescent, never villous, with thin unmargined edges, calvous or usually biaristate.— Annuals or sometimes perennials, with at least the lower leaves opposite, and usually paniculate heads. Type species *S. ficifolia* Pers. and *S. amplexicaulis* (Cav.) Pers., both reducible to *S. foetida*.— About 22 species of western America, from the arid southwestern part of the United States to Argentina; one species in Jamaica.

Simsia Pers. Syn. ii. 478 (1807), excl. *S.? heterophylla* which = *Iostephane heterophylla* (Cav.) Benth.

Armania Bert. in DC. Prod. v. 576 (1836).

Barrattia Gray & Engelm. Am. Journ. Sci. ser. 2. iii. 274 (Mar. 1847).

KEY TO THE SPECIES OF SIMSIA.

A. Rays purple . . . . 22. *S. sanguinea.*
A. Rays yellow or wanting.
B. Leaves silky beneath.
C. Achenes glabrous . . . . 19. *S. Ghiesbreghtii.*
C. Achenes appressed-pubescent . . . . 20. *S. sericea.*
B. Leaves densely canescent-tomentose beneath . . . . 8. *S. Sodiroi.*
9. *S. pubescens.*
B. Leaves neither silky nor densely canescent-tomentose beneath.
C. Petiole-bases connate, forming foliaceous disks.
D. Involucral scales subequal . . . . 1. *S. calva.*
D. Scales 3–4-seriate.
E. Leaves hastately lobed; stems glandular-setose . 2. *S. setosa.*
E. Leaves unlobed; stems merely glandular . . . . 3. *S. tenuis.*
C. Petiole-bases not connate into foliaceous disks.
D. Involucral scales 3–4-seriate, the outer ovate.
E. Achenes glabrous and awnless.
F. Petioles not auriculate . . . . 4. *S. exaristata.*
5. *S. submollicoma.*
F. Petioles auriculate . . . . 6. *S. eurylepis.*
E. Achenes pubescent, 2-awned.
F. Pales and scales purple-tipped; achene 5 mm. long.
7. *S. lagascaeformis.*
F. Pales scarcely purple-tipped; achenes 6–7 mm. long.
10. *S. Chaseae.*
D. Scales subequal, or the outer if shorter lance-oblong to linear-lanceolate.
E. Leaves 3-lobed, the lobes narrow, sessile by a margined base; achene 5–6 mm. long . . . . 21. *S. triloba.*
E. Leaves unlobed, or the lobes if present broad, and the achene 4–4.5 mm. long.
F. Achenes 6–7.3 mm. long.
G. Leaf blade broadly deltoid, abruptly narrowed to a broadly margined clasping base; Mexican.
18. *S. megacephala.*
G. Leaves not clasping, grayish beneath with tuberculate-based hairs; South American . . 14. *S. hirsuta.*
15. *S. Dombeyana.*
G. Leaves not clasping, densely glandular-dotted both sides; Jamaican . . . . 13. *S. jamaicensis.*
F. Achenes 4–5.3 (–6) mm. long.
G. Whole plant densely covered with stalked glands.
12. *S. adenophora.*
G. Glands less prominent.
H. Involucral scales herbaceous, scarcely striate; leaves mostly lobed, generally auriculate-clasping; Mexican & Guatemalan . . . . 11. *S. foetida.*
H. Scales corky-ribbed at base; heads large, disk 2–3 cm. in diameter; Central American.
16. *S. grandiflora.*
H. Scales not corky-ribbed; heads smaller, disk 1.2–1.7 cm. in diameter; Central American.
17. *S. polycephala.*

Synopsis of Species.

* Petiole-bases connate into conspicuous foliaceous stipule-like appendages; tuberous-rooted perennials, at least the first species.

+– Involucral scales subequal; heads solitary, long-pedunculate, terminating the branches.

1. S. calva (Gray & Engelm.) Gray. Erect, much branched, from a thick woody tuberlike root, harsh with white bristle-like hairs with swollen bases intermixed with a fine puberulence, the stem and branches striate-grooved; leaves opposite usually to tips of the branches, very harsh with a double pubescence like that of stem, lance-deltoid in outline, acute at apex, somewhat cordate at base, crenate-dentate, unlobed or hastately eared or deeply trilobed, 3–5 cm. long, 1.5–4 cm. wide, on short margined petioles 7–12 mm. long, their bases united into broad entire or lobed foliaceous disks; heads solitary on long naked peduncles terminating the branches, the disk 1–2.2 cm. wide; involucre 7–9 mm. high, the scales linear-lanceolate to linear-oblong, in about 3 rows, subequal or the outer series slightly shorter, densely hispid and covered with a fine puberulence, somewhat green-nerved; rays 20–30, yellow, somewhat livid without, oval, 8 mm. long, pubescent on tube and nerves of back; disk-corollas yellow, becoming purplish upwardly, puberulent, 6.5 mm. long, the tube very short; pales 8 mm. long, puberulent on back and sub-herbaceous tip; achene glabrous, emarginate, awnless, mottled with black and gray, 4.5 mm. long, 2.5 mm. broad.

Barrattia calva Gray & Engelm. Am. Journ. Sci. ser. 2. iii. 275 (1847).

Simsia calva Gray, Pl. Lindh. ii. 228 (1850).

Encelia (*Barrattia*) *calva* Gray, Proc. Am. Acad. viii. 658 (1873).

Specimens examined: Texas: rocky bluffs, Baird, Apr. 1882, *Reverchon* (N); rocky terraces of limestone hills, under stunted live oaks, Comanche Spring, June 1849, *Lindheimer* 60 (G); Comanche Spring, June 1849, *Lindheimer* 900 (FGN); Brazos, *Sutton Hayes* 47 part (F); dry hills, local, Austin, 22 Oct. 1891, *J. E. Bodin* 192 (N); rocky soil, in open woods between the headwaters of the Guadalupe and Pedernales Rivers, Oct. 1845, *Lindheimer* 432 (G, type); top of dry hills, Pedernales, 1847, *Lindheimer* 39 (G); summit of rocky hills, Upper Guadalupe, 1846, *Lindheimer* 142 (G) [?= *Lindheimer* III 433: GN]; San Antonio, 28 July 1882, *Letterman* 5 (N); common in woods, San Antonio, 18 Sept. 1901, *Bush* 837 (N); Spring Creek, Gillespie Co., *G. Jermy* (F); Kerrville, alt. 487–610 m., June 1894, *Heller* 1860 (N); Bexar Co., *Jermy* (N); foot of limestone hill, near Bracken, Bexar Co., 28 July 1903, *B. H. A. Groth* 143 (FG); rocky bluffs, Crockett Co., May 1885, *Reverchon* (FN); rocky bank, Devils River, 22 July 1900, *Earle* 439 (N); Pinto, 6 Sept. 1900, *Earle* 464 (N); Roma, 1889, *G. C. Neally* (N); southwestern Texas, 1879–80, *Palmer* 610 (GN), 611 (GN), 613 part (G); without definite locality, 1849, *Wright* 330 part (GN); "cultivated," *Hall* (N); *Mexican Boundary Survey under Emory*, 560 in part (N). NUEVA LEON: Sierra Madre, above Monterey, alt. 915 m., 17 Aug. 1903, *Pringle* 8739 (FGN); without definite locality, 1880, *Palmer* 615 part (N); TAMAULIPAS: Matamoros, April 1831, *Lindheimer* 2290 = 870 (G). Mexico, without definite locality, 1848–9, *Gregg* 85 (G).

1β. S. CALVA (Gray & Engelm.) Gray var. **subaristata** (Gray) Blake, n. comb. Similar in all respects to the last, except that the achenes are appressed-pubescent (sometimes only on the margin above) and usually provided with a pair of upwardly hispidulous awns, which may be reduced to mere traces.

Simsia subaristata Gray, Pl. Fendl. 84 (1849).

Encelia subaristata Gray in Hemsl. Biol. Centr.-Am. Bot. ii. 185 (1881).

Specimens examined: TEXAS: Brazos, *Sutton Hayes* 447 part (G); dry hills, Austin, 20 May 1872, *Hall* 339 (GN); Guadalupe, 20 miles W. of San Antonio, 1880, *Palmer* 612 (GN); Eagle Pass, San Antonio, &c., 1882, *Havard* 57 (G); San Angelo, 18–19 May 1899, *W. L. Bray* 356 (N); without definite locality, 1849, *Wright* 330 part (G); southwestern Texas, 1880, *Palmer* 613 part (N); *Mex. Bound. Surv. under Emory*, 560 part (N). TAMAULIPAS: Matamoros, Apr. 1836, *Lindheimer* 3010 = 1510 part (G); San Fernando to Jimeney, 26–27 Feb. 1902, *Nelson* 6603 (GN); NUEVA LEON: Bishops Hill near Monterey, 6 Feb. 1847, *Gregg* 47 (G, TYPE); Pico Chico, near Monterey, 18 Mar. 1900, *Canby, Sargent, & Trelease* 135 (N); COAHUILA: Caracol Mts., S. E. of Manclova, 1880, *Palmer* 615 part (G).— Often occurs mixed with *S. calva* on herbarium sheets.

+ + Scales 3–4-seriate, the outer much shorter; heads paniculate.

++ Leaves hastately lobed; stems bristly-hairy as well as glandular: heads 1.5 cm. high, 1 cm. broad.

2. S. **setosa** Blake, n. sp., herbacea (inferiore caulis parte ignota) ubique dense glandulosa et parce setosa pilis albis patentibus; foliis ovato-lanceolatis basi sinu lato leviter cordatis et hastato-lobatis dentato-crenatis maximis 9 cm. longis 6 cm. latis; petiolis anguste marginatis 3.5–4 cm. longis basi in orbes dentatas latas conjunctis,

summis alternis denique minimis linearibus; capitulis paniculatis; disco 11–15 mm. alto 8–12 mm. diametro; involucri squamis 4-seriatis exterioribus sensim brevioribus striatis margine et apice minute glanduloso-strigosis lanceolatis interioribus lanceolato-attenuatis disco subaequalibus; radiis ca. 12 oblongo-ovalibus flavis 6.5 mm. longis; disci flosculis 8 mm. longis (tubulo 1.5 mm.) puberulo-glandulosis flavidis denique purpurascentibus; paleis 11.5 mm. longis lateraliter sublaceratis dorso et apice subherbaceo acuminato glanduloso-pubescentibus; acheniis 5 mm. longis 2.4 mm. latis appresse pubescentibus interdum subglabris supra ciliatis calvis vel cum aristis 2 tenuibus sursum ciliatis ad 2.2 mm. longis.

Specimens examined: SONORA: Alamos, 16–30 Sept. 1890, *Palmer* 741 (COTYPES in Gray Herb. and Nat. Herb., no. 46131).— Distributed as *Encelia mexicana*, from which it is very distinct.

++ ++ Leaves unlobed; stems merely glandular; heads 9–10 mm. high, 5–7 mm. wide.

3. S. **tenuis** (Fernald) Blake, n. comb. Base and lowest leaves unknown; stem finely and thickly glandular, only the ultimate branches of the inflorescence somewhat setose; leaves deltoid-ovate, cordate with broad shallow sinus, acute, crenate, glandular-puberulent and somewhat strigose, on glandular and setose barely margined petioles 1–2.5 cm. long, with their bases united into rounded crenate foliaceous disks; inflorescence much branched, the branches slender, the very slender peduncles 2–10 cm. long; heads short-cylindric; scales 3–4-seriate, distinctly graduated, the outer ovate, the inner lance-acuminate, glandular-puberulent, ciliate above, striate-veined; rays about 8, yellow, oval, 6 mm. long, barely bidentate, hairy on tube and slightly pubescent on back; disk-corollas 5.8 mm. long, glandular-puberulent on tube and pubescent above, yellow becoming purplish; pales 7–8 mm. long, glandular-puberulent; achenes dark-mottled, appressed-pubescent, awnless, 5 mm. long, 3 mm. wide.

Encelia tenuis Fernald, Proc. Am. Acad. xxxiii. 94 (1897).

Specimens examined: GUERRERO; rather scarce on edge of a cornfield, Nov. 1894, *Palmer* 96 (FGN, TYPE COLLECTION).

* * Petioles sometimes auriculate, not connate at base into disks; annuals or perennials.

+ Rays yellow.

++ Involucral scales distinctly unequal, the outer ovate, 3–4-seriate.

= Achenes glabrous and awnless.

× Petioles not auriculate.

4. S. EXARISTATA Gray. Annual, erect, much branched above, the stem and branches striate, covered with a fine glandular pubescence

intermixed with long hairs; leaves mostly opposite, ovate, acute, cuneate-truncate to cordate at base, subentire, crenate, or sometimes rather sharply toothed, tuberculate-strigillose with some longer hairs intermixed, the blade 5–11 cm. long, 2.5–8.5 cm. wide, on nearly naked petioles 1–7 cm. long, glandular-puberulent and fringed with long white hairs; heads numerous, paniculate, the bracts linear-lanceolate or broader; disk 1–1.3 cm. high, about as broad; involucre nearly or quite equaling the disk, about 3-seried, the inner scales lance-acuminate, the outer ovate or ovate-oblong and half their length, all striate, glandular on the back, ciliate particularly toward the tip, purple-tipped, the inner mostly purple; rays about 8, oval-oblong, under 8 mm. long, subentire, yellow, apparently sometimes wanting; disk-corollas 7 mm. long, glandular-puberulent particularly on tube and teeth, yellow turning purplish with age; pales 9–10 mm. long, narrow, glandular-puberulent and ciliate on the back and tip; achene blackish, glabrous and awnless, nearly truncate at apex, 4.5 mm. long, 2.3–2.8 mm. broad.

Simsia exaristata Gray, Pl. Wright. ii. 87 (1853).

Encelia exaristata Gray in Hemsl. Biol. Centr.-Am. Bot. ii. 183 (1881).

Specimens examined: TEXAS: valleys in the mountains east of El Paso, 1849, *Wright* 331 (GN); ARIZONA: Mexican Boundary Line, south of Bisbee, 14 Sept. 1892, *Mearns* 873 (N); southern part, 1881, *Lemmon* 575 (G); NEW MEXICO: southern part, *Mex. Bound. Surv. under Emory*, 561 (N). SONORA: valley of a tributary of the San Pedro, Sept. 1851, *Wright* 1224 (GN, TYPE COLLECTION); sandy places, Sept. 1851, *Thurber* 953 (G); CHIHUAHUA: valley near Chihuahua, 19 Sept. 1885, *Pringle* 321 (FGN); VERA CRUZ: Orizaba, *Botteri* 804 (G).

5. S. **submollicoma** Blake, n. sp., herbacea caule ramisque striatis puberulo-pilosis ad nodos canescentibus; foliis fere omnibus oppositis ovato-deltoideis cordatis sinu lato breveque vel superioribus truncatis crenato-dentatis paullulo hastato-lobatis acutis infra dense submolliterque pubescentibus pilis brevibus patentibus in venis longioribus supra subcanescentibus pube subsimile autem appressa asperaque 4–6 cm. longis 3–5.5 cm. latis, petiolis immarginatis puberulo-pilosis 1–3 cm. longis; capitulis subcorymbosis ramos terminantibus juventate ut gemmis dense pilosis maturitate 10–14 mm. alto 9–11 mm. diametro; involucro 1 cm. altitudine squamis acutis purpurascentibus triseriatis puberulis subdense pilosisque interioribus oblongis exterioribus ovatis duplo brevioribus: radiis (an semper) 0; corollis disci

6.5 mm. longis (tubulo 2 mm.) infra glanduloso-puberulis supra glanduloso-pilosis flavis denique obscurantibus; paleis latis 9 mm. longis appresse pubescentibus dorso et apice ciliatis; acheniis atris truncatis glabris calvis 4.5 mm. longis 2.8 mm. latis.

Specimens examined: TAMAULIPAS: weed in waste places, growing in clusters, vicinity of Tampico, alt. 15 m., 10 Mar.–19 Apr. 1910, *Palmer* 250 (TYPE in Gray Herb.).

× × Petioles auriculate.

6. S. **eurylepis** Blake, n. sp., herbacea foliis inferioribus et basi invisis; caule striato sparse puberulo et pilis longis mollibus nonnullis; foliis remotis maxima ex parte alternis ovatis acutis basi subtruncatis subintegris vel infra dentatis utrimque pubescentibus pilis brevibus patentibus basi tuberculatis venas secundum subhispidis, petiolo vix marginato puberulo-glanduloso ad basin latas in auriculas integras ampliato; capitulis plerumque glomeratis in pedunculis longis nudis terminalibus, eradiatis 1.2–1.4 cm. altitudine 1–1.5 cm. diametro; involucro discum subaequante 3–4-seriato squamis sensim gradatis interioribus oblongis 7–9.5 mm. longis 2–2.5 mm. latis extimis ovalibus 4.5 mm. longis 2.5 mm. latis acutis striatis puberulo-glandulosis et subpilosis supra purpureo-brunneis; corollis disci flavis denique purpurascentibus 8 mm. longis (tubulo 2.2 mm.) infra et in dentibus puberulis; paleis 8.5 mm. longis apice pilosis a latere subdentatis; acheniis nigricantibus emarginatis calvis glabris 5.5 mm. longis 3.4 mm. latis.

Specimens examined: SAN LUIS POTOSI: district Cuidad del Maiz near Gallinas, Feb. 1888, *C. & E. Seler* 684 (TYPE in Gray Herb.).

= = Achenes pubescent and biaristate.

× Tips of pales and inner scales purple; leaves not tomentose beneath; Mexican.

7. S. LAGASCAEFORMIS DC. Annual, erect or rarely somewhat spreading, usually much branched; stems and branches striate, glandular-pubescent and pilose particularly near the nodes; leaves opposite below, alternate in the inflorescence, broadly deltoid-ovate, unlobed or rarely hastately 3-lobed, the margin crenate or subentire, acute, the base truncate or cordate with broad sinus, beneath granular-puberulent, pilose along the veins, or when young almost tomentose, above strigillose, the hairs tuberculate-based, intermixed with longer looser ones, the blades 3–12 cm. long, 2–13.5 cm. wide, on purplish naked glandular-puberulent pilose-fringed petioles 1.5–5 cm. long; heads numerous, cymose-panicled, the branches somewhat pilose and stipitate-glandular; heads cylindric becoming hemispheric, 10–12 mm. high, 5–8 mm.

wide; involucre nearly equaling the disk, its scales 3-ranked, striate, purple-tipped, glandular-puberulent, ciliate, the outer ovate, the inner oblong-lanceolate, all acute; rays 5–8, small, oval to oval-oblong, 5 mm. long, yellow; disk-corollas 5.5–6 mm. long, glandular-puberulent, yellow becoming purplish; pales truncate or retuse and mucronate, ciliate on back and tip, glandular, purplish above, striate, 6–8 mm. long; achene appressed-pubescent, mottled with brownish-gray and black, bearing 2 slender upwardly pubescent fimbriate-based awns, 5 mm. long, 2.5 mm. wide.— A photograph of the type of *Simsia lagascaeformis*, kindly sent me by M. C. de Candolle, who also writes that the achenes are hairy and biaristate (not glabrous as originally described), proves that this long-misunderstood species is identical with *E. pilosa* Greenm., which in all technical characters is the same as *E. purpurea* Rose. The latter species, known only from two plants collected by Palmer in 1891, and represented by sections in the Gray and National herbaria, seems to be merely a peculiarly branched and perhaps somewhat teratological condition, with very numerous capitula and somewhat flattened branches.

Simsia lagascaeformis DC. Prod. v. 577 (1836).

Encelia (*Simsia*) *purpurea* Rose, Contr. U. S. Nat. Herb. i. 336 (1895).

Encelia pilosa Greenm. Proc. Am. Acad. xxxix. 111 (1903).

Specimens examined: San Luis Potosi: alt. 1830–2440 m., 1878, *Parry & Palmer* 472 part (G); Colima: in a creek bottom, Colima, 9 Jan.–6 Feb. 1891, *Palmer* 1105 (GN, type collection of *E. purpurea*); Puebla: maize fields, Rio de San Francisco, Aug. 1909, *Purpus* 3826 (FGN); Tehuacan, 7 Nov. 1903, *Holway* 5340 (G); Oaxaca: Las Sedas, alt. 1830 m., Sept. 1894, *C. L. Smith* 277 (N); between Coixtlahuaca and Tamazulapam, alt. 2000–2500 m., 12 Nov. 1894, *Nelson* 1937 (GN); valley of Etla, alt. 1700 m., 23 Oct. 1895, *L. C. Smith* 854 (G); Ocotlan, Dec. 1901, *Conzatti & González* 1263 (G); without definite locality, alt. 1750 m., July–Aug. 1900, *Conzatti & González* 1002 (G); 25 Oct. 1899, *Holway* 3740 (G); 17 Oct., 1899, *Holway* 3747 (G).

× × Pales and scales not purple-tipped, or else leaves densely canescent-tomentose beneath; plants of Yucatan, Columbia, and Ecuador.

o Leaves canescent-tomentose beneath; scales pilose.

8. S. **Sodiroi** (Hieron.) Blake, n. comb. Said to be suffrutescent and 2 m. high; stem and branches striate, short-pubescent and somewhat glandular; leaves all but the uppermost opposite, ovate-lanceolate, acute, truncate or subcordate at base, roughish with appressed

hairs above, canescent-tomentose beneath, crenate-serrate, 4–7.5 cm. long, 3–6 cm. wide, on puberulent-pilose wingless earless petioles 1–1.5 cm. long; heads rather closely corymbed at tips of branches, the short peduncles densely pubescent; heads 1.2 cm. high; involucre triseriate, the outer scales ovate, densely pilose, half the length of the oblong less pilose inner ones, all acute and striate; rays few, yellow, oblong, 8 mm. long;[39] disk-corollas 7 mm. long, pilose on tube and teeth, yellowish becoming darker; pales 9 mm. long, laterally somewhat toothed, pilose on back and tip; achenes 4.3 mm. long, 1.9 mm. wide, blackish, more or less appressed-pubescent on sides and tip, bearing 2 slender slightly ciliate awns.

Encelia mexicana Klatt, Engl. Bot. Jahrb. viii. 43 (1887), not Mart.

Encelia fruticulosa Hieron. l. c. xix. 54 (1894), not *Hopkirkia fruticulosa* Spreng. Sys. iii. 444 (1826), fide Hieron. Engl. Bot. Jahrb. xxix. 43 (1900).

Encelia Sodiroi Hieron. Engl. Bot. Jahrb. xxix. 43 (1900).

Specimen examined: COLUMBIA: open savannas of the Rio Dagua, Cauca, alt. 800 m., 15 July 1883, *Lehmann* 2964 (G). Reported also by Hieronymus from Ecuador, along the Guallabamba R. (type locality).

9. S. PUBESCENS Triana. "Suffrutex erectus; ramis gracilibus, multi-angulatis, minute puberulo-hirtis; foliis inferioribus oppositis, superioribus alternis ovato-lanceolatis acutis dentato-serratis, supra pubescentibus, subtus dense pubescenti-canescentibus, deorsum subabrupte in petiolum longum attenuatis, petiolo basi auriculato-amplexicauli; involucri squamis striatis, dorso tenuiter pubescentibus, exterioribus ovatis, interioribus oblongo-lanceolatis, utrisque acutis; capitulis pedunculatis, laxe corymbosis; acheniis atris, alatis, oblongis, undique decumbenti-pilosis.

"Crescit altitudine 1400 metr. inter *Tena* et *El Colegio*, in devexis occidentalibus Andium Bogotensium."

Simsia pubescens Triana, Ann. Sci. Nat. Bot. sér. 4. ix. 40 (1858).

This species, not since recognized, appears to differ from *S. Sodiroi* only in its auriculate-amplexicaul petioles and perhaps in the less pubescent scales of the involucre, and may be identical with that species, but in the absence of specimens it seems unwise to combine them.

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[39] Described by Hieronymus as stylose, but neutral in the Lehmann plant which he refers to *E. Sodiroi*.

o o Leaves merely puberulent beneath except along the veins; scales glandular-hispid.

10. S. **Chaseae** (Millsp.) Blake, n. comb. Herbaceous, the base unknown; stem and branches striate, glandular-hispid particularly in the inflorescence; only the lower leaves opposite, thin, ovate-deltoid, acute, broadly wedge-shaped at base, crenate-dentate with blunt teeth, above granular-scabrous, beneath rather softly puberulent and somewhat pilose along the veins, 5–6 cm. long, 3–5.5 cm. wide, narrowed into margined glandular-setose petioles 1.5 cm. or less long, the upper sessile and oblong-lanceolate; heads corymbose-paniculate, 1–1.2 cm. high; scales somewhat triseriate, the outer ovate-lanceolate, glandular-hispid, shorter than the lance-oblong inner ones which are glandular on back and ciliate toward tip; rays sometimes wanting, when present 8–10, yellow, elliptic, 4–7 mm. long; disk-corollas 5.5 mm. long, pubescent, yellow; pales 8–9 mm. long, broad, the margin denticulate, green-ribbed, hispidulous on the keel; achenes 6–7 mm. long, 3.5–4 mm. broad, appressed-pubescent and short-ciliate, bearing two upwardly ciliate awns about half their length.

Encelia Chaseae Millsp. in Millsp. & Chase, Field Col. Mus. Pub. Bot. iii. 125, pl. (1904).

Specimens examined: YUCATAN: ruins of Kobah, 26 Nov. 1865, *Schott* 911 (TYPE no. 176020, Field Mus.); "herb, 5 feet high, common at Izamal, Oct.," *Gaumer* 910 (FG); Chichankanab, *Gaumer* 2045 (F); San Anselmo, *Gaumer* 2046 (F).

++ ++ Involucral scales subequal, or the outer if shorter linear-lanceolate to lance-oblong.

= Leaves ovate or ovate-deltoid, unlobed except in *S. foetida*, the lobes when present broad.

× Leaves not silky-pubescent beneath.

o Involucral scales herbaceous throughout, scarcely striate; the usually lobed leaves generally broadly auriculate-clasping at base of petiole; achenes small, 4–4.5 mm. long.

11. S. **foetida** (Cav.) Blake, n. comb. Annual, erect, often much branched, the stem usually purplish, glandular-puberulent and hispid with tuberculate-based hairs; lower leaves opposite, the upper often alternate, ovate or deltoid, often 3-lobed, particularly the upper, crenate-dentate, acute at apex, broadly cuneate or cordate at base, hispid with tuberculate-based hairs longer along the veins, the blade 5–14 cm. long, 3–12 cm. wide, the petioles usually margined, often broadly so, and generally auriculate-clasping at base; heads numerous, panicled, the peduncles glandular-hispid; heads 1 cm. high, radiate;

involucre equaling disk, the scales subequal, about 2-rowed, lance-acuminate, glandular-hispid, herbaceous nearly throughout, slightly or not at all striate, the inner sometimes purplish; rays about 10, yellow, oval, pubescent on tube, faintly bidentate, 10 mm. long; disk-corollas 6 mm. long (tube less than 1 mm.), puberulent, yellow turning purple; pales 9 mm. long, purplish and glandular above; achenes black or mottled, appressed-pubescent, 4–4.5 mm. long, 2–2.4 mm. wide, the 2 upwardly pubescent awns fimbriate at base.— Very variable in leaf-form, even on the same plant, but not satisfactorily divisible even into formae.

Coreopsis foetida Cav. Icon. i. 55. t. 77 (1791).
Ximenesia foetida Spreng. Sys. iii. 606 (1826).
Encelia foetida Hemsl. Biol. Centr.-Am. Bot. ii. 183 (1881).
Simsia ficifolia Pers. Syn. ii. 478 (1807).
Coreopsis amplexicaulis Cav. Descrip. 226 (1802).
Simsia amplexicaulis Pers. l. c. (1807).
Encelia amplexicaulis Hemsl. l. c. (1881).
Ximenesia cordata HBK. Nov. Gen. iv. 228 (1820).
Simsia cordata Cass. Dict. lix. 137 (1829).
Encelia cordata Hemsl. l. c. (1881).
Ximenesia heterophylla HBK. l. c. 227. t. 380 (1820).
Simsia heterophylla DC. Prod. v. 577 (1836), not Pers. l. c.
Simsia Kunthiana Cass. l. c. (1829).
Encelia heterophylla Hemsl. l. c. 184 (1881).
Simsia auriculata DC. l. c. v. 577 (1836).
Encelia mexicana Mart. in DC. l. c. 578 (1836), as syn.; Gray, Proc. Am. Acad. xix. 8 (1883).
Ximenesia hirta Mart. l. c. 578 (1836), as syn.
Helianthus amplexicaulis DC. l. c. 589 (1836).
Simsia Schaffneri Sch. Bip. in Gray, Proc. Am. Acad. xix. 8 (1883).

Specimens examined: CHIHUAHUA: southwestern part, 1885, *Palmer* 440 part (N); COAHUILA: Saltillo, Sept. 1898, *Palmer* 422 (FG); Parras, 16 May 1847, *Gregg* (G); Parras, Oct. 1898, *Palmer* 427 (FGN); without definite locality, 1880, *Palmer* 493 (GN); DURANGO: Durango, Sept. 1896, *Palmer* 657 (FGN); SAN LUIS POTOSI: Villa de Guadalupe, Sept. 1855, *Schaffner* 19 (G); sandy places near San Luis Potosi, Aug. 1876, *Schaffner* 265 (G); woods about San Luis Potosi, 1876, *Schaffner* 389b (G)[40]; sandy cultivated grounds, 1880, *Schaffner* 389a (G)[40]; alt. 1830–3050 m., 1878, *Parry & Palmer*

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[40] "'Lamyrote' incolarum."

471 (GFN); alt. 1830–2440 m., 1878, *Parry & Palmer* 472 part (G); fields, Cardenas, 3 Nov. 1891, *Pringle* 5090 (G); JALISCO: Bolaños, Sept. 1897, *Rose* 2875 (G); Guadalajara, Oct. 1886, *Palmer* 622 (GN); fields near Guadalajara, 1 Nov. 1893, *Pringle* 4622 (FGN); GUANAJUATO: Leon, *Mendez* (G, fragments of type of *Helianthus amplexicaulis* DC.); 1895, *Dugès* 455 (G)[41]; QUERETARO: alt. 1800 m., 12 Dec. 1898, *Deam* (F); HIDALGO: fields near Dublan, 2074 m., 19 Sept. 1902, *Pringle* 9897 (FGN); fields, Pachuca, Sept. 1905, *Purpus* 1542 (FGN); VERA CRUZ: La Luguna, 22 Jan. 1906, *Greenman* 33 (F); Orizaba, *Botteri* 805, 808 (G); on a volcanic mountain, near Tantoyca, 1858, *L. C. Ervendberg* 378 (G); near Orizaba, *Botteri & Sumichrast* 122 (G); MICHOACAN: Patzcuaro, 29 Oct. 1895, *C. & E. Seler* 1185 (G); MEXICO: fields, Salto de Aqua, Oct. 1905, *Purpus* 1541 (FGN); Ixtaccihautl, Jan. 1903, *Purpus* (N); near Mexico City, *Berlandier* 927 (G, type collection of *S. auriculata* DC.); Valley of Mexico, 1848–9, *Schmitz* (G); fields, Valley of Mexico, Sept.–Oct. 1865–6, *Bourgeau* 850 (GN); Valley of Mexico, Aug. 1856, *Schaffner* 162 (G, type of *S. Schaffneri* Sch. Bip.); near San Angel, *Schaffner* (G); Tacubaya, 20 June 1865–6, *Bourgeau* 155 (G); plaza, Chalchicomula, 27 July 1901, *Rose & Hay* 5808 part (G); PUEBLA: Mt. Orizaba, alt. 2440 m., 14 Aug. 1891, *Seaton* 329 (FGN); Cholula, 1 Jan. 1899, *Deam* 73 (FG); without definite locality, alt. 2000 m., Nov. 1895, *Conzatti* 131 (G); OAXACA: near Puebla, alt. 2135 m., 8 Nov. 1895, *L. C. Smith* 909 (G); CHIAPAS: plains, 1864–70, *Ghiesbreght* 540 (G). Mexico, without locality, *Hartweg* 145 (G); *Coulter* 359 (G); "Mexico Commu," 21 Sept. 1865–6, *Bourgeau* 959 (G); 1905, *Lemmon* (G). GUATEMALA: Dept. Quiché, alt. 1372–3660 m., Apr. 1892, *Heyde & Lux* 3396 (FG); Laguna Amatitlan, Dept. Amatitlan, alt. 1250 m., Feb. 1890, *Heyde & Lux* 2408 (F).

11β. S. FOETIDA (Cav.) Blake var. **decipiens** Blake, n. var., pappo nullo, achenio glabro. As the typical form, but achene glabrous and pappusless; leaves mostly entire, but at least the upper auriculate-petioled.— It is barely possible that the specimen is a hybrid with *S. exaristata,* with which species it was identified by Dr. Gray; but the heads and leaf-bases are quite those of *S. foetida.*

Specimen examined: CHIHUAHUA: southwestern part, Aug.–Nov. 1885, *Palmer* 440 part (TYPE in Gray Herb.).

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[41] "Vulg. Mirasol amarillo ou Lampotillo" (i. e. yellow turnsol or little fire).

o o Scales somewhat corky basally, generally strongly striate; leaves very rarely lobed; achenes usually larger, 6–7 mm. long (except in nos. 12 and 17).

+ Whole plant densely covered with stalked glands; petioles never auriculate; achene 5.3 mm. long or less.

12. S. **adenophora** (Greenm.) Blake, n. comb. Erect annual, branched above, 1–2.5 m. high, setose and yellowish with dense stalked glands; lower leaves opposite, deltoid-ovate, rarely 3-lobed, acute, truncate or shallowly cordate at base, crenate-dentate, 6.5–12 cm. long, 5–13 cm. wide, setose and glandular both sides, on marginless densely glandular petioles 1–8 cm. long, never auriculate, the upper lance-acuminate, gradually reduced to linear-lanceolate bracts; heads panicled, rather numerous, 10–17 mm. high; involucre slightly exceeding disk, 3-rowed, the scales densely glandular-hispid, subequal, lanceolate, striate and somewhat corky-ribbed at base, the narrow herbaceous tips loosely spreading; rays when present yellow, about 12, 7–8 mm. long, 3.5 mm. wide; disk-corollas pubescent, 6 mm. long, yellow turning purplish above, the tube only 0.6 mm. long; pales 9 mm. long, laterally dentate, appressed-pubescent above; achenes blackish, 5–5.3 mm. long, 2.9 mm. wide, appressed-pubescent and slightly ciliate, with 2 finely and upwardly pubescent awns 3–4 mm. long.

Encelia adenophora Greenm. Proc. Am. Acad. xxxix. 109 (1903).

Specimens examined: JALISCO: fields and copses, Tequila, Sept.–Oct. 1893, *Pringle* 4602 (FGN, TYPE COLLECTION); Etzatlan, 2 Oct. 1903, *Holway* 5092 (G); MORELOS: limestone hills near Yautepec, alt. 1372 m., 21 Oct. 1902, *Pringle* 9898 (FGN); GUERRERO: between Tlapa and Tlaliscatilla, alt. 1190–1372 m., 5 Dec. 1894, *Nelson* 2045 (GN); OAXACA: Monte Alban, near Oaxaca City, alt. 1677–1830 m., 8 Oct. 1894, *C. L. Smith* 236 (N); hills of Soledad de Etla, alt. 1830 m., 19 Nov. 1895, *L. C. Smith* 894 (G); Hacienda Guadalupe, alt. 1600 m., 7 Oct. 1906, *Conzatti* 1529 (F); without definite locality, 10 Nov. 1903, *Holway* 5360 (G).

+ + Usually less glandular, the glands mostly sessile; petioles sometimes auriculate; achene usually 6–7 mm. long.

— Lower leaves (at least) not with broadly margined clasping bases; the upper sometimes cordate-clasping.

A. Achenes 6–7.5 mm. long, or if smaller, the bracts strongly ribbed.

B. Leaves rather densely glandular-dotted both sides; mature heads 1.5 cm. in diameter; achene 7 mm. long; Jamaican.

13. S. **jamaicensis** Blake, n. sp., herbacea erecta ramosa 1.3–2.5 m. alta foliis infimis et radice invisis; caule ramisque multistriatis hirsutis dense glandulosis praecipue in inflorescentia ubi glandulae pedatae sunt; foliis inferioribus late ovatis crenato-dentatis acutis basi cuneato-

truncatis 8–15 cm. longis 5.5–15 cm. latis utrobique pilis basi glandulosis tectis venas paginae inferioris secundum et supra ubique hirsutis, glandulis inferiore in superficie conspicuoribus quam eis specierum affinium, petiolis immarginatis 2–4.5 cm. longis infra glandulari-hirsutis supra complanatis pilosis saepe basi subauriculatis; foliis summis sensim lanceolatas ad bracteas sessiles reductis; capitulis non paucis cymoso-paniculatis pedunculis 2–6 cm. longis dense glandulosis et patenti-hirsutis; disco 15–17 mm. alto maturitate 1.5 cm. aetate 18–24 mm. diametro; involucri biseriati squamis subaequalibus vel interioribus paullo longioribus oblongo-lanceolatis subobtusis striatis glandulosis et strigoso-hirsutis; radiis vel nullis vel ca. 10 ovalibus parvis flavis; corollis disci 6.5 mm. longis (tubulo 2 mm.) flavis puberulo-glandulosis; paleis 9–12 mm. longis ad apicem glandulosis viridi-carinatis cuspidatis; acheniis 6–7.5 mm. longis 2.4–3.5 mm. latis nigricanti-maculosis appresse aliquid pubescentibus aristis 2 subaequalibus 4–4.5 mm. longis.

Specimens examined: JAMAICA: vicinity of Kingston, alt. 152 m., 29 Jan.–4 Feb. 1900, *Clute* 2 (COTYPES in Gray Herb. and Field Mus. no. 83001: distributed as "*Verbesina gigantea* Jacq.?"); Kings House Grounds, alt. 183 m., 17 Nov. 1897, *Harris* 6953 (F); Hope Grounds, alt. 198 m., 4 Dec. 1901, *Harris* 8228 (F); Hope Road, 13 Jan. 1898, *Harris* 6989 (F); Long Mountain Road, alt. 91 m., 19 Nov. 1907, *Harris* 10001 (FN).

B B. Leaves grayish with short tuberculate-based hairs especially beneath; mature heads 1.5–2 cm. broad; achene 7 mm. long; South American.

14. S. **hirsuta** (Ktze.) Blake. n. comb. Erect branched annual 1–1.5 mm. high, the stem and branches substriate, glandular-puberulent and setose; lower leaves ovate, acute, subcordate or cuneate-truncate at base, repand-dentate to subentire, setose along the veins, roughish both sides with short white hairs with swollen bases, some gland-like particularly on the upper surface, 8 cm. long, 5 cm. wide, on puberulent-setose marginless petioles 1 cm. long, the uppermost sessile and lance-oblong; heads rather few, cymose-paniculate, on glandular-puberulent and setose peduncles 2.5–14 cm. long; heads chiefly discoid; involucre 10–13 mm. high, equaling the disk, its scales subequal, 2-rowed, lanceolate-subulate, striate, glandular-pubescent and hispid; disk-corollas 5 mm. long (tube 0.6 mm.), glandular-puberulent, yellow; pales 1 cm. long, glandular-puberulent above; achene mottled, appressed-pubescent all over, oblong, obcordate, 6.5–7 mm. long, 2.7–3.6 mm. wide, the upwardly pubescent awns 2.5 mm. long.

Encelia hirsuta Ktze. Rev. Gen. iii. pt. 2. 145 (1898).

Encelia hirsuta Ktze. f. *radiata* Ktze. l. c.

Specimens examined: ARGENTINA: Dique near Cordoba, Dec. 1891, *Kuntze* (N, COTYPES of *E. hirsuta* Ktze.). Also reported by Kuntze from Sierra de Cordoba, Argentina, *Lorentz;* Peru (f. *radiata,* leg. *Dombey*); and Cartagena, Columbia, *Billberg.*

15. S. DOMBEYANA DC. "Caule terete sparse hispido et inter setas minute puberulo-glanduloso, foliis superioribus alternis petiolatis late ovatis irregulariter repando-dentatis hinc inde sublobatis acutis utrinque setis hispidulis et puberulo-glandulosis, capitulis paucis breviter pedicellatis, invol. squamis lineari-lanceolatis acuminatis disco longioribus, ligulis paucis minimis, achaeniis obcordatis biaristatis margine ciliolatis.— in Amer. austr. verisim. in Peruvia legit Dombey. Petioli valde hispidi, setis ut in tota planta longis patulis mollibus. (v. s. comm. à Mus. reg. Par.)"

Simsia Dombeyana DC. Prod. v. 578 (1836).

M. Casimir de Candolle, to whom I sent fragments of Kuntze's cotypes of *E. hirsuta* for comparison with the Prodromus type of *S. Dombeyana,* states that the latter is distinguished by its more hairy achenes with awns nearly as long as the paleae, and by the more long-triangular leaves with the long hairs thinner and the short ones nearly pulverulent.

B B B. Leaves glandular-puberulent; heads large, 2–3 cm. in diameter, the rays rather prominent; the scales broader and more corky-ribbed than in any related species; achene 5–6 mm. long; Central American.

16. S. GRANDIFLORA Benth. Erect annual, 1.6 m. high, subsimple or branched above; stem and branches setose, striate, rather sparsely glandular-puberulent; lower leaves opposite, broad-ovate, acute, truncate at base, crenate or crenate-serrate, setose along the veins, glandular-puberulous both sides, the glands more prominent on the lower surface, 7–14.5 cm. long, 4.5–10 cm. wide, on naked glandular-setose petioles 1.5–8 cm. long; the upper decurrent into winged amplexicaul bases, the uppermost sessile, subentire, lance-ovate; heads few, axillary and terminal, hemispheric even in anthesis, the disk 1.5–2 cm. high, 2–3 cm. wide, equaled or slightly surpassed by the involucre; peduncles setose and glandular, 2–12 cm. long; involucre triseriate, the inner scales slightly longer, lance-ovate to ovate-oblong, subacute, glandular-hispid, with about 4 conspicuous light thickened ribs in the lower half, the central pair most prominent, the upper part herbaceous; rays oval, about 20, yellow, 9 mm. long, 4.2 mm. wide, rather prominent, usually present; disk-corollas 5.5–6 mm. long (tube 1.3 mm.), yellow, glandular; pales 9–10 mm. long, puberulent on keel

and margin above; achene blackish or dark-mottled, ovate-oblong, subobcordate, appressed-pubescent, 5–6 mm. long, 2.5 mm. wide, bearing 2 awns 4–5.5 mm. long.

Simsia grandiflora Benth. Vidensk. Medd. Kjöbenh. for 1852. 92 (1853).

Encelia grandiflora Hemsl. Biol. Centr.-Am. Bot. ii. 184 (1881).

Specimens examined: NICARAGUA: very common on the high plateaus toward the Pueblos, Niquinohomo, Dept. of Granada, 13 Feb. 1903, *Baker* 2419 (GN); waste land near Granada, alt. 40 m., Jan. 1870, *P. Levy* 355 (Bot. Mus. Copenhagen); COSTA RICA (?): sunny pastures near Ojos de Agua,— 352 & 382 (G, ex herb. Klatt). There seems to be no place of this name in Costa Rica, and the town so called in Honduras may have been intended. Bentham's type (*Oersted* 100) came from Volcan el Viejo, Nicaragua.

A A. Achene 5 mm. long; bracts not strongly ribbed; Central American.

17. S. POLYCEPHALA Benth. Base not seen; stem and the numerous branches densely glandular and pilose-hispid; middle leaves ovate or ovate-lanceolate, acuminate, truncate or subcordate at base, crenate-dentate, glandular-hispid both sides, 4–7 cm. long, 3.5–5 cm. wide, on nearly marginless densely glandular-hispid petioles 1.5–2.5 cm. long, with slightly enlarged bases; heads numerous, cymose-paniculate, on glandular-setose peduncles 1–7 cm. long; disk 11–13 mm. high, 12–17 mm. broad; involucre nearly as tall, its scales triseriate, subequal or the outer a little shorter, glandular-hispid, oblong-lanceolate, bluntish, only slightly striate-ribbed; rays 5–7 mm. long, yellow; disk-corollas 6 mm. long (tube 1.5 mm.), yellow, glandular below, pubescent on the teeth; pales 8–10.5 mm. long, pubescent above, greenish toward the apex; achenes blackish, appressed-pubescent chiefly on margin and middle, 5 mm. long, 2.6–3.5 mm. wide, the awns 3.2 mm. long.

Simsia polycephala Benth. Vidensk. Medd. Kjöbenh. for 1852. 93 (1853).

Encelia polycephala Hemsl. Biol. Centr.-Am. Bot. ii. 184 (1881).

Specimens examined: GUATEMALA: Cerro Redondo, Dept. Santa Rosa, alt. 1372 m., Oct. 1894, *Heyde & Lux* 6160 (FGN); Chupadero, Dept. Santa Rosa, alt. 1600 m., Oct. 1892, *Heyde & Lux* 3810 (FG).

— — Even the lower leaves abruptly contracted to broadly margined clasping bases.

18. S. **megacephala** Sch. Bip. in herb., herbacea supra ramosa glandulari-pubescens et praecipue in inflorescentia piloso-

hirsuta; foliis inferioribus deltoideis vel ovato-deltoideis utrimque glandulari-hirsutis venas tres secundum praecipue, lamina 7–10 cm. longa lataque acuta infra abrupte fere truncate contracta in portionem petioliformem late marginatam media in parte 1.2–1.7 cm. basi 1.7–3.5 cm. latam, superioribus sensim ad bracteas ovatas reductis; capitulis non paucis axillaribus et terminalibus in pedunculis 4–14 cm. longis; disco 1.3–1.5 cm. alto 1.7–2.2 cm. diametro; involucro quam eo paullo breviore squamis 2–3-seriatis exterioribus paullo brevioribus acutis lanceolatis glandulari-hirsutis valde costatis; radiis ca. 20 flavis ovalibus 6.5 mm. longis; corollis disci flavis 5.5–6 mm. longis (tubulo 1 mm.) infra et in dentibus puberulis; paleis 9 mm. longis supra hirsutulis; achenio nigricante dense appresse pubescente 7.2 mm. longo 3.3 mm. lato aristis 2 sursum ciliatis basi fimbriatis 4 mm. longis.

Specimens examined: GUANAJUATO: near cultivated gardens, Cerro de Cuarto, Sept. 1903, *Dugès* 12 (G). Also the following cultivated specimens: Harvard Botanic Garden, 1866, "e sem. Hort. Par."; Botanic Garden of Deidesheim, Rhine-Palatinate, 25 Nov. 1859, *Schultz Bipontinus* (TYPE in Gray Herb.).

× × Leaves silky beneath.

○ Achenes glabrous, awnless; branches puberulent and pilose.

19. S. **Ghiesbreghtii** (Gray) Blake, n. comb. Lower part of stem unknown; branches apparently few, opposite, like the stem purplish, glandular-puberulent and pilose; leaves all opposite, ovate-lanceolate, acuminate, truncate or subcordate at base, serrate, canescent above with a short glandular pubescence, silky with dense appressed hairs beneath, 4.5 cm. long, 2–2.5 cm. wide, on densely pilose marginless petioles a centimeter long; heads few or solitary towards tips of branchlets, the disk 10–11 mm. high, 10 mm. wide, equaled by the periclinium, this triseriate, the outer scales a little shorter, all subulate-linear, glandular-puberulent and pilose-hirsute; rays about 10, pale yellow, purplish-tinged outside, oblong, 11 mm. long, 4.6 mm. wide; disk-corollas yellow, puberulent particularly on tube, 6–6.5 mm. long (tube 1.2 mm.); pales 9.5 mm. long, pubescent on the subherbaceous keel above, very acute; immature achene 3.5 mm. long, oblong, glabrous and awnless.

Encelia (*Barrattia*) *Ghiesbreghtii* Gray, Proc. Am. Acad. viii. 658 (1873).

Encelia Ghiesbreghtiana Hemsl. Biol. Centr.-Am. Bot. iv. 57 (1887), clerical error for *E. Ghiesbreghtii* Gray.

Specimen examined: CHIAPAS: mountain forests, 1864–70, *Ghiesbreght* 568 (TYPE in Gray Herb.).

o o Achenes appressed-pubescent, biaristate; branches puberulent.

20. S. **sericea** (Hemsl.) Blake, n. comb. Herbaceous?, erect, the stem and nearly opposite branches slender, terete, striate, densely glandular-puberulent; leaves mostly opposite, lance-ovate, acuminate, truncate or slightly rounded at base, remotely and obscurely serrulate, densely glandular-strigillose above, softly silvery-silky beneath, 4–9 cm. long, 1.5–3.5 cm. broad, on densely glandular short-pilose petioles about 1 cm. long; heads rather few, corymbed toward tips of branches, on peduncles 2–5.5 cm. long; disk 1 cm. high, about as broad, exceeding the involucre; the latter triseriate, the scales graduated, the inner lance-attenuate, the outer ovate-lanceolate, glandular-puberulent and hispid-ciliate; rays "about 7," 1 cm. long; disk-corollas 6.5 mm. long, puberulent, yellow; pales 8 mm. long, stiff, scarious, acuminate, glandular-pubescent on keel and tip, laterally lacerate-dentate; achene 3.2 mm. long, 1 mm. wide, densely appressed-pubescent, the awns ampliate and lacerate toward the base, 1.5 mm. long.

Encelia (*? Simsia*) *sericea* Hemsl. Biol. Centr.-Am. ii. 185 (1881).

Specimens examined: GUATEMALA: Antigua, dept. Sacatapéquez, 13 Feb. 1905, *Kellerman* 4982 (GN) (distr. as *Viguiera helianthoides* HBK.). Hemsley's type (*Salvin & Godman* 133, in Kew Herb.) came from the Motagua Valley in Guatemala.

= = Leaves three-lobed, sessile, the upper reduced to lance-linear bracts; inflorescence few-headed, the peduncles very long.

21. S. **triloba** Blake, n. sp., verisimiliter herbacea; caule ad 1 m. alto tenui supra pauciramoso, ramis longissimis 1–3-capitulatis, utrisque purpureis substriatis glandulari-pubescentibus et praecipue in inflorescentia sparse pilosis; foliis inferioribus oppositis ovatis 1 cm. supra basin trilobatis, lobo medio 2.5–4 cm. longo alteris 1 cm. sive minoribus, omnibus oblongis subdentatis vel integris, sessilibus basi subamplexicaule 1 cm. longo latitudine medium lobum aequante, glandulari-scabris et praecipue supra hirsutis pilis basi tuberculatis; foliis superioribus oblongo-lanceolatis amplexicaulibus sensim ad bracteas ovatas vel lineares reductis; capitulis paucis longe pedunculatis pedunculis nudis vel pluribracteatis; disco 12–13 mm. alto 13–15 mm. diametro quam involucro paullo longiore; squamis circa triseriatis striatis glandulari-pubescentibus subhirsutis linear-subulatis exterioribus brevioribus; radiis ca. 10 flavis ovalibus 7.5 mm. longis

latis; corollis disci 6.5–7.5 mm. (tubulo 1 mm.) longis flavis denique apice purpurascentibus glandulosis praecipue in tubulo; acheniis maculosis appresse pubescentibus pilis ferrugineis 5–6 mm. longis 2.8–3 mm. latis; aristis 2, 4 mm. longis.

Specimens examined: PUEBLA: rocky soil, Cerro de Paxtle, Sept. 1908, *Purpus* 3022 (FGN, COTYPES). Distributed as *Encelia heterophylla* (= *S. foetida*), from which it differs in size of achene, leaf-base, and whole character.

+– +– Rays purple.

22. S. SANGUINEA Gray. Stems several, erect from a woody root, 1 m. high or more, branched above, usually purplish, glandular-setose; lower leaves opposite, variable, hastately 3-lobed, with broadly margined cordate-clasping petiolar bases, crenate-dentate or sometimes cut-lobed, scabrous and glandular-setose both sides, 3–18.5 cm. long including petiole, about as wide across the lobes, the upper reduced to linear-lanceolate bracts; heads numerous, panicled, on peduncles 1–11 cm. long; disk 1–1.5 cm. high, mostly slightly surpassing the involucre; this 3-seried, the scales linear-subulate to lanceolate, striate, densely glandular, setose, the outer distinctly shorter; rays about 10, rich purple, oblong-oval, 6–10 mm. long; disk-corollas 5–8 mm. long, purple above, glandular-pubescent below, hairy on the teeth; pales 8.5–13 mm. long, narrow, glandular-pubescent toward the acute tip; achenes blackish, oval, barely obcordate, 4.5–6.5 mm. long, 2.5–3.5 mm. wide, more or less densely appressed-pubescent, bearing 2 teeth or 1 short smooth awn or 2 upwardly pubescent awns as much as 3 mm. long.

Simsia sanguinea Gray, Pl. Wright. i. 107 (1852).

Encelia sanguinea Hemsl. Biol. Centr.-Am. Bot. ii. 185 (1881).

Simsia erythranthema Sch. Bip. in Gray, Proc. Am. Acad. xix. 9 (1883), as syn.

Specimens examined: JALISCO: dry grassy slopes of the barranca near Guadalajara, 5 Nov. 1888, *Pringle* 1738 part (F: intermediate between this and the next); VERA CRUZ: hillsides, Chavarillo, 7 Sept. 1906, *Barnes, Chamberlain, & Land* 6 (F); Mirador, Consoquitla, Aug. 1841, *Liebmann* 492 (G); Mirador, without date, *Sartorius* (G); OAXACA: hills, Las Sedas, alt. 1830 m., 11 Aug. 1894, *Pringle* 5756 (G); El Parian, Etla, alt. 1200 m., Nov. 1898, *Conzatti & González* 899 (G); Santa Catarina, alt. 1000 m., 26 Dec. 1906, *Conzatti* 1652 (F); La Carbonera, alt. 2135 m., 20 Sept. 1895, *L. C. Smith* 817 (G); Monte Alban, alt. 1800 m., 18 Aug. 1897, *Conzatti & González* 403 (G);

valley of Oaxaca, alt. 1677–2287 m., 20 Sept. 1894, *Nelson* 1445 (N); same data, *Nelson* 1426 (GN). Temperate Mexico, without locality, mountains, *Ghiesbreght* 305 (TYPE in Gray Herb.).

22β. S. SANGUINEA Gray var. **Palmeri** (Gray) Blake, n. comb. Similar in size, habit, pubescence, and inflorescence; leaves ovate-lanceolate, acuminate, contracted below the middle to a clasping base, subentire or coarsely dentate, unlobed or slightly three-lobed; heads mostly 1.5 cm. high, the scales generally linear-lanceolate and fully equaling the disk; achenes variable as in the last as to pubescence and awns, sometimes quite glabrous; rays mostly paler, violet to pale purple.

Encelia (*Simsia*) *sanguinea* Hemsl. var. (?) *Palmeri* Gray in Wats. Proc. Am. Acad. xxii. 427 (1887).

Specimens examined: JALISCO: thickets on sides of cañons, Rio Blanco, Sept. 1886, *Palmer* 602 (GN, TYPE COLLECTION); barranca of Guadalajara, alt. 1372 m., 29 Sept. 1903, *Pringle* 11513 (FGN); dry grassy slopes of barranca near Guadalajara, 5 Nov. 1894, *Pringle* 1738 part (GN: intergrading with the species).

DOUBTFUL AND TRANSFERRED SPECIES.

Simsia canescens Gray, Pl. Fendl. 85 (1849) = GERAEA CANESCENS T. & G.

Simsia frutescens Gray in Torr. Bot. Mex. Bound. 89 (1859) = ENCELIA FRUTESCENS Gray.

Simsia ? heterophylla Pers. Syn. ii. 478 (1807) = IOSTEPHANE HETEROPHYLLA (Cav.) Benth.

Simsia hispida (HBK.) Cass. Dict. Sci. Nat. lix. 137 (1849). *Ximenesia hispida* HBK. Nov. Gen. iv. 227 (1820). *Encelia hispida* Hemsl. Biol. Centr.-Am. Bot. ii. 184 (1881). This species, with "foliis alternis, sessilibus, ovato-oblongis, obsolete serratis, supra piloso-, subtus sericeo-hispidis," and hispidulous stem three-flowered at apex, has not since been recognized. The description points to a poorly developed *S. foetida*.

Simsia pastoensis Triana, Ann. Sci. Nat. sér. 4. ix. 40 (1858), from Columbia, has not since been identified. It seems to be related to *S. pubescens* Triana and *S. Sodiroi* (Hieron.) Blake. The original description reads thus: "Suffrutex, ramis teretibus sparse molliter pilosis et inter pilos puberulo-glandulosis asperulis; foliis summis alternis et subbracteiformibus, inferioribus oppositis breviter petiolatis ovatis acutis serratis, supra sparse decumbenti-pilosis et glandu-

4 mm. loso-scabris, subtus secus nervos pilosis, inter nervos sparse pubescentibus, petiolo basi auriculato amplexicauli longeque pilis longiusculis villoso; involucri squamis lanceolatis, acutis, extus pilis longiusculis villoso-canescentibus, squamis exterioribus brevioribus; capitulis petiolatis [sic], corymbo laxo subpaniculato; acheniis decumbentipilosulis, margine alatis.

"Crescit prope *Ortega,* altitudine 1200 metr. in prov. Pasto."

Simsia scaposa Gray, Pl. Wright. ii. 88 (1853) = ENCELIA SCAPOSA Gray.

Errata.

Page 336, line 20; for *calliopisidea* read *calliopsidea.*
" 353, " 37; for +— Heads larger, rays 2–4.5 cm. read
+— +— Heads larger, rays 2–4.2 cm.

PLATE 1.

A. *Enceliopsis* (Gray) A. Nels. Figures 1, 2, 3, 4, 5, 6, whole plant, head, disk-floret, achene, pale, and style-tip of *E. nudicaulis* (Gray) A. Nels. Figure 1 is from a cotype of *E. tuta* A. Nels.

B. *Geraea* Torr. & Gray. Figures 1, 2, 3, 4, 5, 6, whole plant, head, disk-flower, achene, pale, and style-branches of *G. canescens* T. & G.

C. *Encelia* Adans. Fig. 1, portion of plant of *E. californica* Nutt. Figures 2, 3, 4, 5, 6, head, disk-floret, achene, pale, and style-tip of *E. farinosa* Gray.

D. *Simsia* Pers. Figures 1, 3, 4, 5, 6, whole plant, disk-floret, achene, pale, and style-tip of *S. foetida* (Cav.) Blake. Fig. 2, head of *S. exaristata* Gray.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY
OF THE MUSEUM OF COMPARATIVE ZOÖLOGY
AT HARVARD COLLEGE.—No. 239.

ON THE SIZE OF LITTERS AND THE NUMBER OF NIPPLES IN SWINE.

By G. H. Parker and C. Bullard.

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Received June 12, 1913.

La vache s'écarte, à cet égard, d'une manière bien remarquable des lois générales de la nature, qui a proportionné dans toutes les espèces le nombre des mamelles des mères à celui de leurs petits; elle a quatre mamelles, quoiqu' elle ne porte qu'un veau et bien rarement deux, parce que ces deux mamelles superflues étoient destinées à être les nourrices du genre humain. La truie, à la vérité, n'en a que douze, et elle nourrit jusqu'à quinze petits. Ici la proportion paroît défectueuse. Mais si la première a plus de mamelles qu'il n'en faut à sa famille, et si la seconde n'en a pas assez pour la sienne, c'est que l'une devoit donner à l'homme la surabondance de son lait, et l'autre celle de ses petits. Saint-Pierre. Études de la Nature, Paris, 1792, tome I, p. 412.

Introduction.

As the accompanying quotation from the author of "Paul and Virginia" shows, naturalists have long been of opinion that an intimate relation exists between the size of the litter in any given species of mammal and the number of nipples possessed by its females. This relation has likewise come to the attention of anatomists; thus Gegenbaur (1898, p. 130) in his account of the mammary glands, declares that "die Zahl der Zitzen steht in inniger Beziehung zur Menge der Jungen." It is the chief object of this paper to discuss the relation of the size of litters to the numbers of nipples in the domesticated swine, Sus scrofa Linn.

The data for this investigation were collected from litters of unborn but nearly mature pigs that were available at one of the large slaughtering establishments in Cambridge, Massachusetts. The sows from which these litters were obtained, came chiefly from the corn-belt of the Mississippi Valley and were in general about two years old. They were of various breeds. Each set of records was taken from the contents of a single uterus. On opening the uterus, the number of pigs and their relative position in each horn was recorded, as well as the sex of each pig and the number and arrangement of its nipples. In all 1000 litters were examined, the records of which are given in an appendix at the end of this paper. We are under obligations to Mr. H. D. Fish for assistance in calculating and checking the results.

In dealing with the relation of the size of the litter to the number of nipples, we shall take up, first, the composition of the litters, then the number and arrangement of the nipples, and, finally, a comparison of the numbers of young in the litters with the numbers of nipples in the females.

Composition of the Litters.

Numbers of pigs in a litter. As already stated, the total number of litters that we examined was 1000, and the number of pigs in a litter varied from 1 to 15. The maximum number of 20 in a litter, as assumed by Aristotle (1883, p. 164), or any excess of that number, as noted by Gilbert White (1870, p. 222), was not met with by us. The frequency with which litters of different numbers occurred is given in Table 1.

TABLE 1.

A thousand Litters classified according to the Number of Pigs in each.

Number of pigs in a litter	1	2	3	4	5	6	7	8
Number of litters	15	50	74	134	137	189	164	116
Number of pigs in a litter	9	10	11	12	13	14	15	Total
Number of litters	61	32	15	6	3	3	1	1000

Since the thousand litters contained in all 5970 pigs, the mean number of pigs per litter is 5.97 which is very close to the mode, 6, as shown in Table 1. These numbers are smaller than those given by Surface (1909), who found litters of Poland China pigs to vary from

1 to 15, with a mean of 7.435 and a mode of 7, and litters of Duroc Jersey pigs to vary from 1 to 19, with a mean of 9.337 and a mode of 9, differences that may have been due to the fact that in many instances the litters in our records were the first litters produced by unselected females and may therefore have been smaller than those of females taken for breeding purposes.

Sexes. Of the 5970 pigs, 2946, or 49+ percent, were females and 3024, or 51— percent, were males; that is, for every 100 females there were 102.6+ males. This proportion agrees closely with that observed by Wentworth (1912, p. 548) according to whom the males exceed the females by about 2.02 percent. It is somewhat less than the number given by Darwin (1895, p. 247), who stated, on the authority of Mr. Wier, that there are about 7 males to 6 females, that is, the males exceed the females by about 16.7 percent.

The distribution of the sexes in the litters of different sizes is given in Table 2.

TABLE 2.

NUMBERS AND PROPORTIONS OF MALE AND FEMALE PIGS IN 1000 LITTERS OF DIFFERENT SIZES.

Numbers of pigs in litter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Totals
Total numbers of females	6	46	112	264	354	556	538	449	280	172	87	41	15	19	7	2946
Total numbers of males	9	54	110	272	331	578	610	479	269	148	78	31	24	23	8	3024
Percentage frequency of males	60	54	50−	51−	48+	51−	54−	51+	49−	46+	47+	43+	62−	55−	54−	51−

An inspection of Table 2 will show that in no class does the percentage of males rise above 62 nor fall below 43, but it is close to the general average, 51— percent, in all the classes of litters except the extremely small one (1) and the large ones (10 to 15). In the former it is somewhat above the general average; in the latter it is below in litters of 10 to 12 pigs and above in litters of 13 to 15. These relations, however, even in the extreme cases, are so nearly uniform that we may conclude with reasonable assurance that there is no intimate relation between sex and the size of the litter.

The thousand litters that we worked upon, then, were composed of from 1 to 15 pigs each, most frequently of 6, and the sexes were distributed almost indiscriminately among them in the ratio of about 1026 males to 1000 females.

The Number and Arrangement of the Nipples.

Introductory. The nipples on pigs form two series situated on either side of the ventral median line and extending from the pectoral to the inguinal regions. In some pigs the nipples in these two series are so exactly paired, right and left, as to give the appearance of extreme segmental regularity. In others they are grouped on the two sides with so much diversity in number and arrangement as to seem to be quite irregular. These relations have been discussed by Bateson (1894, p. 190) and especially by Wentworth (1913), who has developed a somewhat extended classification to include the chief variations. This aspect of the subject is not essential to our problem and we have, therefore, not attempted to deal with it. In collecting our data, we have incidentally recorded the arrangement of the nipples as either regular or irregular. By a regular arrangement we mean one in which the nipples are grouped in pairs right and left and we have used this term to include cases were certain members of assumed pairs were absent. By an irregular arrangement we mean one in which the members of the right and left series are so disposed as to show no obvious pairing. Most examples fall naturally under one or other of these heads, but the process of classifying is at times somewhat artificial. These two classes do not agree with the two chief classes established by Wentworth (1913) on the basis of what he calls the symmetrical and the asymmetrical patterns. These patterns have not been clearly defined, but apparently they refer to the even or odd numbers of nipples possessed by the animal in the sense that when one side exhibits the same number of nipples as the other, the arrangement is called symmetrical; when otherwise, it is designated asymmetrical. The asymmetrical pattern may be due to a "suppressed nipple" or to the presence of a "triangular pattern" and the latter may exhibit several subordinate variations. Although some such classification as that proposed by Wentworth is necessary for a treatment of the subject from the standpoint of inheritance, our own observations have shown us that the variations in the arrangements of the nipples are so irregular that any classification of them must be more or less artificial. Even in what we have called the regular plan, we are not disposed to admit of so close a comparison of organs as to favor the institution of a system of homologies for nipples such as Wentworth has suggested, but we are more inclined to the opinion advanced by Bateson (1894, p. 192) that the milk series as a whole is variously divided in different individuals or on the two sides of the same individual, and thus we have attempted to avoid that unreality to which extreme forms of morphological comparison lead. In discussing the arrangement of the nipples in swine, we shall, therefore, not lay much stress on close comparisons, but content ourselves with the division of the material into the two chief classes already designated as regular and irregular. In a few of the pigs that came under our observation nipples were found so near the median line as to appear to be median in position. Such nipples, however, can usually be shown to be really lateral in position, and, as a matter of fact, all have been classed as belonging to either the right side or the left.

TABLE 3.

THE PIGS OF 1000 LITTERS, IN ALL 5970, CLASSIFIED ACCORDING TO THE NUMBER OF NIPPLES IN EACH.

Numbers of nipples	8	9	10	11	12	13
Numbers of cases	1	7	697	1134	2094	962
Frequency per mill	1−	1+	117−	190−	351−	161+

Numbers of nipples	14	15	16	17	18	Total
Numbers of cases	738	208	111	14	4	5970
Frequency per mill	124−	35−	19−	2+	1−	1002

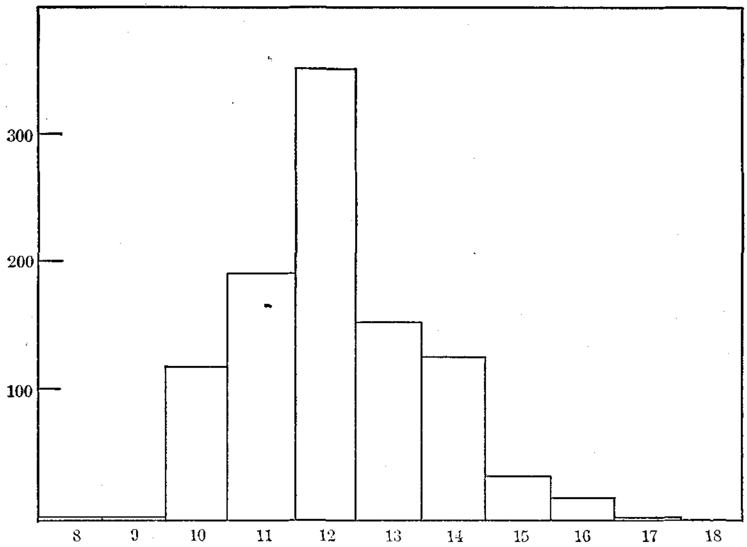
Numbers of nipples in pigs. Gegenbaur (1898, p. 129) makes the statement that swine possess from 8 to 10 nipples. Bateson (1894, p. 190) records for young pigs 12 to 16, and Wentworth (1913, p. 268) gives 8 to 16. In the thousand litters that we examined, as shown in Table 3, the smallest number of nipples present was 8, and the largest 18. The mode was 12, which was very near the mean number for the whole population, 12.2−.

In the pigs studied by Wentworth (1913, p. 269) the mode, which is even more marked than in our series, fell upon 10, instead of 12, a difference probably due to a difference in breeds.

Plotting the records contained in Table 3 in the form of a frequency polygon, Table 4, it is clear that the groups representing even numbers of nipples stand relatively high as compared with those representing odd numbers, a condition which indicates that the arrangement of nipples tends to be in pairs. This is also evident from the table given for Duroc Jersey swine by Wentworth (1913, p. 269), who has shown that the nipples are more probably inherited in pairs than laterally.

TABLE 4.

Frequency Polygon showing the Distribution of 5970 Pigs (1000 Litters) According to the Number of Nipples on each Pig. The Numbers of Nipples are Plotted as Abscissae and the Number of Variates as Ordinates (Table 3).



Numbers of nipples in male and female pigs. The numerical distribution of the nipples in male and in female pigs is given in Table 5. In the 3024 male pigs the nipples ranged from 9 to 18, with a mean number of 12.4+ and a mode of 12. the standard deviation was 0.6906 ±.0060. In the 2946 female pigs the nipples ranged from 8 to 18 with a mean of 11.9+ and a mode of 12. The standard deviation was

0.7905 ± .0069. Thus the females are over 14 percent more variable than the males.

The similarity in the numbers of nipples in the males and in the females, as exhibited in Table 5, is noteworthy. When it is considered that the mammary glands, so far as is known, are only normally functional in the females and probably never have been functional in the males, even in the most primitive mammals, it is remarkable that the two sexes in pigs should at the outset of life show such considerable similarity. The presence of embryonic nipples and the other rudiments of the mammary apparatus in male mammals must be dependent upon a continual process of inheritance from the female side, a process which is apparently the outcome of high female specialization, whereby traits which have arisen in that sex are, so to speak, forced upon the other. The result is that an organ functionless both in the past history and the present state of a sex is nevertheless continuously exhibited by that sex.

TABLE 5.

Pigs from 1000 Litters, 3024 Males and 2946 Females, Classified According to the Number of Nipples on each Pig.

Number of nipples	Males		Females	
	Observed numbers	Numbers per mill	Numbers per mill	Observed numbers
8	0	0.0	0.3	1
9	2	0.7	1.7	5
10	274	90.6	143.2	423
11	492	162.7	217.9	642
12	1004	332.0	370.3	1090
13	507	167.7	154.4	455
14	494	163.4	82.8	244
15	149	49.3	20.0	59
16	90	29.8	7.1	21
17	9	3.0	1.7	5
18	3	1.0	0.3	1
Totals	3024	1000.2	999.7	2946

Numbers of nipples on right and left sides. The numbers of nipples on the right and on the left sides of pigs are recorded in Table 6.

The nipples on the left side range from 4 to 9 with a mean number of 6.1— and a mode of 6, and on the right side from 4 to 10 with a mean of 6.1+ and a mode of 6. This striking similarity is also shown in the percent of individuals in which the two sides agree in numbers, or

TABLE 6.

Numbers of Pigs from 1000 Litters Classified According to the Numbers of Nipples on the Right and Left sides.

Numbers of nipples	Left Side		Right Side	
	Observed numbers	Numbers per mill	Numbers per mill	Observed numbers
4	5	0.8	0.8	5
5	1461	244.7	198.1	1182
6	2938	492.1	529.6	3162
7	1324	221.8	228.9	1367
8	233	39.0	39.7	237
9	9	1.5	2.7	16
10	0	0.0	0.2	1
Totals	5970	999.9	1000.0	5970

differ by one, two, or three nipples. No cases were found where the two sides differed by as much as four nipples. This form of comparison for the whole population as well as for the separate sexes is given in Table 7.

TABLE 7.

Percentages of Pigs in 1000 Litters in which the Nipples of the Right and Left Sides agree in Number, or differ by one, two, or three. The Percentages are given for the Whole Population (5970 individuals), the Males (3024 Individuals), and the Females (2946 Individuals).

	Whole population	Males	Females
Agreement	58.6−	59.1−	58.1−
Difference of 1	38.9+	38.3−	39.6−
Difference of 2	2.5−	2.6−	2.4−
Difference of 3	0.03+	0.07−	0.0

The correlation of the nipples of the two sides of the pigs is given in Table 8.

The coefficient of correlation of the right and left sets of nipples is 0.6063 ± .0055, a somewhat lower degree of correlation than was found by Davenport and Bullard (1896, pp. 96) for the Müllerian glands on the fore legs of swine.

The correlation of the nipples of the two sides as exhibited by the two sexes is shown in Table 9 for the males and in Table 10 for the females.

TABLE 8.

Correlation between the Nipples of the Right and Left Sides of 1000 Litter of Pigs, 5970 Individuals.

		Nipples of Right Side						
		4	5	6	7	8	9	10
Nipples of Left Side	4	1	4	0	0	0	0	0
	5	3	696	696	64	2	0	0
	6	1	438	1986	500	13	0	0
	7	0	44	460	705	112	3	0
	8	0	0	20	96	106	10	1
	9	0	0	0	2	4	3	0
	10	0	0	0	0	0	0	0

In the male pigs, as shown in Table 9, the nipples of the right side range from 4 to 10 with a mean of 6.2+ and a mode at 6; those of the left side range from 4 to 9 with a mean at 6.2— and a mode at 6. The coefficient of correlation in the males is 0.6359 ± .0073.

TABLE 9.

Correlation between the Nipples of the Right and Left Sides of the Male Pigs in 1000 Litters, 3024 Individuals.

		Nipples on Right Side						
		4	5	6	7	8	9	10
Nipples of Left Side	4	0	1	0	0	0	0	0
	5	1	274	307	26	2	0	0
	6	0	185	957	271	11	0	0
	7	0	21	234	467	77	1	0
	8	0	0	16	72	87	8	1
	9	0	0	0	2	1	2	0
	10	0	0	0	0	0	0	0

In the female pigs, as shown in Table 10, the nipples of the right side range from 4 to 9 with a mean of 6.0 — and a mode at 6; those of the left side also range from 4 to 9 with a mean of 5.9+ and a mode at 6. The coefficient of correlation in the females is 0.5465 ± .0087. Thus the females show slightly less correlation in respect to their two series of nipples than the males, a condition somewhat different from that found by Davenport and Bullard (1896, p. 96) for the Müllerian glands, which were about equally correlated in the two sexes.

TABLE 10.

Correlation between the Nipples of the Right and Left Sides of the Female Pigs in 1000 Litters, 2946 Individuals.

		Nipples of Right Side						
		4	5	6	7	8	9	10
Nipples of Left Side	4	1	3	0	0	0	0	0
	5	2	422	389	38	0	0	0
	6	1	253	1029	229	2	0	0
	7	0	23	226	238	35	2	0
	8	0	0	4	24	19	2	0
	9	0	0	0	0	3	1	0
	10	0	0	0	0	0	0	0

Regular and irregular plans. The proportions of pigs in which the nipples are arranged on the regular and the irregular plans (see p. 402) as seen in the whole population and in the sexes separately are given in Table 11.

TABLE 11.

Proportions in 1000 Litters of the Whole Population, of Males, and of Females, that Exhibited the Regular and the Irregular Arrangement of the Nipples.

	Whole population		Males		Females	
	Number	Percent	Number	Percent	Number	Percent
Regular	3559	59.6+	1848	61.1+	1711	58.1−
Irregular	2411	40.4−	1176	38.9−	1235	41.9+

From Table 11 it is clear that in the population as a whole the regular arrangement occurs more frequently than the irregular one in the ratio of about 6 to 4. The proportion of instances of the regular arrangement is very slightly greater in the males than in the females.

The details of the various combinations of nipples under the two general plans, regular and irregular, are given for the population as a whole and for the sexes separately in Table 12.

What we have called the regular plan in the arrangement of the nipples has led to the assumption on the part of some investigators that the mammary glands are segmentally arranged, an opinion especially advocated by Williams (1891). But the conditions presented by many of the lower mammals do not seem to us to favor this interpretation and we are inclined to regard the signs of segmentation as of quite secondary origin, perhaps the result of the segmental tendencies of the trunk exerting themselves on what would have been otherwise an irregular series. From this point of view we regard the irregular arrangement as the more primitive and the regular one as of later origin. Physiologically the best arrangement for the nipples would probably be one of alternation in the two rows such as is partially met with in what Wentworth has called the triangular pattern, for this would admit of the greatest freedom of access to the nipples for the young pigs as the sow lies on her side. Such an arrangement of the whole series of nipples is extremely rare, however, though what we have called the irregular plan approaches this more nearly than the regular one.

Relation of the Size of the Litters to the Number of Nipples in Individual Pigs.

We have now gathered the necessary data for a comparison of the size of the litter with the number of nipples in individual pigs. These relations are shown in Table 13 in which the sizes of the litters are plotted against the numbers of nipples in the females.

The number of pigs in a litter studied by us ranged from 1 to 15 with 6 as the mode. The number of nipples in female pigs ranged from 8 to 18 with a mode of 12. These figures show at once that no simple and close correlation is to be expected between the two series of records and this opinion is confirmed by the coefficient of correlation calculated from Table 13, namely 0.0035 ± .0124. This lack of correlation is also represented in the last column in Table 13, in which is given the mean number of nipples for litters of different sizes. An inspection of this column shows at once that there is no obvious tendency for the number of nipples to vary with the size of the litters; in other words, in swine, as in sheep (Bell, 1904, p. 768), the number of nipples is not obviously related to fecundity.

The relation of the size of the litters to the number of nipples can also be shown in superimposed frequency polygons, Table 14. Although there may be as few nipples as 8 and as many pigs in a litter as 15, it is quite evident from the nature of the overlapping of the polygons that disadvantageous combinations in which the number of young pigs outrun the provision for milk, cannot be of frequent occurrence and that in general, as indicated by the mode of 6 for the litters and 12 for the nipples, there are about twice as many nipples as young.

TABLE 12.

Frequency and Mode of Representation of the Regular and Irregular Arrangement of Nipples in 5970 Pigs (1000 Litters) Distributed for the Sexes Separately and for the Population as a Whole.

Numbers of nipples	Regular			Irregular			Grand Totals
	Male	Female	Total	Male	Female	Total	
Equal							
4–4	0	1	1	0	0	0	1
5–5	256	391	647	18	31	49	696
6–6	824	819	1643	133	210	343	1986
7–7	386	191	577	81	47	128	705
8–8	66	13	79	21	6	27	106
9–9	0	0	0	2	1	3	3
Differ by one							
4–5	0	0	0	1	3	4	4
5–4	0	0	0	1	2	3	3
5–6	75	93	168	232	296	528	696
6–5	64	81	145	121	172	293	438
6–7	52	36	88	219	193	412	500
7–6	72	60	132	162	166	328	460
7–8	13	5	18	64	30	94	112
8–7	17	8	25	55	16	71	96
8–9	3	2	5	5	0	5	10
9–8	1	1	2	0	2	2	4
Differ by two							
4–6	0	0	0	0	0	0	0
6–4	0	0	0	0	1	1	1
5–7	4	3	7	22	35	57	64
7–5	9	4	13	12	19	31	44
6–8	2	0	2	9	2	11	13
8–6	3	2	5	13	2	15	20
7–9	1	1	2	0	1	1	3
9–7	0	0	0	2	0	2	2
8–10	0	0	0	1	0	1	1
10–8	0	0	0	0	0	0	0
Differ by three							
5–8	0	0	0	2	0	2	2
8–5	0	0	0	0	0	0	0
Totals	1848	1711	3559	1176	1235	2411	5970

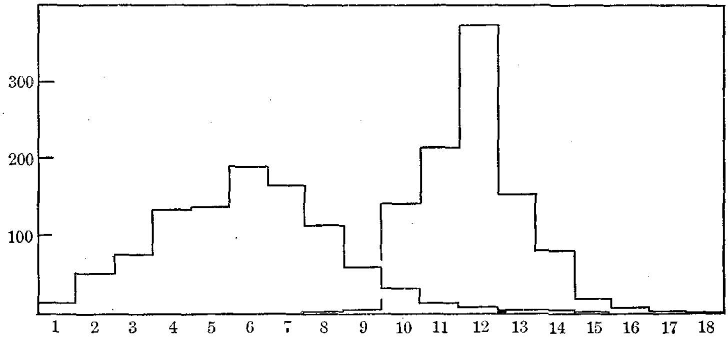
TABLE 13.

Correlation between the Size of Litters of Pigs, as seen in 1000 Litters, and the Numbers of Nipples in Females, as seen in 2946 Individuals.

Numbers in Litters	Numbers of Nipples in Females												Mean Number of Nipples per Litter
	8	9	10	11	12	13	14	15	16	17	18	Totals	
1	0	0	0	1	3	2	0	0	0	0	0	6	12.17 –
2	0	0	6	14	19	4	3	0	0	0	0	46	11.65 +
3	0	0	12	26	36	24	10	3	0	0	1	112	12.08 +
4	0	0	48	59	91	41	20	2	1	2	0	264	11.80 –
5	0	0	53	84	138	49	25	4	1	0	0	354	11.79 –
6	0	2	75	115	219	84	49	9	2	1	0	556	11.91 +
7	0	1	73	118	200	78	47	15	4	0	0	538	11.94 +
8	0	1	57	93	169	72	41	10	5	1	0	449	12.00 –
9	1	0	46	65	101	37	16	9	4	1	0	280	11.84 +
10	0	0	26	34	63	30	13	4	2	0	0	172	11.94 +
11	0	0	20	13	31	14	7	2	0	0	0	87	11.78 +
12	0	0	6	10	8	10	4	1	2	0	0	41	12.17 +
13	0	0	0	4	7	4	0	0	0	0	0	15	12.00
14	0	1	1	6	5	3	3	0	0	0	0	19	11.90 –
15	0	0	0	0	0	3	4	0	0	0	0	7	13.57 +
Totals	1	5	423	642	1090	455	244	59	21	5	1	2946	11.91 –

TABLE 14.

Superimposed Frequency Polygons Illustrating the Distribution of the Numbers of Pigs in Litters (Left) and the Numbers of Nipples on the Females (Right). The Numbers of Pigs and of Nipples are Plotted as Abscissae and the Numbers of Variates as Ordinates.



Summary.

1. 1000 litters of unborn pigs contained 5790 individuals; the mean number of pigs per litter was 5.97, the mode 6.

2. Of the 5790 pigs, 3024 were males and 2946 were females; there were 102.6+ males for every 100 females.

3. In the whole population the nipples ranged from 8 to 18 with a mean of 12.2+ and a mode of 12.

4. In the male pigs the nipples ranged from 9 to 18 with a mean of 12.4+ and a mode of 12, and a standard deviation of 0.6906 ± .0060; in the females the range was from 8 to 18 with a mean of 11.9+, a mode of 12, and a standard deviation of 0.7905 ± .0069.

5. On the left side the nipples ranged from 4 to 9 with a mean of 6.1—, and a mode of 6; on the right side they ranged from 4 to 10 with a mean of 6.1—, and a mode of 6. The coefficient of correlation for the two sides was 0.6063 ± .0055.

6. In the whole population the arrangement of the nipples fell under the regular plan (p. 402) in 3559 cases and under the irregular plan in 2411 cases.

7. There is no obvious relation between the size of the litter and the number of nipples in the females. Though there may be as few nipples as 8 and as large litters as 15, disadvantageous combinations of large litters borne by females with few nipples cannot be of frequent occurrence. Commonly there are about twice as many nipples, 12, as young, 6.

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Appendix.

Records of 1000 litters of unborn pigs showing the relative position of the pig in the uterus, its sex, and the number and arrangement of its nipples. The table begins with litters of one pig and proceeds in sequence to the largest litter, one of fifteen. The litters are numbered from 1 to 1000. The vertical heavy line indicates the line of separation between the right and left horns of the uterus; the records at the reader's left from this line represent the pigs in the left horn of the uterus, at the reader's right in the right horn. Each pig is represented by a letter and two figures: m indicating male, f indicating female, and the figures indicating the numbers of nipples on the left and on the right side of the pig. The sides of the pig correspond to the reader's right and left. When the sex letter is Roman, the arrangement of the nipples was regular (p. 402), when italic, it was irregular. Thus *m* 76 indicate a male pig with 7 nipples on the left and 6 on the right arranged irregularly, and f 76 indicates a female pig with the same numbers of nipples on each side as the male, but regularly arranged. The records for each litter, occupying a single line, form a series corresponding to that of the pigs in the uterus, in that the extreme left-hand record was taken from the pig nearest the left ovary, the extreme right from the one nearest the right ovary.

1		*m* 76
2		f 76
3		*m* 78
4		m 77
5		*m* 56
6		m 77
7	f 66	
8	m 66	
9	m 55	
10	m 66	
11	*f* 65	
12	f 66	
13	*f* 76	
14	m 76	
15	f 66	
16		f 65 *f* 56
17		*m* 76 *f* 67
18		*m* 55 f 55
19		*m* 56 f 55
20		*f* 67 *f* 66
21		*m* 76 m 66
22		*f* 76 m 88
23	f 65	*f* 56
24	*m* 66	*f* 65
25	*m* 76	*f* 75
26	m 77	m 77
27	f 55	m 66
28	m 77	m 77
29	f 66	m 56
30	m 66	*f* 56
31	m 76	m 65
32	m 66	f 77
33	f 66	f 66
34	f 65	*m* 65
35	m 55	m 55
36	*m* 66	*m* 54
37	*f* 66	*f* 76
38	m 55	*f* 66
39	*f* 77	f 77
40	f 66	*m* 76
41	m 66	*m* 67
42	*m* 76	f 66
43	*m* 68	*f* 56
44	f 56	m 66
45	m 66	*m* 55
46	*f* 66	m 55
47	f 66	m 77
48	*m* 56	*m* 56
49	m 66	*f* 75
50	*m* 77	*m* 76
51	f 66	*f* 56
52	f 55	*f* 56
53	m 66	m 66
54	m 55	*m* 77
55	*m* 56 m 66	
56	f 66 f 66	
57	m 66 *m* 67	
58	*f* 56 m 55	
59	*m* 76 f 66	
60	m 77 m 76	
61	f 56 m 66	
62	f 55 m 55	
63	*f* 56 f 66	
64	m 56 f 66	
65	f 66 f 55	
66		f 67 m 76 f 56
67		*f* 66 f 66 *m* 65
68		f 66 *f* 56 m 77
69		m 56 m 75 f 66
70		*m* 67 f 66 f 67
71		f 55 *f* 55 m 66
72		f 55 f 66 f 65

73	f 55	*m* 77	*m* 76	
74	f 77	*f* 66	m 66	
75	f 76	m 88	m 78	
76	m 66	*m* 76	m 65	
77	*m* 56	*m* 65	*f* 56	
78	m 66	f 55	*m* 65	
79	*f* 65	*f* 76	*m* 67	
80	m 55	f 66	f 67	
81	f 76	f 66	f 55	
82	m 66	f 66	m 66	
83	*m* 97	*f* 56	*f* 78	
84	*f* 77	*m* 65	m 66	
85	m 55	m 66	*f* 56	
86	f 65	m 55	*f* 67	
87	*f* 65	*f* 66	*m* 66	
88	f 66	*f* 67	*m* 99	
89	f 67	*m* 65	*f* 56	
90	m 67	*m* 56	*m* 56	
91	m 66	m 87	m 88	
92	f 66	*m* 65	f 55	
93	*f* 99	*f* 67	f 87	
94	*f* 56	*f* 56	m 66	
95	m 55	m 55	m 66	
96	*m* 67	f 66	m 66	
97	*m* 76	*m* 78	f 66	
98	*f* 65	m 66	*m* 56	
99	m 66	m 66	m 66	
100	m 55	*m* 56	*m* 66	
101	*f* 56	*m* 67	*m* 65	
102	f 66	m 65	*m* 66	
103	*f* 66	*f* 76	f 66	
104	f 66	*f* 56	m 55	
105	*m* 67	f 77	*f* 66	
106	m 66	*f* 66	m 55	
107	f 77	f 66	*f* 76	
108	m 68	f 76	f 66	
109	m 55	*m* 66	*m* 56	
110	m 56	*f* 65	m 77	
111	*m* 67	*f* 76	*f* 65	
112	m 66	*f* 67	f 66	
113	f 55	*m* 78	m 77	
114	*f* 65	m 77	f 77	
115	*m* 56	*f* 76	*m* 45	
116	m 55	m 66	m 77	
117	*f* 56	f 66	f 66	
118	m 55	*f* 65	f 65	
119	f 66	f 66	f 55	
120	m 77	f 66	*f* 66	
121	*f* 76	*f* 77	*f* 76	
122	f 55	m 55	f 65	
123	m 66	*m* 67	m 88	
124	f 77	f 77	m 76	
125	*m* 76	f 66	f 77	
126	*f* 66	*m* 65	f 66	
127	*m* 56	m 65	*f* 67	
128	*f* 66	*f* 67	*f* 65	
129	*m* 65	m 55	*m* 66	
130	m 66	f 65	f 55	
131	m 66	m 77	m 66	
132	*m* 88	m 66	m 66	
133	f 56	m 76	*m* 65	
134	*m* 66	f 77	f 78	
135	*f* 67	*f* 76	*m* 67	
136	m 77	f 66	*f* 76	
137	m 66	m 77	f 56	
138	*m* 66	*f* 57	m 77	
139	*f* 76	f 55	m 66	
140	m 55	m 55	*m* 56	f 55
141	m 88	m 88	*f* 66	m 88
142	m 66	m 77	*f* 67	*f* 67
143	m 66	m 88	m 56	m 77
144	m 66	*m* 67	f 56	*m* 65
145	m 66	m 65	*f* 65	*f* 77
146	*m* 87	m 55	f 66	m 88
147	*f* 56	*f* 68	f 55	*m* 76
148	m 55	f 56	f 66	f 55
149	f 76	f 77	f 66	m 66
150	f 65	*f* 66	m 65	f 55
151	*m* 76	f 66	f 66	f 55
152	m 77	m 77	m 87	m 77
153	*m* 66	m 66	f 77	*m* 66
154	f 66	*f* 67	*f* 56	f 66
155	f 65	m 55	f 55	f 65
156	m 66	m 76	*f* 67	f 66
157	f 66	m 55	*m* 56	*f* 55
158	m 66	f 66	f 66	m 67
159	*f* 56	f 55	f 55	m 55
160	*m* 78	*f* 67	f 67	*m* 87
161	f 55	*f* 57	f 66	*m* 67
162	*f* 65	*m* 66	f 56	f 76
163	*f* 56	m 66	m 77	*f* 76
164	m 66	f 56	*m* 55	m 66
165	*m* 76	m 66	m 65	f 66
166	m 66	*f* 65	f 66	*m* 87
167	*f* 66	f 66	m 66	m 66
168	*m* 55	m 66	f 55	f 55
169	*m* 67	m 56	*f* 66	*m* 67
170	f 66	m 65	*m* 78	f 66
171	f 66	m 66	*m* 66	f 66
172	*f* 76	*m* 66	*m* 76	m 66
173	f 67	f 66	m 66	*f* 67
174	f 66	m 75	*m* 56	*f* 56
175	m 66	*f* 56	*f* 56	m 66
176	m 77	*m* 56	f 56	*m* 67
177	*f* 57	m 66	f 55	*f* 67
178	*m* 66	f 77	f 66	*m* 55
179	f 55	*m* 77	*f* 65	f 66
180	m 55	m 66	*f* 66	m 55
181	*f* 67	m 56	m 77	m 66

182	f 77	f 89	f 88	f 89	
183	f 66	m 56	m 66	f 66	
184	*m* 99	m 98	*f* 66	m 77	
185	m 88	f 77	f 66	*m* 56	
186	*f* 76	m 66	f 75	*m* 56	
187	*m* 78	m 77	f 66	m 77	
188	m 66	m 66	f 66	f 66	
189	*m* 65	m 77	*m* 77	f 77	
190	f 66	m 66	f 66	m 66	
191	f 76	m 76	*f* 67	m 76	
192	*m* 76	*f* 67	m 56	*f* 66	
193	f 65	*f* 56	f 55	*f* 56	
194	m 66	m 55	f 66	f 55	
195	f 67	*f* 67	*f* 66	m 76	
196	f 56	f 66	m 76	*f* 66	
197	m 66	m 77	f 77	*m* 76	
198	m 66	m 66	m 66	m 77	
199	f 66	f 66	f 55	*f* 56	
200	*f* 67	m 76	m 66	f 55	
201	m 66	*m* 67	*f* 67	*m* 67	
202	*m* 76	*f* 77	f 77	*m* 87	
203	*f* 57	m 66	*f* 56	m 66	
204	*m* 67	f 66	m 66	m 76	
205	*f* 56	*f* 56	*m* 56	m 77	
206	m 66	*f* 67	*m* 56	m 66	
207	f 56	*f* 56	f 55	*m* 56	
208	f 55	*m* 56	*m* 65	*f* 65	
209	m 66	m 66	*f* 56	f 55	
210	*f* 76	m 66	m 77	m 77	
211	f 55	m 66	*f* 66	*f* 66	
212	*f* 56	m 66	m 55	m 55	
213	m 65	*f* 67	f 66	*f* 67	
214	f 55	m 66	*f* 66	m 55	
215	*m* 66	m 65	*m* 87	m 66	
216	*m* 56	f 66	*m* 88	*m* 66	
217	m 55	f 77	f 55	f 55	
218	*f* 65	m 55	f 55	*f* 56	
219	*f* 56	f 66	*m* 65	m 77	
220	m 65	*f* 65	f 65	m 55	
221	*m* 78	*f* 67	*f* 66	*f* 65	
222	*m* 67	*m* 56	m 66	f 66	
223	*f* 76	*m* 67	f 56	f 66	
224	m 66	f 56	*f* 65	m 55	
225	m 66	f 66	*f* 56	*f* 66	
226	f 55	f 66	*f* 76	*m* 67	
227	f 55	*f* 65	*m* 67	m 56	
228	*f* 57	*m* 65	*f* 55	*m* 56	
229	f 66	m 66	f 66	m 66	
230	m 77	*m* 67	*m* 67	f 66	
231	m 66	*f* 56	f 66	f 55	
232	m 55	*f* 65	m 77	*m* 66	
233	m 77	f 77	*f* 77	f 65	
234	f 66	f 66	*m* 67	*m* 76	
235	m 66	f 55	f 66	*f* 57	
236	m 55	m 55	f 57	m 66	
237	f 77	*m* 78	m 76	m 88	
238	*f* 65	*f* 56	m 66	*m* 56	
239	m 66	f 66	f 66	*f* 67	
240	m 66	f 66	*f* 66	*f* 56	
241	*m* 66	f 55	*f* 65	m 66	
242	*m* 76	f 66	*f* 76	*m* 65	
243	f 66	m 77	*m* 65	f 66	
244	m 76	*m* 65	*m* 56	m 65	
245	f 55	m 55	m 56	m 55	
246	f 77	m 88	m 88	*f* 77	
247	f 55	*f* 56	m 55	f 55	
248	*f* 56	f 55	f 55	f 55	
249	m 77	m 66	f 55	*m* 67	
250	*f* 76	*f* 67	m 57	*m* 56	
251	m 66	*f* 65	*m* 65	*f* 66	
252	f 77	f 87	*m* 87	f 77	
253	f 55	m 55	f 55	f 55	
254	*m* 89	f 66	m 76	*m* 67	
255	*f* 65	*f* 76	f 66	*f* 67	
256	*f* 75	m 55	*m* 66	f 56	
257	*m* 68	m 66	m 88	*f* 67	
258	*m* 66	*m* 65	*f* 65	*f* 66	
259	f 66	*m* 66	m 66	f 66	
260	m 77	f 55	*f* 76	m 66	
261	m 65	m 66	m 66	*m* 56	
262	*m* 56	m 77	m 55	f 55	
263	f 76	*m* 67	*f* 76	f 77	
264	f 66	*m* 76	f 65	*f* 56	
265	*m* 67	*m* 66	*m* 56	*m* 76	
266	*f* 65	*m* 67	m 67	m 77	
267	*m* 67	m 66	m 56	m 66	
268	*m* 76	*f* 67	*m* 89	*f* 66	
269	*f* 78	*m* 56	m 66	*f* 66	
270	m 66	*f* 76	m 77	f 66	
271	m 66	*m* 67	*m* 76	*m* 56	
272	f 55	f 55	f 55	f 55	
273	m 66	f 66	f 66	m 77	
274	m 66	m 77	m 66	f 56	m 66
275	f 66	m 66	*m* 56	m 66	*m* 65
276	*f* 76	m 66	f 66	m 66	m 66
277	*m* 76	*m* 78	*f* 66	*f* 65	m 66
278	f 66	f 66	f 66	*f* 65	f 56
279	*m* 57	f 56	f 66	f 65	f 77
280	m 76	m 87	*f* 76	m 66	*f* 66
281	f 77	f 66	f 66	m 77	*f* 56
282	*f* 66	*m* 56	m 76	f 55	*m* 67
283	*f* 65	f 66	f 77	m 77	*m* 76
284	m 66	*f* 65	m 66	m 76	*m* 76
285	*m* 56	*m* 56	f 66	f 66	m 66
286	f 66	*f* 65	*m* 75	f 66	f 55
287	m 65	*f* 56	*m* 66	*f* 65	*m* 57
288	f 66	m 66	*m* 76	*f* 67	m 66
289	m 77	*f* 65	f 55	*m* 77	*f* 56
290	f 55	m 77	f 55	m 66	f 55
291	m 56	m 56	*f* 65	m 66	*m* 56

292	f 55	*f* 65	f 55	f 55	f 66	**347**	*f* 66	f 56	*m* 56	*m* 56	m 55
293	*m* 87	f 66	*m* 67	f 55	f 66	**348**	f 55	f 56	m 65	f 56	f 66
294	*f* 75	f 55	f 55	m 55	f 55	**349**	*f* 56	f 66	f 77	*f* 56	*f* 66
295	*f* 56	*m* 77	*f* 66	*m* 77	m 67	**350**	f 66	m 66	m 55	f 66	f 55
296	m 66	f 76	m 65	*f* 67	f 66	**351**	*f* 66	m 77	f 66	f 79	m 77
297	m 66	f 66	m 66	*f* 66	f 66	**352**	m 88	*m* 65	*f* 76	m 77	m 87
298	*m* 66	f 55	*m* 55	m 56	m 56	**353**	*f* 56	m 66	*m* 56	f 77	f 66
299	*m* 56	m 55	*m* 67	f 55	f 65	**354**	*f* 67	m 66	*f* 77	f 55	*f* 65
300	f 66	*m* 67	m 67	f 66	*m* 65	**355**	m 55	*f* 55	*m* 67	f 55	*m* 56
301	f 65	m 66	m 66	*f* 56	m 66	**356**	f 65	f 65	*m* 76	f 66	m 66
302	f 56	*f* 66	m 65	m 66	m 56	**357**	f 55	f 55	f 55	f 66	m 65
303	m 66	f 66	*m* 67	f 67	f 66	**358**	*f* 77	f 66	*f* 78	f 66	m 87
304	*f* 67	f 66	f 66	m 67	*f* 66	**359**	*m* 67	m 77	m 66	*m* 87	f 66
305	m 66	f 66	f 66	m 86	*f* 65	**360**	f 66	*f* 66	*f* 66	*f* 67	*m* 67
306	m 67	f 67	*f* 56	m 66	m 66	**361**	f 66	m 55	m 55	*f* 66	f 55
307	m 77	f 66	m 77	*f* 56	*m* 56	**362**	m 55	m 55	m 55	m 55	f 55
308	m 77	*m* 75	f 66	*f* 66	m 77	**363**	m 77	m 77	*m* 66	*f* 67	*f* 76
309	f 66	m 66	*f* 67	*f* 66	m 66	**364**	*m* 66	*f* 56	m 55	f 55	*m* 56
310	m 77	f 65	*m* 57	*m* 76	m 77	**365**	*f* 67	f 66	*m* 75	*f* 67	m 66
311	*f* 65	*f* 75	m 55	f 66	m 66	**366**	*f* 65	f 66	*f* 67	m 66	*f* 67
312	*f* 66	*f* 66	f 55	m 55	m 66	**367**	*f* 67	m 66	*f* 77	m 77	*f* 67
313	f 55	f 66	m 66	f 55	*m* 65	**368**	*f* 76	*f* 66	f 66	m 76	f 66
314	*m* 67	*f* 67	*f* 56	f 66	m 66	**369**	f 76	f 66	m 67	f 66	m 77
315	f 66	*f* 76	f 66	*f* 56	f 66	**370**	f 56	*m* 65	*f* 66	f 66	*f* 56
316	*m* 67	f 66	m 77	f 66	*m* 76	**371**	*f* 65	m 66	m 66	f 56	f 55
317	*f* 76	*m* 55	m 66	*f* 56	*m* 66	**372**	m 56	f 66	*m* 66	m 66	*f* 65
318	*f* 65	f 55	*f* 65	*f* 56	f 55	**373**	f 55	f 55	f 77	f 66	m 77
319	m 66	f 66	m 55	m 66	*m* 75	**374**	*m* 65	m 55	f 76	f 65	*f* 66
320	*f* 56	m 77	*m* 65	*m* 67	*m* 65	**375**	*m* 76	f 56	m 77	f 56	m 77
321	f 67	m 77	*m* 66	f 77	*f* 65	**376**	m 76	f 77	m 77	f 66	m 76
322	*m* 56	*m* 65	f 66	m 55	m 76	**377**	m 66	f 65	m 77	m 66	f 66
323	*f* 56	f 67	*m* 67	m 66	m 75	**378**	m 66	m 66	m 66	f 65	m 67
324	m 77	*m* 87	*f* 77	f 66	*m* 67	**379**	f 66	f 55	*m* 65	m 66	*m* 67
325	*f* 78	m 66	*f* 67	*m* 77	*m* 67	**380**	m 66	f 66	*m* 67	m 66	f 77
326	*f* 56	*m* 65	*f* 56	*f* 56	m 77	**381**	f 66	*f* 55	f 67	*f* 56	m 55
327	m 77	m 66	m 77	m 77	f 77	**382**	m 76	m 55	*f* 76	f 66	*f* 66
328	m 55	f 65	f 55	*m* 56	f 76	**383**	*f* 56	f 55	m 66	f 55	f 55
329	*m* 67	*f* 56	*m* 76	*m* 78	f 77	**384**	m 77	*f* 76	f 66	*m* 78	*m* 67
330	m 55	m 76	f 66	*f* 56	*f* 66	**385**	m 66	*f* 76	*f* 57	*f* 56	f 65
331	m 66	*f* 67	*m* 56	m 66	m 66	**386**	m 56	f 66	m 66	f 77	f 66
332	m 66	m 66	*m* 75	m 66	f 55	**387**	f 66	m 56	*m* 65	f 66	m 55
333	*f* 67	m 66	*m* 65	*m* 67	f 66	**388**	f 55	f 66	m 66	*m* 56	m 66
334	*f* 66	*f* 55	*f* 75	m 66	f 55	**389**	f 77	m 77	f 66	f 66	f 66
335	m 87	*f* 67	*f* 55	*m* 76	*f* 56	**390**	*f* 56	m 77	f 76	m 77	m 66
336	*f* 67	*m* 87	m 77	f 76	m 77	**391**	f 65	*m* 87	*m* 78	*f* 78	f 77
337	*m* 66	*m* 67	f 65	f 65	m 67	**392**	f 66	*f* 65	f 66	m 56	*f* 56
338	m 77	f 66	*f* 56	m 66	*f* 75	**393**	f 66	*f* 66	f 66	m 66	f 66
339	f 66	*m* 56	f 77	f 66	m 66	**394**	*m* 56	f 55	f 55	*f* 56	*m* 65
340	m 66	m 88	*m* 67	m 77	*f* 56	**395**	*f* 56	f 66	*m* 87	m 55	m 55
341	f 66	m 66	*m* 56	*m* 55	f 55	**396**	*f* 66	*m* 65	*f* 65	*f* 66	m 66
342	f 77	f 66	*m* 67	*m* 67	f 66	**397**	*m* 67	f 66	m 66	f 66	m 66
343	*m* 56	f 66	m 76	m 66	*m* 77	**398**	m 67	m 77	f 67	m 65	*m* 67
344	*f* 76	*f* 67	*f* 87	m 77	m 66	**399**	f 55	f 66	*f* 55	m 55	m 55
345	m 55	f 55	m 56	m 55	*m* 65	**400**	*m* 76	*f* 56	f 55	*f* 67	m 55
346	*m* 56	*m* 65	*m* 67	*f* 66	*m* 76	**401**	f 66	m 76	m 66	m 66	f 66

402	m 75	f 77	m 77	f 66	*m* 87	
403	f 56	m 66	*f* 56	f 65	*f* 67	
404	m 76	m 65	f 66	f 66	*m* 66	
405	*f* 56	f 55	f 77	f 66	*m* 76	
406	*f* 76	*f* 66	*f* 67	f 66	m 66	
407	*m* 76	f 67	f 66	m 66	m 66	
408	m 65	f 77	f 77	*m* 65	*m* 76	
409	m 66	m 66	*m* 66	*f* 65	m 56	
410	f 86	*m* 76	*f* 67	*m* 87	*m* 77	
411	f 66	m 66	f 66	f 66	m 55	m 66
412	*m* 67	m 55	m 66	*f* 56	m 66	f 76
413	f 56	f 66	*m* 65	f 55	f 55	*m* 65
414	*m* 56	f 56	f 66	f 77	*f* 66	m 55
415	m 66	m 55	m 56	f 55	f 55	f 55
416	f 55	*f* 56	*f* 56	f 55	f 55	m 55
417	*f* 66	*f* 66	*f* 67	*f* 56	m 66	*f* 56
418	*m* 65	*m* 66	*f* 66	m 66	m 77	f 66
419	*m* 56	f 66	*m* 55	*f* 65	m 55	f 86
420	*m* 56	f 55	m 66	*f* 76	m 66	m 66
421	m 77	*m* 66	f 66	*f* 56	m 76	m 56
422	f 66	m 55	*m* 56	m 55	m 66	f 65
423	f 66	*f* 65	*m* 56	f 66	m 66	f 76
424	*f* 67	*f* 56	f 66	f 66	*f* 56	*f* 67
425	*f* 56	*f* 67	m 65	f 55	*m* 56	m 77
426	f 77	*m* 67	f 66	*f* 76	*f* 76	f 77
427	f 66	f 66	m 66	*m* 56	f 66	*m* 67
428	f 66	*f* 67	*f* 76	f 66	m 66	m 66
429	f 55	m 55	*f* 65	*m* 56	m 66	*f* 66
430	m 66	m 76	f 77	f 66	m 77	m 77
431	f 66	*m* 66	f 56	f 66	m 56	f 66
432	*m* 76	f 56	*m* 67	f 77	*f* 76	*f* 55
433	*m* 56	*f* 57	m 66	*f* 56	*m* 76	*f* 56
434	*f* 56	m 56	f 55	f 66	m 77	*m* 76
435	m 66	*m* 65	f 65	f 66	m 55	*m* 77
436	f 55	m 66	m 66	f 66	*f* 56	f 66
437	*f* 56	m 66	*m* 67	m 77	m 67	f 66
438	f 66	m 66	*m* 65	m 66	*m* 76	*m* 77
439	*m* 78	m 66	*m* 65	*m* 77	m 66	m 77
440	*m* 75	*m* 65	f 55	*m* 67	*f* 45	f 55
441	m 66	m 77	f 77	m 76	f 77	m 77
442	f 66	*m* 65	m 65	*m* 57	*f* 67	*f* 67
443	m 77	*m* 67	*f* 66	m 78	f 77	m 77
444	*f* 56	m 77	*m* 67	*f* 56	*f* 66	f 55
445	m 88	f 66	*f* 76	*m* 88	m 66	*f* 76
446	*f* 67	m 77	f 55	m 66	f 66	m 56
447	*f* 76	*m* 66	m 66	m 77	f 66	m 77
448	f 55	m 66	m 66	*f* 67	m 66	*f* 66
449	f 55	m 56	m 56	*f* 66	*f* 55	*m* 76
450	*m* 67	*f* 56	*m* 77	*f* 77	m 67	m 77
451	f 65	*f* 65	*f* 66	*f* 55	*m* 56	m 66
452	*f* 66	*f* 65	*f* 56	*f* 67	*f* 66	m 66
453	m 55	*f* 66	m 66	*m* 67	f 66	f 66
454	f 55	*m* 67	*m* 66	f 66	f 66	m 65
455	m 66	m 77	*m* 65	f 66	m 66	f 66
456	*m* 56	*m* 56	*m* 56	*m* 56	f 66	*m* 76
457	*f* 67	m 55	m 77	f 66	*f* 45	f 57
458	*m* 66	f 66	m 66	f 66	f 66	m 66
459	f 66	m 77	*m* 77	*m* 75	m 66	*f* 56
460	m 66	m 66	*m* 67	f 56	f 66	f 66
461	*m* 65	*m* 65	f 77	m 65	f 76	m 66
462	m 66	m 77	f 66	f 76	*f* 76	*m* 77
463	*m* 56	m 76	f 55	*m* 67	*m* 65	f 66
464	f 66	m 77	f 66	m 77	m 65	*m* 56
465	*f* 76	m 66	*f* 75	m 66	*m* 86	*m* 67
466	m 66	*m* 55	*f* 56	m 55	m 55	m 77
467	*f* 66	m 66	f 55	m 66	m 66	*f* 56
468	*f* 98	*m* 76	*f* 65	m 88	f 77	f 77
469	m 66	m 66	f 66	m 66	m 66	m 66
470	*f* 66	f 77	f 66	f 55	f 66	*f* 56
471	*f* 67	*m* 67	*f* 76	*m* 78	f 76	*f* 77
472	*m* 66	m 66	m 77	f 66	*m* 88	m 88
473	m 88	m 66	*f* 67	*m* 57	m 66	m 88
474	m 66	*m* 56	m 66	m 66	*f* 65	f 55
475	*f* 56	*f* 56	*f* 55	*m* 67	*m* 57	m 66
476	f 65	*f* 56	f 65	f 66	*f* 55	*m* 65
477	*f* 65	*m* 67	m 87	f 66	m 66	f 66
478	*f* 65	*m* 65	*m* 56	*f* 66	*m* 56	*m* 55
479	m 66	*m* 76	*m* 66	*m* 65	f 66	*f* 56
480	m 65	*m* 66	*m* 66	m 66	m 66	*m* 67
481	*m* 78	*m* 76	*m* 76	*f* 65	f 66	f 76
482	*m* 66	m 55	*m* 65	f 66	f 55	m 66
483	m 66	m 76	f 66	m 55	f 55	*m* 67
484	f 56	f 66	m 66	m 78	f 66	f 65
485	m 66	f 66	f 66	m 66	f 66	f 66
486	m 66	f 66	*f* 66	m 66	*m* 67	*m* 66
487	*f* 66	f 66	f 66	*m* 87	m 66	m 66
488	*m* 76	*f* 65	*m* 65	*f* 66	m 88	*m* 56
489	m 66	f 56	m 56	*f* 56	f 55	f 55
490	*f* 76	f 66	m 77	m 77	*f* 66	m 66
491	*f* 56	*m* 66	*f* 56	f 55	m 55	*f* 65
492	m 66	*m* 56	f 66	m 66	*m* 67	*f* 67
493	f 77	f 55	*m* 66	f 65	*f* 56	*f* 65
494	f 66	f 66	*m* 76	f 66	m 66	m 66
495	f 55	m 66	*m* 67	f 88	*m* 67	f 77
496	m 66	f 65	*m* 77	m 66	m 76	m 66
497	*m* 56	m 66	f 66	f 56	*m* 66	m 66
498	f 55	*f* 56	m 55	m 55	m 56	f 65
499	*m* 67	*m* 77	f 66	f 66	m 66	*m* 76
500	*f* 56	f 66	f 66	*m* 77	f 66	*m* 78
501	m 77	*m* 66	*f* 66	m 56	m 66	f 55
502	m 77	f 66	m 67	m 66	f 66	f 66
503	f 55	*f* 56	m 66	f 66	m 77	m 66
504	*m* 78	*f* 67	f 66	f 66	m 88	m 88
505	m 55	m 55	*f* 57	m 66	m 66	m 56
506	m 56	m 55	m 66	m 55	m 66	m 66
507	m 55	*m* 67	m 55	m 66	*f* 65	*f* 76
508	*m* 66	*m* 65	f 55	f 66	f 55	m 56
509	*f* 77	m 88	*f* 56	f 66	m 66	*f* 65
510	m 66	*m* 66	*m* 67	f 55	f 66	*m* 65
511	f 76	*m* 67	f 66	f 77	*m* 76	f 66

512	*f* 76	f 66	*m* 66	m 78	*m* 76	m 77	
513	f 65	m 65	m 66	*f* 67	m 76	m 66	
514	*f* 67	m 77	m 66	f 66	m 76	*f* 66	
515	m 66	f 77	*m* 76	*f* 67	*f* 67	*m* 87	
516	*f* 66	f 66	*f* 66	*m* 87	*m* 87	f 66	
517	m 77	m 88	*m* 56	*f* 66	m 67	f 88	
518	f 66	*m* 56	f 76	f 76	*f* 67	*f* 65	
519	m 66	*f* 66	*m* 76	f 66	f 66	*f* 66	
520	f 77	f 66	*f* 65	m 66	m 77	f 66	
521	m 77	*f* 78	*f* 66	*f* 67	*m* 67	*f* 56	
522	m 66	m 67	f 66	*f* 78	*f* 67	*f* 76	
523	f 55	*f* 66	m 55	m 66	m 66	f 55	
524	m 77	m 66	m 66	*m* 67	m 55	*f* 56	
525	m 66	m 66	f 56	m 55	m 55	*f* 65	
526	m 66	m 66	m 66	f 65	f 77	m 77	
527	f 77	*m* 77	m 77	f 66	f 66	f 77	
528	*m* 77	*f* 56	f 77	f 77	*m* 76	m 66	
529	*f* 55	*m* 56	f 55	f 55	m 66	m 66	
530	*m* 76	*m* 77	*f* 87	f 66	*f* 87	*f* 77	
531	*f* 65	f 66	*f* 56	*f* 56	m 65	f 76	
532	m 66	*m* 56	f 65	m 87	f 66	f 66	
533	*m* 57	f 66	*f* 56	*f* 56	m 66	m 66	
534	m 67	m 66	*f* 76	f 66	*f* 67	f 66	
535	m 66	f 56	f 56	f 66	f 66	*f* 66	
536	*m* 67	f 66	f 66	*f* 67	m 77	m 77	
537	f 66	m 66	m 55	m 66	*m* 65	f 66	
538	f 66	m 66	m 66	m 66	f 66	f 65	
539	m 77	*m* 56	*m* 56	*f* 67	f 66	*m* 57	
540	f 66	f 55	m 55	*m* 56	*f* 55	m 66	
541	f 66	*m* 56	m 66	f 66	m 66	f 66	
542	m 65	*f* 55	f 56	m 66	m 66	m 67	
543	*m* 78	m 77	f 66	*f* 77	f 67	*f* 66	
544	*f* 67	*f* 76	m 66	f 66	m 65	m 55	
545	*m* 66	m 66	f 66	*f* 56	m 65	m 56	
546	*m* 76	m 88	m 66	*f* 78	*m* 76	m 77	
547	*f* 65	f 55	*f* 56	m 76	*m* 65	m 76	
548	m 56	*f* 57	*f* 67	*f* 67	m 66	*f* 76	
549	*f* 76	*m* 66	*f* 87	*m* 87	*f* 86	*m* 76	
550	*m* 77	m 65	m 66	*m* 56	m 77	*f* 65	
551	*f* 66	m 66	m 77	*m* 76	m 67	f 66	
552	*m* 86	*f* 67	*m* 65	*f* 67	*f* 76	*f* 76	
553	*f* 76	*m* 67	*m* 76	f 66	*m* 88	f 66	
554	f 66	f 66	m 66	*f* 65	f 65	f 66	
555	f 55	f 77	*f* 77	*f* 66	f 66	f 66	
556	*f* 55	m 77	f 55	*m* 65	*m* 67	f 66	
557	*m* 88	*f* 67	*f* 56	*m* 66	*f* 77	m 77	
558	m 55	f 66	*f* 55	f 55	*m* 67	*f* 56	
559	*f* 65	m 66	*m* 87	*f* 56	*m* 66	*m* 56	
560	m 77	m 88	m 56	f 77	*f* 56	f 66	
561	*m* 65	*m* 56	*m* 57	f 55	*m* 87	*f* 56	
562	f 66	*m* 67	m 76	m 66	*f* 76	m 66	
563	f 55	*f* 67	f 55	*f* 66	m 66	*f* 56	
564	m 77	*f* 56	*m* 57	m 66	*f* 76	*f* 76	
565	f 77	m 78	m 66	*f* 66	*m* 76	m 77	
566	*m* 78	*f* 66	*f* 77	*f* 67	f 77	m 77	
567	*m* 77	f 77	*m* 78	m 77	*f* 67	f 77	
568	*f* 77	m 66	*f* 66	*f* 67	*m* 77	f 66	
569	f 66	*f* 56	*m* 57	*m* 56	m 66	m 55	
570	m 65	*m* 65	m 66	f 55	f 55	f 55	
571	f 66	*m* 65	m 66	*m* 88	*f* 65	*f* 67	
572	*m* 76	m 66	f 66	f 55	f 66	*f* 77	
573	m 56	m 66	f 66	f 56	*m* 56	f 55	
574	*f* 66	f 66	f 77	*f* 66	*m* 66	m 55	
575	*f* 78	f 77	f 66	m 55	f 76	*f* 67	
576	*m* 67	f 66	f 65	f 66	f 66	*f* 65	
577	*m* 56	*f* 67	m 77	*f* 56	f 66	f 66	
578	*m* 67	*f* 55	m 78	*m* 67	f 55	*m* 57	
579	*f* 56	f 55	f 66	*f* 66	m 66	m 66	
580	m 55	*f* 56	f 66	m 55	*f* 75	*f* 67	
581	*m* 65	m 76	f 66	*m* 88	*f* 55	f 65	
582	m 77	m 77	f 66	m 67	*m* 87	m 66	
583	*m* 56	m 77	f 55	f 66	*m* 56	*m* 76	
584	*f* 66	m 77	f 66	f 55	f 55	m 66	
585	*f* 78	*f* 77	f 77	m 66	f 77	*m* 76	
586	m 66	*m* 56	*m* 67	*f* 67	*m* 56	*m* 65	
587	f 66	f 55	*m* 77	*m* 78	m 76	*m* 65	
588	*m* 76	m 66	m 77	m 55	*f* 75	*f* 67	
589	*f* 56	m 56	m 76	f 55	*f* 76	f 77	
590	m 55	*f* 75	*m* 67	f 77	*m* 66	*f* 76	
591	f 66	m 66	f 66	*m* 56	*m* 56	m 55	
592	*f* 56	f 66	*m* 67	m 55	*m* 76	m 66	
593	f 66	m 55	f 65	*m* 56	f 76	*f* 66	
594	m 66	f 55	m 66	m 66	m 77	f 56	
595	m 77	f 66	*m* 76	*m* 87	m 77	*m* 66	
596	m 56	*m* 67	f 56	m 66	m 56	*m* 76	
597	*f* 65	f 66	*f* 67	*m* 66	f 55	f 66	
598	f 77	*f* 77	*f* 66	*f* 66	*f* 76	m 66	
599	m 67	*m* 78	m 77	*f* 67	*f* 78	*f* 56	
600	*f* 78	m 77	f 66	f 55	m 65	*m* 56	*f* 57
601	*f* 65	m 66	*m* 66	m 77	m 55	f 66	f 66
602	*m* 66	*m* 65	f 56	f 66	m 55	f 56	f 66
603	f 66	*m* 57	m 77	m 77	*f* 67	f 65	f 77
604	f 56	*f* 65	m 66	f 55	m 76	*m* 66	*m* 56
605	*m* 57	*f* 56	f 55	m 66	*m* 56	*m* 67	*f* 65
606	m 76	*m* 87	m 77	*f* 66	f 88	m 66	*f* 76
607	m 66	f 56	f 55	*m* 65	*f* 56	f 55	m 55
608	m 56	*f* 76	m 66	m 77	*m* 77	f 66	m 66
609	f 55	f 66	*f* 77	m 66	f 66	f 66	*f* 65
610	f 55	*m* 67	m 55	m 66	m 65	f 55	*f* 56
611	f 66	m 66	f 66	*m* 66	f 66	m 66	f 66
612	*m* 66	f 66	m 77	m 66	*m* 56	*m* 56	*m* 56
613	*f* 66	m 66	f 55	f 55	*m* 67	m 66	*m* 67
614	m 66	*m* 76	*m* 56	*m* 65	*m* 56	m 65	*m* 56
615	*f* 76	m 66	*f* 76	m 77	m 76	m 66	*f* 78
616	f 55	*m* 76	m 66	*f* 56	f 66	*m* 66	*m* 67
617	m 77	*f* 66	f 66	*m* 67	*f* 67	m 66	*m* 66
618	*m* 77	*f* 67	f 87	f 77	*m* 77	*m* 77	*f* 77
619	m 66	*f* 65	*f* 56	f 56	m 66	m 87	f 66
620	f 67	m 66	m 76	f 65	f 77	f 66	m 66
621	f 87	*m* 67	f 77	*m* 66	m 76	*m* 76	m 66

622	m 66	*f* 56	m 66	*f* 65	m 77	f 66	*m* 56
623	*m* 78	*m* 56	f 66	f 56	f 66	*m* 67	m 76
624	*m* 56	*f* 65	m 55	*f* 65	*m* 56	*f* 66	m 55
625	f 55	m 66	f 55	*f* 56	f 56	f 65	f 66
626	m 66	*f* 65	m 88	f 66	*f* 67	*m* 67	f 77
627	f 66	f 66	f 66	m 66	f 77	*m* 67	m 55
628	m 55	*m* 56	m 66	*m* 67	f 66	m 66	*m* 76
629	*m* 55	m 66	m 77	f 66	f 66	m 66	f 66
630	*m* 78	f 76	*m* 56	*m* 56	*m* 66	*m* 56	*f* 56
631	*m* 76	*m* 56	m 77	*m* 56	*m* 66	m 66	f 66
632	f 55	f 55	m 55	*f* 65	f 55	f 55	m 55
633	*f* 56	f 66	f 66	*f* 76	f 67	f 66	m 66
634	*m* 67	m 55	m 66	f 65	*m* 65	f 55	m 55
635	f 55	m 66	m 55	*m* 76	f 65	m 77	m 77
636	m 55	*f* 56	m 66	f 55	*m* 56	f 55	*m* 56
637	m 77	m 66	f 77	f 66	f 57	*f* 76	*f* 76
638	m 66	m 77	f 55	*f* 77	*f* 66	*m* 67	m 77
639	*f* 56	*f* 66	*f* 55	*f* 56	m 65	m 66	*f* 66
640	f 55	f 66	*m* 77	*m* 67	f 66	*f* 67	*m* 77
641	m 66	f 77	f 55	m 56	*m* 56	f 55	*f* 45
642	f 66	m 65	f 66	*m* 76	f 65	f 66	*m* 56
643	*f* 76	f 66	f 66	m 88	*f* 67	*f* 66	*f* 76
644	*m* 76	m 77	f 66	f 66	*m* 76	f 66	m 77
645	f 55	f 55	*m* 65	f 55	*f* 56	f 55	m 55
646	m 66	f 66	m 66	f 65	f 66	f 66	m 66
647	m 66	m 77	m 66	f 78	*f* 66	*m* 76	m 78
648	m 55	*m* 56	m 77	*m* 56	*f* 66	m 77	*f* 56
649	*m* 56	*f* 67	f 55	m 66	*f* 65	f 55	*f* 65
650	*m* 56	f 66	m 77	m 56	*m* 67	*m* 67	f 77
651	*m* 67	m 77	f 77	*m* 67	*m* 78	m 77	m 77
652	f 66	m 66	f 56	m 66	f 66	m 66	m 66
653	*f* 65	f 66	f 66	f 55	*m* 77	f 66	f 66
654	*m* 67	m 77	*f* 76	*m* 66	m 66	f 66	*m* 65
655	*m* 67	*f* 56	f 55	f 55	f 77	f 66	f 66
656	*f* 66	m 77	f 66	*m* 56	m 66	f 66	m 66
657	m 66	*m* 56	f 66	f 77	*m* 65	m 66	m 66
658	m 65	m 66	*m* 65	*m* 56	f 66	*m* 56	m 55
659	*f* 65	*f* 66	*f* 56	f 55	*f* 65	f 66	*f* 66
660	*m* 67	m 66	m 66	m 77	f 77	f 77	m 77
661	m 66	*m* 76	f 66	f 55	*m* 76	*f* 56	m 66
662	*f* 66	*f* 76	*f* 66	*m* 77	m 66	m 67	m 76
663	f 66	f 66	f 67	f 66	f 66	*m* 77	m 87
664	m 66	m 87	*f* 56	*f* 56	f 66	*f* 56	m 66
665	m 56	m 77	*m* 67	m 66	m 66	m 77	f 66
666	*m* 56	*m* 67	*m* 87	*m* 76	m 66	m 77	m 66
667	m 77	*f* 65	*m* 88	m 77	*m* 87	f 66	*f* 76
668	f 55	*f* 56	*f* 67	m 66	*m* 76	*m* 56	*m* 66
669	f 56	f 55	f 55	m 55	m 66	f 66	m 55
670	f 66	*f* 78	*f* 56	*f* 66	m 66	*m* 56	*f* 67
671	f 66	f 77	m 77	*m* 76	*m* 76	f 77	m 66
672	f 77	f 76	m 77	*f* 78	*m* 67	*f* 87	*m* 76
673	*f* 67	f 67	*m* 88	*f* 66	f 66	m 66	*m* 66
674	m 77	f 55	m 66	*f* 65	*f* 67	*m* 66	*m* 66
675	m 66	m 76	f 77	m 66	m 87	m 66	*m* 67
676	f 55	m 66	*m* 56	*f* 56	f 66	*f* 65	f 66

677	*m* 56	f 65	f 55	*f* 65	*f* 67	f 65	*m* 56
678	*m* 76	m 55	m 66	m 55	f 76	*f* 56	m 66
679	*f* 67	f 77	*f* 66	*m* 65	*f* 67	*m* 86	m 66
680	m 55	*m* 66	*f* 76	m 66	*f* 67	*m* 56	*f* 76
681	*f* 66	m 66	m 66	m 55	*m* 87	f 66	m 66
682	*m* 87	*f* 65	m 88	f 75	*m* 76	m 66	m 55
683	m 55	f 66	m 87	m 77	f 77	*m* 87	f 66
684	*m* 76	*m* 76	f 66	f 66	f 66	f 55	m 55
685	*m* 76	*f* 66	*m* 76	*f* 87	m 66	f 77	f 66
686	f 55	m 66	*m* 67	m 77	m 66	m 66	f 55
687	m 66	m 66	*f* 67	m 66	*m* 56	m 66	*f* 65
688	m 66	m 66	m 67	m 66	m 66	m 66	m 66
689	*m* 67	*m* 86	*m* 78	m 66	*f* 66	*f* 76	f 66
690	m 77	m 88	m 77	f 76	*m* 67	*m* 78	f 77
691	*m* 87	*f* 66	*f* 56	m 66	m 66	*m* 67	*m* 67
692	f 77	f 66	m 67	f 77	*f* 56	f 77	*f* 66
693	m 65	*m* 67	*f* 76	m 66	*f* 88	f 55	m 77
694	m 55	*m* 67	*f* 65	m 66	*m* 76	m 66	*m* 56
695	*m* 66	m 66	*m* 66	f 67	m 66	f 66	m 66
696	m 66	f 66	m 66	f 77	*f* 66	m 56	*f* 57
697	*f* 76	f 55	f 55	*f* 66	*f* 76	*f* 56	*m* 56
698	*f* 66	*f* 76	f 66	m 88	f 66	m 77	f 76
699	*m* 67	f 55	*m* 56	m 55	m 66	m 66	f 66
700	m 66	f 66	m 76	*m* 77	*f* 67	*m* 77	*m* 66
701	f 66	m 66	*f* 66	m 66	*m* 77	*f* 66	m 65
702	*f* 66	*f* 67	m 77	m 77	*m* 66	m 77	*m* 65
703	*m* 56	*f* 76	*m* 67	*m* 88	m 66	m 66	*f* 76
704	m 55	*f* 65	f 66	m 66	*m* 56	m 55	*m* 66
705	m 66	m 66	*m* 68	f 66	m 66	f 66	f 55
706	f 77	f 66	m 66	*f* 67	*f* 67	*m* 56	*f* 75
707	f 66	*m* 65	*m* 76	f 66	m 66	f 66	m 66
708	m 77	*f* 78	f 77	*f* 68	m 66	f 67	*m* 67
709	f 66	m 77	f 77	m 67	*f* 56	*m* 76	m 76
710	f 55	m 77	f 66	*f* 67	*m* 56	m 66	m 55
711	m 66	*m* 65	m 66	*m* 56	*f* 65	m 66	*f* 66
712	m 76	f 55	*m* 67	*f* 56	m 67	m 55	*f* 56
713	m 66	*m* 67	*f* 56	*f* 65	f 66	*m* 65	*m* 56
714	f 76	f 55	*m* 56	m 66	f 66	m 66	f 55
715	*m* 89	*f* 78	*f* 88	m 88	*m* 810	*m* 87	m 89
716	m 66	f 65	m 55	m 65	m 66	m 66	f 56
717	f 87	*m* 76	m 76	*f* 66	*f* 66	*f* 65	*m* 58
718	*m* 66	f 66	m 66	m 66	m 56	f 77	f 66
719	*f* 56	*f* 67	*m* 77	f 77	*m* 77	*f* 65	*m* 78
720	f 77	m 67	m 67	m 66	m 78	f 88	m 77
721	*m* 66	m 66	m 56	f 66	*f* 66	m 66	m 66
722	*m* 67	*f* 76	*f* 76	f 66	*m* 67	m 79	*f* 76
723	*m* 66	*m* 77	*f* 65	*f* 67	*f* 65	*f* 56	m 66
724	*m* 88	*f* 77	*f* 56	f 66	m 77	f 77	m 77
725	f 55	f 66	*f* 56	*f* 65	*f* 56	m 77	m 66
726	*m* 76	m 77	f 87	m 66	*m* 65	f 77	*f* 66
727	m 77	*f* 76	m 76	m 66	*f* 77	*f* 76	f 76
728	m 55	m 66	m 66	*m* 56	m 66	f 66	*f* 56
729	f 66	m 66	f 55	m 56	f 77	*m* 66	m 66
730	*m* 66	*m* 56	m 66	*f* 66	f 55	*m* 65	*m* 76
731	m 66	f 55	m 55	*m* 65	m 55	*f* 65	m 55

732	*f* 65	m 66	*f* 76	f 55	f 66	m 77	*m* 77	
733	m 66	m 55	f 56	*m* 66	*m* 67	m 77	m 66	
734	m 77	*f* 67	*f* 57	*m* 77	*f* 76	m 66	f 66	
735	*f* 67	f 77	f 66	*m* 76	m 66	*m* 87	*m* 56	
736	f 66	f 66	*m* 57	f 56	f 66	*f* 56	*m* 76	
737	*f* 66	*m* 78	*f* 65	f 55	f 66	m 65	*m* 66	
738	f 66	f 66	f 66	m 77	*m* 67	*f* 76	*f* 56	
739	f 66	f 55	m 66	f 77	*m* 86	m 66	m 77	
740	m 65	m 66	f 66	f 66	m 77	*m* 76	f 66	
741	*f* 67	f 66	*f* 57	*m* 77	*m* 76	*m* 67	*m* 87	
742	f 66	m 66	*f* 65	*f* 66	*m* 55	*m* 56	*f* 56	
743	*f* 56	m 77	f 67	*f* 56	m 78	*f* 86	m 77	
744	m 77	f 65	*f* 76	f 66	m 66	m 76	f 66	
745	f 66	m 66	*f* 55	m 66	*m* 66	*m* 76	*m* 56	
746	*f* 66	*m* 67	m 88	*m* 67	f 56	m 77	f 77	
747	f 77	m 67	*f* 76	m 66	*f* 78	*m* 67	f 66	
748	m 66	f 66	*f* 56	*f* 56	f 66	*f* 56	*m* 65	
749	f 66	m 66	f 55	*f* 76	f 66	m 77	m 68	
750	*m* 66	f 65	*m* 67	m 66	*m* 56	*f* 56	f 66	
751	f 66	f 66	*m* 66	m 77	*m* 78	*m* 77	*f* 66	
752	m 66	f 55	*f* 65	m 66	f 66	f 56	m 66	
753	m 77	m 66	m 67	m 77	*f* 56	*m* 67	f 66	
754	*f* 56	m 56	f 67	m 66	*f* 66	m 76	f 66	
755	f 77	f 66	*m* 77	*m* 77	*f* 76	*f* 76	*m* 65	
756	f 55	*f* 56	*f* 56	*f* 56	m 77	*m* 86	*f* 65	
757	m 66	m 77	m 66	f 77	*f* 76	m 66	m 77	
758	f 66	f 66	*f* 76	m 77	f 66	m 55	f 66	
759	m 55	f 55	m 55	f 55	m 65	f 55	f 55	
760	m 55	*f* 65	*f* 56	*m* 65	f 55	*m* 56	m 55	
761	m 66	f 55	f 66	f 56	m 66	*m* 66	*f* 76	
762	m 66	m 88	*m* 67	*f* 78	f 66	*f* 56	m 66	
763	*f* 77	m 77	f 66	f 66	*f* 56	f 55	f 66	
764	*f* 76	*m* 67	m 66	f 66	f 66	m 66	m 77	*f* 56
765	*f* 78	f 66	m 88	m 77	*m* 77	*f* 66	*m* 78	*m* 87
766	*m* 56	m 66	*m* 67	m 66	*m* 56	m 66	m 55	*m* 56
767	*m* 66	m 67	*f* 76	m 77	m 77	*f* 67	f 65	*m* 66
768	f 66	*f* 57	*m* 56	m 66	f 67	f 66	*m* 75	f 56
769	*m* 66	*f* 56	f 66	*f* 75	m 65	m 66	f 66	*f* 65
770	m 66	m 66	*f* 78	f 66	f 66	m 77	f 77	m 77
771	m 66	*f* 76	f 66	f 66	*m* 56	*f* 76	*m* 56	m 66
772	*m* 65	*m* 68	*f* 67	f 66	m 66	*m* 67	f 55	*m* 67
773	f 66	*f* 67	*m* 76	f 66	f 77	f 66	*f* 76	m 67
774	*f* 66	*m* 56	*f* 76	*f* 66	*m* 76	*m* 76	m 66	m 66
775	*m* 77	*f* 66	f 66	m 67	*m* 67	*m* 76	f 55	f 55
776	f 55	*m* 87	*f* 76	m 66	*f* 55	*f* 56	m 77	m 66
777	f 66	m 66	f 66	f 66	m 67	f 55	*f* 76	m 66
778	*m* 66	m 66	*m* 65	m 66	*m* 65	m 56	m 66	m 66
779	f 55	*m* 56	f 77	*m* 65	f 55	m 55	m 66	*f* 76
780	m 55	*f* 56	m 66	m 66	f 55	m 66	f 56	f 56
781	*m* 87	*f* 66	*f* 56	*m* 78	*m* 56	f 56	*f* 77	*f* 67
782	f 66	m 66	f 55	*f* 65	m 66	m 66	f 66	*f* 67
783	*m* 56	f 66	f 66	*f* 55	f 55	f 65	m 55	*m* 67
784	*m* 67	m 66	m 55	f 66	*m* 76	m 66	f 66	f 77
785	m 66	*f* 87	*f* 57	*f* 98	*f* 67	*f* 78	*m* 65	f 77
786	*f* 56	*m* 56	m 66	f 66	m 66	f 55	m 66	*f* 56

787	m 66	m 56	f 66	m 66	*m* 56	f 66	*f* 56	*f* 67
788	*m* 56	f 77	*m* 56	m 66	m 65	m 77	m 66	*m* 56
789	f 55	*m* 56	*f* 66	*m* 67	m 77	*m* 56	m 76	f 66
790	m 55	*f* 56	*f* 66	m 65	*m* 56	f 77	*f* 56	*f* 65
791	m 77	m 76	m 77	m 66	f 66	f 76	m 77	f 76
792	f 77	m 66	f 76	m 66	m 77	*f* 66	f 66	f 66
793	*f* 65	m 66	m 66	*f* 56	m 55	m 66	m 66	*m* 56
794	f 55	f 66	*f* 56	m 77	m 77	m 66	m 66	*f* 66
795	*m* 56	*f* 56	f 65	m 77	m 56	*m* 67	m 66	f 56
796	*f* 66	f 55	*m* 66	*m* 87	f 66	m 66	*m* 67	m 66
797	f 55	*f* 76	f 77	m 55	*m* 77	f 66	*f* 56	f 77
798	*m* 75	f 55	*f* 57	m 55	*m* 66	m 56	f 55	*m* 66
799	m 66	*f* 87	f 66	m 77	m 56	f 66	f 77	f 65
800	*m* 57	f 66	*f* 76	f 66	*m* 56	*m* 57	m 66	m 66
801	*f* 65	m 56	*f* 56	m 66	*f* 56	*f* 66	*f* 65	f 66
802	*m* 77	m 56	*m* 67	f 55	f 66	m 66	f 55	m 55
803	f 66	*m* 87	m 77	m 77	f 88	m 77	*m* 67	m 67
804	f 66	f 66	*f* 66	*f* 66	*m* 65	*m* 56	m 66	f 66
805	*m* 56	f 56	m 55	m 55	f 66	*m* 56	*f* 56	m 66
806	f 66	m 77	f 66	*m* 89	*f* 67	f 66	f 77	m 77
807	m 66	*f* 56	*m* 76	m 66	f 66	*f* 65	f 55	*m* 76
808	*f* 78	m 77	*f* 67	f 77	*m* 78	*m* 78	*f* 77	*f* 57
809	*f* 76	*f* 77	*f* 56	m 66	f 76	f 66	*m* 67	m 77
810	*m* 67	*f* 67	f 76	*f* 76	*f* 67	f 66	f 77	m 66
811	*f* 57	m 66	*f* 65	*f* 56	*f* 56	f 56	*m* 56	*f* 66
812	m 77	f 55	*f* 67	*f* 66	f 77	m 86	*m* 67	*f* 75
813	*m* 78	f 55	*f* 55	m 55	f 55	*m* 57	f 66	m 56
814	m 67	f 66	*m* 65	m 55	m 77	*m* 87	f 66	m 77
815	*m* 76	m 77	m 66	*f* 56	m 56	m 66	f 67	f 77
816	*f* 56	*f* 56	*m* 56	*m* 56	*f* 65	*m* 76	*f* 66	*m* 76
817	*f* 56	*m* 78	m 77	f 66	m 57	f 66	m 77	f 66
818	m 77	*f* 65	f 66	*f* 76	*f* 56	*f* 66	*f* 66	*m* 87
819	m 66	f 66	f 77	*f* 66	*m* 86	f 66	m 66	m 66
820	f 67	f 66	f 66	m 66	*m* 66	*f* 66	*f* 66	*f* 76
821	*f* 75	*m* 56	*m* 77	*m* 76	f 66	f 56	*f* 66	*m* 67
822	*m* 67	*m* 76	*m* 88	*f* 77	*m* 76	*m* 87	f 66	f 66
823	m 65	f 55	f 55	f 55	m 55	m 55	m 66	m 66
824	*m* 66	*m* 56	m 77	m 67	*f* 67	*f* 76	*f* 56	*m* 88
825	m 77	m 88	m 77	*f* 76	f 66	m 67	*m* 78	*f* 76
826	*f* 76	*m* 76	m 66	f 66	f 66	m 66	f 66	m 66
827	m 66	f 55	f 66	f 56	*f* 65	m 56	m 55	m 77
828	m 56	*f* 65	*m* 77	*f* 56	*f* 67	m 88	*m* 78	m 77
829	*m* 76	m 77	m 55	f 77	f 66	m 66	f 76	*f* 67
830	m 56	m 77	*m* 65	f 66	*f* 65	f 66	*m* 76	f 66
831	m 77	m 78	f 77	m 88	f 88	*m* 7[illegible]	*m* 78	f 88
832	m 76	f 66	m 66	m 77	*m* 88	*m* 67	*f* 66	m 66
833	*m* 78	*f* 76	f 77	*f* 76	f 76	*m* 67	f 66	*f* 66
834	f 65	m 77	m 77	m 66	m 65	m 77	*f* 66	m 55
835	m 66	m 66	m 66	*f* 76	f 77	*f* 65	*f* 76	*f* 66
836	*m* 66	f 55	m 65	f 66	m 55	*f* 56	*m* 65	*m* 76
837	*f* 65	*f* 56	m 55	m 55	f 55	*f* 66	m 55	m 55
838	m 66	*f* 56	m 55	m 66	f 55	f 55	*m* 56	m 66
839	f 66	*m* 89	*m* 67	m 77	m 66	f 66	f 66	f 77
840	f 65	m 66	*f* 57	*f* 66	f 66	f 66	*f* 65	f 76
841	*m* 75	f 55	f 66	m 77	f 55	f 55	f 55	*m* 65

842	m 55	*f* 76	*m* 78	f 77	f 66	f 77	m 66	*m* 66	
843	m 56	*f* 66	*f* 56	*f* 56	*m* 66	*f* 65	m 77	*m* 56	
844	*f* 54	*f* 56	*m* 56	*f* 65	f 55	*f* 75	*f* 55	*f* 65	
845	m 66	m 55	m 76	*m* 67	f 66	*m* 56	*f* 66	m 55	
846	m 55	f 66	m 66	*f* 66	f 66	m 65	m 77	m 65	
847	*f* 76	m 66	f 66	*f* 56	m 66	f 66	f 66	*f* 57	
848	f 66	f 56	m 77	*m* 66	*m* 66	*f* 66	m 66	m 66	
849	*f* 76	f 66	f 55	*f* 67	*m* 86	m 66	m 66	m 55	
850	f 87	*m* 67	*f* 66	*m* 65	*f* 66	*m* 77	m 77	f 76	
851	f 56	m 66	m 66	m 66	m 66	m 66	m 66	f 66	
852	f 77	*m* 87	*f* 57	m 77	*m* 76	*m* 66	m 66	*f* 76	
853	m 55	f 56	f 55	f 76	m 55	m 65	*f* 65	f 56	
854	*m* 86	m 77	m 77	*m* 76	m 77	m 66	*m* 67	*m* 76	
855	*f* 65	m 66	m 77	*m* 56	m 66	m 56	f 66	m 66	
856	m 66	m 55	m 67	m 66	f 66	m 67	m 67	f 55	
857	f 66	f 66	*f* 56	m 66	*m* 56	m 66	*m* 65	f 55	
858	f 55	f 77	f 66	m 88	m 66	f 66	*f* 65	f 77	
859	f 76	*f* 77	*m* 67	*f* 77	*f* 77	*f* 76	*m* 66	*m* 66	
860	*m* 68	*m* 58	f 77	*f* 55	*f* 67	*f* 55	*f* 67	f 67	
861	f 66	f 66	*m* 76	*f* 56	*f* 65	*m* 67	m 66	*m* 56	
862	*m* 56	f 56	*m* 67	m 55	f 66	m 66	f 56	f 66	
863	*f* 65	m 77	m 66	*f* 67	*f* 57	*f* 65	f 66	m 66	
864	m 66	*f* 67	m 77	m 77	m 77	f 77	*f* 87	f 78	
865	m 55	m 77	*m* 67	*f* 56	f 66	*f* 66	f 55	*f* 56	
866	*m* 77	m 87	*f* 67	*m* 87	*f* 67	m 77	m 66	m 66	
867	*m* 55	*f* 56	*m* 67	*f* 56	*f* 65	m 77	*f* 65	f 55	
868	*f* 76	m 77	m 77	m 55	*f* 66	f 66	f 66	*m* 77	
869	m 77	*m* 67	m 66	m 66	f 76	m 66	m 67	m 77	
870	*m* 77	*f* 78	f 66	*m* 66	*m* 78	*f* 88	*f* 88	*m* 78	
871	*m* 87	m 66	*m* 78	*f* 66	*m* 56	*f* 67	*m* 77	f 66	
872	f 65	*m* 67	m 66	m 66	*m* 78	f 55	f 55	*m* 76	
873	f 77	*m* 67	m 66	m 66	*m* 67	m 77	*f* 56	f 66	
874	*f* 76	*m* 76	m 77	*f* 76	m 77	m 77	m 77	*f* 76	
875	m 88	*f* 77	*m* 77	*m* 67	f 56	*m* 78	*f* 77	m 88	
876	f 55	f 77	f 66	m 55	f 77	m 77	f 55	*m* 66	
877	f 55	f 55	f 55	m 66	m 66	f 55	*f* 65	*m* 76	
878	m 77	m 56	f 66	*m* 78	f 66	*f* 66	m 76	f 66	
879	*f* 66	f 66	m 66	f 66	m 55	m 66	f 76	m 66	
880	*m* 77	m 87	f 77	m 88	f 98	*f* 88	*m* 88	*m* 78	m 77
881	f 55	f 55	f 44	*f* 56	m 76	f 55	f 55	m 65	m 55
882	*f* 87	f 77	*f* 78	*m* 76	f 88	*m* 66	*f* 77	*f* 57	f 67
883	m 88	m 77	m 66	*m* 76	f 66	f 65	m 77	f 66	m 56
884	f 66	f 66	*m* 87	m 66	f 78	f 77	m 66	*f* 67	m 66
885	*m* 76	m 55	*f* 75	m 55	*f* 76	m 56	f 66	*f* 56	*f* 65
886	*f* 76	*m* 87	m 75	f 75	*f* 66	m 67	m 77	f 65	*f* 67
887	m 65	*m* 56	f 66	m 55	f 66	m 66	f 77	*f* 76	m 77
888	m 66	f 66	m 66	*f* 56	m 66	m 55	*m* 56	*f* 56	*m* 56
889	f 66	*f* 56	*f* 56	*m* 77	m 77	f 66	f 76	m 88	m 88
890	f 66	m 66	f 66	*m* 76	*m* 65	f 76	m 66	*m* 78	*f* 67
891	m 66	m 66	*m* 65	*m* 67	*f* 65	m 55	f 55	f 55	f 65
892	*f* 56	m 55	m 66	m 55	m 55	m 55	m 66	f 55	*m* 56
893	m 55	f 55	*f* 65	*f* 56	f 55	*m* 56	f 55	*m* 65	*f* 57
894	m 55	f 56	*f* 66	m 67	f 66	*f* 56	m 55	m 66	f 66
895	*f* 76	f 66	f 66	*f* 87	*m* 67	*m* 56	*f* 78	*m* 67	*f* 67
896	*f* 76	f 66	*f* 67	m 66	*m* 77	m 78	f 66	f 55	*f* 76

897	f 66	*m* 78	f 76	m 77	m 77	*f* 67	*f* 67	f 66	*m* 67	
898	*m* 77	*m* 88	f 88	f 88	*m* 78	*m* 88	m 88	*f* 87	m 77	
899	m 77	*f* 66	f 76	m 55	m 66	f 55	m 67	m 77	f 66	
900	f 55	m 75	f 66	m 55	*m* 76	*f* 55	f 55	*f* 57	*f* 67	
901	f 66	m 77	*f* 65	*f* 76	f 55	m 66	f 66	m 66	*m* 66	
902	f 77	m 66	f 87	f 77	m 77	*f* 67	*f* 67	*f* 87	f 77	
903	f 55	m 66	m 77	m 77	m 77	*f* 66	f 66	*f* 65	m 66	
904	*f* 56	*f* 56	f 66	m 77	f 66	*f* 67	m 76	m 77	m 66	
905	f 66	f 66	f 66	*m* 67	m 77	*m* 78	f 66	f 66	*m* 76	
906	m 55	*f* 56	*m* 76	*f* 57	*m* 56	f 55	f 55	m 55	m 67	
907	f 55	*f* 66	*m* 57	*m* 65	*f* 57	f 55	*m* 65	f 77	f 55	
908	*m* 68	*m* 78	m 88	m 77	m 77	m 66	*f* 67	*m* 78	*m* 67	
909	f 55	m 77	f 55	f 55	m 66	f 55	m 66	*f* 56	*m* 56	
910	m 56	*m* 67	f 55	*f* 65	*f* 56	m 66	*f* 65	*m* 76	*m* 65	
911	f 66	*m* 55	m 66	*m* 66	*m* 66	m 77	f 66	f 66	*m* 56	
912	*f* 55	*m* 55	*f* 56	f 66	*m* 56	*m* 86	*m* 67	m 66	*m* 56	
913	m 88	*f* 76	f 66	m 75	m 66	*f* 65	*f* 67	m 55	f 66	
914	f 66	m 66	m 66	m 87	m 55	f 55	f 55	m 55	f 66	
915	m 77	m 66	*f* 65	f 55	m 66	f 65	m 66	*f* 67	f 66	
916	f 55	m 55	f 66	f 66	f 66	f 77	m 66	f 66	f 66	
917	m 76	m 67	m 88	*m* 87	*m* 77	*m* 87	m 67	*m* 76	f 66	
918	*f* 56	*f* 56	*f* 56	*f* 56	f 56	*f* 65	*f* 65	*f* 56	m 55	
919	*f* 57	f 66	f 65	m 88	f 55	*m* 67	*f* 66	*m* 66	*m* 77	
920	f 66	f 66	*m* 66	m 77	m 77	f 66	m 66	m 88	m 77	
921	*m* 76	f 66	*m* 78	f 77	*f* 55	m 77	m 66	*m* 78	*m* 67	
922	*m* 56	f 66	f 55	m 55	*m* 56	f 65	m 66	*f* 65	m 66	
923	f 77	m 87	*m* 67	*f* 87	f 65	*m* 77	m 88	*m* 86	m 77	
924	f 56	f 55	m 66	*f* 65	*m* 76	*f* 76	f 66	m 66	*f* 56	
925	m 66	m 66	m 65	*m* 76	m 66	*m* 75	*f* 66	m 66	f 66	
926	m 88	f 66	f 66	*f* 65	*m* 87	m 76	*m* 67	f 66	m 77	
927	*f* 66	*f* 55	f 66	*f* 66	*f* 66	*f* 65	m 65	m 67	*f* 66	
928	f 56	m 66	m 55	*m* 56	*m* 56	*f* 56	*m* 56	f 56	*m* 65	
929	m 56	f 66	f 66	m 66	*f* 56	m 66	f 66	f 66	f 66	
930	f 76	m 57	*m* 66	f 65	*m* 67	f 66	*f* 57	m 66	f 66	
931	m 55	m 55	*m* 76	f 55	f 55	f 55	m 55	m 55	f 55	
932	*f* 67	f 77	m 65	*m* 77	f 55	*m* 76	*m* 77	f 77	*f* 67	
933	m 77	m 66	m 66	*f* 76	f 66	*f* 67	f 65	f 66	f 65	
934	f 66	f 77	f 56	m 66	f 66	m 66	f 66	*m* 76	f 66	
935	*f* 65	m 77	*f* 66	m 66	f 66	*f* 76	f 55	m 66	f 66	
936	*m* 76	*m* 66	*m* 78	f 66	m 66	*m* 88	f 66	*f* 76	m 77	
937	m 55	*f* 56	f 56	*m* 56	*m* 56	f 56	f 76	f 67	f 65	
938	*f* 56	*f* 66	*f* 66	*m* 55	*f* 57	*m* 67	*m* 66	m 76	m 86	
939	m 66	*m* 77	m 66	*f* 56	f 66	m 88	*f* 56	f 77	f 55	
940	f 56	f 55	f 55	f 56	m 55	m 66	m 77	m 77	*f* 56	
941	f 66	m 65	m 66	m 66	m 66	f 55	f 66	m 66	f 66	f 66
942	*f* 67	m 66	m 66	*m* 56	m 55	f 66	m 56	*m* 75	*f* 76	*m* 67
943	*m* 67	f 66	*f* 56	f 66	*m* 65	f 55	*m* 88	m 55	f 55	*f* 76
944	f 66	*f* 76	*f* 78	m 77	f 77	*f* 67	m 77	f 66	*f* 67	m 77
945	f 66	f 66	*m* 66	f 66	*f* 66	f 66	f 66	*f* 76	*m* 56	*f* 56
946	m 88	*m* 67	m 77	f 66	*f* 57	*m* 66	m 55	*m* 76	*f* 65	f 66
947	m 77	*f* 66	*f* 67	*f* 56	*m* 77	f 66	*m* 77	*m* 76	*m* 66	m 77
948	f 66	m 55	m 77	m 76	*m* 86	*f* 65	f 55	f 56	*f* 67	m 66
949	m 66	*f* 67	f 66	m 66	f 66	m 66	*f* 65	*m* 66	m 66	*m* 56
950	f 55	*f* 65	f 66	f 66	m 55	*m* 76	*m* 67	f 66	*m* 65	m 66
951	*f* 56	*f* 56	f 55	m 77	*m* 66	*m* 56	m 77	m 57	*f* 65	*f* 65

952	f 66	*m* 66	*m* 56	m 66	f 66	f 77	f 55	m 65	m 66	*m* 65					
953	m 77	*f* 65	*f* 76	m 77	m 77	f 66	m 66	*m* 65	f 55	f 66					
954	f 65	*f* 66	f 66	*m* 65	f 66	f 55	m 77	m 77	m 75	*f* 56					
955	f 66	*f* 65	*m* 65	*m* 66	f 66	*m* 65	*f* 66	*f* 57	m 66	*m* 56					
956	*f* 87	*f* 79	*f* 78	*f* 78	m 88	m 88	m 77	f 77	*f* 66	*m* 87					
957	*m* 65	f 66	f 55	*m* 56	m 77	*m* 76	*m* 67	m 66	f 66	m 88					
958	*f* 76	m 88	*m* 88	*m* 87	*f* 67	f 56	f 77	*m* 86	f 77	m 77					
959	m 66	*f* 65	m 76	f 56	f 55	f 56	m 77	*m* 66	*f* 64	m 56					
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961	f 55	m 55	f 55	m 66	f 66	m 66	f 55	*f* 66	f 66	*f* 56					
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963	f 77	*f* 67	*f* 66	f 76	*m* 76	f 66	f 76	f 66	f 77	*f* 77					
964	*m* 57	*f* 65	*f* 66	*m* 56	f 66	f 66	*m* 66	m 66	f 66	*f* 77					
965	m 55	f 55	m 55	*m* 67	f 55	f 55	*m* 56	*m* 56	m 55	f 55					
966	m 66	f 55	f 55	f 76	m 66	m 77	f 75	f 66	f 67	f 55					
967	m 77	f 77	*f* 56	*f* 67	*f* 66	*m* 76	m 77	f 67	*f* 88	*m* 88					
968	*m* 78	*f* 56	f 66	*f* 76	*m* 56	*f* 67	m 66	m 77	f 66	*m* 67					
969	f 66	*f* 56	*m* 77	f 55	*m* 67	f 66	*f* 65	*f* 65	*f* 56	f 66					
970	*m* 67	f 66	*f* 67	*f* 56	*m* 77	*f* 56	*m* 66	m 66	*m* 76	m 77					
971	f 66	f 66	m 76	m 66	*f* 67	m 66	*f* 56	*m* 67	*f* 66	f 55					
972	*f* 56	*f* 77	*f* 56	m 66	f 77	m 78	f 55	f 55	*m* 87	*f* 76					
973	*m* 67	*m* 76	m 77	*f* 76	*m* 87	m 77	m 77	f 78	f 66	*m* 88	f 77				
974	f 66	*f* 56	f 66	f 56	*f* 57	*m* 67	*f* 76	m 55	*m* 56	f 66	*m* 77				
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976	f 66	f 66	f 66	m 66	*f* 56	m 66	*f* 66	f 66	*m* 66	f 66	*f* 76				
977	*f* 56	f 66	*f* 57	*m* 55	*f* 56	*m* 76	m 77	f 67	*m* 67	*m* 65	f 55				
978	f 55	f 55	f 55	f 55	m 66	f 55	m 55	f 55	m 66	m 66	m 55				
979	*f* 76	m 66	*f* 66	*f* 66	*m* 67	*m* 56	m 66	f 55	m 66	*f* 76	f 77				
980	*f* 67	f 66	m 77	f 66	m 66	m 67	f 66	m 66	f 66	f 65	m 66				
981	*f* 56	f 66	*f* 67	f 66	f 77	m 66	f 77	f 66	f 66	*m* 77	m 55				
982	m 66	m 77	f 55	f 55	f 55	f 77	m 55	f 55	f 55	f 55	m 77				
983	*m* 76	*m* 65	*f* 76	m 77	*m* 67	*f* 67	*m* 68	f 66	f 66	m 66	m 77				
984	*f* 56	*f* 67	m 65	m 55	f 56	f 66	f 66	m 66	m 55	m 66	*m* 56				
985	m 66	m 76	*m* 65	*f* 67	m 87	f 77	*f* 56	*m* 78	*m* 57	*f* 56	f 76				
986	m 66	m 55	f 55	f 66	m 55	m 66	f 55	f 55	m 55	f 55	m 66				
987	m 77	f 55	*m* 67	f 55	*f* 56	m 77	m 77	m 66	f 66	m 76	f 66				
988	*f* 87	f 77	m 88	*f* 67	*f* 67	*m* 77	*m* 67	f 88	f 56	f 66	*m* 97	*f* 77			
989	m 66	*m* 67	m 55	f 66	f 55	m 55	m 66	*f* 65	m 55	*m* 65	f 55	m 55			
990	*f* 56	*f* 67	f 66	m 56	*f* 65	*m* 65	f 56	f 66	f 66	f 55	f 56	*f* 56			
991	*m* 77	*f* 66	f 76	*m* 56	f 77	*f* 67	*f* 76	f 65	m 78	m 77	*f* 76	*m* 76			
992	f 67	*f* 56	f 77	f 67	f 55	*m* 87	m 66	m 66	m 66	m 66	f 66	f 88			
993	m 66	*m* 56	*f* 65	m 66	f 66	*m* 57	f 55	m 77	m 77	m 66	*f* 76	f 55			
994	*f* 66	m 77	m 77	m 77	m 66	m 77	*m* 76	m 56	*m* 56	f 66	m 66	*m* 65	m 66		
995	m 55	*m* 78	*f* 67	f 76	m 66	*f* 67	*m* 66	m 66	m 75	*f* 65	m 77	*m* 67	f 66		
996	*m* 78	f 66	*f* 75	*f* 56	f 66	*m* 67	m 77	*f* 65	f 66	*f* 65	*f* 76	*m* 67	*m* 67		
997	f 56	m 77	f 55	m 77	m 66	*f* 65	f 66	*f* 54	*m* 66	*f* 66	m 56	*f* 56	*f* 56	m 66	
998	f 77	*m* 76	f 76	m 66	*f* 76	f 67	*f* 77	m 67	m 77	*m* 77	*f* 77	f 66	*m* 78	*f* 75	
999	f 66	m 65	m 67	m 88	*f* 56	m 77	m 67	*m* 56	m 66	*f* 65	m 67	m 77	m 66	m 77	
1000	*f* 77	m 66	m 77	*f* 77	*m* 68	*m* 76	m 89	f 76	f 77	m 77	*f* 67	*m* 77	f 77	m 89	*f* 67

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XLII.

I. A Key to the Genera of the *Compositae-Eupatorieae*. By B. L. Robinson.

II. Revisions of *Alomia, Ageratum,* and *Oxylobus*. By B. L. Robinson.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.—NEW SERIES, No. XLII.

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I. A GENERIC KEY TO THE COMPOSITAE-EUPATORIEAE.

By B. L. Robinson.

It is now more than twenty years since the genera of the *Eupatorium* tribe were keyed by Hoffmann in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5 (1890). During this interval several genera have been added to the tribe, some few have been definitely removed from it to other tribes of the *Compositae*, two (*Brachyandra* and *Addisonia*) have been reduced, and certain sections of genera have come to appear worthy of generic rank within the tribe. In consequence the key of Hoffmann, though excellent for its time, is now unsatisfactory and far from complete. The one here offered, though drawn up after some years' study of the group, is put forth rather as a convenient working hypothesis than a finished or monographic product.

In the *Eupatorium* tribe, as elsewhere in the *Compositae*, generic distinctions, though essential for classification, often seem pretty artificial and the more precisely they are stated the greater of necessity becomes the artificial or arbitrary element. A re-examination of the technical characters relied upon by the older authors discloses many exceptions and transitions. On the other hand persistent attempts to secure a more natural classification by relying in larger measure upon habital traits have proved even more disappointing. These, while fairly convincing among plants of a circumscribed area, quickly lose any statable definiteness in dealing with the species of the world. One is in consequence forced to a restatement of the distinctions of pappus, achenes, anther-tips, and involucral-bracts as yielding after all the most practical basis for classification in the group.

In order to employ these more technical features effectively some explanation and definitions are necessary.

The pappus of the *Eupatorium* tribe offers a wide variety, including nearly all forms found in the *Compositae*. Scales, distinct or connate, blunt or bristle-tipped, as well as hairs truly capillary or setiform or awn-like, simple, barbellate, or plumose, rarely clavate or even gland-tipped, all occur within the tribe.

With rare exceptions it has been found possible to distinguish pretty readily between the truly capillary pappus of such genera as *Eupatorium, Mikania,* or *Brickellia* and the also terete but stiffer bristle-formed pappus characteristic of *Agrianthus* and of certain species to be segregated from *Ageratum.* On the other hand it is believed that this bristle-formed pappus can in practically all cases be distinguished from the scale-pappus of the true Ageratums and that it forms a useful basis of separation for some elements long classified with *Ageratum.* More difficult is the sharp distinction of plumose pappus from forms in which the setae are merely barbellate, and while this traditional character seems almost necessary at times it has been employed as sparingly as possible.

A distinction which is more important and which seems never to have been adequately studied even by close students of the *Compositae* is the difference between a very short cup-shaped, saucer-shaped, or coroniform pappus, such as occurs in *Ageratum* § *Coelestina,* and the similar low and often slightly angulate or toothed annulus which often crowns the mature achene in genera like *Alomia,* which lack all true pappus. When seen on fully ripe achenes from which the corolla has fallen away these structures often appear considerably alike, yet even in this late stage differences are usually to be noted. The true pappus, even when very rudimentary, inclines to be cup-shaped and possesses a thin edge. The annulus is a mere low usually thickish cartilagineous or fleshy ring with a blunt edge. If examined in a younger stage, while the corolla is still in place, it will be seen that the true pappus, however rudimentary, is distinctly exterior to the base of the corolla, while the annulus is merely the ringlike base from which the mature corolla disarticulates. This base sometimes enlarges slightly after the corolla falls and has in such species as *Ageratum echioides* or more properly *Alomia echioides* been taken for a true and much reduced pappus. In the peculiar genus *Jaliscoa* the annulus takes on a saucer shape and has frayed quasi ciliolate edge, yet when carefully examined in a young state it can be seen to be a sub-corollar rather than an extra-corollar structure.

To add to the complexity an intra-corollar disk is often present in various rudimentary forms, as for instance in minute fleshy or glandular papillae or in a fleshy ovoid, depressed-globose, or napiform enlargement at the base of the style, capable of some persistence in the mature achene.

The presence or absence of a true (though often much reduced) pappus becomes especially important in distinguishing *Ageratum* § *Coelestina* and *Alomia* and leads, as will be seen elsewhere in this paper, to some readjustment of generic lines. The character is believed to be a good one and to lead to a real distinction between groups which have hitherto been very poorly delimited.

To see clearly what is here meant it is only necessary to compare *Ageratum micropappum,* possessing a true but excessively reduced pappus, with *Ageratum heterolepis, A. echioides, A. microcephalum,* or *A. microcarpum,* all of which so far as observed by the writer have no real pappus but merely a sub-corollar annulus and in consequence are to be referred to *Alomia.*

The involucre in the *Eupatorium* tribe takes on many forms, all more or less intergrading. In *Eupatorium* itself, though the involucre goes through the whole gamut of variation, the number of species is so great that grouping by involucral differences has never appeared to be sharp enough to permit any satisfactory generic segregation of the elements concerned. It is very easy to refer certain marked species to *Osmia* Sch. Bip. and others to *Kyrstenia* Necker, but transitions are innumerable. On the other hand in many of the smaller genera involucral characters furnish distinctions of sectional or even generic value. Three types of involucre may be recognized as follows: 1) in which the chief scales are definite or subdefinite in number (four in *Mikania* and *Kanimia,* five or six in *Stevia)* often surrounded at the base by 1–3 considerably reduced scales i. e. calyculate. 2) the chief scales of indefinite number (though rarely very numerous) and subequal, appearing to be in 1–3 series and often accompanied by a very few much reduced outer ones. This is the common form of involucre in *Ageratum, Kuhnia,* etc. 3) scales of indefinite number (usually numerous) conspicuously unequal and gradually diminishing outward, forming apparently several to many series, though of course in reality spirally arranged. These three types of involucre, while sometimes confluent, are in general pretty readily distinguished.

There are also considerable differences in the texture of the involucral scales and to some extent these may be used, at least as supplementary aids, in distinguishing genera. Thus in *Ageratum,* most Alomias, and several other genera the scales are prevailingly of rather firm texture and pretty definitely 1–3-costate, while in *Brickellia, Hofmeisteria, Podophania, Oaxacania,* etc. the scales are prevailingly thin, flat, and finely striate.

One other character of special classificatory significance in the

Eupatorium tribe needs restatement with greater detail, namely the apical appendage of the anthers. It has long been customary to separate the subtribe *Piquerinae* on the ground that this apical appendage was there absent, yet to this subtribe have been referred several genera, such as *Adenostemma, Gymnocoronis, Hartwrightia, Podophania, Decachaeta, Helogyne,* and *Eupatoriopsis,* which exhibit such an apical appendage in various degrees of development, while in *Eupatorium* proper there are several species in which the appendage is decidedly rudimentary. It will thus be seen that the distinction between the *Piquerinae* and the *Ageratinae* breaks down completely unless it can be supplemented by other characters or restated with proper qualification. Persistent search has failed to discover any correlated differences, so it is necessary to make the most of the anther-appendage.

Examination of the typical *Piquerinae,* such as *Piqueria, Ophryosporus,* etc., shows that the anthers are entirely destitute of apical appendage, nor is the connective upwardly thickened or expanded in a way to cover the cells. If, however, the doubtful genera above mentioned are examined, it becomes evident that the rudimentary appendage assumes several distinguishable forms. In *Decachaeta* and *Podophania* it consists merely of an exceedingly short cushion-like expansion of the dilated summit of the connective. The connective is thus without change of texture broadened out until it partially or entirely covers the apices of the loculi, the whole anther being decidedly blunt or even slightly retuse. It is to be noted that there is here no membranaceous appendage in the stricter sense, and it seems best to retain these genera in the *Piquerinae.*

The next stage in the development of the apical appendage is seen in the three genera *Adenostemma, Gymnocoronis,* and *Hartwrightia.* In these there appears always to be at least a very short and decidedly retuse membranous appendage. This may be a single or simple structure or in *Adenostemma* it is sometimes so deeply divided in the middle that it virtually becomes two small membranous tips crowning and slightly prolonging the two loculi. In other respects these three genera manifest striking similarities, such as the form of the corolla and especially in the uniformly glandular faces of the achenes. Both on account of the seemingly close relationship among themselves and in order to permit a more precise definition of the *Piquerinae* it seems desirable to class these three genera in a new co-ordinate subtribe, the *Adenostemmatinae.*

In *Helogyne* and *Eupatoriopsis* the apical appendage assumes the ovate-oblong form, fairly characteristic of other portions of the *Eupatorium* tribe, and these genera may well be transferred to the *Ageratinae* as has been earlier pointed out. See Proc. Am. Acad. xlii. 27 (1906). With these changes it is believed that the *Piquerinae, Adenostemmatinae,* and *Ageratinae* will be found fairly clear as subtribes. The fourth subtribe of the *Eupatorieae,* characterized by its more numerously ribbed achenes, has been known as the *Adenostylinae* from its (as assumed) typical genus, *Adenostyles* Cass. This genus, however, is one which has always been very dubiously placed with the *Eupatorieae* if classed with them at all. To the writer, after repeated examinations of the genus from various points of view, it seems clear that its real affinity is with the *Senecioneae.* Arguments for this view can be found in many minor details of habit, which taken together become convincing. The elongated style-branches alone would suggest a relationship with the *Eupatorieae,* but even these do not appear really eupatorioid. They tend to an attenuate rather than a clavellate form in the first place, and in the second they are inclined to be recoiled through a much greater arc than is usual among the *Eupatorieae.* Finally the unbranched portion of the style in *Adenostyles* is at maturity commonly exserted, while this would in the *Eupatorium* tribe be highly exceptional. With the removal from the *Eupatorium* tribe of the genus *Adenostyles* the remaining portion of the subtribe hitherto known as the *Adenostylinae* must in accordance with Art. 52 of the International Rules of Botanical Nomenclature be renamed. It may be called the *Kuhniinae.*

The genus *Mallinoa* Coult. Bot. Gaz. xx. 47 (1895) was doubtfully ascribed by its author to the *Inuleae,* its anthers being described as sagittate and being figured (l. c. t. 5, f. 4) as having acute auricles at the base of the anther-cells. Soon after its publication Hoffmann in Engl. & Prantl, Nat. Pflanzenf. Nachtr. 322 (1897) placed *Mallinoa* in the *Eupatorieae,* where there can be no doubt it in reality belongs. For some reason not made clear, Hoffmann, though recognizing the eupatorioid nature of *Mallinoa* placed it next *Trichogonia,* with which it has no close habital resemblance nor striking likeness of involucre or pappus. To the writer *Mallinoa* seems to be merely a species of the genus *Eupatorium,* exceedingly close to the long known *E. bellidifolium* Benth. In foliage, gesture, and inflorescence these plants possess a resemblance amounting almost to identity. *Mallinoa,* however, is readily distinguished (specifically) by its decidedly broader, blunter, and much smoother involucral bracts. In the light of excellent material of both plants secured by Mr.

Pringle, the writer finds that the anther-bases in *Mallinoa* are cordate rather than sagittate, the basal auricles being decidely rounded. In *Eupatorium bellidifolium* furthermore the anther-bases are also retuse, the condition being very similar and the difference merely a trifling one of degree and in no sense one of kind. The degree of difference is closely comparable to that shown between fig. 2 and fig. 3 on plate 9 of Bentham's classical "Notes" (Jour. Linn. Soc. xiii.). It will be noticed that Bentham, whose experience and judgment upon these matters have probably never been surpassed, admits the occurrence of anthers of both these types in the *Eupatorium* tribe (l. c. 360). If admitted to the *Eupatorium* tribe at all, *Mallinoa* cannot be kept out of the genus *Eupatorium* as at present delimited.

Regarding the transfer of *Apodocephala* Bak. to the *Vernonieae,* see Proc. Am. Acad. xlii. 32 (1906), and of *Lepidesmia* Klatt to the *Heliantheae,* see Proc. Am. Acad. xlvii. 210 (1911).

Further study, especially of the larger genera, may well reveal profitable generic segregations not as yet clear. This is especially likely to be the case among the numerous and as yet very imperfectly known South American members of the tribe. It is also by no means improbable that when these are more satisfactorily represented in herbaria some new and more convincingly natural re-adjustment of generic lines will become possible. Furthermore, it is to be remembered by users of the following key that disconcerting exceptions and anomalies occur here as elsewhere among the *Compositae,* as for instance four-, six-, or even seven-angled achenes among the normally 5-angled ones in the *Ageratinae,* or occasionally calvous achenes in certain heads of the normally pappus-bearing genera *Trichogonia* and *Ageratum.*

Subtr. I. PIQUERINAE. Antherae apice omnino exappendiculatae vel a connectivo sursum leviter incrassato et supra loculos plus minusve lateraliter expanso truncato vel retuso terminatae. Achaenia non glandulari-papillifera.

a. Pappus nullus vel rarissime brevissime obsoleteque setosus. . 1. *Piqueria*
a. Pappus setosus *b.*
  b. Pappi setae laeves vel modice scabratae *c.*
    c. Pappi setae saepissime 10 sursum clavellatae. Folia pleraque alterna 3-lobata vel non lobata. Capitula parva numerosa subsessilia paniculata. . . . . 2. *Decachaeta.*
    c. Pappi setae tenues non clavellatae. Folia opposita dissecta. Capitula axillaria longe pedunculata. . . . . 3. *Podophania.*
  b. Pappi setae plumosae. . . . . 4. *Ophryosporus.*

a. Pappus squamosus *d.*

d. Pappi squamulae breves fimbriatae. Ind. Occ. . . . . . . .5. *Phania.*

d. Pappi squamulae longae integrae apice in aristam attenuatae. Mex. . . . . .6. *Ageratella.*

Subtr. II. **Adenostemmatinae,** subtr. nov. Antherae apice breviter appendiculatae, appendice membranacea aut simplici et retusa aut bifida et apices loculorum coronante. Corollae tubus brevis et fauces infundibulares. Achaenia obovoidea glandulari-papillosa.

a. Pappus e setis brevibus 3–5 clavatis ab annulo patentim ascendentibus compositus. . . . . .7. *Adenostemma.*

a. Pappus nullus vel brevissime obsoleteque setosis, setulis numquam clavatis *b.*

b. Receptaculum nudum. Folia opposita dentata. Mex., Am. Aust. 8. *Gymnocoronis.*

b. Receptaculum margine paleiferum. Folia alterna integra. Fla. 9. *Hartwrightia.*

Subtr. III. Ageratinae. Antherae apice cum appendice membranacea ovata vel oblonga integra vel retusa instructae. Achaenia normaliter 5(rarissime 4–6)-angulata prismatica vel subprismatica rariter compressa eglandularia vel rariter glandulari-atomifera.

a. Pappus e squamis distinctis vel basi connatis aut ex aristis deorsum squamiforme expansis aut ex aristis et squamellis aut e corona dentata vel fimbriata vel integra compositus aut omnino deficiens *b.*

b. Involucri squamae 5–6 uniseriales subaequales cum vel absque 1–3 squamulis extimis multo brevioribus. .10. *Stevia.*

b. Involucri squamae ∝ saepius numerosa 2–∝-seriatae *c.*

c. Pappus evolutus e squamis aut distinctis tenuibus vel rare indurescentibus aut in cupulam coroniformem tenuem scariosam dentatam vel integram connatis compositus rarissime (apud *Agerati* spp.) obsoletus *d.*

d. Involucri squamae valde inaequales ∝-seriatim imbricatae *e.*

e. Pappus coronam crateriformem margine laciniatam formans. Folia bipinnatifida. Bras. . . . . .11. *Lomatozona.*

e. Pappus e squamulis brevibus obovatis vel oblongis obtusis compositus *f.*

f. Receptaculum pilosiusculum. Pappi paleae apicem versus denticulatae. Folia penninervia. Bras. . . . . . . .12. *Carelia.*

f. Receptaculum paleaceum. Pappi paleae breviter fimbriatae. Folia 3-nervia. Mex. . . . . .13. *Aschenbornia.*

e. Pappus ex aristis cum squamulis alternantibus aut ex aristis basi squamiforme dilatatis compositus *g.*

g. Folia sessilia linearia subintegra. Herba annua. Calif. infer. . . . . .14. *Malperia.*

g. Folia longiuscule petiolata dentata vel lobata vel dissecta. Frutices vel suffrutices. . . . . .15. *Hofmeisteria.*

d. Involucri squamae subaequales 2–3-seriales cum vel absque squamulis 1–3 extimis multo brevioribus *h.*

h. Pappi squamae distincte indurescentes. Folia verticillata. Am. Bor. atlant. . . . . .16. *Sclerolepis.*

h. Pappus tenuis non indurescens. Folia opposita vel alterna *i.*
  i. Corollae tubus bene distinctus subfiliformis in fauces discrepitante ampliatas campanulatas subito transiens. Folia subcoriacea. Suffrutices. Mex...17. *Oxylobus.*
  i. Corollae tubus saepissime brevis, faucibus vix distinctis vel solum modice ampliatis. Folia membranacea............................18. *Ageratum.*
c. Pappus verus nullus, achaeniis tamen cum annulo sub corolla numquam extra corollam orienti coronatis *j.*
  j. Achaenia 4–5-angulata. Involucrum non foliaceum *k.*
    k. Achaenia prismatica *l.*
      l. Herbae vel frutices non repentes...............19. *Alomia.*
      l. Herba repens. Columbia................20. *Tuberostyles.*
      l. Frutex. Caulis fistulosus et maturitate clathratus. Mex. 21. *Jaliscoa.*
    k. Achaenia compressiuscula. Frutex, foliis alternis suborbicularibus sinuato-lobulatis. Mex..............22. *Oaxacania.*
  j. Achaenia 6-angulata. Involucri squamae exteriores subfoliaceae. Bras.......................................23. *Planaltoa.*
a. Pappus setosus, setis capillaribus vel firmiusculis, liberis vel basi in annulum coalitis *m.*
  m. Pappi setae plumosae *n.*
    n. Achaenia compressa. Pappi setae brevissimae. Bras. 24. *Eupatoriopsis.*
    n. Achaenia prismatica *o.*
      o. Pappi setae firmiusculae saepissime breves non rite capillares *p.*
        p. Capitula sessilia. Folia squamiformia imbricata 25. *Agrianthus.*
        p. Capitula pedicellata. Folia cum lamina normali 27. *Trichogonia?* [1]
      o. Pappi setae capillares plumosae *q.*
        q. Corollae fauces subcampanulatae a tubo proprio differentiatae 27. *Trichogonia.*
        q. Corolla graciliter tubulata sine faucibus distinctis *r.*
          r. Folia alterna. Frutices xerophyticae. Am. Aust. occ. 28. *Helogyne.*
          r. Folia opposita. Herba. Am. Bor. aust.-occ. et Mex. 29. *Carminatia.*
  m. Pappi setae capillares simplices laeves vel modice scabratae *s.*
    s. Achaenia valde compressa. Pappi setae 2. Mex. 30. *Schaetzellia.*
    s. Achaenia prismatica. Pappi setae 5–∝ *t.*
      t. Pappi setae longitudine valde conspicueque inaequales. Am. Aust. 31. *Dissothrix.*
      t. Pappus uniformis vel subaequalis *u.*
        u. Pappi setae brevissimae paucae longitudine crassitiem achaenii vix aequantes..........................32. *Trichocoronis.*
        u. Pappi setae achaenii longitudinem subaequantes *v.*
          v. Pappi setae caducae *w.*
            w. Involucri squamae valde inaequales ∝-seriatim imbricatae. Folia opposita. Mex..................33. *Piptothrix.*
            w. Involucri squamae subaequilongae ca. 2–3-seriatim laxe imbricatae. Folia alterna. Bras.......34. *Leptoclinium*

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[1] Hic expeterentur species quaedam adhuc dubiae ex *Agerato* ob pappo setiformi expulsae et ad *Trichogoniam* valde approximantes.

- *v.* Pappus persistens *x.*
  - *x.* Pappi setae basi in annulum crassiusculum coalitae. Folia coriacea. Bras. . . . . 35. *Symphyopappus.*
  - *x.* Pappi setae basi liberae vel levissime inconspicueque coalitae. Folia saepissime membranacea *y.*
    - *y.* Involucri squamae ut flosculi ∞ *z.*
      - *z.* Pappi setae subdefinitae saepissime 5. 36. *Fleischmannia.*
      - *z.* Pappi setae ∞ numerosae. Receptaculum nudum vel obscure setiferum . . . . 37. *Eupatorium.*
      - *z.* Pappi setae ∞ numerosae. Receptaculum paleiferum. Folia magna. Mex. . . . . 38. *Eupatoriastrum.*
    - *y.* Involucri squamae definitae 4. Flosculi etiam 4. Herbae vel frutices saepissime volubiles. . 39. *Mikania.*

Subtr. IV. **Kuhniinae,** nom. nov. Antherae apice cum appendice membranacea ovata vel oblonga munitae. Achaenia 10–20 (rariter 6–9)-costata.— *Adenostylinae* auct. excl. *Adenostyle* genere ipsissimo nominiferente.

- *a.* Involucri squamae definitae 4. Flosculi etiam 4. . . . . 40. *Kanimia.*
- *a.* Involucri squamae ∞ saepissime numerosae *b.*
  - *b.* Pappus setosus *c.*
    - *c.* Receptaculum nudum *d.*
      - *d.* Achaenia compressa. Foliorum dentes saepissime setiferae. Mex. 41. *Barroetea.*
      - *d.* Achaenia prismatica vel teretia *e.*
        - *e.* Involucri squamae valde inaequales pluriseriatim imbricatae *f.*
          - *f.* Involucri squamae tenues striatae vix herbaceae. Pappi setae uniseriatae laeves vel barbulatae. Folia opposita vel alterna 42. *Brickellia.*
          - *f.* Involucri squamae herbaceae vel coloratae vel scariosae non conspicue striatae. Pappi setae uniseriatae saepissime plumosae rariter barbulatae. Herbae perennes, caule saepius simplici basi tuberoso . . . . 43. *Liatris.*
          - *f.* Involucri squamae herbaceae. Pappi setae 2–3-seriatae, exterioribus brevioribus. Frutex foliis alternis obovatis. Fla. 44. *Garbera.*
        - *e.* Involucri squamae subaequales 2–3-seriatae. Folia saepissime alterna *g.*
          - *g.* Pappi setae capillares conspicue plumosae. . . . 45. *Kuhnia.*
          - *g.* Pappi setae firmiusculae vix barbulatae. . . . . . 46. *Trilisa.*
    - *c.* Receptaculum paleiferum. Folia alterna. . . . . 47. *Carphephorus.*
  - *b.* Pappus ex aristis basi squamiforme dilatatis compositus. Folia opposita. Am. Bor. aust.-occ. et Mex. . . . . 48. *Carphochaete.*

II. REVISIONS OF ALOMIA, AGERATUM, AND OXYLOBUS.

By B. L. Robinson.

The following treatment has been based chiefly upon an intensive study of the pertinent material in the Gray Herbarium, but much aid has been derived from a series of photographs, taken at various European herbaria in 1905 and 1910, representing the types of nearly all the recognized species of *Alomia* and *Ageratum* not originally described from the Gray Herbarium. There has been no opportunity to test the keys and descriptions by a re-examination of the extensive material in foreign collections, but through the kindness of Messrs. Coville and Maxon the writer has been permitted to borrow and study the representation of *Alomia* and *Ageratum* from the National Museum at Washington, a considerable privilege for which he would express grateful appreciation. The writer is also much indebted to Mr. John Donnell Smith for the loan of *Ageratum* and *Alomia* from his rich personal herbarium, and to Mr. A. B. Rendle of the British Museum of Natural History for the critical comparison establishing beyond doubt the identity of *Ageratum Houstonianum* Mill. Miss M. A. Day and Miss E. M. Vincent of the Gray Herbarium staff have given bibliographical aid.

It may be said in a general way that the species of these genera do not tend to serious intergradation, except in the case of *Ageratum conyzoides* L. and *A. latifolium* Cav. (not Hemsl.), where separation though easily made seems pretty artificial, and on the other hand the several species which are closely allied to the highly variable *A. corymbosum* Zuccag. and are distinguished chiefly by such characters as leaf-contour, pubescence, etc.

It must be frankly admitted that the treatment of the South American species is very sketchy, being derived from wholly inadequate material. It is highly probable that further exploration of northern South America and of Brazil will bring to light many further species, and perhaps considerably modify our present views as to the distinctness of those already known.

1. Revision of the Genus Alomia.

The tropical American genus *Alomia* is a convenient rather than a convincingly natural group of species. Depending for its separation from *Ageratum* and *Trichogonia* solely on the absence of pappus, the genus may well have a composite — at least a double — origin. Either a *Trichogonia,* on the loss of its plumose pappus-bristles (a condition known to occur in *T. menthaefolia* and *T. salviaefolia* Gardn.), or an *Ageratum,* on the complete abortion of its often obsolescent scale-pappus, would become as to technical characters an *Alomia,* and the fact that the genus *Alomia,* as now circumscribed, contains species of a wide range of habit, extending on the one hand from the original *A. ageratoides,* with distinctly ageratoid habit, to *A. dubia,* on the other, which except for the lack of pappus would certainly be placed in *Trichogonia,* it appears by no means unlikely that the elements now grouped in *Alomia* may have come in part from an ageratoid and in part from a *Trichogonia*-like ancestry.

Although from these considerations *Alomia* may seem an artificial group, its components certainly have close affinity and our present knowledge does not permit any improvement of the situation either by dividing the genus on trifling traits of habit or by merging it bodily with any of the neighboring genera.

The three species here grouped as a new subgenus, *Geissanthodium,* possessing softer much imbricated involucral scales (striate in the manner of *Brickellia*), form an interesting strain, a small presumably natural group. After careful comparison of *A. alata* Hemsl. and *Ageratum callosum* Wats. it is impossible to see any grounds for their generic separation. Either *Ageratum callosum* must be referred to *Alomia* or the genus *Alomia* must be transferred to *Ageratum.* The close relationship of the third species of this subgenus, the South American *A. Regnellii* Malme, although less convincing, seems highly probable to judge from Malme's description and figures. Some specious arguments might be advanced for the separation of these three species as a new genus, but their distinctions from other species of *Alomia* are not strong. Involucral characters in nearly related genera are seen to be highly inconsistent and it is to be noticed that the imbrication of the scales is by no means so striking in the Brazilian *A. Regnellii* as in the Mexican species. Nor is the undifferentiated corolla-tube a strong character. It seems best therefore to treat the group merely as a subgenus.

Alomia tenuifolia (the genus *Lycapsus* of Philippi), a xerophytic shrub confined to a small island off the coast of Chili, is still obscure. It does not appear to be represented in the leading European or North American herbaria. Bentham, judging it merely by the description and crabbed little figures of Philippi, placed it in *Alomia.*

This may be its real affinity, but it is to be noted that Philippi represents the style-branches as being recoiled through more than 360°. This would be highly exceptional in the *Eupatorieae.* Furthermore, the style-branches as shown in Philippi's figures are unusually short for the tribe. Until material of this rare and local plant can be obtained and subjected to further study the species would better be left here, where Bentham placed it, but it would seem almost as likely to prove one of the *Heliantheae,* perhaps near *Isocarpha.*

As explained elsewhere in this paper, the rudimentary annular or coroniform "pappus," accredited in the past to several species hitherto placed in *Ageratum,* proves not to be a true pappus, i. e. a calycular structure exterior to the corolla, but only an annulus upon which the corolla itself is borne and which after the disarticulation and fall of the corolla sometimes is slightly accrescent. These species, destitute of a true pappus, must certainly be transferred to *Alomia* if that genus is to be kept distinct from *Ageratum* and they are so treated in the following revision.

While in nearly all of the species concerned the presence or absence of a pappus is upon careful observation sufficiently evident and constant to permit a pretty ready separation of species into those which should be referred to *Alomia* on the one hand and those which would better be placed in *Ageratum* on the other, there are two exceptional species, *Ageratum littorale* Gray and *A. maritimum* HBK., in which the distinction breaks down absolutely. Here the pappus may be entirely wanting, it may consist of minute teeth, very short and slightly exterior to the corolla, or finally it may develop into a perfectly definite and conspicuous scale-pappus. These marked variations in pappus occur in individuals of precisely similar habit, and so far as can be ascertained are accompanied by no concomitant changes of structure. This wide intra-specific variation presents, of course, a technical difficulty in delimiting the genera *Ageratum* and *Alomia.* However, it is to be remembered that calvous forms in normally pappus-bearing *Compositae* are by no means rare and must be accepted as one of the inherent difficulties of the group. It would be highly artificial to transfer to *Alomia* the calvous forms of *Ageratum littorale* and *A. maritimum,* nor does it seem best to unite with the otherwise consistently pappus-bearing genus *Ageratum* the consistently calvous genus *Alomia* because in certain exceptional species an abortion of the pappus occurs inconstantly, as it does also in *Trichogonia menthaefolia* Gardn., *Calea peduncularis* HBK., and various other *Compositae* in which the loss of pappus can in no sense be regarded as having generic significance.

Alomia was confused by Gardner with *Piqueria* and *Gymnocoronis*, but may be readily distinguished by its apically appendaged anthers. *Orsinia* Bert., associated with his *Piqueria* § *Eupiqueria* by Gardner in Hook. Lond. Jour. Bot. vi. 430 (1847), and with *Alomia* by Baker in Mart. Fl. Bras. vi. pt. 2, 189 (1876), and even by Dalla Torre & Harms, Gen. Siph. 526 (1905), is a *Clibadium*, as was indicated by Bentham, Gen. ii. 240 (1873).

ALOMIA HBK. (Nomen ex ἀ privativo et λῶμα, *margo*, defectionem pappi alludens.) Capitula homogama parva vel mediocra saepe numerosa laxe paniculata vel in apicibus ramorum corymbulosa 20–70-flora; involucri campanulati vel subturbinati squamis vel subaequalibus ca. 2-seriatim imbricatis lanceolato-linearibus saepissime acutis 1–3-costatis firmiusculis vel valde inaequalibus 3–∞-seriatim imbricatis tenuioribus striatulis; receptaculo plano vel conico nudo vel plus minusve paleifero, paleis saepe angustis. Corollae albae vel roseae vel purpureae saepissime extus glandulis subsessilibus conspersae rarius in tubo vel limbo hirsutae vel tomentosae rarissime glabrae. Antherae oblongae apice appendiculatae basi rotundatae haud rarenter vix connatae. Styli rami filiformi-clavellati purpurei vel flavi vel aurei rectiusculi vel leviter recurvantes vel per exceptionem (Subg. 3) spiraliter recurvati. Achaenia prismatica 5-angularia glanduloso-atomifera vel hispidula vel glabra fusca vel nigrescentia basi callosa apice annulo cartilagineo vel carnoso integerrimo vel angulato conspicuo vel tenuissimo vel omnino obsoleto coronata; pappo nullo.— Nov. Gen. et Spec. iv. 151, 312, t. 354 (1820); Cass. Dict. Sci. Nat. xxvi. 227 (1823); Less. Syn. Comp. 154 (1832); DC. Prod. v. 105 (1836); Endl. Gen. 366 (1838); Benth. & Hook. f. Gen. ii. 240 (1873); Pfeiff. Nom. i. 116 (1873); Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 135 (1890); Dalla Torre & Harms, Gen. Siph. 526 (1905), excl. syn. *Orsinia*. *Piqueria* sect. *Alomia* (HBK.) Gardn. in Hook. Lond. Jour. Bot. vi. 430 (1847), excl. spp. *attenuata, eupatorioides, latifolia, subcordata;* Walp. Ann. i. 393 (1848), ex parte. *Lycapsus* Phil. Bot. Zeit. xxviii. 499, t. 8 (1870). *Adenostemma* sect. *Alomia* Baill. Hist. Pl. viii. 131 (1882).— Herbae annuae vel perennes vel suffrutescentes rarissime frutices. Folia ovata vel rhomboidea vel lanceolata vel linearia pleraque serrata vel dentata rarius integriuscula rarissime profunde pinnatifida. Inflorescentia corymbosa vel paniculata. Habitus *Agerati* aut *Trichogoniae*. Species 12 quarum 3 mexicanae, 1 (ulterius inquirenda) insulam chilensem incolens, ceterae Brasiliam centrali-meridionalem habitantes.

Clavis subgenerum.

Involucri squamae 3–∞-seriatim imbricatae tenues striatulae. Corolla in tubum proprium et fauces distinctas non differentiata. Herbae Subg. 1. GEISSANTHODIUM.
Involucri squamae ca. 2-seriatim (apud *A. heterolepidem* pluriseriatim) imbricatae saepius firmiusculae 1–3-costatae. Corolla cum tubo proprio graciliore definito et faucibus plus minusve ampliatis.
  Styli rami rectiusculi vel modice recurvantes. Folia numquam profunde pinnatifida.................................Subg. 2. EUALOMIA.
  Styli rami spiraliter recurvati. Folia pinnatifida lobis linearibus Subg. 3. LYCAPSUS.

Subg. 1. **Geissanthodium**, subg. nov. (Nomen ex γεῖσσόν, *protectum* sensu imbricatum, et *anthodium* i. e. involucrum, ab ἀνθώδης, derivatum.) Involucri squamae (more earum *Brickelliae*) multiseriatim imbricatae tenues virides aut purpurascentes saepe tenuiter albido-striatae, exteriores gradatim breviores. Corollae graciliter tubulatae, tubo proprio haud discreto, faucibus distinctis nullis, limbo brevi quinquifido. Styli rami filiformi-clavellati rectiusculi vel leviter curvati saepissime flavi vel aurei. Achaenia per coronam carnosam albidam integerrimam coronata.— Herbae nunc perennes nunc verisimiliter annuae pubescentes. Folia longiuscule petiolata saltim inferiora opposita superiora saepius alterna. Species 2 calciphilae mexicanae necnon 1 brasiliana.

Clavis specierum.

Folia crenato-serrata. Species mexicanae.
  Caulis glanduloso-puberulus nec longius pilosus. Petioli sursum anguste alati. Folia late cordata.............................1. *A. alata.*
  Caulis glanduloso-puberulus et lanato-pilosus. Petioli exalati. Folia non cordata...2. *A. callosa.*
Folia grosse inciseque dentata. Species brasiliana..........3. *A. Regnellii.*

1. A. ALATA Hemsl. herbacea perennis gracilis decumbens vel saepe a rupibus pendula laxe ramosa; caulibus 4–6 dm. longis teretibus striatulis tenuissime glanduloso-puberulis; foliis deltoideo-ovatis late cordatis crenato-serratis tenuibus utrinque viridibus supra leviter pubescentibus subtus praesertim in nerviis venisque pilosis 2.5–5.5 cm. longis 1.5–5 cm. latis, petiolis 1–7 cm. longis saepissime apicem versus anguste alatis; capitulis laxe et irregulariter corymboso-paniculatis, pedicellis filiformibus saepius brevibus; involucri turbinato-campanulati squamis lineari-lanceolatis viridibus vel purpurascentibus albido-striatulis apice attenuatis extimis puberulis ceteris glabriusculis; corollis 3.6 mm. longis graciliter tubulatis viridescenti-albis plus minusve atomiferis limbum brevissimum versus subconstrictis; achaeniis 2.2 mm. longis basi callosis apice annulo turgido carnoso coronatis a

glandulis subsessilibus sparse conspersis.— Biol. Cent.-Am. Bot. ii. 79 (1881).— MEXICO: in rupibus abruptis umbrosis vallicularum prope Cuernavacam, *Bourgeau*, n. 1216 (hb. Gray.), *Bilimek*, n. 579 f. Hemsl. l. c., *Pringle*, nn. 6229, 9846 (hb. Gray., hb. U. S. Nat. Mus.); Guadalupe, *Bilimek*, n. 488 (hb. Gray.).

2. **A. callosa** (Wats.), comb. nov., herbacea gracilis; caulibus teretibus saepe purpureo-brunneis brevissime glanduloso-puberulis et conspicue griseo-pilosis sublanatis quamquam aetate glabriusculis 3–6 dm. longis oppositirameis; foliis oppositis ovatis obtusiusculis vel acutis crenato-serratis basi obtusis vel subtruncatis 3-nerviis tenuibus utrinque laxe pubescentibus 2–6 cm. longis 1–4 cm. latis, petiolo 1–3 cm. longo exalato; capitulis laxe corymbosis; involucri campanulati squamis multiseriatim imbricatis anguste lanceolato-oblongis acutis mucronulatis erosis; corollis graciliter tubulatis a basi ad limbum sensim ampliatis sed sine faucibus ullis distinctis viridescenti-albis 3.2 mm. longis glandulis minutis subsessilibus conspersis; achaeniis 1.2 mm. longis atropurpureis in costis 5 glanduloso-puberulis apice coronam carnosam integerrimam crateriformem gerentibus.— *Ageratum callosum* Wats. Proc. Am. Acad. xxv. 153 (1889).— MEXICO: in rupibus humidis prope Guadalajaram, *Pringle*, nn. 2166 (hb. Gray.), alt. 1525 m., 4739 (hb. U. S. Nat. Mus., hb. J. D. Sm.), n. 9353 (hb. Gray., hb. U. S. Nat. Mus.).

3. A. REGNELLII Malme, herbacea perennis ad 3 dm. alta; caule tereti subsimplici molliter patentimque piloso et etiam glanduloso-puberulo; foliis oppositis vel superioribus alternis tenuibus deltoideis vel late ovatis glanduloso-pilosis usque ad 4.5 cm. longis et ad 4 cm. latis acutis grosse inciseque dentatis basi rotundatis vel ad insertionem petioli plus minusve cuneatis utrinque viridibus supra parce glandulosis vel glabriusculis subtus glandulosis et in nerviis molliter pilosis, petiolo 1.5–3 cm. longo; capitulis laxe cymosis ca. 40-floris, pedicellis bracteolisque glanduloso-puberulis, involucri campanulati squamis valde inaequalibus ovato-lanceolatis vel lanceolatis vel interioribus lineari-oblanceolatis viridibus pilosis apicem versus ciliatis; receptaculo plano nudo; corollis albis caerulescentibus ca. 3 mm. longis a basi ad limbum sensim ampliatis tubo extus intusque glabro sed dentibus limbi extus plus minusve pilosiusculis; achaeniis ca. 2.5 mm. longis subprismaticis vel graciliter turbinatis nigris nitidis glabris vel basi et sub apice pilis brevissimis paucis caducis munitis.— Kongl. Sv. Vet. Akad. Handl. xxxii. n. 5, 32, t. 2 (1899).— Brasiliae prov. MATTO GROSSO, in fissuris rupium praeruptarum, Serra da Chapada, Buriti, locos subumbrosos praesertim habitans, *Malme*, n. 1678. Sp. non visa, solum e descriptione et icone optimis cl. Malmei interpretata.

Subg. 2. **Eualomia,** subg. nov. Involucri squamae (more *Agerati* earum) subaequales saepissime firmiusculae 1–3-costatae. Corollae cum tubo proprio saepe brevi et faucibus plus minusve cylindraceis munitae. Styli rami filiformes vel leviter clavellati patente recurvati saepius purpurascentes. Achaenia ecoronata vel coronam tenuem cartilagineam integerrimam ferentia.— Herbae annuae vel perennes vel suffrutescentes. Folia rhomboideo-ovata vel lanceolata vel linearia. Species 9, quarum una mexicana est ceterae Brasiliam centrali-meridionalem incolunt.

Clavis specierum.

a. Receptaculum paleiferum *b.*

b. Herbae annuae *c.*

c. Folia anguste linearia. Capitula 8 mm. diametro.....4. *A. Pohlii.*

c. Folia lanceolata. Capitula 5 mm. diametro..........5. *A. foliosa.*

b. Herbae perennes vel frutices *d.*

d. Paleae receptaculi apice obtusiusculae vel acutae saepius erosae vel pilosae numquam induratae *e.*

e. Folia lineari-oblanceolata tenuia inciso-dentata 6. *A. fastigiata.*

e. Folia late rhomboidea inciso-dentata submembranacea 7. *A. myriadenia.*

e. Folia lanceolata vel oblonga, serrata vel integra, coriacea *f.*

f. Capitula 6–8-flora........................8. *A. longifolia.*

f. Capitula ca. 36-flora......................9. *A. heterolepis.*

d. Paleae receptaculi apice subulato-attenuatae plus minusve induratae glabrae *g.*

g. Caulis per fere totam longitudinem foliosus *h.*

h. Folia subtus glandulis numerosissimis crebre punctata cum pilis sparsissimis vel nullis munita *i.*

i. Involucri squamae costatae *j.*

j. Involucri squamae glanduloso-atomiferae aliter glabrae lineares rigidiusculae. Folia lanceolata 10. *A. microcephala.*

j. Involucri squamae fere eglandulosae sed sub lente conspicue pubescentes et ciliatae lanceolatae. Folia fere linearia................11. *A. guatemalensis.*

i. Involucri squamae planae latiusculae ovatae ecostatae 12. *A. platylepis.*

h. Folia subtus velutino-pilosa.......13. *A. isocarphoides.*

g. Caulis superne nudiusculus. Folia subtus pilosa 14. *A. echioides.*

a. Receptaculum nudum *k.*

k. Folia angusta, linearia vel lanceolata vel oblanceolata *l.*

l. Capitula 30–35-flora. Folia sublinearia. Pedicelli breves (3–4 mm. longi). Corolla ca. 2 mm. longa..................15. *A. cinerea.*

l. Capitula ca. 65-flora. Folia oblanceolata. Pedicelli 6–8 mm. longi. Corolla ca. 4 mm. longa...........................16. *A. dubia.*

k. Folia latiora, cordato- vel rhomboideo-ovata *m.*

m. Capitula laxe scorpioideo-cymosa patentim paniculata. Mex. 17. *A. ageratoides.*

m. Capitula dense corymbosa *n.*
  n. Capitula 12–15-flora. Folia irregulariter grosse dentata. Annua. Bras. . . . . 18. *A. angustata.*
  n. Capitula 60–70-flora. Mex., Am. Cent., Venez. *o.*
    o. Perennis. Corymbi laxi patentes planiusculi. Folia acute acuminata et serrata. Mex. . . . . 19 *A. Wendlandii.*
    o. Annua. Capitula in cymis congestis valde convexis vel subglobosis disposita. Folia crenata saepissime obtusa. Costa Rica et (dubitative) Venez. . . . . 20. *A. microcarpa.*

4. A. Pohlii (Sch. Bip.) Bak., annua erecta vel ascendens parce ramosa; radice fibrosa; caule glabriusculo 3–4.5 dm. alto, ramis divergentibus pilosiusculis; foliis plerisque oppositis supremis solum alternis linearibus 4–7 cm. longis vix 2 mm. latis planis utrinque viridibus et pilosiusculis obscure arguteque serratis apice basique acutatis uninerviis; capitulis ca. 8 mm. diametro ca. 35-floris ad apices ramorum in corymbis parvis confertis graciliter pedicellatis; involucri subhemisphaerici squamis subaequalibus oblongis obtusis viridibus vel apicem versus purpureis extus glanduloso-puberulis margine et praesertim apice membranaceo expanso translucido conspicue ciliatis; receptaculo conico cum paleis angustissime oblanceolati-linearibus instructo; corollis 2.8 mm. longis subglabris vix basin versus cum glandulis paucis subsessilibus conspersis, tubo proprio vix ullo, faucibus fere ab apice achaenii ampliatis; achaeniis nigris saepe curvatis cum glandulis parce conspersis neque basi callosis nec apice annulo coronatis ca. 1.9 mm. longis.— Bak. in Mart. Fl. Bras. vi. pt. 2, 190 (1876). *Coelestina Pohlii* Sch. Bip. ex Bak. l. c.— Brasilia: "inter Manoël Souza et Mideroz," *Pohl,* n. 358 (hb. Berol., phot. et fragm. in hb. Gray.); Lagoa Santa prov. Minas Geraës, *Warming.*

5. A. foliosa (Gardn.) Benth. & Hook. f., annua 2–3 dm. alta decumbens vel suberecta griseo-pilosa vel subtomentosa; radice fibrosa; caule paucirameo foliosissimo; foliis parvis 3-nerviis 1.5–2 cm. vel ultra longis ca. 6 mm. latis lanceolatis vel oblongo-lanceolatis oppositis arcte sessilibus tenuibus apice basique acutis acute serratis utrinque griseo-pilosis; capitulis in apicibus ramorum paucis aggregatis parvis 5 mm. solum diametro; involucri hemisphaerici vel campanulati squamis subaequalibus biseriatis herbaceis oblongis griseo- vel praesertim ad apicem obtusum plus minusve translucidum purpurascenti-pilosis et ciliatis 2.8–3.4 mm. longis; receptaculo conico cum paleis parvis angustis deciduis instructo; corollis 2.5 mm. longis roseis vel pallide purpureis subglabris, dentibus limbi papillosis, tubo proprio vix ullo in fauces confestim ampliato; achaeniis nigris prismaticis parce glandulosis 1.8 mm. longis.— Gen. ii. 240 (1873); Bak. in Mart. Fl. Bras. vi. pt. 2, 190 (1876). *Isocarpha foliosa* Gardn. in

Hook. Lond. Jour. Bot. v. 457 (1846); Walp. Rep. vi. 703 (1847). *Piqueria foliosa* Gardn. l. c. vi. 432 (1847). *Emmallocalyx chamaedrifolius* Pohl ex Bak. l. c. (1876).— BRASILIA: prov. Minas Geraës ad ripas inundatas fluminis Urucuyae prope San Romão, *Gardner,* n. 4838; sine loco indicato, *Claussen, Pohl,* n. 669 (hb. Berol., phot. et fragm. in hb. Gray.); prov. Bahia ad Utinga, *Blanchet,* n. 2754, fide Bak. l. c.

6. A. FASTIGIATA (Gardn.) Benth., suffruticosa fastigiatim ramosa 4–9 dm. alta foliosissima; ramis gracilibus teretibus pallide brunnescentibus crispe puberulis vel glabriusculis; foliis oblanceolatis vel lineari-lanceolatis incise serratis vel integriusculis basi attenuatis apice acutatis vel attenuatis utrinque viridibus subtus vix pallidioribus puncticulatis 2–8 cm. longis 4–8 mm. latis saepius alternis et fasciculatis; capitulis parvis 20–40-floris 3.5 mm. diametro subdense corymbosis; involucri campanulati squamis lanceolatis viridibus pilosiusculis; receptaculo convexo vel planiusculo cum paleis oblongis paucis instructo; corollis albis (Gardner) vel rubris (Baker) vel lilaceis (Ktze.) 1.6 mm. longis, tubo proprio granuloso fauces cylindratas glabras subaequante; achaeniis graciliter prismaticis glabris acutangularibus 1.4 mm. longis.— Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 192 (1876). *Isocarpha fastigiata* Gardn. in Hook. Lond. Jour. Bot. v. 455 (1846). *Piqueria fastigiata* Gardn. l. c. vi. 431 (1847). *P. polyphylla* Sch. Bip. ex Bak. l. c. 191 (1876). *Alomia polyphylla* (Sch. Bip.) Bak. l. c. (1876). *Coelestina linearifolia* Sch. Bip. ex Bak. l. c. (1876).— BRASILIA: prov. Minas Geraës in regione Adamantium, locis humidis, *Gardner,* n. 4837 (hb. Kew., phot. in hb. Gray.); Contendas, *Kuntze* (hb. U. S. Nat. Mus.); in eadem provincia, *Riedel,* n. 711, *Pohl,* n. 371, *Sello,* nn. 120, 122; in Brasilia australi locis exactis non datis, *Pohl,* n. 535, *Riedel,* n. 959, *Beyrich;* prov. Rio de Janeiro, *Brunet,* n. 24 (hb. U. S. Nat. Mus.), Tijuca, *Lund* (hb. Havn., fragm. in hb. Gray.), *Ule,* n. 3904 (hb. Berol.).

NOTA.— *A. polyphylla* (Sch. Bip.) Bak. ob foliis angustioribus subintegris separata videtur solum forma inconstans nullo modo subtiliter disjungenda.

7. A. MYRIADENIA (Sch. Bip.) Bak., suffruticosa erecta ascendente ramea; caule tereti brunneo glabrato folioso; ramulis brevissime griseo-puberulis et cum glandulis subsessilibus conspersis; foliis rhomboideis oppositis vel ramealibus alternis graciliter petiolatis tenuibus acuminatis incise irregulariterque serratis basi cuneata subintegris juventate plus minusve pilosiusculis maturitate glabratis; capitulis in corymbis parvis subcongestis dispositis ca. 35-floris; involucri campanulati squamis subaequilongis obovati-oblongis herbaceis 4 mm. longis apice

rotundatis scarioso-marginatis ciliato-erosis dorso pilosiusculis; receptaculo conico paleaceo, paleis squamis involucri similibus angustioribus persistentibus; corollis 2.2 mm. longis tubo proprio ca. 0.3 mm. longo extus patentim hirtello, faucibus leviter ampliatis ca. 1.5 mm. longis glabris, limbi dentibus deltoideis vix 0.3 mm. longis patentibus; achaeniis subclavellati-prismaticis 2 mm. longis nigris glabris.—Bak. in Mart. Fl. Bras. vi. pt. 2, 192 (1876). *Piqueria myriadenia* Sch. Bip. ex Bak. l. c. (1876).— BRASILIA: prov. Minas Geraës, *Sello*, n. 119; Lagoa Santa, *Warming*, n. 402 (hb. Kew., phot. in hb. Gray.); Rio de Janeiro, *Glaziou*, n. 10,980 (hb. Berol., fragm. in hb. Gray.).

NOTA.— Forma corollae cum tubo proprio brevissimo hirtello et faucibus dentes limbi valde superantibus ut ab auctore observata cum descriptione cl. Bakeri male quadrat.

8. **A. longifolia** (Gardn.), comb. nov., fruticosa 1.8–2 m. alta; ramis glabriusculis teretibus striatis; foliis oppositis petiolatis oblongo-lanceolatis penninerviis serratis coriaceis basi acutis apice attenuatis 1–2 dm. longis 1–2 cm. latis glabris reticulato-venosis; capitulis 6–12-floris corymbosis; involucri squamis paucis (6–8) inaequalibus oblongis obtusis 2–3-seriatim imbricatis; receptaculo paleaceo, paleis linearibus apice plus minusve pilosis; corolla glabra; achaeniis prismaticis 5-angulatis glabris, pappo proprio nullo, annulo parvo vix angulato.— *Decachaeta longifolia* Gardn. in Hook. Lond. Jour. Bot. v, 462 (1846); Walp. Rep. vi. 705 (1847). *Ageratum longifolium* (Gardn.) Benth. in Benth. & Hook. f. Gen. ii. 242 (1873) ex Bak. in Mart. Fl. Bras. vi. pt. 2, 197 (1876). *Carelia longifolia* (Gardn.) Ktze. Rev. Gen. i. 325 (1891).— BRASILIA: prov. Minas Geraës in regione Adamantium locis calcareis, *Gardner*, n. 4863 (hb. Kew.).

9. **A. heterolepis** (Bak.), comb. nov., fruticosa; ramis virgatis foliosis apicem versus puberulis; foliis oppositis coriaceis lanceolatis ca. 1 dm. longis 2.2 cm. latis apice basique attenuatis utrinque reticulato-venosis supra viridibus glabriusculis subtus griseo-tomentellis; capitulis ca. 36-floris in corymbis congestis dispositis; involucri campanulati 5 mm. diametro squamis oblongis arcte appressis apice rotundatis dorso paullo carinatis ecostatis, interioribus subaequalibus extimis paucis gradatim brevioribus; receptaculo convexo usque ad mediam partem paleifero, paleis oblongis concavis obtusis; corollis 3.7 mm. longis glabris tubulosis, faucibus vix ampliatis; achaeniis 3 mm. longis glabris basi callosis; pappo proprio nullo.— *Ageratum heterolepis* Bak. in Mart. Fl. Bras. vi. pt. 2, 198 (1876).— BRASILIA: prov. Bahia, *Blanchet*, n. 3123 (hb. Berol., fragm. et phot. in hb. Gray.).

10. **A. microcephala** (Hemsl.), comb. nov., herba perennis; ramis virgatis teretibus glabris; foliis oppositis late lanceolatis vel ovato-lanceolatis serratis 3-nerviis 5–14 cm. longis 2–4 cm. latis firmiusculis apice basique acuminatis supra viridibus scabridis vel laevibus subtus pallidioribus cum glandulis minutis flavis densissime punctatis omnino epilosis; corymbis terminalibus multicapitulatis densis convexis; capitulis ca. 6 mm. diametro; involucri campanulati viridis squamis lanceolatis attenuatis saepissime 2-costatis glabriusculis vel parce glanduloso-atomiferis; paleis apice indurato-subulatis glabris; corollis glanduloso-atomiferis, tubo proprio fauces subaequante; achaeniis glabris acute 5-angulatis.— *Ageratum microcephalum* Hemsl. Biol. Cent.-Am. Bot. ii. 82, t. 43, ff. 1–5 (1881). *Carelia microcephala* (Hemsl.) Ktze. Rev. Gen. i. 325 (1891).— MEXICO: Oaxaca in agris et silvis, alt. 2135 m., *Galeotti*, n. 2098 (hb. Kew., phot. in hb. Gray.); prope Choapam, alt. 1170–1380 m., *E. W. Nelson*, n. 858 (hb. U. S. Nat. Mus., fragm. et icon. simplici in hb. Gray.).

11. **A. guatemalensis**, spec. nov., herbacea perennis erecta 3 dm. vel ultra alta; caulibus teretibus brunneo-pupureis gracilibus virgatis superne crispe griseo-puberulis basin versus glabratis; internodiis perlongis (7–13 cm.) folia superantibus; foliis oppositis lanceolato-linearibus remote serratis vel integriusculis 3-nerviis utrinque obscure et tenuissime pilosis subtus flavido-viridibus glanduloso-*punctatis 5–8 cm. longis 4–7 mm. latis longe attenuatis basi acutis* breviter petiolatis; corymbis terminalibus longipedunculatis; densis 10–20-capitulatis saepius subglobosis, pedicellis ca. 3 mm. longis tomentellis; capitulis 5 mm. altis 6 mm. diametro ca. 80-floris; involucri campanulati squamis subaequalibus oblongo- vel lineari-lanceolatis acutis ciliolatis dorso 2-costatis pilosiusculis; receptaculo conico ubique paleifero; paleis angustis apice subulato-acutis pallidis corollas subaequantibus; corollis 3.2 mm. longis glabris, tubo proprio fauces paullo vel modice ampliatas aequanti; achaeniis nigrescentibus glabris lucidis acute angularibus basi callosis apice cum annulo cartilagineo humillimo obscure angulato coronatis.— *Ageratum salicifolium* Coult. in J. D. Sm. Enum. Pl. Guat. iv. 72 (1895), non Hemsl.— GUATEMALA: Canchón, Depart. Santa Rosa, alt. 610 m., Oct. 1894, *Heyde & Lux*, n. 6153 a cl. J. D. Smithio distributa (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.).

12. **A. platylepis**, spec. nov., verisimiliter herbacea perennis; caule tereti primo puberulo maturitate glabrato purpureo folioso; foliis majusculis oppositis breviter petiolatis ovato-lanceolatis caudato-attenuatis subremote serratis basi rotundatis vel acutatis utrinque

glabris viridibus subtus vix pallidioribus minute glanduloso-punctatis 7–10 cm. longis 2.5–3.8 cm. latis supra basin 3-nerviis; corymbis in ramis terminalibus densis pauci(5–8)-capitulatis longiuscule pedunculatis; bracteolis parce pubescentibus linearibus; pedicellis 3–5 mm. longis rectis; capitulis pro genere majusculis 7–9 mm. diametro; involucri turbinato-campanulati squamis ovati- vel obovati-oblongis planis ecostatis acutis erosis pallidis subaequalibus (extimis paucis multo angustioribus exceptis); receptaculo undique paleifero; paleis corollas subaequantibus apice subulatis pallidis induratis; corollis 2.2 mm. longis glabris limbum versus caeruleo-purpureis, tubo proprio faucibus paullo ampliatis longiore; achaeniis nigris glabris 5-angularibus saepe arcuatis basi callosis apice cum annulo parvo albido subintegro coronatis.— GUATEMALA: prope Nenton, alt. 900–1225 m., 13 Dec. 1895, *E. W. Nelson*, n. 3528 (hb. Gray., sub nomine *Isocarpha echioides* distributa).

13. **A. isocarphoides** (DC.), comb. nov., suffruticosa; caule tereti hispido-pubescenti usque ad inflorescentiam terminalem compositam foliato; foliis oppositis 3-nerviis lanceolatis integriusculis vel crenato-dentatis subsessilibus supra scabridis subtus velutino-villosis acuminatis basi cuneato-angustatis; inflorescentia oppositi-ramea; capitulis ca. 25-floris in corymbis densis dispositis.— *Coelestina isocarphoides* DC. Prod. v. 107 (1836), incl. *β dentata*. *Ageratum isocarphoides* (DC.) Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881). *Carelia isocarphodes* [DC.] Ktze. Rev. Gen. i. 325 (1891).— MEXICO, sine loci indicio accuratiori, *Haenke* (hb. DC., phot. in hb. Gray.); in cultis prope Orizabam, *Botteri & Sumicrast*, n. 524 (hb. Gray., immatura et dubitativa). Species minus cognita, an cum sequenti conjungenda?

14. **A. echioides** (Less.), comb. nov., herbacea perennis; radice fibrosa; caulibus solitariis vel paucis virgatis suberectis simplicibus vel rariter ramosis saepius ad mediam partem solum foliosis hirsuto-villosis; foliis lanceolati-oblongis 3-costatis serrato-dentatis vel saepius integriusculis basi cuneatis apice gradatim angustatis sed vix vero acutis 2.5–9 cm. longis 8–18 mm. latis utrinque villoso-hirsutis subtus paullo pallidioribus glanduloso-punctatis; corymbis terminalibus saepius densis rarius trichotomis et laxioribus; bracteis parvis linearibus; capitulis 7 mm. diametro ca. 50-floris; involucri campanulati squamis lanceolato-oblongis viridibus vel purpurascentibus ciliolatis saepius 2-costatis apicem versus subinduratis pallidioribus; receptaculo valde elevato subcylindrato-conico undique paleifero; corollis subcylindricis sparse pubescentibus 3 mm. longis caeruleo-purpureis, tubo proprio fauces vix ampliatas fere aequante; achaeniis nigris

argute 5-angulatis glabris lucidulis ca. 2 mm. longis.— *Isocarpha echioides* Less. Linnaea, v. 141, t. 2, ff. 14–16 (1830); DC. Prod. v. 107 (1836); Hemsl. Biol. Cent.-Am. Bot. ii. 167 (1881). *Ageratum echioides* (Less.) Hemsl. Biol. Cent.-Am. Bot. ii. 81 (1881). *Carelia echiodes* [Less.] Ktze. Rev. Gen. i. 325 (1891).— Mexico: Vera Cruz, in graminosis prope Hacienda de la Laguna, *Schiede,* n. 304 (hb. Berol., icon. simplici et fragm. in hb. Gray.); prope Mirador, *Sartorius* (hb. Gray.), *Liebmann,* n. 143 (f. Hemsl. l. c.), n. 144 (hb. Havn.); *Linden* n. 1156 (f. Hemsl. l. c.); Jalapa, alt. 915 m., *Galeotti,* n. 2200; Orizaba, *Botteri,* n. 623 (hb. Gray.), *Müller,* n. 1129 (f. Hemsl. l. c.), *Bourgeau,* n. 2393 (hb. Gray.), *Sallé* (f. Hemsl. l. c.), *Thomas* (hb. Gray.); Escamella, *Bourgeau,* n. 3207 (hb. Gray.); Zacuapan, *Purpus,* n. 2199 (hb. Gray., hb. U. S. Nat. Mus.).

15. A. cinerea (Gardn.) Benth., suffruticosa griseo-puberula; caulibus pluribus teretibus striatis glanduloso-tomentellis foliosis alterni-rameis apicem versus corymboso-ramosis; foliis alternis saepe fasciculatis patentibus linearibus 4–7.5 cm. longis 3–6 mm. latis obscure crenulatis apice rotundatis basi angustissimis sessilibus utrinque griseo-tomentellis; capitulis ca. 35-floris breviter pedicellatis modice numerosis; involucri campanulati 4 mm. diametri squamis subaequalibus anguste lanceolati-oblongis obtusis dorso glanduloso-puberulis apicem versus longius ciliatis; corollis roseis extus dense hirsutis ca. 2 mm. longis in fauces ampliatis; achaeniis prismaticis 5-angularibus nigrescentibus glabris calvis basi callo conspicuo munitis.— Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 191 (1876). *Piqueria cinerea* Gardn. in Hook. Lond. Jour. Bot. vi. 432 (1847).— Brasilia: prov. Goyaz, in campis siccatis altis prope Villam de Arrayas, *Gardner,* n. 3810 (hb. Kew., phot. in hb. Gray.).

16. A. dubia Robinson, herbacea perennis glandulari-tomentosa 4–7 dm. alta; caudice erecto; caulibus teretibus striatulis foliosissimis usque ad inflorescentiam corymbosam simplicissimis; foliis (infimis caducis suboppositis exceptis) alternis oblanceolatis 2–3 cm. longis 5–9 mm. latis obtusis basin versus attenuatis vix petiolatis supra rugulosis subtus reticulatis utrinque sed praesertim in pagina inferiori glandulari-granulosis et sordide pubescentibus; capitulis ca. 1 cm. diametro 65-floris in corymbis laxis dispositis; involucri campanulati squamis sub-biserialibus herbaceis lanceolato-linearibus dorso dense glanduloso-pulverulis et apicem attenuatam versus plus minusve lanato-pilosis; corollis roseis 4 mm. longis tubo proprio gracili extus glanduloso-hirtello, faucibus turbinato-campanulatis, limbo purpurascenti-tomentello; achaeniis nigris 5-angularibus glabris basin

versus attenuatis apice annulo integerrimo cartilagineo coronatis.— Brasilia: prov. Goyaz, *Glaziou*, n. 21,579 (hb. Berol., hb. Kew., fragm. in hb. Gray.).

17. A. ageratoides HBK., herbacea perennis suberecta diffuse ramosa 3 dm. vel ultra alta; ramis oppositis teretiusculi-tetragonis primo obscure hirtellis, tardius glabratis foliatis saepissime brunnescentibus; foliis oppositis petiolatis ovatis acutis acute serratis vel subintegris 1.5–3.5 cm. longis 1–1.4 cm. latis trinerviis primo sparse hirtellis mox glabrescentibus subtus distincte pallidioribus; petiolis brevibus 2–4 mm. longis plus minusve pilosis ciliatisque; capitulis parvis ca. 6 mm. diametro ca. 25-floris laxe cymoso-paniculatis, ramulis paniculae sympodialibus scorpioideis, pedicellis filiformibus rectiusculis vel leviter arcuatis 1–3 cm. longis, bracteolis parvis 1–3 subulati-linearibus instructis; involucri campanulati squamis ca. 30 sub-biserialibus lanceolatis acutis saepissime 2-costatis; corollis purpureis 1.8 longis glabris vel glabriusculis, tubo proprio faucibus subcylindratis breviori; achaeniis glabris nitidis nigrescentibus acute 5-angulatis saepe leviter curvatis 1.5 mm. longis.— Nov. Gen. et Spec. iv. 152, t. 354 (1820); Hemsl. Biol. Cent.-Am. Bot. ii. 79 (1881); Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 134, f. 78 A (1890). *Ethulia ageratoides* (HBK.) Spreng. Syst. iii. 458 (1826). *Phalacraea Lindenii* Sch. Bip. ex Benth. & Hook. f. Gen. ii. 240 (1873).— Mexico: Guerrero prope Mescala, *Humboldt & Bonpland*, n. 3949 (hb. Par., phot. in hb. Gray.); Puebla ad Matlala, *Liebmann*, n. 84 (hb. Havn., hb. Gray.); Hidalgo ad Cazadera, *Liebmann*, n. 82 (hb. Havn., fragm. in hb. Gray.), sine loco indicato, *Liebmann*, n. 83 (hb. Havn., fragm. in hb. Gray.). Duo ultima sunt forma dubia inflorescentiis densioribus et ut apud *Ageratum littorale* vel *A. maritimum* longe pedunculatis.

18. A. angustata (Gardn.) Benth., annua erecta pauciramea 4–6 dm. alta; radice fibrosa; caule tereti striato cum pilis glanduloso-viscosis instructo; foliis plerisque alternis longe graciliterque petiolatis ovato-deltoideis vel rhomboideis grosse inaequaliterque dentatis vel lobatis utrinque pilosis 3-nerviis 6–7.5 cm. longis basi subtruncatis, dentibus sinubusque obtusis, illis mucronulatis; petiolis et pedunculis viscoso-pilosis; capitulis 12–15-floris ad apices ramorum in corymbulis irregulariter dichotomis dispositis; involucri squamis ca. 12 glanduloso-pilosis herbaceis acuminatis 3-nerviis 5 mm. longis sub-biseriatis ciliatis; corollae tubo proprio glandulari-puberulo, faucibus modice ampliatis; achaeniis prismaticis nigris calvis.— Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 190 (1876). *Piqueria angustata* Gardn. in Hook. Lond. Jour. Bot. vi. 432 (1847).— Brasilia: prov. Goyaz, in

locis saxosis silvarum umbrosarum, prope Villam de Arrayas, *Gardner*, n. 3809 (hb. Kew., phot. in hb. Gray.); prov. Minas Geraës, ad Lagoa Santa, *Warming* f. Bak. l. c.

Nota.— Folia usque adhuc ab auctore observata leviter vel modice dentato-lobata sunt sed numquam profunde pinnatifida ut a cl. Bakero, l. c., descripta. Specimen typicum est Gardneri n. 3809 nec ut in Fl. Bras. 3089 datum.

19. **A. Wendlandii** (Sch. Bip.), comb. nov., perennis 3 dm. vel ultra altitudine; caule tereti viridi striatulo pubescenti; internodiis paucis elongatis ca. 1 dm. longitudine; foliis oppositis ovatis tenuibus argute serratis ca. 6 cm. longis ca. 4 cm. latis apice acuminatis basi obtusiusculis supra tenuiter pubescentibus subtus paullo pallidioribus molliter griseo- vel cinereo-tomentosis; petiolo gracili 1–2.8 cm. longo dense pubescenti; corymbis longiuscule pedunculatis trichotomis planiusculis densis multicapitulatis ca. 5 cm. diametro; bracteis filiformibus; pedicellis filiformibus 2–8 mm. longis; capitulis erectis 60–70-floris 6–6.5 mm. altis 6 mm. diametro; involucri ovoideo-turbinati squamis oblanceolatis attenuatis sub-biseriatim imbricatis subaequalibus saepius 2-costatis dorso griseo-puberulis et cum glandulis minutis obscure nigro-punctatis; corollis saltim limbum versus caeruleis 2.7 mm. longis, tubo proprio glanduloso-hispidulo, faucibus subaequilongis parce glanduloso-atomiferis; achaeniis nigrescentibus acute 5-angulatis 1.6 mm. longis saepius curvatis apice cum annulo integro cartilagineo coronatis basi callosis.— *Phalacraea Wendlandii* Sch. Bip. ex Klatt, Leopoldina, xx. 74 (1884), in synon. et sine char. ?*Ageratum Wendlandii* Hort. ex Vilm.Fl. de pl. terre, Suppl. 2 (1884).— Mexico: ad "Gualulu" verisimiliter lapsu pennae pro Guatulco (seu Huatulco) in civitate Oaxaca, *Liebmann*, n. 147 (hb. Havn., fragm. in hb. Gray.); Etzatlan in civitate Jalisco, *Pringle*, n. 11,819 (hb. Gray.).

Nota.— *Ageratum Wendlandii* Hort. huc dubitanter locatum variat flosculis albis f. Vilm. l. c.

20. **A. microcarpa** (Benth.), comb. nov., herbacea annua erecta vel decumbens undique molliter pilosa 3–6 dm. alta; radice fibrosa; caulibus teretibus saepius flexuosis medullosis usque ad inflorescentiam foliatis viridibus vel purpureis; foliis oppositis petiolatis deltoideo-ovatis saepius cordatis vel subcordatis 3–6 cm. longis 2–4 cm. latis crenato-serratis obtusis vel vix acutiusculis membranaceis subconcoloribus utrinque pilosis; petiolo 8–18 mm. longo saepius hirsuto; corymbis terminalibus et ex axillis superioribus oriuntibus densis multicapitulatis; capitulis ca. 60-floris; involucri campanulati squamis anguste lanceolatis saepius 2-costatis attenuatis viridibus plus

minusve pilosis; corollis limbum versus caeruleo-purpureis, tubo proprio gracili glanduloso-atomifero fauces cylindratas glabras distincte ampliatas fere aequanti; achaeniis nigris glabris argute 5-angulatis prismaticis 1.1 mm. longis.— *Coelestina microcarpa* Benth. ex Oerst. Vidensk. Meddel. 1852, p. 72 (1852). *Ageratum microcarpum* (Benth.) Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881).— Costa Rica: in graminosis, etc., Cartago, *Oersted,* nn. 241, 247, 248 (omnia in hb. Kew., phot. in hb. Gray.); in pascuis ad Général, *Pittier,* n. 3415 (hb. Gray.); Turrialba in pascuis, alt. 1200 m., *Pittier,* n. 4139 (hb. Gray.); Aguacaliente secundum viam ferriam, alt. 1300 m., *Pittier,* n. 2390 (hb. Gray., hb. J. D. Sm.); Poaz, alt. 2200 m., *Tonduz,* n. 10,816 (hb. Gray.); in sepis secundum vias, San José, *Tonduz,* n. 7281 (hb. Gray., hb. J. D. Sm.); in terris cultis et incultis, San José, alt. 1135 m., *Pittier,* n. 3533 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); in pascuis, San Francisco de Guadeloupe, alt. 1170 m., *Tonduz,* n. 8479 (hb. U. S. Nat. Mus., hb. J. D. Sm.); in agro ad Juan Viñas, alt. 1000 m., *Cook & Doyle,* n. 277 (hb. U. S. Nat. Mus.). Venezuela: ad Caracas, *Linden,* n. 349 (hb. Kew.), forma dubia parvifolia.

Subg. 3. **Lycapsus** (Phil.), subg. nov. Involucri squamae 6–8 subuniseriales subaequales angustae herbaceae enerviae. Receptaculum paleaceum. Corollae fauces distincte ampliatae. Styli rami pro tribu breviusculi valde spiraliter recurvati.— *Lycapsus* Phil. Bot. Zeit. xxviii. 499, t. 8A (1870).— Frutex xerophyticus. Folia alterna pinnatipartita. Species 1 chilensis.

21. A. tenuifolia (Phil.) Benth. & Hook. f., fruticosa ramosa glaberrima; foliis alternis crassiusculis pinnatipartitis, rhachi laciniisque filiformibus; petiolo ca. 17 mm. longo rhachin subaequanti, laciniis utrinque 3–4 oppositis vel alternis usque ad 9 mm. longis 1.3 mm. crassis; capitulis corymbosis pedicellatis; bracteolis 1–2 lineari-setaceis 3–4 mm. longis; involucri hemisphaerici squamis 3 mm. longis lanceolati-linearibus herbaceis subaequalibus vix imbricatis corollas aequantibus; corollis verisimiliter albis; tubo proprio glandulari-atomifero fauces glabras cylindratas aequanti; limbi dentibus 5 patentim recurvatis; achaeniis 2 mm. longis granulosis deorsum decrescentibus.— Benth. & Hook. f. Gen. ii. 240 (1873), sine combinatione definitiva sed ex Reicheo, Fl. de Chil. iii. 260 (1901).— Chili: in insula San Ambrosio.

Nota.— Haec planta minime cognita a navarcho innominato navis bellicae chilensis in petaso suo collecta fide cl. Philippii (l. c. 501) in insula San Ambrosio inveniebatur, nec ut dicit cl. Reiche (l. c.) in insula San Felix.

Species reducta vel exclusa.

A. Armani (Balbis) Bak. in Mart. Fl. Bras. vi. pt. 2, 191 (1876). *Eupatorium Armani* Balbis, Pl. Rar. Hort. Turin, 1810, p. 27, t. 6 (1810). *Orsinia Eupatoria* DC. Prod. v. 104 (1836). *Piqueria Eupatorium* (DC.) Gardn. in Hook. Lond. Jour. Bot. vi. 430 (1847). *Clibadium rotundifolium* DC. Prod. v. 505 (1836); Bak. l. c. vi. pt. 3, 152 (1884), ubi syn. alia. = CLIBADIUM ARMANI (Balbis) Sch. Bip. ex Bak. l. c. (1884).

A. polyphylla (Sch. Bip.) Bak. = A. FASTIGIATA (Gardn.) Benth.

A. spilanthoides D. Don ex Hook. & Arn. Comp. Bot. Mag. i. 238 (1835) = GYMNOCORONIS SPILANTHOIDES (D. Don) DC. Prod. vii. 266 (1838).

2. REVISION OF THE GENUS AGERATUM.

The genus *Ageratum* L. has not been subjected to any general revision since its treatment in DeCandolle's Prodromus in 1836. For many years it has been made to include plants of a considerable range of habit and, what is more noteworthy, though chiefly defined by its pappus, has been allowed to contain species of widely divergent character in just this matter. While the more typical species have a pappus of five distinct scales, others have a cup-like crown of very short and connate scales; still others have been admitted into the genus which instead of scales of definite number have short or long, slender or slightly thickened, smooth or plumose bristles of indefinite number ranging from 8 to 20 or more. Finally certain species have been included from similarity of habit which possess no true pappus whatever but merely a sort of annulus beneath rather than exterior to the corolla.

To render the genus properly natural and compact, as well as to permit its more precise definition, it seems best to refer to *Alomia* the species destitute of pappus, and to exclude also those species which have a bristle pappus. The latter group consists of six South American species, namely *A. Agrianthus* Hoffm. (*Agrianthus corymbosus* DC.), *A. alternifolium* (Gardn.) Bak., *A. campuloclinioides* Bak., *A. confertum* (Gardn.) Benth., *A. melissaefolium* DC., and *A. Pohlianum* Bak. At first, it seemed likely that these species could be appropriately separated as a distinct genus. Schultz-Bipontinus seems to have planned such a segregation in his undescribed *Melissopsis*. The species, however, differ much among themselves, both as to habit and technical characters, and several are scarcely to be separated from *Trichogonia* except by their slightly shorter and thicker pappus-bristles. Their satisfactory generic disposition must await further study, for which there is no adequate material as yet available in the North American herbaria.

The other two elements traditionally included in *Ageratum*, namely, the true Ageratums with pappus of distinct scales, and *Coelestina* Cass. with cuplike or crownlike pappus, are groups of very close affinity, by no means sharply separable. It is believed that they are best treated merely as sections of the genus *Ageratum*.

The writer has not had many opportunities to follow up the various forms of *Ageratum* which have received horticultural names. So far as seen these have proved merely cultural improvements of the common annuals, *A. conyzoides* L. and especially *A. Houstonianum* Mill. *A. rubens* Viviani, early described from cultivated material, has never been recognized. *A. Lasseauxii* Carr., to judge from a supposably authentic specimen in the Gray Herbarium, is clearly a species of *Eupatorium*. *A. conspicuum* Hort. is generally believed to have been *Eupatorium glechonophyllum* Less. *Ageratum Wendlandii* of Vilmorin's Fleurs de pleine terre, Suppl. 2 (1884), with uncharacteristic figure and mere horticultural description, was presumably founded on the same plant as the one to which Schultz-Bipontinus applied the manuscript name *Phalacraea Wendlandii*, later published in synonymy by Klatt. If this is the case Vilmorin's plant was the one described elsewhere in this paper as *Alomia Wendlandii* and came from the uplands of southern central Mexico. It would seem probable that both *Ageratum* and *Alomia* would repay further horticultural attention, there being several other species quite as promising as those already brought into cultivation.

AGERATUM L. (Nomen ab antiquis et graece et latinice ad plantam aliquem non viescentem non certe cognitam fortasse ut dicitur *Achilleam* applicatum.)— Capitula homogama tubuliflora. Involucrum plerumque campanulatum rariter turbinato-subcylindratum vel hemisphaericum, squamis angustis longitudine subaequalibus 2–3-seriatim imbricatis (cum vel absque squamulis 1–3 extimis multo brevioribus) plerisque lanceolato-linearibus acutis vel attenuatis saepius 2(1–4)-costatis. Receptaculum planum vel convexum vel conicum nudum vel paleiferum. Corolla 5-dentata limbum versus saepius caerulea vel purpurea vel alba rarius rosea, tubo proprio superne sensim in fauces subcylindratas plus minusve ampliato.

Antherae oblongae vel lineares apice cum appendicem membranaceam ovatam vel oblongam munitae basi rotundatae. Achaenia 5-angulata prismatica vel deorsum paullo decrescentia. Pappus e squamis apice setiferis vel muticis distinctis vel basi connatis vel in coronam crateriformem integram vel dentatam compositus.— Sp. Pl. ii. 839 (1753); DC. Prod. v. 108 (1836); Benth. & Hook. f. Gen. ii. 241 (1873), excl. syn. *Decachaeta* et *Oxylobus;* Bak. in Mart. Fl. Bras. vi. pt. 2, 193 (1876), pro parte; Hemsl. Biol. Cent.-Am. Bot. ii. 80 (1881), pro parte majori; Gray, Syn. Fl. i. pt. 2, 93 (1884); Hoffm. in Engl. & Prantl. Nat. Pflanzenf. iv. Ab. 5, 137 (1890), excl. syn. *Oxylobus;* Dalla Torre & Harms, Gen. Siph. 527 (1905), excl. syn. *Oxylobus*, *Melissopsis*, *Decachaeta*. *Carelia* Adans. Fam. ii. 123 (1763); Ktze. Rev. Gen. i. 325 (1891). *Coelestina* Cass. Bull. Soc. Philom. Par. 1817, p. 10 (1817); DC. Prod. v. 107 (1836), partim. *Caelestina* Cass. Dict. Sci. Nat. vi. suppl. 8, t. 93 (1817).— Herbae annuae vel perennes vel suffrutices vel frutices. Folia saepius opposita rariter alterna saepissime ovata vel lanceolata crenata vel serrata rariter integra sessilia vel petiolata saepe glandulari-punctata. Inflorescentia plerumque terminalis corymbosa vel cymosa saepius composita.

Genus extraneis expurgatis e speciebus 27 sistens quarum 19 regionem mexicano-centrali-americanam incolunt, aliae inter Floridam australem et insulas Indiae Occidentalis et Americam Australem calidiorem distributae, una africana, una in terris calidioribus latissime dispersa.

Sect. I. EUAGERATUM DC. Pappi squamae omnino distinctae vel imo paullulo connatae nunc longae et apice setiferae corollam subaequantes nunc mutices corolla multo breviores rariter (apud *A. maritimi* et *A. littoralis* formas nonnullas) omnino desunt.— Prod. v. 108 (1836). *Ageratum verum* Bak. in Mart. Fl. Bras. vi. pt. 2, 194 (1876), pro parte.— Species 13 pleraeque herbaceae saepius annuae.

Clavis specierum.

a. Caulis procumbens vel prostratus repens *b.*
b. Folia parva suborbicularia profunde crenata. Capitula solitaria 1. *A. domingense.*
b. Folia majora lanceolata subintegra. Capitula cymosa 2. *A. radicans.*
a. Caulis erectus vel plus minusve decumbens *c.*
c. Folia sessilia integra *d.*
d. Folia subtus griseo-incana. Afr. . . . . 3. *A. polyphyllum.*
d. Folia subtus glabra viridia. Am. Cent. . . . . 4. *A. Peckii.*
c. Folia petiolata crenata vel dentata *e.*
e. Pappi paleae omnes vel nonnullae apice setiferae corollam aequantes *f.*

f. Capitula subcongestim cymoso-corymbosa *g.*
  g. Decumbens superne nudum. Sp. maritima 13. *A. littorale*, f. *setigerum.*
  g. Erectum vel suberectum saepius usque ad inflorescentiam foliosum *h.*
    h. Annuum. Pars pappi palearum squamiforme dilatata setam fere aequans *i.*
      i. Involucri squamae lanceolato-lineares integrae ciliolatae longe in apicem peracutam attenuatae saepius coloratae dorso hirsutae........5. *A. Houstonianum.*
      i. Involucri squamae oblongae subabrupte acuminatae ciliatae saepius margine plus minusve dentatae vel erosae dorso parce pubescentes vel glabriusculae 6. *A. conyzoides.*
    h. Suffruticosum. Pars pappi palearum squamiforme dilatata quam seta triplo brevior......7. *A. suffruticosum.*
f. Inflorescentia laxa; capitulis omnibus longiuscule (1–4 cm.) pedicellatis..............................8. *A. Gaumeri.*
e. Pappi paleae omnes muticae corollae tertiam partem longitudine vix aequantes *j.*
  j. Erectum vel suberectum. Folia 2–8 cm. longa. Sp. non maritimae *k.*
    k. Perenne. Folia serrata longe attenuata acuta. Guatemala 9. *A. rugosum.*
    k. Perenne. Folia crenata obtusa vel obtuse acuminata. Petiolo obcompressi late planiusculi. Caulis paullo lignescens. Involucrum basi saepius umbonatum. Mex. 10. *A. platypodum.*
    k. Annuum omnino herbaceum. Petioli subteretes. Involucrum basi plus minusve acutatum......11. *A. latifolium.*
  j. Decumbens. Folia 1–1.8 cm. longa. Sp. maritimae *l.*
    l. Caulis usque supra mediam partem foliosus. Folia ovato-oblonga vel anguste deltoidea regulariter crenata. Corymbi 3–5-capitulati...............12. *A. maritimum.*
    l. Caulis vix ad mediam partem foliosus. Folia rhomboideo-ovata vel infimis breviter lateque deltoidea. saepissime inciso-dentata. Corymbi 4–13-capitulati 13. *A. littorale.*

1. A. DOMINGENSE Spreng., herbaceum pusillum prostratum reptans; caule gracili ad nodos radicantes folia et ramos suberectos basi foliatos apice floriferos et etiam scapos nudos unicapitatos gerenti; foliis oppositis suborbicularibus 6–12 mm. diametro tenuibus grosse crenato-lobulatis cordatis, petiolis 3–16 mm. longis; capitulis solitariis terminalibus 5–8 mm. diametro; involucri turbinati squamis viridibus tenuibus vix costatis oblanceolatis acutis laxe imbricatis subaequilongis dorso laxe pilosis; pedunculis vel scapis flexuosis saepe nutantibus 3–7.5 cm. longis; corollis albis 2 mm. longis glanduloso-atomiferis, tubo proprio gracili faucibus campanulatis brevioribus; achaeniis glabris nigris lucidis deorsum decrescentibus basi substipitati-callosis; pappi squamulis oblongis fimbriatis quam achaenio bis brevioribus.— Syst. iii. 446 (1826). *Phania domingensis*

(Spreng.) Griseb. Cat. Pl. Cub. 145 (1866). *Carelia domingensis* (Spreng.) Ktze. Rev. Gen. i. 325 (1891). *Eupatorium planellasianum* Maza & Molt. An. Hist. Nat. Madr. xix. 271 (1890).— SANTA DOMINGO: *Bertero* (hb. DC., icon. simpl. in hb. Gray.). CUBA: in ripis fluminis Sta. Catalina, *Wright,* n. 2798 (hb. Gray.); in saxis prope San Diego de Tapia, *Wright* (hb. Gray.); sub fruticibus in collibus petrosis Bahia Honda, prov. Pinar del Rio, *Wilson,* n. 9405 (hb. Gray.).

NOTA.— Quamquam planta domingensis a cl. Sprengel glabra descripta sit ea cubensis tamen distincte pubescens se exhibet.

2. A. RADICANS Robinson, herbaceum glabrum prostratum reptans; caule tereti crassiusculo medulloso; ramis patentibus ad nodos saepe radicantibus apicem versus ascendentibus; foliis oppositis lanceolatis vel anguste elliptico-oblongis 3-nerviis integriusculis utroque angustatis apice obtusis utrinque glabris supra viridibus subtus vix pallidioribus impunctatis 4–8 cm. longis 5–15 mm. latis breviter petiolatis; pedunculis nudis cymos parvos 2–3-capitulatos terminales vel etiam laterales gerentibus; capitulis breviter pedicellatis 8–10 mm. diametro; involucri squamis lanceolatis vel linearibus fere a basi ad apicem attenuatis 2-costatis peracutis glabris; receptaculo parvo convexo nudo; corollis glabris limbum versus purpureis; achaeniis acute 5-angularibus 1.2 mm. longis glabris; pappi paleis 5 ovatis scariosis apice in setam productis corollam aequantibus.— Proc. Am. Acad. xlvii. 192 (1911).— BALIZE: in aquis dulcis prope Manatee Lagoon, *Peck,* n. 99 (hb. Gray.).

3. A. POLYPHYLLUM Bak., herbaceum perenne foliosissimum; caulibus erectis simplicibus pubescentibus; foliis sessilibus oppositis lanceolatis integris 1–2.5 cm. longis margine revolutis supra glabris viridibus subtus griseo-incanis tomentosis; corymbis densis; capitulis ca. 5 mm. diametro multifloris; involucri campanulati squamis lanceolatis pauciseriatim imbricatis extimis gradatim brevioribus; corollis rubris tubo proprio pubescenti; achaeniis glabris; pappi paleis paucis obtusis quam corolla triplo brevioribus.— Kew Bull. 1898, p. 148 (1898).— AFRICA CENTRALIS ANGLICA: ad Nyikam, alt. 1800–2150 m., *Whyte,* n. 252. Non vidi.

4. A. PECKII Robinson, herbaceum annuum erectum 5 dm. altum fastigiatim ramosum glabrum; radice fibrosa; caule subtereti basin versus crassiusculo nodoso; foliis lineari-oblongis integris 3-nerviis 2.5–4.5 cm. longis 2.5–6 mm. latis acutiusculis sed in apice vero obtusis basi petioliforme attenuatis margine revolutis; capitulis paucis

3.5–4 mm. diametro ca. 20-floris in cymis dichotomis dispositis; bracteolis lineari-subulatis 2–4 mm. longis; involucri squamis lineari-lanceolatis attenuatis saepius 2-costatis acutissimis longitudine subaequalibus glabris; corollis glabris caeruleo-purpureis; achaeniis nigris 5-angularibus in angulis parce hispidulis in faciebus obscure granulosis; pappi paleis ovatis scariosis argenteis plus minusve laciniatis apice setiferis corollam aequantibus.— Proc. Am. Acad. xlvii. 191 (1911).— BALIZE: in apertis arenosis in collibus piniferis prope Manatee Lagoon, *Peck,* n. 80 (hb. Gray.).

5. A. HOUSTONIANUM Mill., annuum robustum erectum vel decumbens; caule tereti medulloso per totam longitudinem folioso-patentim piloso plus minusve ramosum; foliis oppositis late deltoideo-ovatis saepius crenatis hinc inde (in eodem individuo) acute serratis basi saepissime late cordatis rarius truncatis vel paullo acutatis apice saepius obtuse acuminatis 3.5–12 cm. longis 1.5–9 cm. latis tenuibus utrinque viridibus hirsutis subtus paullo pallidioribus impunctatis; corymbis terminalibus saepe compositis densis valde convexis multicapitulatis glanduloso-hirtellis et cum pilis moniliformibus albis longius hirsutis; capitulis ca. 8 mm. diametro ca. 75-floris; involucri squamis anguste lanceolatis vel lanceolato-linearibus integris herbaceis longe continenterque attenuatis acutissimis ciliolatis apicem versus saepius purpureis dorso 2-costatis glandulari-puberulis et conspicue hirsutulis; corollis gracillimis ca. 3 mm. longis limbum versus caeruleis, tubo proprio parce glandulari-puberulo fauces cylindratas glabriusculas aequanti vel paullo superanti, limbi dentibus extus hispidulis; achaeniis nigris nitidis saepius in angulis et etiam plus minusve in faciebus sursum hispidulis ca. 1.2 mm. longis; pappi paleis 5 lanceolatis margine fimbriatis apice longe setiferis, setis sursum scabridis corollas subaequantibus.— Dict. ed. 8, n. 2 (1768). *A. mexicanum* Sims, Bot. Mag. t. 2524 (1825); Sweet, Brit. Fl. Gard. t. 89 (1825), a copia eadem ac ea Simsii descripta, pappi squamulis a cl. Sweetio per errorem nimium brevioribus depictis; Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881), pro parte. *A. conyzoides,* var. *mexicanum* (Sims) DC. Prod. v. 108 (1836). *A. conyzoides* Hemsl. l. c. 81 (1881), pro parte, non L. *Carelia Houstoniana* (Mill.) Ktze. Rev. Gen. i. 325 (1891).— MEXICO: praesertim in civitate Vera Cruz: Cordova, *Bourgeau,* n. 1557 (hb. Gray., hb. U. S. Nat. Mus.); Orizaba, alt. 1220 m., *Seaton,* n. 55 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); Jalapa, *Schiede,* f. Hemsl. l. c., alt. 1220 m., *Pringle,* no. 8065 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); Fortin, *Kerber,* n. 302 (hb. Havn.); Mirador, *Sartorius* (hb. Gray.); Coatzacoalcos, *C. L. Smith,* n. 145 (hb. Gray.), n. 979

(hb. U. S. Nat. Mus.); Huasteca, Tantoyuca, *Ervendberg*, n. 100 (hb. Gray.); Tepic, *Palmer* (anno 1892), n. 2066 (hb. U. S. Nat. Mus.). COSTA RICA: San Rafael de Cartago, *Pittier*, n. 6995 (hb. Gray.). GUATEMALA: Alta Vera Paz, *Goll*, n. 238 (hb. U. S. Nat. Mus.). CUBA: in agro nicotianae, Pinar del Rio, *Palmer & Riley*, n. 65 (hb. U. S. Nat. Mus.), fl. roseis. JAMAICA: *Eggers*, n. 3445 (hb. J. D. Sm.); Cinchona, alt. 1525 m., *Clute*, n. 190 (hb. U. S. Nat. Mus.); in marginibus silvarum, Morce's Gap, alt. 1525 m., *Nichols*, n. 33 (hb. Gray., hb. U. S. Nat. Mus.). MARTINIQUE: cultum et erratum, *Duss*, n. 4681 (hb. U. S. Nat. Mus.), partim. BRASILIA: Rio de Janeiro, *Rudio* (hb. Gray., hb. U. S. Nat. Mus.). ST. HELENA: *Brown & Brown*, n. 254 (hb. U. S. Nat. Mus.). ASSAM: Mangaldai, *Chatterjee* (U. S. Nat. Mus.).

NOMEN VULGATUM hispanice (Mex.) "Yerba de zopilote" ex cl. Kerbero.
NOTA.— Haec species pro genere optima, origine certe austro-mexicana est diu ob floribus laete caeruleis vel albis perornatis ubique culta et aliter ex hortis errata vel fortuite dispersa saepe in locis incultis vel ruderatis reparatur.

6. A. CONYZOIDES L., annuum pubescens oppositi-rameum normaliter erectum rarius decumbens 2.5–9 dm. altum; radice fibrosa; caule molli tereti patentim pubescenti saepius purpurascenti per totam longitudinem folioso; ramis patentim ascendentibus; foliis ovatis plerisque obtusis basi rotundatis vel breviter cuneatis rarius late cordatis crenatis vel rarius serratis tenuibus utrinque parce pilosis subtus dilutiore viridibus 2–8 cm. longis 1.5–6 cm. latis a basi 3(–5)-nerviis impunctatis vel subtus glandulis paucis inconspicuis conspersis; petiolis 0.5–3 cm. longis hirsutis; corymbis in caule ramisque terminalibus saepe compositis (3–)8–40-capitulatis valde convexis breviter pedunculatis; pedicellis filiformibus 3–7 mm. longis glandulosis vel hispidulis; bracteolis setaceis; capitulis 50-floris ca. 6 mm. diametro; involucri campanulati squamis oblongis viridibus saepius 2-costatis apice subabrupte acuminatis margine scariosis sub acumine saepius erosis et ciliatis dorso parce pilosis vel glabriusculis; receptaculo nudo; corollis limbum versus caeruleis ca. 2 mm. longis, tubo proprio gracili fauces modice ampliatas paullo superanti glabriusculo vel glandulari-puberulo; achaeniis nigris nitidis in angulis saepius minute sursum hispidulis; pappi paleis 5 lanceolatis margine fimbriolatis apice setiferis corollam subaequantibus.— Sp. Pl. ii. 839 (1753). *A. hirtum* Lam. Dict. i. 54 (1783). *A. conyzoides*, *β hirtum* (Lam.) DC. Prod. v. 108 (1836). *A. hirsutum* Poir. Suppl. i. 242 (1810), sphalm. pro *hirtum*. *A. odoratum* Vilm. Fl. Pl. Terre, ed. 2, 42 (1866).

Cacalia mentrasto Vell. Fl. Flum. 339 (1825), viii. t. 69 (1827). *Carelia conyzodes* [L.] Ktze. Rev. Gen. i. 325 (1891), cum varietatibus vegetivis *robusta, umbrosa, pusilla, coerulea.*— MEXICO: Tepic, *Palmer* (anno 1892), n. 1850 (hb. Gray., hb. U. S. Nat. Mus.), n. 1890 (hb. Gray.). GUATEMALA: *Heyde,* n. 530 (hb. U. S. Nat. Mus.); Santa Rosa, alt. 915 m., *Heyde & Lux* (distrib. J. D. Sm.), n. 3781 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); Coban, alt. 1310 m., *v. Tuerckheim* (distrib. J. D. Sm.), n. 488 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.), alt. 1350 m., *v. Tuerckheim,* n. II 999 (hb. U. S. Nat. Mus.); Gualan, *Deam,* n. 330 (hb. Gray., hb. U. S. Nat. Mus.); Alta Vera Paz, *Cook & Griggs,* n. 560 (hb. U. S. Nat. Mus.); prope Nenton, alt. 915–1220 m., *Nelson,* n. 3542 (hb. J. D. Sm.); in agris gossypii vulg., prope Mazatenango, alt. 350 m., *Maxon & Hay,* n. 3487 (hb. U. S. Nat. Mus.); El Paxte, in monte igniv. Ipala, alt. 1000 m., *Pittier,* n. 1891 (hb. U. S. Nat. Mus.); Las Ánimas, alt. 200 m., *Shannon* (distrib. J. D. Sm.), n. 600 (hb. J. D. Sm.). HONDURAS: Rio Permejo, alt. 180 m., *Thieme* (distrib. J. D. Sm.), n. 5323 pro parte (hb. J. D. Sm.). COSTA RICA: Cartago, alt. 1300 m., "Santa Lucia incolarum," *Cooper* (distrib. J. D. Sm.), n. 5824 (hb. J. D. Sm.). NICARAGUA: Chinandega, *Baker,* n. 2021 (hb. Gray., hb. U. S. Nat. Mus.). SALVADOR: *Renson,* n. 190 (hb. U. S. Nat. Mus.). CUBA: prope villam Monte Verde dictam, *Wright,* 1310 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); in ripis, Cienfuegos, *Combs,* n. 59 (hb. Gray.). INSULA PINORUM: in silvis pinorum, *Palmer & Riley,* n. 1068 (hb. U. S. Nat. Mus.). PORTO RICO: *Sintenis,* nn. 59 (hb. U. S. Nat. Mus., hb. J. D. Sm.), 3661 (hb. J. D. Sm.), 5816 (hb. J. D. Sm.); *Heller,* nn. 542 (hb. U. S. Nat. Mus.), 6143 (hb. Gray., hb. U. S. Nat. Mus.); *Britton & Cowell,* n. 927 (hb. U. S. Nat. Mus.); *Underwood & Griggs,* nn. 871, 872 (hb. U. S. Nat. Mus.). ST. THOMAS: *Eggers,* n. 303 (hb. Gray., hb. J. D. Sm.). ST. CROIX: *Ricksecker,* nn. 430 (hb. Gray., hb. U. S. Nat. Mus.), 250 (hb. U. S. Nat. Mus.). MONTSERRAT: *Shafer,* n. 207 (hb. U. S. Nat. Mus.). GUADELOUPE: *Duss,* n. 2520 (hb. U. S. Nat. Mus.). MARTINIQUE: *Duss,* nn. 934, 4681 partim (hb. U. S. Nat. Mus.); *Hahn,* n. 1190 (hb. Gray., hb. U. S. Nat. Mus.). ST. VINCENTS: *H. H. & G. W. Smith,* n. 543 (hb. Gray.). GRENADA: *Broadway* (hb. Gray., hb. U. S. Nat. Mus.). BARBADOS: n. 124 legulo innom. (hb. Gray., hb. U. S. Nat. Mus.). COLUMBIA: Santa Marta, alt. 305 m., *H. H. Smith,* n. 523 (hb. Gray., hb. U. S. Nat. Mus.). VENEZUELA: prope Tovar, alt. 1980 m., *Fendler,* n. 652 (hb. Gray.). GUIANA ANGLICA: in terris litoreis, *Jenman,* n. 5398 (hb. J. D. Sm.). BOLIVIA: Mapiri, alt. 1525 m., *Rusby,* nn. 1642 (hb. U. S. Nat. Mus.), 1643 (hb. Gray., hb. J. D. Sm.), *Buchtien*, n. 1456 (hb. Gray.); Yungas, *Bang*, n. 407 (hb. U. S. Nat. Mus.). BRASILIA: in pascuis ruderatisque prope Rio de Janeiro, *Beyrich* (hb. Gray.); *Rudio* (hb. Gray., hb. Berol.); Petropolis, *Binot*, n. 15 (hb. Berol.); ex umbrosis juxta Tijuca, *Ball* (hb. U. S. Nat. Mus.); juxta Santos, *Ball* (hb. Gray.); Minas Geraës, *Regnell*, n. III 675 (hb. J. D. Sm.); Saõ Paulo, *Everett*, n. 16 (hb. Gray.); sine loco indicato, *Martius*, n. 672 (hb. Gray.), *Burchell*, n. 854 (hb. Gray.), *Luschnath* (hb. Gray.), *Riedel*, n. 1346 (hb. Gray.). INS. ST. HELENA: *Wilkes* (hb. U. S. Nat. Mus.). SIERRA LEONE: Freetown, *W. H. & A. H. Brown*, nn. 31a, 51 (hb. U. S. Nat. Mus.). AEGYPTO: prope Luxorem, *Kralik* (hb. Gray.), prope Girgeh, *Joad* (hb. U. S. Nat. Mus.). NUBIA: in insula Tutli ad urb. Chartum, *Kotschy*, n. 327 (hb. Gray.). AFR. ORIENT. GERMAN.: Kilima Njaro, *Abbott* (hb. U. S. Nat. Mus.); Usambara, *Holst*, n. 8860 (hb. U. S. Nat. Mus.). NYASSALAND: *Buchanan*, n. 740 (hb. U. S. Nat. Mus.). SANSIBAR: *Stuhlmann* (hb. Gray.). MADAGASCAR: Imerina, *Hildebrandt*, n. 3500 (hb. Gray., hb. U. S. Nat. Mus.); Nossi-be, *Hildebrandt*, n. 3304a (hb. J. D. Sm.); *Shufeldt*, n. 101 (hb. U. S. Nat. Mus.). CHINA: Meng-Tsze, *Henry*, n. 9094 (hb. U. S. Nat. Mus.); Canton, *Williams* (hb. Gray.); Hong Kong, *Wright* (hb. U. S. Nat. Mus.). IND. OR.: Sikkim, reg. subtrop., *Hooker f.* (hb. Gray.), Khasia, reg. subtrop., *Hooker f. & Thompson* (hb. Gray.); in locis incultis prope Mangalor, *Hohenacker*, n. 1 (hb. Gray.). ASSAM: Dumar Dallang, *Watt*, n. 10, 431 (hb. U. S. Nat. Mus.). SIAM: Bangkok, *Zimmermann*, n. 70 (hb. U. S. Nat. Mus.). BURMA SUPERIOR: a legulo cl. Kingii, n. 11 (hb. Gray.). MALACCA: *Cuming*, n. 2362 (hb. Kew., phot. in hb. Gray.). JAVA: *Zollinger*, n. 23 (hb. Gray.). INS. PHILIPPENSES: *Cuming*, n. 2419 (hb. Gray.); Manila, *Merrill*, n. 35 (hb. Gray.); Luzon, *Merrill*, n. 1955 (hb. Gray.), *Ramos*, n. 2086 (hb. Gray.); Cution, *Merrill*, n. 563 (hb. Gray.). INS. VITENSES: *Seemann*, n. 267 (hb. Gray.). INS. TONGENSES: *Moore*, n. 469 (hb. U. S. Nat. Mus.). INS. HAWAIIENSES: Oahu, *Remy*, n. 229 (hb. Gray.), *Drill* (hb. Gray.), *Heller*, n. 1999 (hb. Gray., hb. U. S. Nat. Mus.).

Forma **album** (Willd.), comb. nov., formae typicae habitu, foliis, etc., simillimum differt solum corollis albis.— *A. album* Willd. ex Steud. Nom. 18 (1821). *Carelia conyzodes* [L.] Ktze., α *robusta* Ktze., var. *alba* (Willd.) Ktze. Rev. Gen. i. 325 (1891), etiam γ *pusilla* Ktze., var. *alba* Ktze., l. c.— Cum forma typica late dispersa tamen multo rarior, e. g. Porto Rico: in graminosis ad "Cacoa," *Sintenis*, n. 5874 (hb. J. D. Sm.).

Var. INAEQUIPALEACEUM Hieron., formae typicae habitu, foliis, etc., simillimum differt pappi paleis valde inaequilongis, saepius 1–3 setiferis corollas subaequantibus, ceteris brevioribus squamiformibus muticis.— Hieron. in Engl. Bot. Jahrb. xix. 44 (1894).— COLUMBIA: ad Fusagasuga, *Holton* (hb. Kew.); prope Popayan, *Lehmann*, n. 4666 (hb. Berol., fragm. in hb. Gray.); in campis prope Chapa in prov. Cauca, alt. 1860 m. (fl. albis), *Lehmann*, n. 3600 (hb. Gray., hb. J. D. Sm.), n. 3601, fl. caeruleis (hb. Gray.). BOLIVIA: Yungas, *Bang*, n. 407 (hb. Gray.). PERUVIA: Lima, *Wilkes* (hb. U. S. Nat. Mus., hb. Gray.). INS. BAHAMENSES: ad Nassau, *Curtiss*, n. 77 (hb. Gray., hb. U. S. Nat. Mus.); Harbor Ins., *E. G. Britton*, n. 6375 (hb. U. S. Nat. Mus.). INSULA PINORUM: *Taylor*, n. 191 (hb. Gray., hb. U. S. Nat. Mus.). SANTO DOMINGO: in ripis fluminis Jaina, *Wright, Parry & Brummel*, n. 250 (hb. U. S. Nat. Mus.), n. 264 (hb. Gray., hb. U. S. Nat. Mus.).

NOTA.— Haec varietas in loco intermedio stat inter *A. conyzoidem* et *A. latifolium* et occurrit solum in regionibus ubi utraque species reparantur, numquam ut videtur in Africa, Asia, insulis Pacifici Maris, ubi *A. latifolium* deest. An forma hybrida?

7. A. SUFFRUTICOSUM Regel, perenne suffruticosum canescenti-pilosum; caule ramoso ad inflorescentiam folioso dense pubescenti; foliis oppositis vel supremis alternis rhomboideo-ovatis serrato-dentatis 3-nerviis attenuatis sed in apice vero obtusis basi abrupte contractis ad insertionem petioli tamen plus minusve cuneatis; corymbis trichotomis planiusculis multicapitulatis; capitulis multifloris; involucri squamis subaequalibus; corollis caeruleis; achaeniis argute angulatis leviter arcuatis in angulis minute sursum hispidulis; pappi paleis 5 ovato-lanceolatis dentatis in aristas ca. 3-plo longioribus disinentibus. — Gartenfl. iii. 389, t. 108 (1854). *Eupatorium nanum* Hort. ex Regel, l. c. 389. *Ageratum nanum* Hort. ex Sch. Bip. in Koch & Fint. Wochenschr. i. Garten-Nachtr. 26 (1858).— Species minus cognita a nobis non visa in hortis gallicis primo cultis origine ignota.

8. A. GAUMERI Robinson, annuum ad 4 dm. altum erectum vel reclinatum ramosissimum laxe pubescens; caule tereti; foliis oppositis graciliter petiolatis oblongo-ovatis crebre et regulariter serratis vel crenatis acuminatis basi rotundatis 3.5–7 cm. longis 2.5–5 cm. latis utrinque viridibus laxe subadpresseque villosis; inflorescentia perlaxa, pedicellis 2–4 (usque ad 6) cm. longis bracteolis paucis parvis subulatis munitis; capitulis numerosis pro genere majusculis oblate sphaericis 7–9 mm. diametro; involucri squamis linearibus attenuatis viridibus 2-costatis subaequalibus; receptaculo conico nudo; corollis glabris limbum versus caeruleo-purpureis, tubo proprio gracili fauces

distincte ampliatas cylindratas subaequanti; achaeniis nigris vix in angulis obscure hispidulis 1.2 mm. longis; pappi squamis 5 aliis saepissime brevioribus muticis aliis in aristam disinentibus et achaenium longitudine subaequantibus.— Proc. Am. Acad. xlvii. 191 (1911). *A. intermedium* Millsp. Field Columb. Mus. Pub. Bot. Ser. iii. 90 (1904), non Hemsl.— YUCATAN: Izamal, *Gaumer*, n. 395 (hb. Gray., hb. U. S. Nat. Mus.); Merida, *Valdez*, n. 13 (hb. Gray., hb. U. S. Nat. Mus.).

9. A. RUGOSUM Coult., erectum verisimiliter perenne; caule tereti sordide pubescenti purpurascenti ad inflorescentiam folioso superne ramoso 4 dm. vel. ultra alto; foliis ovatis attenuatis acutis serratis 3(–5)-nerviis 6–8 cm. longis 3.5–4.3 cm. latis supra scabriusculis rugulosis vel planis tenuiter pubescentibus subtus paullo pallidioribus pubescentibus et puncticulatis basi rotundatis; petiolo usque ad 1.5 cm. longo; corymbis terminalibus multicapitulatis 3–6 cm. diametro; capitulis 7 mm. diametro ca. 65-floris; involucri turbinato-campanulati squamis subaequalibus lanceolato-linearibus attenuatis uncinatis dorso 2-costatis hirsutis; receptaculo conico plerumque nudo margine solo paleis paucissimis instructo; corollis caeruleo-purpureis 2.5 mm. longis glandulari-granulosis supra mediam partem in fauces turbinato-campanulatas paullo ampliatis; achaeniis argute 5-angulatis deorsum decrescentibus nigris glaberrimis lucidis basi albido-callosis; pappo e paleis 5 saepius deltoideis brevibus (ca. 0.3 mm. longis) acutis vel obtusis et denticulatis basi per tertiam partem longitudinis connatis.— Bot. Gaz. xx. 42 (1895); J. D. Sm. Enum. Pl. Guat. iv. 72 (1895). *A. latifolium* Hemsl. Biol. Cent.-Am. Bot. i. 82 (1881) quoad pl. guatem.— GUATEMALA: Santa Rosa, alt. 915 m., *Heyde & Lux* (distrib. J. D. Sm.), n. 4243 (hb. J. D. Sm.); basi montis igniv. Fuego, *Salvin & Godman*, n. 48 (hb. Kew.); San Cristobal, alt. 1400 m., *v. Tuerckheim*, n. II 2051 (hb. Gray., hb. J. D. Sm.); alt. 1130 m., *Deam*, n. 6169 (hb. Gray.) et dubitatim n. 6208 (hb. Gray.)— forma foliis multo minoribus.

NOTA.— Ob pappo e squamulis basi connatis transitionem ad Subg. *Coelestinam* formans. Plantae nonnullae perplexantes occurrunt, e. g., San Salvador, *Velasco* (distrib. J. D. Sm.), n. 8967 (hb. J. D. Sm.), cum habitu passim *A. rugosi* sed cum pappo subintegro *A. corymbosi;* Guatemala, Alta Verapaz, *Goll.* n. 114 (hb. U. S. Nat. Mus., hb. J. D. Sm.), cum pappo *A. rugosi* sed foliis minoribus subtus dense canescenteque tomentosis.— Formae ultius inquirendae.

10. **A. platypodum**, spec. nov., robustum subglabrum; caule (basi ignoto) superne suberecto paucirameo folioso tereti purpurascenti crassiusculo paullo lignescenti tamen medulloso obscure puberulo tardius glabrato; foliis magnis ovatis obtuse acuminatis grosse crena-

tis oppositis supra viridibus obscure adpresseque pilosis subtus pallidioribus glaucescentibus in veniis parcissime pilosis 5–8 cm. longis 3–5 cm. latis basi subcuneatis in petiolum subdecurrentibus; petiolo 1–2 cm. longo obcompresso latiusculo supra late canaliculato paullo adpresse piloso vel subglabro; corymbis in caule et ramis terminalibus pedunculatis 7–12-capitulatis; pedicellis crispe puberulis 7–17 mm. longis; bracteolis setaceis; capitulis majusculis 8–9 mm. diametro; involucri late campanulati basi umbonati squamis oblongo-lanceolatis 1–2-costatis adpresse pilosulis; receptaculo nudo; corollis limbum versus purpureis extus hispidulis, tubo proprio gracili papilloso in fauces cylindratas subglabras sensim dilatato; achaeniis atrobrunneis vel nigrescentibus glabris nitidis 1.8 mm. longis basi callosis; pappi paleis 5–8 oblongis obtusis dentatis vel laciniatis 0.5 mm. longis.— MEXICO: ad Guadalajaram, *Palmer* (anno 1886), n. 437, pro parte (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.).

NOTA.— Species bene distincta cum *A. Houstoniano* tamen a legulo commixta et sub nomine *A. conyzoides*, var. *mexicanum* distributa, sed facile ob pappi paleis multo brevioribus, foliorum forma, etc., distinguenda.

11. A. LATIFOLIUM Cav., annuum erectum vel decumbens laxe pubescens 3–5 dm. altum; radice fibrosa; caule tereti striato viridi vel purpureo saltim ad nodos piloso; ramis arcuato-ascendentibus nudiusculis corymbiferis; foliis ovatis vel ovato-oblongis crenato-serratis obtusis basi rotundatis 2–5 cm. longis 1–3.5 cm. latis tenuibus utrinque parce pilosis subtus paullo pallidioribus impunctatis; petiolis 1–1.7 cm. longis pubescentibus; corymbis 5–8-capitulatis inaequaliter sed saepius longe pedunculatis, bracteis primariis foliaceis, bracteolis setaceis, pedicellis 2–6 mm. longis; capitulis ca. 6 mm. diametro ca. 40-floris; involucri squamis lanceolato-oblongis acutis 2-costatis subaequalibus viridibus vel purpurascentibus subglabris; corollis violaceis vel albis subglabris a media parte sensim ampliatis; achaeniis nigris in angulis minute sursum hispidulis vel glabris; pappi paleis oblongis vel ovatis vel lanceolatis muticis.— Ic. iv. t. 357 (1797). *Ageratum brachystephanum* Regel, Gartenfl. iii. 245, t. 108, fig. c (1854). *A. muticum* Griseb. Fl. Brit. W. Ind. 356 (1861). *A. maritimum* β Sch. Bip. ex Griseb. l. c. *Calea densiflora* Klatt, Leopoldina, xx. 96 (1884). *Carelia brachystephana* (Regel) Ktze. Rev. Gen. i. 325 (1891). *C. mutica* (Griseb.) Ktze. l. c.— MEXICO: Tepic, *Palmer* (anno 1892), n. 1834 (hb. Gray.). COSTA RICA: in sepi ad Turrialbam, alt. 200 m., *Tonduz*, n. 4139 (hb. J. D. Sm.). CUBA: in rupibus montium Farallonium, *Wright*, n. 1631 (hb. Gray.). INS. BAHAM.: Little Harbor Cay, *Britton & Millspaugh*, n. 2240 (hb. U. S. Nat. Mus.); Harbor Ins.,

E. G. Britton (hb. U. S. Nat. Mus.); secundum vias et in ruderis, *Wight,* n. 26 (hb. Gray.). SANTO DOMINGO: alt. 50 m., ad Paradis, *Fuertes,* n. 458 (hb. U. S. Nat. Mus.). JAMAICA: *March* (hb. Kew., phot. in hb. Gray.). COLUMBIA: *Triana,* n. 1157 (hb. Kew.). VENEZUELA: Caracas, *Wagener,* f. Regel., l. c. PERUVIA: prope Limam, *Née,* f. Cav. l. c., *Gaudichaud,* n. 106 (hb. Gray.).

NOMEN VULGATUM peruviorum *Teatina* ex Cav. l. c.

NOTA.— Ab vulgato *A. conyzoide* praecipue pappo breviori mutico ab auctoribus successivis sub nominibus diversis sejunctum quam species fortasse videndum, tamen in *A. conyzoidem,* var. *inaequipaleaceum* facillime et saepissime transiens.

NOTA.— Ad hanc speciem praeterea *Coelestinam parvifoliam* DC. Prod. v. 108 (1836), plantam minus cognitam ob speciminis typici axi frusto praecipue ab axibus lateralibus parvifoliatis descriptam confidenter refero.

Var. **galapageium,** var. nov., valde decumbens; foliis tenuissimis; capitulis plerisque minoribus 3–5 mm. diametro ca. 20-floris; pappi squamulis saepius (non semper) brevissimis vix 0.2 mm. longis.— *A. conyzoides* Hook. f. Trans. Linn. Soc. xx. 207 (1847); Anderss. Stockh. Akad. Handl. 1853, p. 175 (1854), et Om Galap. Veg. 67 (1857); Robinson & Greenman, Am. Jour. Sci. ser. 3, l. 146 (1895); Stewart, Proc. Calif. Acad. Sci. ser. 4, i. 148 (1911); non L. *Coelestina latifolia* Anderss. ll. cc. quoad plantam, nec Benth. *Ageratum latifolium* Robinson, Proc. Am. Acad. xxxviii. 209 (1902), quoad plantam, nec Hemsl.— INS. GALAPAGOS: Charles Insula, *Darwin, Andersson* (hb. Gray.), *Baur, Snodgrass & Heller,* n. 423 (hb. Gray.). Verisimiliter etiam in insulis Chatham et Albemarle, tamen pl. Chierchiae ex illa numquam vidi et pls. Stewartii ex utraque non critice revidi.

12. A. MARITIMUM HBK., herbaceum decumbens a basi ramosum parce pilosus 2.5–3.5 dm. altum annuum vel perenne; caule ramisque teretibus juventate viridibus striatulis per majorem partem foliosis aetate a cortice flavido-griseo exfolianti tectis; foliis oppositis ovato-oblongis vel anguste deltoideis obtusiusculis regulariter crenatis basi integris abrupte contractis subtruncatis vel subcordatis 1.5–3 cm. longis 9–22 mm. latis subglabris; petiolo 9–15 mm. longo gracili villoso-hirsutulo; pedunculis 2–11 cm. longis aut terminalibus aut lateralibus; cymis 3–5-capitulatis; capitulis ca. 7 mm. diametro ca. 75-floris; involucri squamis lanceolati-oblongis acuminatis viridibus 2-costatis glabriusculis; receptaculo leviter convexo nudo; corollis 3 mm. longis fere tubulatis in fauces vix ampliatis parce pubescentibus limbum versus violaceis (Kunth), caeruleis aut albis (Shafer), caeruleis aut roseis (Palmer & Riley), purpureis (Wright), breviter hispidulis; achaeniis nigris acute 5-angularibus 2.1 mm. longis glabris; pappo e

squamulis saepius 5 ovatis laciniatis acutis vix 0.3 mm. longis basi hinc inde plus minusve connatis.— Nov. Gen. et Spec. iv. 150 (1820). *Carelia maritima* (HBK.) Ktze. Rev. Gen. i. 325 (1891).— CUBA: arenosis maritimis prope Havanam, *Humboldt & Bonpland*, n. 1273 (hb. Par., phot. in hb. Gray.); *Curtiss*, n. 650 (hb. Gray., hb. U. S. Nat. Mus.); prope Mariel, Pinar del Rio, *Palmer & Riley*, n. 712 (fl. caeruleis, hb. U. S. Nat. Mus.), n. 739 (fl. roseis, hb. U. S. Nat. Mus.); Bay Corrientes, *Wright*, n. 1631 (hb. Gray.); Cayo Sabinal, Camaguey, *Shafer*, n. 1099 pro parte (hb. Gray., hb. U. S. Nat. Mus.).

Forma **calvum**, forma nova, achaeniis omnino epapposis ab annulo sub corolla nec extra corollam oriunti coronatis, aliter nullo modo a forma typica discretum.— CUBA: Cayo Sabinal, Camaguey, *Shafer*, n. 1099 pro parte (hb. Gray.); YUCATAN: in insula Cozumel, *Gaumer*, n. 20 (fl. albis, hb. Berol., hb. Gray.).

Var. **intermedium** (Hemsl.), comb. nov., formae typicae habitu foliis etc. simillimum differt pedunculis paullo longioribus (6–18 cm. longitudine); pappi squamulis distincte longioribus (ca. 0.7 mm. longitudine) aliis (1–3) setiferis aliis acutis sed muticis.— *A. intermedium* Hemsl. Biol. Cent.-Am. Bot. iv. 102 (1887).— YUCATAN: in insula Cozumel, *Gaumer*, n. 93 (hb. Kew., hb. Gray.).

NOTA.— *A. maritimum* quamquam a Hook. f. & Jacks. Ind. Kew. i. 58 (1895) ad vulgatissimum *A. conyzoidem* incaute reductum apparet speciminibus numerosis examinatis bene distinctum.

13. A. LITTORALE Gray, decumbens a basi ramosissimum 3–4 dm. altum subglabrum verisimiliter perenne; ramis teretibus pallide brunneis vel purpurascentibus basi saepius decumbentibus deinde erectis fere ad mediam partem foliosissimis superne nudis apice corymbiferis; foliis oppositis saepe fasciculatis rhombeis basi attenuatis integris aliter inciso-dentatis apice acutiusculis 11–27 mm. longis 10–14 mm. latis glaberrimis subcarnosis 3-nerviis saepe conduplicatis; petiolis gracillimis 9–25 mm. longis apicem versus paullo dilatatis; pedunculis 6–16 cm. longis; corymbis parvis densis 4–13-capitulatis; pedicellis brevibus cum bracteolis subulatis parvis munitis; capitulis 5 mm. diametro; involucri squamis anguste lanceolatis attenuatis acutissimis glabris 2-costatis; corollis 2.7 mm. longis limbum versus caeruleo-purpureis, tubo proprio minute puberulo fauces cylindratas pallidas subaequanti; achaeniis nigris glabris lucidis 1.3 mm. longis prismaticis basi callosis apice cum annulo humili cartilagineo saepius integro coronatis; pappo saepius vero nullo rarius e dentibus 2–5 brevissimis obscuris composito.— Proc. Am. Acad. xvi. 78 (1880), Syn. Fl. i. pt. 2, 93 (1884). *Coelestina maritima* Torr. & Gray, Fl. ii.

64 (1841), non *Ageratum maritimum* HBK. *Carelia litorale* [Gray] Ktze. Rev. Gen. i. 325 (1891).— In insulis parvis FLORIDAE AUSTRALIS: in litoribus corallinis, Key West, *Bennett; Blodget* (hb. Gray.); *Palmer* (anno 1874), n. 192; *Garber*, (hb. Gray, hb. U. S. Nat. Mus.); *Tweedy*, n. 312 (hb. U. S. Nat. Mus.); *Pollard, Collins & Morris*, n. 12 (hb. U. S. Nat. Mus.); Boca Chica Key, *Curtiss*, n. 1163 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); No Name Key, *Simpson*, n. 246 (hb. U. S. Nat. Mus.); Few-fish Key, *Curtiss*, n. 5446 (hb. U. S. Nat. Mus.); *Chapman*, n. 49 (hb. J. D. Sm.).

Var. **hondurense**, var. nov., formae typicae multis simillimum differt foliis paullo majoribus usque ad 34 mm. longis et 20 mm. latis potius regulariter crenato-dentato nec incisis; involucri squamis paullulo villosis.— In insulis sinus hondurensis: RUTAN, *Gaumer*, n. 1 (hb. Berol., hb. U. S. Nat. Mus.). MUGERES, *Gaumer* (hb. Berol., hb. U. S. Nat. Mus.).

Forma **setigerum**, forma nova, a forma precedenti statura habitu foliis etc. nullo modo discretum, pappo tamen bene evoluto squamulis 5 lanceolatis attenuatis aliis (saepius 3) longe setiferis corollam longitudine aequantibus aliis plus minusve brevioribus.— In insula MUGERES hondurensi, *Gaumer* (hb. Berol., hb. U. S. Nat. Mus.) cum precedenti commixtum et pappo solo distinguendum.

Sect. II. COELESTINA (Cass.) Gray. Pappus e corona sistens margine integra vel plus minusve angulata vel dentata (rarissime setam unicam gerenti) hinc inde valde reductus.— Syn. Fl. i. pt. 2, 93 (1884). *Coelestina* Cass. Bull. Soc. Philom. Par. 1817, p. 10 (1817). DC. Prod. v. 107 (1836), pro parte. *Caelestina* Cass. Dict. Sci. Nat. xvi. 10 (1820), lx. 585 (1830). *Coelestinia* Endl. Gen. 366 (1838). *Ageratum* subg. *Coelestina* (Cass.) Bak. in Mart. Fl. Bras. vi. pt. 2, 197 (1876), pro parte.— Species 14 omnes neontogeae pleraeque suffrutices.

Clavis specierum.

a. Receptaculum paleaceum *b.*
b. Paleae apice firmiusculae subulato-attenuatae *c.*
c. Corollae purpureo-caeruleae. Folia subtus tomentosa et cum glandulis flavis vel aureo-brunneis creberrime tecta. Mex. 14. *A. paleaceum.*
c. Corollae albae. Folia subtus tomentosa sed sine glandulis conspicuis. Mex. . . . . 15. *A. albidum.*
b. Paleae oblongae vel lineares apice obtusae vel acutae neque induratae nec subulato-attenuatae. Bras. . . . . 16. *A. micropappum.*
a. Receptaculum nudum *d.*
d. Rhizoma bene evolutum horizontali repens. Inflorescentia laxe dichotoma scorpoideo-cymosa . . . . 17. *A. scorpoideum.*

d. Rhizoma brevissimum vel nullum. Inflorescentia corymbosa, capitulis rariter solitariis terminalibus *e*.
e. Folia a basi alterna ....18. *A. stachyofolium.*
e. Folia saltim pleraque opposita *f*.
f. Maritimum basi valde ramosum decumbens glabriusculum 13. *A. littorale.*
f. Non maritima rariter basi ramosa. Caules pubescentes vel puberuli *g*.
g. Annuae *h*.
h. Folia quamquam membranacea tamen firmiuscula subtus distincte pallidiora et glanduloso-punctata. Am. Cent. et Venez. ....19. *A. Oerstedii.*
h. Folia tenuissima subconcoloria subtus eglandulosa 11. *A. latifolium.*
g. Herbae perennes vel frutices *i*.
i. Folia sesqui vel bis (rariter 2½-plo) longiora quam lata *j*.
j. Folia subtus incano-tomentosa......20. *A. tomentosum.*
j. Folia opaca utrinque griseo-pubescentia vel -tomentosa. Capitula saepius parvula *k*.
k. Involucrum 6–8 mm. diametro et altitudine. Achaenia 2.1–2.4 mm. longa *l*.
l. Folia basi valde obliqua. Involucri squamae in linea media ciliatae aliter glabrae extimis calloso-obtusis. Costa Rica.............21. *A. riparium.*
l. Folia basi aequalia. Involucri squamae crispe puberulae acutissimae. Mex.....22. *A. corymbosum.*
k. Involucrum ca. 4 mm. diametro vix 3 mm. altitudine. Achaenia 1.6 mm. longa. Guatemala 23. *A. elachycarpum.*
j. Folia supra laete viridia modice lucida fere glabra subtus vix pallidiora saepius scabriuscula. Capitula pro genere majuscula *m*.
m. Petioli 3–4 cm. longi.............24. *A. petiolatum.*
m. Petioli breviores *n*.
n. Herbaceum vel lignescens. Folia crenata. Corollae caeruleae ....25. *A. scabriusculum.*
n. Fruticosum. Folia acute serrata. Corollae albae 26. *A. lucidum.*
i. Folia angustiora triplo usque ad quintuplo longiora quam lata 27. *A. salicifolium.*

14. A. PALEACEUM (Gay) Hemsl., herbaceum vel basi lignescens erectum ramosum; caule tereti brevissime crispeque griseo-tomentello; ramis oppositis virgatis foliatis saepius atropurpureis griseo-tomentellis; foliis oppositis breviter petiolatis ovato-lanceolatis crassiusculis integriusculis vel obscure dentatis acutiusculis basi etiam attenuatis supra scabris rugosis subtus reticulatis in nerviis et venis griseo-tomentosis in superficie cum glandulis perpluribus aureis vel flavido-brunneis obtectis; corymbis terminalibus multicapitulatis densis; pedicellis brevibus saepius rectis densissime glandulari-puberulis; capitulis ca. 25-floris 5 mm. diametro; involucri squamis anguste lanceolatis firmiusculis sed laxe imbricatis apice subulato-attenuatis

albidis dorso viridibus vel atropurpureis; receptaculo ubique paleifero; paleis apice subulato-attenuatis firmiusculis albidis corollas fere aequantibus; corollis extus glandulis aureis sessilibus conspicue conspersis limbum versus caeruleo-purpurascentibus, tubo proprio gracili fauces subaequanti; achaeniis glabris nigris 1.8 mm. longis; pappo crateriformi albido margine subintegro vel plus minusve dentato (hinc inde in setam expanso f. DC.).— Biol. Cent.-Am. Bot. ii. 83 (1881). *Coelestina paleacea* Gay ex DC. Prod. v. 107 (1836). *Carelia paleacea* (Gay) Ktze. Rev. Gen. i. 325 (1891). *Ageratum rhytidophyllum* Robinson, Proc. Am. Acad. xxxvi. 476 (1901).— MEXICO: circa Oaxacam, *Andrieux*, n. 287 (hb. DC., phot. in hb. Gray.); in convalli Oaxacanae, *E. W. Nelson*, n. 1446 (hb. Gray.); Sierra de San Filipe, alt. 2150 m., *Pringle*, n. 5675 (hb. Gray.), *C. L. Smith*, n. 594 (hb. U. S. Nat. Mus.), alt. 2440 m., *Pringle*, n. 6177 (hb. U. S. Nat. Mus., hb. J. D. Sm.); in montibus San Juan del Estado, *L. C. Smith*, n. 277 (hb. Gray.).

15. A. ALBIDUM (DC.) Hemsl., herbaceum perenne 3–6 dm. altum; caudice crassiusculo paullo lignescenti; radicibus fibrosis longis firmis; caulibus 1–3 vel pluribus teretibus striatulis viridibus vel purpurascentibus erectis vel plus minusve decumbentibus crispe griseo-tomentellis usque ad mediam partem foliosis superne nudis; foliis ovatis vel ovato-oblongis 3-nerviis crenato-serratis plerisque obtusis basi plus minusve angustatis 2.5–6 cm. longis 1.5–3.8 cm. latis supra viridibus rugosis saepius scabris subtus griseo-tomentellis vel -pubescentibus plus minusve reticulatis, glandulis paucis inconspicuis argenteis vel nullis; petiolis 3–5 mm. longis; pedunculo terminali elongato 1–3 dm. longo; bracteolis saepius parvis setaceis; corymbis leviter convexis 8–40-capitulatis 4–7 cm. diametro; capitulis 6–9 mm. diametro ca. 60-floris; involucri squamis lanceolatis acutis subaequalibus (extimis paucis brevioribus) viridibus dorso pubescentibus et hinc inde minutissime nigro-punctatis saepius 2-costatis apice albidis subulatis; receptaculo conico ubique paleifero; paleis linearibus apice subulatis albidis; corollis albis 2.7 mm. longis, tubo proprio gracili hispidulo fauces cylindratas glabriusculas subaequanti; limbi dentibus extus hispidulis; achaeniis 1.8 mm. longis saepius arcuatis nigris glabris lucidis; pappo crateriformi albo ca. 0.4 mm. alto margine denticulato.— Biol. Cent.-Am. Bot. ii. 81 (1881). *Coelestina albida* DC. Prod. v. 107 (1836). *Carelia albida* (DC.) Ktze. Rev. Gen. i. 325 (1891).— MEXICO: Oaxaca, inter urbem Oaxacam et Mitlam, *Andrieux*, n. 543 (hb. DC., phot. in hb. Gray.); in valle Etla, *Alvarez*, n. 751 (hb. Gray.); in collibus supra Oaxacam, alt. 1830 m., *Pringle*, n. 4816 (hb. Gray., hb.

U. S. Nat. Mus., hb. J. D. Sm.); in monte Alban prope urbem Oaxacam, alt. 1680–1830 m., *C. L. Smith*, n. 365 (hb. U. S. Nat. Mus.); Tecrango, alt. 2075 m., *L. C. Smith*, n. 424 (hb. Gray.); Cerro de San Felipe, alt. 2000 m., *Conzatti & González*, n. 542 (hb. Gray.); in collibus aridis vallis Oaxacae, alt. 1550–1770 m., *Nelson*, n. 1208 (hb. Gray.).

Var. **Nelsonii**, var. nov., caulibus altius foliosis; foliis majoribus tenuioribus late ovatis basi rotundatis utrinque viridibus subtus vix pallidioribus molliter pubescentibus nec tomentosis 6–11 cm. longis 2.5–4.5 cm. latis; petiolis usque ad 2 cm. longis; involucro, paleis, pappo, etc. ut apud formam typicam; corollis ut videtur caerulescentibus.— MEXICO: in civitate Oaxaca inter oppida Zanatepec et Papana, alt. 200 m., *E. W. Nelson*, n. 2822a (hb. Gray.). Varietas vegetior fortasse quam species distincta videnda tamen sine characteribus validis.

16. A. MICROPAPPUM Bak., fruticosum; ramis virgatis striato-angulatis crispe puberulis et glanduloso-atomiferis, internodiis 5–11 cm. longis; foliis oppositis obovato-oblongis coriaceis argute serrato-dentatis apice rotundatis basi attenuatis 4–8 cm. longis 2–4 cm. latis supra viridibus glabriusculis laevibus subtus reticulato-venosis pallidioribus griseo-tomentellis; petiolo ca. 6 mm. longo supra canaliculato; corymbis terminalibus densissimis convexis ca. 4 cm. diametro; capitulis ca. 20-floris ca. 3 mm. diametro; involucri campanulati squamis oblongis apice rotundatis dorso saepissime 2-costulatis margine tenui ciliatis; paleis lineari-oblongis obtusis apice ciliolatis vel erosis; achaeniis glabris nigrescentibus prismaticis deorsum levissime decrescentibus basi callosis; pappo brevissimo coroniformi dentato; corollis fere exacte cylindratis glabris, faucibus vix ullis.— Bak. in Mart. Fl. Bras. vi. pt. 2, 198 (1876).— BRASILIA: prov. Bahia, *Blanchet*, n. 3700 (hb. Kew., hb. Gray.).

17. A. SCORPIOIDEUM Bak., herbaceum perenne subglabrum; rhizomate horizontali valde repente ad nodos radicante; caulibus erectis 3–4 dm. altis per totam longitudinem foliosis; foliis lanceolatis tenuibus penniveniis crenatis basi apiceque angustatis obtusis ca. 5 cm. longis 1 cm. latis; inflorescentiis terminalibus semel vel bis dichotomis deinde scorpioideo-cymosis; capitulis parvis graciliter pedicellatis vel supremis subsessilibus 4–5 mm. diametro 18–31-floris; involucri squamis late oblanceolatis acutis glabris tenuibus subcostatis; receptaculo nudo; corollis 1.8 mm. longis glabris, tubo proprio brevissimo faucibus campanulatis duplo breviori; achaeniis 1.3 mm. longis nigris glabris prismaticis deorsum paullo decrescentibus basi callosis;

pappo coroniformi dentato tenui quam achaenio ter vel quater breviori.— Bak. in Mart. Fl. Bras. vi. pt. 2, 197 (1876); Robinson, Proc. Am. Acad. xlii. 34 (1906). *Coelestina repens* Sch. Bip. in Schomb. Faun. et Fl. Guy. 1134 (1848), sine char.— GUIANA ANGLICA: in graminosis verisimiliter plus minusve paludosis "savannas" dictis, *Rob. Schomburgk,* n. 353 f. Bak. l. c., *Rich. Schomburgk,* n. 488 (hb. Berol., fragm. et phot. in hb. Gray.).

18. A. STACHYOFOLIUM Robinson, herbaceum perenne 5–6 cm. altum strictum; caule simplici erecto tereti rubescenti griseo-tomentello foliosissimo; foliis alternis ovato-oblongis vel ellipticis crenulatis 1.5–3.2 cm. longis 1–1.4 cm. latis basi rotundatis sessilibus vel brevissime petiolatis apice obtusis vel rotundatis paullo supra basin 3-nerviis supra parce pilosis subtus paullo pallidioribus laxe pubescentibus et glandulari-punctatis; corymbo terminali leviter convexo ca. 10-capitulato 4 cm. diametro; pedicellis 1.5–1.8 cm. longis tomentellis; bracteolis spatulato-filiformibus 8–11 mm. longis; capitulis ca. 1 cm. diametro ca. 100-floris involucri squamis linearibus acutis viridibus pubescentibus saepius 4-costatis; corollis albis, tubo proprio longiusculo puberulo, faucibus turbinatis; styli ramis pallidis clavellatis; achaeniis atrobrunneis 2 mm. longis basin versus paullo decrescentibus; pappo crateriformi albido margine integro.— MEXICO: Oaxaca prope La Parada, alt. 2310–2620 m., *Nelson,* n. 991 (hb. Gray.). Species ob foliis omnibus alternis pedicellis longis capitulis majusculis praesertim ob involucri bracteis 4-costatis bene distincta.

19. **A. Oerstedii,** nom. nov., herbaceum annuum erectum simplex vel patentim paucirameum glabriusculum vel saltim ad nodos hirsutulum 3–5 dm. altum; radice fibrosa; foliis ovatis vel ovato-oblongis crenatis obtusis vel obtuse acuminatis basi abrupte contractis utrinque sparse pilosis supra viridibus subtus pallidioribus glanduloso-punctatis 4–7 cm. longis 2–5 cm. latis, petiolo 1–2 cm. longo piloso; corymbis longipedunculatis caulem ramosque terminantibus laxiusculis paucicapitulatis; capitulis 7.5 mm. diametro; involucri squamis viridibus sparse hirtellis vel glabriusculis anguste lanceolatis attenuatis; corollis caeruleo-purpureis glabris ca. 2.2 mm. longis, tubo proprio fauces cylindratas paullo ampliatas vix aequanti; achaeniis nigris glabris; pappo minimo coroniformi dentato obscuro sed sub lente forte manifestim extra corollam locato.— *A. latifolium* (Benth.) Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881), quoad pl. costaricense, non Cav. *Coelestina latifolia* Benth. in Oerst. Vidensk. Meddel. 1852, p. 71 (1852). *Carelia latifolia* (Benth.) Ktze. Rev. Gen. i. 325 (1891).—COSTA RICA: in Monte Aguacate, *Oersted,* n. 251 (hb. Kew., phot. in hb. Gray.);

in apertis silvarum, Cerro de San Isidro prope San Ramon, alt. 1300 m., *Brenes*, n. 14,491 (hb. Gray.).

20. A. TOMENTOSUM (Benth.) Hemsl., herbaceum perenne vel suffruticosum basi ramosum; ramis virgatis vel iterum ramosis puberulis vel tomentellis; foliis oppositis ovatis vel deltoideis crenatis plerisque obtusis basi rotundatis vel truncatis ad insertionem petioli plus minusve cuneatis supra saepius viridibus tenuiter pubescentibus vel tomentellis subtus canescenti-tomentosis 1.3–3.5 cm. longis 1–2.5 cm. latis margine saepe revolutis; petiolis 5–16 mm. longis; corymbis longe pedunculatis convexis densis subsimplicibus vel compositis; capitulis ca. 5 mm. diametro; involucri campanulati squamis anguste lanceolatis tomentellis viridibus 2-costatis; corollis caeruleo-purpureis vel albis plerisque glandulari-atomiferis et apicem versus hispidulis; achaeniis atrobrunneis glaberrimis 2 mm. longis; pappo albido coroniformi margine plus minusve angulato vel dentato.— Biol. Cent.-Am. Bot. ii. 82 (1881). *Coelestina tomentosa* Benth. in Oerst. Vidensk. Meddel. 1852, p. 71 (1852). *Carelia tomentosa* (Benth.) Ktze. Rev. Gen. i. 325 (1891).— COSTA RICA: Candelaria, alt. 1525 m., *Oersted.* MEXICO: Chiapas, in silvis pinorum, *Ghiesbreght*, nn. 111, 547 (hb. Gray.), forma foliis majusculis; Orizaba, Maltrata, alt. 1680 m., *Seaton*, n. 346 (hb. Gray., hb. U. S. Nat. Mus.); Esperanza, alt. 2440 m., *Seaton*, n. 358 (hb. Gray., hb. U. S. Nat. Mus.); Vera Cruz, *Linden*, n. 1185 (hb. Kew., phot. in hb. Gray.); silvis prope Jalapam, *Galeotti*, n. 2202 (hb. Kew., phot. in hb. Gray.); Puebla, in declivibus calcareis, Tehuacan, alt. 1700–1800 m., *Pringle*, n. 6754 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.), n. 9522 (hb. Gray., hb. U. S. Nat. Mus.), *Purpus*, n. 1179 (hb. Gray.); Zacuapan, in graminosis aridis, *Purpus*, n. 1870 (hb. Gray.). Sierra de la Yerba, *Purpus*, n. 2547 (hb. Gray., hb. U. S. Nat. Mus.), forma receptaculo paleis paucis instructo; Oaxaca, Tomellin Cañon, alt. 915 m., *Pringle*, n. 5786 (hb. Gray.); collibus aridis alt. 1550–1775 m., *Nelson*, n. 1213 (hb. Gray.); Jayacatlan, alt. 1300 m., *L. C. Smith*, 285 (hb. Gray.); in Monte Alban prope urbem Oaxacam, alt. 1680 m., *Pringle*, n. 6267 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); San Luis Potosi, Minas de San Rafael, *Purpus*, n. 4814 (hb. Gray., hb. U. S. Nat. Mus.).

NOTA.— Species foliis et pubescentia variabilis, formis mexicanis a forma typica costaricense minus cognita adhuc ut videtur numquam reperta fortasse distinguendis.

21. **A. riparium**, spec. nov., caule (parte inferiori ignota) superne medulloso crassiusculo glabello tactu laevissimo purpurascenti, internodiis longis folia superantibus; foliis oppositis crassiusculis

ovatis longiuscule acuminatis sed ad apicem verum obtusis basi plus minusve angustatis valde inaequalibus et obliquis margine undulatis supra scabriusculis subglabris verucosis subtus paullo pallidioribus punctatis et breviter praesertim in veniis pilosulis ca. 8 cm. longis ca. 4 cm. latis; petiolis ca. 1 cm. longis; corymbis in caule ramisque terminalibus 4–8-capitulatis densis; capitulis 7 mm. diametro 70-floris; involucri campanulati squamis ca. 23 subaequalibus lanceolati-linearibus 2-costatis viridibus in linea media (inter costas) ciliatis aliter subglabris extimis calloso-obtusis interioribus attenuatis; receptaculo conico nudo; corollis albis glaberrimis eglandulosis, tubo proprio gracili fauces cylindratas subaequanti; achaeniis nigrescentibus saepius curvatis glaberrimis vel basin versus in angulis obscure hispidulis; pappi corona ad mediam partem 5-lobata, lobis plerisque deltoideo-ovatis muticis margine denticulatis.— *A. tomentosum* Klatt, Bull. Soc. Bot. Belg. xxxi. 185 (1892), pro parte, non Hemsl. (a quo longe distat).— Costa Rica: in arenosis secundum flumen Ceibo, *Pittier*, n. 4914 (hb. Gray.).

Nota.— Haec planta minus cognita (an abnormalis) differt ab *A. corymboso* caule ut videtur herbaceo, foliis basi obliquis, involucri squamis laevioribus extimis calloso-obtusis, corollis glaberrimis, ab *A. scabriusculo* foliis multo majoribus basi obliquis, corollis albis, characteribus involucri, ab *A. tomentoso* toto coelo.

22. A. corymbosum Zuccag., suffruticosum vel vero fruticosum erectum vel saepe decumbens 3–7 dm. altum simplex vel ramosum plus minusve griseo-pubescens vel -tomentosum; foliis oppositis vel supremis alternis firmiusculis ovatis vel rhomboideo-lanceolatis plerisque acutis basi rotundatis vel cuneatis margine grosse crenatis vel dentatis rarius serratis a basi 3(–5)-nerviis saepe reticulatis 3–6 cm. longis 1–3.5 cm. latis utrinque pallide viridibus supra saepius scabris puberulis subtus molliter crispe puberulis vel plus minusve tomentosis; petiolis 4–8 mm. longis; corymbis in caule ramisque terminalibus saepe pluribus simplicibus vel compositis multicapitulatis; pedicellis filiformibus 3–12 mm. longis glandulari-tomentellis; capitulis ca. 7 mm. diametro; involucri campanulati vel hemisphaerici squamis lanceolato-linearibus attenuatis hispidulis 2-costatis apicem versus saepe coloratis; receptaculo nudo; corollis extus parce glandulari-atomiferis et saepius plus minusve pubescentibus limbum versus caeruleis et vulgo hispidulis, tubo proprio gracili fauces paullo ampliatas subaequanti; achaeniis 2.2 mm. longis atrobrunneis vel nigrescentibus glabris; pappo crateriformi 0.3 mm. longo albido margine subintegro vel paullo irregulariterque dentato.— Zuccag. ex Pers. Syn. ii. 402 (1807). *Spargano-*

phorus ageratoides Lag. Elench. Hort. Matr. 25 (1815) f. Hemsl. Biol. Cent.-Am. Bot. ii. 81 (1881) et Lag. Nov. Gen. et Spec. 25 (1816). *Ageratum coelestinum* Sims, Bot. Mag. t. 1730 (1815). *Caelestina coerulea* Cass. Dict. vi. Suppl. 8 (1817). *Coelestina corymbosa* (Zuccag.) DC. l. c. *C. suffruticosa* Sweet, Hort. Brit. 229 (1826). *Coelestinia Lessingiana* Klotzsch ex Walp. Rep. ii. 545 (1843) f. Hook. f. & Jacks. Ind. Kew. i. 370 (1895) sub nomine *Caelestina*. *Coelestina Lessingiana* (Klotzsch) Hemsl. Biol. Cent.-Am. Bot. ii. 81 (1881). *Carelia corymbosa* (Zuccag.) Ktze. Rev. Gen. i. 325 (1891).— TEXAS OCCIDENT.: ad angustas Guadelupe, *Wright*, n. 1129 (hb. Gray., hb. U. S. Nat. Mus.). MEXICO: Nuevo Leon, ad Monclovam, *Palmer* (anno 1880), n. 427 (hb. Gray., hb. J. D. Sm.); Coahuila, Saltillo, *Palmer* (anno 1880), n. 428 (hb. Gray., hb. U. S. Nat. Mus.), *Palmer* (anno 1898), n. 307 (hb. Gray.), in Lorenzo Cañon prope Saltillo, *Palmer* (anno 1904), n. 387 (hb. Gray.), Sierra de Parras, *Purpus*, n. 4651 (hb. Gray., hb. U. S. Nat. Mus.); Chihuahua, ad Cosihuiriachi, *Wislizenus*, n. 181 (hb. Gray.), prope urbem Chihuahuam, *Pringle*, n. 669 partim (hb. U. S. Nat. Mus.), Porral, alt. 1830 m., *Nelson*, n. 5006 (hb. Gray.); San Luis Potosi, alt. 1830–2440 m., *Parry & Palmer*, n. 317 (hb. Gray., hb. U. S. Nat. Mus.) ad var. *jaliscense* spectans, *Ward* (hb. U. S. Nat. Mus.), in umbrosis circa urbem, *Schaffner*, n. 293 (hb. Gray.), ad Santa Maria del Rio, *Palmer* (anno 1904), n. 154½ (hb. Gray.); Guanajuato, *Guillemin-Tarayre* (hb. Gray.), *Dugès*, nn. 24, 426 (hb. Gray.); Durango, in montibus et collibus, *Palmer* (anno 1896), n. 506 (hb. Gray., hb. U. S. Nat. Mus.) ad var. *jaliscense* accedens, Tejamén, *Palmer* (1906), n. 486 (hb. Gray., hb. U. S. Nat. Mus.); Hidalgo, Real del Monte, *Coulter*, n. 247 (hb. Gray.), in petrosis montanis ad El Chico, *Purpus*, n. 1565 (hb. Gray.); Sinaloa, in Cerro Colorado ad Culiacan, *Brandegee* (hb. Gray.), forma glabrescens; Jalisco, ad Balaños, *Coulter*, n. 248 (hb. Gray.), Guadalajara, *Palmer* (anno 1886), n. 289 (hb. Gray. partim, hb. U. S. Nat. Mus., hb. J. D. Sm.); Querétaro, prope Cadereytam, *Rose, Painter & Rose*, n. 9631 (hb. Gray.); Val. de Mexico, *Bourgeau*, n. 719 (hb. Gray., hb. U. S. Nat. Mus.), in terra vulcanica, alt. 2440 m., *Pringle*, n. 9047 (hb. Gray.), prope Tacubayam, *Schaffner*, n. 202 (hb. Gray.); sine loco indicato, *Gregg*, n. 372 (hb. Gray.), *Schumann*, n. 40b (hb. J. D. Sm.).

NOMEN VULGATUM hispanice "cielitos" f. Dugèsii i. e. caelula, ad colorem azureum alludens.

Forma **album**, forma nov., corollis albis aliter formae typicae simile et aequaliter variabile.— MEXICO: Jalisco, prope Huejuquilla, *Rose,* n. 2538 (hb. Gray.); Puebla, in collibus calcareis prope Amozac, alt. 2300 m., *Pringle,* n. 9542 (hb. Gray.) glabriusculum; Chihuahua aust.-occ., *Palmer* (anno 1885), n. 31 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.) griseo-tomentosum, prope urbem Chihuahuam, *Pringle,* n. 669 partim (hb. Gray., hb. J. D. Sm.).

Var. **latifolium** (DC.), comb. nov., erectum saepius simplex; foliis ovatis magnis 7–10.5 cm. longis 4–6 cm. latis gradatim acuminatis acutis acute serratis plerumque tenuibus vix vel haud reticulatis subtus tenuiter molliterque pubescentibus; corollis glandulosis et pilosis caeruleis.— *Coelestina ageratoides* HBK. Nov. Gen. et Spec. iv. 151 (1820)! et var. β *latifolia* DC. Prod. v. 108 (1836)! — MEXICO: sine loco indicato (hb. DC., phot. in hb. Gray.); Michoacan, inter Aguasarco et Ario, *Humboldt & Bonpland* (hb. Par., phot. in hb. Gray.); collibus aridis, Patzcuaro, *Pringle,* n. 3599 (hb. Gray.); Morelos, in valleculis praeruptis montanis prope Cuernavacam, *Pringle,* n. 9843 (hb. Gray., hb. U. S. Nat. Mus.); *Holway,* n. 3509 (hb. Gray.); Yautepec, *Holway,* n. 5235 (hb. Gray.); Sinaloa, Lodiego, *Palmer* (anno 1891), n. 1587 (hb. Gray., hb. U. S. Nat. Mus.).

Forma **albiflorum,** forma nov., precedenti simillimum differt corollis albis.— MEXICO: Morelos, in collibus prope Yautepec, alt. 1220 m., *Pringle,* n. 9842 (hb. Gray., hb. U. S. Nat. Mus.).

Var. **euryphyllum,** var. nov., foliis late ovatis vel rhomboideo-ovatis obtusis grosse crenatis 4–6 cm. longis 3–4 cm. latis; corollis caeruleis glandulosis vix pilosis.— MEXICO: San Luis Potosi, alt. 1830–2440 m., *Parry & Palmer,* n. 315 (hb. Gray., hb. U. S. Nat. Mus.), n. 318 (hb. Gray., hb. U. S. Nat. Mus.), prope Alvarez, *Palmer* (anno 1902), n. 101 (hb. Gray., hb. U. S. Nat. Mus.); Nuevo Leon, Monclova, *Palmer* (anno 1880), n. 427 (hb. U. S. Nat. Mus.); Zacatecas, in montibus prope Plateado, *Rose,* n. 2739 (hb. Gray.); sine loco indicato, *Hartweg,* n. 142 (hb. Gray.).

Var. **jaliscense,** var. nov., foliis elliptico-lanceolatis breviter petiolatis 6–8 cm. longis 2.3–3 cm. latis grosse serratis vel crenato-dentatis vel etiam lobulatis firmiusculis scaberrimis reticulatis; corollis caeruleis glandulosis parce pilosis.— MEXICO: Jalisco ad Chapalam, *Palmer* (anno 1886), n. 715 (hb. Gray., hb. U. S. Nat. Mus.).

Forma **lactiflorum,** forma nov., precedenti simillimum differt corollis albis.— MEXICO: Jalisco, ad Tequilam, *Palmer* (anno 1886), no. 351 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.), ad Guadalajaram, n. 290 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.), via inter Huejuquilla et Mesquitec, *Rose,* n. 2585 (hb. Gray.).

Var. **longipetiolatum,** var. nov., foliis elongatis lanceolatotriangularibus attenuatis basi plus minusve cuneatis integris supra basin grosse dupliceque crenato-dentatis ca. 8–10 cm. longis 3–4 cm. latis tenuibus non reticulatis subtus molliter tomentosis; petiolis usque ad 3 cm. longis; corollis glabriusculis albis.— MEXICO: in Chihuahua aust.-occ., *Palmer* (anno 1885), n. 110 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.).

Var. **subsetiferum**, var. nov., fruticosum ramosum; foliis ovatis 2–4 cm. longis 1.5–2.5 cm. latis obtusis vel acutiusculis crenato-serratis basi cuneato-angustatis brevissime petiolatis utrinque dilute viridibus scabridis; capitulis pro specie longiusculis; corollis caeruleis ex involucro valde exsertis; pappi corona flosculorum nonnullorum cum seta laevi corollam subaequanti instructa.— MEXICO: Zacatecas, prope Conception del Oro, *Palmer* (anno 1902), n. 382 (hb. Gray., hb. U. S. Nat. Mus.).

23. **A. elachycarpum**, spec. nov., fruticulosum gracile; caule tereti 1–2 mm. diametro a cortice griseo-brunnea tecto; ramis virgatis purpurascentibus crispe puberulis usque ad inflorescentiam foliosis; foliis oppositis tenuibus ovatis plerisque attenuato-acuminatis regulariter crenato-serratis 3-nerviis supra badio-viridibus pilosulis subtus pallidioribus griseo-tomentellis ca. 5 cm. longis ca. 2.5 cm. latis; corymbis compositis planiusculis; pedicellis 1–4 mm. longis; bracteolis setaceis; involucri hemisphaerici 4–5 mm. diametro 2.5–3 mm. alti squamis anguste lanceolatis attenuatis 2-costatis dorso crispe puberulis; corollis involucrum vix superantibus glandulari-atomiferis vix vel nullo modo pilosulis apicem versus violaceis; tubo proprio fauces campanulatas subaequilongo; achaeniis nigris 1.6 mm. longis glabris nitidis; pappi corona 0.2 mm. alta fere ad mediam partem 5-dentata albida.— *A. corymbosum* J. D. Sm. Enum. Pl. Guat. iv. 72 (1895), non Benth.— GUATEMALA: Santa Rosa, alt. 915 m., *Heyde & Lux* (distrib. J. D. Sm.), n. 4228 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.). Species *A. corymboso* gracilior differt etiam textura molliori foliorum, capitulis minoribus crassioribus quam altis, achaeniis multo minoribus, etc.

24. A. PETIOLATUM (Hook. & Arn.) Hemsl., perenne herbaceum vel suffruticosum ascendens foliosum; foliis oppositis vel supremis alternis ovatis obtusiusculis firmis glaberrimis crenato-serratis basi angustatis ad petioli insertionem cuneatis 6–9 cm. longis 2.5–4 cm. latis; petiolis gracilibus usque ad 3.6 cm. vel ultra longis; pedunculis longis terminalibus; corymbis parvis pauci-capitulatis; pedicellis erectis ad 2 cm. longis; bracteolis setaceis; capitulis 8 mm. diametro; involucri campanulati squamis anguste lanceolatis; corollis glaberri-

mis; pappo crateriformi dentato, dente uno hinc inde setifero ceteris vel omnibus muticis.— Biol. Cent.-Am. Bot. ii. 83 (1881). *Caelestina petiolata* Hook. & Arn. Bot. Beechy, 433 (1841). *Carelia petiolata* (Hook. & Arn.) Ktze. Rev. Gen. i. 325 (1891).— NICARAGUA: ad Realejo, *Sinclair* (hb. Kew., phot. in hb. Gray.). Species ut videtur numquam reperta, ob dentibus pappi hinc inde setiferis transitionem ad Sect. *Euageratum* monstrat.

25. A. SCABRIUSCULUM (Benth.) Hemsl., perenne herbaceum vel suffruticosum decumbens plus minusve ramosum 1.3–9 dm. altum primo aspectu glaberrimum tamen tactu plus minusve scabrum et vero minute pilosiusculum; caule ramisque gracilibus ascendentibus curvatis vel flexuosis foliosis; foliis oppositis ovatis saepius acuminatis serratis firmiusculis supra laete viridibus lucidis subtus dilutiore viridibus opacis punctatis 2–5 cm. longis 1–3.5 cm. latis; petiolis 5–11 mm. longis glandulari-pubescentibus; corymbis (1–)3–7-capitulatis simplicibus vel compositis longe pedunculatis; pedicellis filiformibus puberulis saepius ca. 1 cm. longis; capitulis ca. 7 mm. diametro ca. 75-floris; involucri squamis angustis acutissimis viridibus 2-costatis dorso crispe puberulis vel glabriusculis; corollis glaberrimis limbum versus caeruleis, tubo gracili fauces cylindratas subaequanti; achaeniis nigrescentibus glabris ca. 2 mm. longis; pappo crateriformi albido saepius 5-dentato 0.3 mm. longo.— Biol. Cent.-Am. Bot. ii. 83 (1881). *Coelestina corymbosa* Benth. Bot. Sulph. 111 (1844), pro parte, non DC. *C. scabriuscula* Benth. ex Oerst. Vidensk. Meddel. 1852, p. 72 (1852). *Carelia scabriuscula* (Benth.) Ktze. Rev. Gen. i. 325 (1891).— MEXICO: inter San Blas et Tepic, *Sinclair* (hb. Kew., phot. in hb. Gray.). GUATEMALA, *Friedrichsthal* f. Oerst. l. c. 73. COSTA RICA: in graminosis declivitatis occidentalis montis ignivomi El Vieja, alt. 610 m., *Oersted* (hb. Kew., phot. in hb. Gray.). NICARAGUA: *Wright* (hb. Gray., hb. U. S. Nat. Mus.), Masaya, *Baker*, n. 2220 (hb. Gray.).

26. A. LUCIDUM Robinson, fruticosum ramosum 4–5 dm. altum decumbens; caulibus nodosis flexuosis maturitate a cortice grisea tectis; ramis curvato-ascendentibus plerisque in media parte praesertim foliosis; foliis oppositis firmis ovatis acutis basi angustatis acute serratis 1.5–4.5 cm. longis 7–20 mm. latis glabriusculis vel plus minusve scabridis supra laete viridibus lucidis veniis prominulentibus albidis subtus dilutiore viridibus punctatis tenuiter reticulatis; corymbis laxis saepe compositis; capitulis pro genere majusculis 9–10 mm. diametro ca. 80-floris; involucri hemisphaerici squamis anguste lanceolatis attenuatis viridibus 2-costatis crispe puberulis; corollis albis

glandulari-atomiferis; achaeniis maturitate nigris 2 mm. longis glaberrimis; pappo crateriformi albido margine subintegro.— Proc. Am. Acad. xxxvi. 475 (1901).— Mexico: in civitate Morelos in declivitatibus muscosis Sierrae de Tepoxtlan, alt. 2300 m., *Pringle,* n. 7851 (hb. Gray.), n. 8362 (hb. Gray., hb. J. D. Sm.), n. 9844 (hb. Gray., hb. U. S. Nat. Mus.), n. 13,022 (hb. Gray., hb. U. S. Nat. Mus.). Species precedenti arcte affinis differt habitu paullo lignescentiore, foliis reticulatis lucidis, corollis albis glandulari-granulosis, pappi corona subintegra.

27. A. salicifolium Hemsl., perenne herbaceum vel distincte fruticosum laxe et subdichotome ramosum gracili 0.6–1.3 m. altum brevissime hirtellum scabriusculum; ramis virgatis; foliis oppositis (vel supremis alternis) anguste lanceolatis tenuibus vel saepius firmiusculis plus minusve reticulatis planis vel saepius conduplicatis 4–11 cm. longis 0.5–3.6 cm. latis obtuse subremoteque serrato-dentatis vel integriusculis utrinque laete viridibus; corymbis terminalibus saepius compositis et laxiusculis; capitulis, flosculis, achaeniis eis *A. corymbosi* simillimis.— Biol. Cent.-Am. Bot. ii. 83 (1881). *Coelestina corymbosa* Benth. Bot. Sulph. 111 (1844), pro parte, nec DC. *Ageratum strictum* Hemsl. l. c. (1881), ubi per errorem (ex sched. Galeottii introductum) annuum dictum. *Carelia salicifolia* et *C. stricta* (Hemsl.) Ktze. Rev. Gen. i. 325 (1891).— Mexico: inter San Blas et Tepic, *Sinclair* (hb. Kew., phot. in hb. Gray.); Jalisco, prope Guadalajaram, *Palmer* (anno 1886), n. 289 partim (hb. Gray.), *Pringle,* n. 11,898 (hb. Gray.); Michoacan, ad Uruapam, *Galeotti,* n. 2451 (hb. Kew., phot. in hb. Gray.), ad Vallecito in terris argillaceis, *Langlassé,* n. 310 (hb. Gray.), forma vegetior; Morelos, in collibus supra Cuernavacam, *Pringle,* n. 6234 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.), nn. 9045, 9845 (hb. Gray., hb. U. S. Nat. Mus.).

Nota.— Species quamquam plerumque facile recognoscenda tamen ab *A. corymboso* foliis longioribus angustioribus saepius viridioribus exceptis vix distinguenda. An potius ei varietas?

Species transferendae vel excludendae vel inquirendae.

A. adscendens Sch. Bip. = Oxylobus adscendens (Sch. Bip.) Robinson & Greenman.

A. Agrianthus O. Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 134 (1890). *Agrianthus corymbosus* DC. Prod. vii. 266 (1838). *Ageratum corymbosum* (DC.) Benth. ["in Benth. & Hook. f. Gen. Pl. ii. 242 (1873)"] ex Bak. in Mart. Fl. Bras. vi. pt. 2, 196 (1876), non

Zuccag. Species ad TRICHOGONIAM approximans tamen ultius scrutanda.

A. album Willd. = A. CONYZOIDES, f. ALBUM (Willd.) Robinson.

A. alternifolium (Gardn.) Bak. in Mart. Fl. Bras. vi. pt. 2, 195 (1876). *Campuloclinum alternifolium* Gardn. in Hook. Lond. Jour. Bot. vi. 438 (1847). Species dubia ultius inquirenda.

A. altissimum L. Sp. Pl. ii. 839 (1753) = EUPATORIUM URTICAEFOLIUM Reichard. Vide Robinson, Proc. Am. Acad. xlii. 46 (1906).

A. angustifolium Spreng. Syst. iii. 446 (1826) = CALEA ANGUSTIFOLIA (Spreng.) Sch. Bip. ex Bak. in Mart. Fl. Bras. vi. pt. 3, 256 (1884).

A. aquaticum Roxb. Hort. Beng. 61 (1814) = ADENOSTEMMA LAVENIA (L.) Ktze. Rev. Gen. i. 304 (1891).

A. arbutifolium HBK. = OXYLOBUS ARBUTIFOLIUS (HBK.) Gray.

A. brachystephanum Regel = A. LATIFOLIUM Cav.

A. caeruleum Hort. Reg. Par. ex Lam. Dict. i. 54 (1783). Species dubia seu A. HOUSTONIANUM Mill. seu A. CONYZOIDES L.

A. callosum Wats. = ALOMIA CALLOSA (Wats.) Robinson.

A. campuloclinioides Bak. in Mart. Fl. Bras. vi. pt. 2, 196 (1876). Species ad TRICHOGONIAM approximans ultius scrutanda.

A. ciliare L. Sp. Pl. ii. 839 (1753). Species valde dubia non recognescenda fortasse forma A. CONYZOIDIS.

A. ciliare L., *β* Lour. Fl. Cochin. ii. 484 (1790). Omnino ignotum.

A. coelestinum Sims. = A. CORYMBOSUM Zuccag.

A. coeruleum Desf. Tab. 98 (1804), nomen.

A. coeruleum Sieber ex Bak. in Mart. Fl. Bras. pt. 2, 345 (1876) = EUPATORIUM MACROPHYLLUM L.

A. confertum (Gardn.) Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 195 (1876). *Decachaeta conferta* Gardn. in Hook. Lond. Jour. Bot. v. 463 (1846). *Carelia conferta* (Gardn.) Ktze. Rev. Gen. i. 325 (1891). Species ob pappo setiformi ex *Agerato* excludenda ultius scrutanda.

A. conspicuum Hort. ex C. Koch & Fint. Wochenschr. i. 33 (1858) = (ut dicitur) EUPATORIUM GLECHONOPHYLLUM Less.

A. conyzoides Sieber ex Steud. Nom. ed. 2, i. 37 (1841) = (f. Steud. l. c.) *Eupatorium repandum* Willd. i. e. E. CORYMBOSUM Aubl.

A. conyzoides α obtusifolium (Lam.) DC. l. c. = *A. coeruleum* Hort. Reg. Par. Vide supra.

A. conyzoides β hirtum (Lam.) DC. l. c. = A. CONYZOIDES L.

A. conyzoides γ mexicanum (Sims) DC. Prod. v. 108 (1836) = A. HOUSTONIANUM Mill.

A. conyzoides δ cordifolium (Roxb.) DC. l. c. Var. non certe constata ex characteribus videtur forma parvi momenti A. CONYZOIDIS L. vel fortasse A. HOUSTONIANI Mill.

A. cordifolium Roxb. Fl. Ind. iii. 415 (1832). Vide supra (*A. conyzoides* δ *cordifolium* (Roxb.) DC.)

A. corymbosum (DC.) Benth. Vide supra (*A. Agrianthus* O. Hoffm.).

A. corymbosum Zuccag., var. *St. Antonii* Sch. Bip. ex Klatt, Leopoldina, xx. 75 (1884). Nomen.

A. echioides (Less.) Hemsl. = ALOMIA ECHIOIDES (Less.) Robinson.

A. febrifugum Sesse ex DC. Prod. v. 104 (1836) = PIQUERIA TRINERVIA Cav.

A. glanduliferum Sch. Bip. = OXYLOBUS GLANDULIFERUS (Sch. Bip.) Gray.

A. glanduliferum Sch. Bip., var. *albiflorum* Sch. Bip. = OXYLOBUS GLANDULIFERUS (Sch. Bip.) Gray, typicus.

A. guianense Aubl. Guian. ii. 800 (1775) = EUPATORIUM MACROPHYLLUM L.

A. heterolepis Bak. = ALOMIA HETEROLEPIS (Bak.) Robinson.

A. hirsutum Poir. Suppl. i. 242 (1810), sphalm. pro *hirtum* = A. CONYZOIDES L.

A. hirtum Lam. = A. CONYZOIDES L.

A. humile Salisb. Prod. 188 (1796) = A. CONYZOIDES L.

A. intermedium Hemsl. = A. MARITIMUM HBK., var. INTERMEDIUM (Hemsl.) Robinson.

A. isocarphoides (DC.) Hemsl. = ALOMIA ISOCARPHOIDES (DC.) Robinson.

A. Lasseauxii Carr. Rev. Hort. xlii. 90 (1870) cum icone pessima. *Conoclinium Lasseauxii* Carr. in Dur. Ind. Sem. Hort. Burdig. 1872, p. 15 (1872) = EUPATORIUM LASSEAUXII Carr. l. c.

A. latifolium (Benth.) Hemsl. = A. OERSTEDII Robinson et A. RUGOSUM Coult.

A. lineare Cav. Ic. iii. 3, t. 205 (1795) = PALAFOXIA LINEARIS (Cav.) Lag.

A. longifolium (Gardn.) Benth. = ALOMIA LONGIFOLIA (Gardn.) Robinson.

A. maritimum HBK. β Sch. Bip. = A. LATIFOLIUM Cav.

A. matricarioides (Spreng.) Less. Syn. Comp. 155 (1832) = PHANIA MATRICARIOIDES (Spreng.) Griseb.

A. melissaefolium DC. Prod. v. 109 (1836), excl. syn. *Chrysocoma pauciflora* Arrab [Vellozo]. Species ad TRICHOGONIAM valde approximans ultius scrutanda.

A. mexicanum Sims, Bot. Mag. t. 2524 (1825) = A. HOUSTONIANUM Mill.

A. microcarpum (Benth.) Hemsl. = Alomia microcarpa (Benth.) Robinson.

A. microcephalum Hemsl. = Alomia microcephala (Hemsl.) Robinson.

A. microphyllum Sch. Bip. in Seemann, Bot. Herald, 298 (1856) = Ageratella microphylla (Sch. Bip.) Gray.

A. muticum Griseb. = A. latifolium Cav.

A. nanum Hort. ex Sch. Bip. in C. Koch & Fint. Wochensch. i. Garten-Nachr. 26 (1858) = (ex Hook. f. & Jacks. Ind. Kew. i. 58) A. suffruticosum Regel.

A. obtusifolium Lam. Dict. i. 54 (1783). Species non constata verisimiliter seu A. conyzoides L. seu A. Houstonianum Mill.

A. odoratum Vilm. Fl. Pl. Terre, ed. 2, 42 (1866) = A. conyzoides L.

A. paniculatum Hort. ex Steud. Nom. ed. 2, i. 609 (1840) = Brickellia paniculata (Mill.) Robinson, Proc. Am. Acad. xlii. 48 (1906).

A. pedatum Ort. Hort. Matr. Dec. 38 (1797) = Florestina pedata (Cav.) Cass.

A. Pohlianum Bak. in Mart. Fl. Bras. vi. pt. 2, 197 (1876). Species minime cognita ob pappi paleis setiformibus et involucri squamis pluriseriatim imbricatis certe ex genere expellenda et ultius scrutanda.

A. punctatum Jacq. Hort. Schönb. iii. 28, t. 300 (1798) = Stevia serrata Cav.

A. punctatum Ort. Hort. Matr. Dec. 37 (1797) = Stevia punctata (Ort.) Pers.

A. purpureum Aubl. Pl. Guian. ii. 800 (1775) Species nunquam recognita verisimiliter *Eupatorii* species ut *A. guianense* Aubl.

A. purpureum Sesse ex DC. Prod. v. 122 (1836) = Stevia viscida HBK.

A. quadriflorum Blanco, Fl. Filip. 624 (1837) = Elephantopus spicatus B. Juss. fide Villarii in Blanco, Fl. Filip. ed. 3, Nov. Append. 114 (1880).

A. rhytidophyllum Robinson = A. paleaceum (Gay) Hemsl.

A. rubens Viviani, Elench. Pl. Hort. Dinegro, 9 (1802). Species neglecta et obscura mihi omnino ignota.

A. serratum Glaziou, Bull. Soc. Bot. Fr. lvi, mém. 3d, 382 (1909), nomen subnudum.

A. sessilifolium Schauer, Linnaea, xix. 715 (1847) = Trichocoronis sessilifolia (Schauer) Robinson, Proc. Am. Acad. xlii. 35 (1906).

A. strictum Hemsl. = A. salicifolium Hemsl.

A. strictum Sims, Bot. Mag. t. 2410 (1823) = Adenostemma Lavenia (L.) Ktze.

A. viscosum Ort. Dec. 36 (1797) = STEVIA SALICIFOLIA Cav.

A. Wrightii Torr. & Gray ex Gray Proc. Am. Acad. i. 46 (1848) = TRICHOCORONIS WRIGHTII (Torr. & Gray) Gray, Pl. Fendl. 65 (1849).

3. REVISION OF THE GENUS OXYLOBUS.

The claims of *Oxylobus* Moc. to generic rank rest quite as much upon habit and peculiar habitat as upon readily stated technical distinctions, yet they seem adequate. This small group of three well marked and obviously related species differ from all the Ageratums in being high alpine plants with thickish evergreen leaves. If a very doubtful report of one of them from Venezuela is excepted, they are confined to certain of the highest mountains of southern Mexico being found about at the timber line in borders of coniferous woods. The commonest of the three, *O. arbutifolius,* first described by Kunth as an *Ageratum,* was placed in *Phania* by DeCandolle, but is clearly distinct from that genus by the presence of a well developed apical appendage on the anthers. DeCandolle associated with it a second supposed species, his *Phania trinervia,* known to him only from one of Mociño's sketches. This second species has long remained vague, though Hemsley, Biol. Cent.-Am. Bot. ii. (1881), surmised its probable identity with the earlier species of Kunth. To the writer it seems that this identity may now be stated with definiteness. The tracing of Mociño's sketch, reproduced in the Calques des Dessins, shows no difference which may not be readily explained by the crudeness of the draftsmanship. The portion of Mexico in question has in recent years been diligently and effectively explored by several highly trained collectors without the discovery of any second species of just this habit. Finally it may be remarked that the eldest DeCandolle appears to have known the Mexican flora chiefly from the collections of Mociño & Sesse, Lagasca, Mendez, Née, and Haenke and on several occasions redescribed unconsciously the species of Kunth, seeming never to have had adequate opportunities to examine the series of plants collected by Humboldt and Bonpland.

The type of *O. glanduliferus* presents a curious confusion of more technical than practical importance. The species was first distinguished and named, though not described, by Schultz-Bipontinus, who on herbarium sheets noted a typical bluish-flowered form and a white-flowered condition, which he labelled var. *albiflorum* Sch. Bip. It was this white-flowered variety which Hemsley later described as *Ageratum glanduliferum* Sch. Bip. and which accordingly becomes ipso facto the type of the species. Further collection may show the two forms sufficiently marked to make it worth while to distinguish them by name, in which event it would be the blue-flowered one which would have to be given a new designation thus reversing the treatment of Schultz, the original author, whose manuscript names, though critical have no nomenclatorial status since unaccompanied by published descriptions. However, the distinction seems to be here scarcely of taxonomic moment. The bluish color appears to be in this genus paler and in drying much less permanent than in *Ageratum* or *Alomia* so that in any but very fresh or recent and carefully dried material a sharp distinction would scarcely be possible.

The technical distinctions of *Oxylobus* are chiefly in the form of the corolla, which has a relatively slenderer proper tube than is usual in *Ageratum* and a more ample and abruptly expanded throat. The lobes of the limb are not so uniformly acute as the generic name would suggest, yet they are longer and more widely spreading than in the nearly related genera. The pappus is uniformly short and its scales are so deeply and irregularly cleft that their number, though not large, becomes indefinite.

OXYLOBUS Moc. (Nomen ab ὀξύς, *acutus*, et λοβός, *lobus*, dentes corollae limbi alludens.) Capitula homogama parva vel mediocria. Involucrum subcylindrati- vel infundibuliformi-campanulatum, squamis ovato- vel lanceolato-oblongis acutis dorso glandulari-tomentellis subherbaceis, costis pluribus obscuris vel nullis. Receptaculum planiusculum nudum. Corollae tubus proprius gracilis, faucibus campanulato-cylindratis abrupte ampliatis, limbi dentibus oblongis vel anguste ovatis plerisque acutiusculis patentibus. Antherae apice cum appendice membranacea ovata munitae basi rotundatae. Achaenia graciliter prismatica 5-angularia deorsum leviter decrescentia in angulis sursum scabrata basi callosa. Pappus e squamulis 5–10 inaequalibus plerisque acutis et fimbriatis compositus achaenio multo brevior.— Moc. ex DC. Prod. v. 115 (1836); Gray, Proc. Am. Acad. xv. 25 (1879); Klatt, Leopoldina, xx. 75 (1884); Robinson & Greenman, Proc. Am. Acad. xli. 272 (1905). *Phania* Sect. *Oxylobus* (Moc.) DC. Prod. v. 115 (1836); Endl. Gen. 367 (1838). Ad *Ageratum* allocatus, Benth. & Hook. f. Gen. ii. 242 (1873); Hemsl. Biol. Cent.-Am. Bot. ii. 80 (1881); Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 137 (1890). In *Careliam* injectus, Ktze. Rev. Gen. i. 325 (1891). — Suffrutices decumbentes ramosi vel simplices. Folia opposita subcoriacea sempervirentia obtusa crenulata tenuiter reticulato-

venosa impunctata. Capitula in cymis compositis congestim vel laxiuscule disposita.— Species 3, omnes in regione alto-alpina montium austro-menicanorum incolae, una ut dicitur etiam in Venezuela reperta sit.

Clavis specierum.

Capitula ca. 20-flora. Folia elliptica subsessilia parva 1–2 cm. longa 1. *O. arbutifolius.*
Capitula ca. 75-flora. Folia oblonga basi abrupte rotundata, inferioribus bene petiolatis....................................2. *O. glanduliferus.*
Capitula ca 35-flora. Folia spatulata basi attenuata.......3. *O. adscendens.*

1. O. ARBUTIFOLIUS (HBK.) Gray, decumbens laxe caespitosus; caulibus flexuosis; ramis curvato-ascendentibus foliosissimis 1–6 dm. longis simplicibus vel ramosis teretibus saepius purpurascentibus glanduloso-tomentellis, internodiis inferioribus brevissimis 2–3 supremis longioribus usque ad 2–5 cm. longitudine; foliis ellipticis 8–19 mm. longis 3.5–10 mm. latis crenulato-serratis plus minusve praesertim marginem versus glandulari-puberulis supra viridibus subtus paullo pallidioribus venulosis; cymis 3–12-capitulatis 3–5 cm. diametro, pedicellis ascendentibus rectis filiformibus 6–20 mm. longis; capitulis ca. 20-floris; involucro subcylindrati-campanulato 4 mm. diametro 5 mm. alto, squamis exterioribus lanceolati-oblongis acutis dorso glandulari-tomentellis, interioribus angustioribus; corollis glabris 3.5 mm. longis, tubo proprio 1.6 mm. longo fauces subcylindratas aequanti; achaeniis plerisque arcuatis basin versus decrescentibus angulis sursum scabratis; pappi squamulis oblongis vel linearibus fimbriatis 0.7 mm. longis.— Proc. Am. Acad. xv. 26 (1879). *Ageratum arbutifolium* HBK. Nov. Gen. et Spec. iv. 149 (1820). *Phania arbutifolia* (HBK.) DC. Prod. v. 115 (1836). *P. trinervia* (Moc.) DC. l. c. (1836); A. DC. Calq. des Dess. t. 527 (1874). *Carelia arbutifolia* (HBK.) Ktze. Rev. Gen. i. 325 (1890).— MEXICO: in monte igniv. Nauhcampatepetl, alt. 3100 m., *Humboldt & Bonpland;* in monte Orizaba, *Galeotti,* n. 2159 (hb. Kew.), *Linden,* n. 1121 (hb. Kew.), alt. 3050 m., *Liebmann,* n. 212 (hb. Gray.), alt. 3300–3600 m., *Liebmann,* n. 76 (hb. Gray.); alt. 3965 m., *Seaton,* n. 237 (hb. Gray.); alt. 4020 m., *Nelson,* n. 286 (hb. Gray., hb. J. D. Sm.); in silvis coniferis, alt. 3500 m., *Pringle,* n. 8555 (hb. Gray., hb. J. D. Sm.); in diclivitate montis Orizabae, *Rose & Hay,* nn. 5738, 5760 (hb. Gray.), alt. 2750–3800 m., *Liebmann,* n. 213 f. Hemsl. l. c., in saxis prope marginem superiorem silvarum, Ixtaccihuatl, alt. 3300–3600 m., *Purpus,* n. 187 (hb. Gray.), in saxis supra silvas, *Purpus,* n. 1567 (hb. Gray.), in saxis humidis, Citlaltepetl, alt. 3050–3350 m., *Purpus,* n.

2767 (hb. Gray.); in rupibus frigidulis Sierrae de las Cruces, *Pringle*, n. 4290 (hb. Gray.); in monte igniv. Popocatepetl in regione pinorum, *Galeotti*, n. 2380 (hb. Kew.).

2. O. GLANDULIFERUS (Sch. Bip.) Gray, suffrutex ramosus 1–1.3 m. altus sordide glanduloso-tomentosus; ramis ascendentibus foliosis, internodiis superioribus 3–5 cm. longis inferioribus vix 1 cm. longis; foliis inferioribus oblongis obtusis basi subtruncatis vel abrupte rotundatis crenatis utrinque viridibus pilosellis margine glandulari-tomentellis 2–3 cm. longis 1–1.8 cm. latis ad 7 mm. longe petiolatis, foliis superioribus basi subcuneato-angustatis subsessilibus; cymis compositis trichotomis paucicapitulatis; capitulis majusculis ca. 75-floris; involucri plus minusve 5-angulati 6 mm. diametro et alti squamis exterioribus ovato-oblongis acuminatis glanduloso-puberulis ca. 7 mm. longis; interioribus oblanceolato-linearibus; corollis 5.6 mm. longis, tubo proprio ca. 2 mm. longo ad junctionem faucium pilosulo, faucibus infundibuliformibus ca. 2 mm. altis, dentibus limbi 1.6 mm. longis oblongis acutiusculis; achaeniis gracilibus 3–3.5 mm. longis; pappi squamulis 5 vel pluribus oblongis 0.7 mm. longis margine laceratis basi plus minusve connatis.— Proc. Am. Acad. xv. 26 (1879), sine char.; Klatt, Leopoldina, xx. 75 (1884). *Ageratum glanduliferum* Sch. Bip. ex Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881), ubi descriptum. *A. glanduliferum*, var. *albiflorum* Sch. Bip. ex Gray, l. c. *Oxylobus glanduliferus*, var. *albiflorus* (Sch. Bip.) Klatt, l. c. *Carelia glandulifera* (Sch. Bip.) Ktze. Rev. Gen. i. 325 (1891). — MEXICO: Sempoaltepec, *Liebmann*, n. 238 (hb. Gray.), specimen typicum, i. e. specimen a quo descriptio principalis a cl. Hemsleyo derivata est (= etiam var. *albiflorum* Sch. Bip.); Oaxaca, prope Sierram de San Felipe, alt. 2900–3350 m., *Nelson*, n. 1112 (hb. Gray.), in summis rupibus, Sierra de San Felipe, alt. 3175 m., *Pringle*, n. 4833 (hb. Gray., hb. J. D. Sm.); sine loco indicato, *Linden*, n. 1155 f. Klatt.

Nota.— Specimen in hb. Kew. ex "Venezuela &c., *Funcke & Schlim*, n. 1155" videtur fortasse lapsu pennae vel transpositione schedularum pro *Linden*, n. 1155, specimen verum hujus species ut videtur ex regione normali mexicana. Idem est sine dubitatione id quod cl. Hemsleyius, l. c. 82, "*Fendler*, n. 1155" per errorem scripsit. Species ergo in Venezuela adhuc non certe demonstrata.

3. O. ADSCENDENS (Sch. Bip.) Robinson & Greenman, perennis decumbens subherbaceus 2.5–5 dm. altus basi prostratus radicans foliosus; caule tereti curvato-ascendenti, internodiis erectis 3–4 longis usque ad 1–2 dm. longitudine glanduloso-puberulis; foliis imis spatulatis 3–4 cm. longis 1–2 cm. latis crenulatis basi in petiolum ad 1.5 cm. longum cuneatim attenuatis supra laete viridibus subtus pallide

viridibus venulosis glabriusculis; foliis caulinis oppositis remotis oblanceolato-oblongis sessibilus margine glanduloso-puberulis; cymis terminalibus compositis densis ca. 4 cm. diametro; capitulis ca. 35-floris; involucri 5–6 diametro aeque alti squamis exterioribus anguste oblongis acutis dorso sordide glanduloso-tomentosis interioribus linearibus; corollis 3–7 mm. longis glabris, tubo proprio 1.5 mm. longo fauces campanulatas aequanti; achaeniis 2.8 mm. longis deorsum leviter decrescentibus in angulis sursum scabratis; pappi squamulis pluribus angustis fimbriatis 0.5 mm. longis.— Proc. Am. Acad. xli. 272 (1905). *Ageratum adscendens* Sch. Bip. ex Benth. & Hook. f. Gen. ii. 242 (1873), nomen; Hemsl. Biol. Cent.-Am. Bot. ii. 80 (1881), ubi descriptum; Klatt, Leopoldina, xx. 75 (1884). *Carelia adscendens* (Sch. Bip.) Ktze. Rev. Gen. i. 325 (1891).— Mexico: in monte Orizaba ad Vaqueriam de Jacal, alt. 3050–3660 m., *Liebmann,* n. 214 (hb. Kew., hb. Gray.); alt. 3660 m., *Linden,* n. 489 (hb. Kew.); Puebla, in monte igniv. Ixtaccihuatl in graminosis prope marginem superiorem silvarum, *Purpus,* n. 1497 (hb. Gray., hb. U. S. Nat. Mus.); in civitate Mexico, in pascuis altis Deserto Viejo, *Bourgeau,* n. 839 (hb. Kew., hb. Gray.), in Serrania de Ajusco in declivitatibus patentibus, alt. 2300 m., *Pringle,* n. 6612 (hb. Gray., hb. U. S. Nat. Mus., hb. J. D. Sm.); Hidalgo, in graminosis humidis Sierrae de Pachuca, alt. 3050., *Pringle,* n. 9841 (hb Gray., hb. U. S. Nat. Mus.); Verz Cruz, in Cordillera, *Galeotti,* n. 2160 (hb. Kew.); sine loco indicato, *Ehrenberg,* n. 463a (hb. U. S. Nat. Mus.).

INDEX EXSICCATARUM.

Ag. = Ageratum. Al. = Alomia. Ox. = Oxylobus.

Abbott, *Ag. conyzoides* L.

Alvarez, 751 = *Ag. albidum* (DC.) Hemsl.

Andersson, *Ag. latifolium* Cav., v. *galapageium* Robinson.

Andrieux, 287 = *Ag. paleaceum* (Gay) Hemsl.; 543 = *Ag. albidum* (DC.) Hemsl.

Baker, 2021 = *Ag. conyzoides* L.; 2220 = *Ag. scabriusculum* (Benth.) Hemsl.

Ball, *Ag. conyzoides* L.

Bang, 407 partim = *Ag. conyzoides* L., partim = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.

Baur, *Ag. latifolium* Cav., v. *galapageium* Robinson.

Bennett, *Ag. littorale* Gray.

Bertero, *Ag. domingense* Spreng.

Beyrich, *Ag. conyzoides* L., *Ag. fastigiata* (Gardn.) Benth.

Bilimek, 488, 579 = *Al. alata* Hemsl.

Binot, 15 = *Ag. conyzoides* L.

Blanchet, 2754 = *Al. foliosa* (Gardn.) Benth. & Hook. f.; 3123 = *Al. heterolepis* (Bak.) Robinson; 3700 = *Ag. micropappum* Bak.

Blodget, *Ag. littorale* Gray.

Botteri, 623 = *Al. echioides* (Less.) Robinson.

Botteri & Sumicrast, 524 = *Al. isocarphoides* (DC.) Robinson.

Bourgeau, 719 = *Ag. corymbosum* Zuccag.; 839 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 1216 = *Al. alata* Hemsl.; 1557 =

Ag. Houstonianum Mill.; 2393, 3207 = *Al. echioides* (Less.) Robinson.

Brandegee, *Ag. corymbosum* Zuccag.

Brenes, 14, 491 = *Ag. Oerstedii* Robinson.

Britton, E. G., 6375 = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.; sine num. *Ag. latifolium* Cav.

Britton & Cowell, 927 = *Ag. conyzoides* L.

Britton & Millspaugh, 2240 = Ag. *latifolium* Cav.

Broadway, *Ag. conyzoides* L.

Brown & Brown, 31a, 51 = *Ag. conyzoides* L.; 254 = *Ag. Houstonianum* Mill.

Brunet, 24 = *Al. fastigiata* (Gardn.) Benth.

Buchanan, 740 = *Ag. conyzoides* L.

Buchtien, 1456 = *Ag. conyzoides* L.

Burchell, 854 = *Ag. conyzoides* L.

Chapman, 49 = *Ag. littorale* Gray.

Chatterjee, *Ag. Houstonianum* Mill.

Claussen, *Al. foliosa* (Gardn.) Benth. & Hook. f.

Clute, 190 = *Ag. Houstonianum* Mill.

Combs, 59 = *Ag. conyzoides* L.

Conzatti & González, 542 = *Ag. albidum* (DC.) Hemsl.

Cook & Doyle, 277 = *Al. microcarpa* (Benth.) Robinson.

Cook & Griggs, 560 = *Ag. conyzoides* L.

Cooper (distrib. J. D. Sm.), 5824 = *Ag. conyzoides* L.

Coulter, 247, 248 = *Ag. corymbosum* Zuccag.

Cuming, 2362, 2419 = *Ag. conyzoides* L.

Curtiss, 77 = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.; 650 = *Ag. maritimum* HBK.; 1163, 5446 = *Ag. littorale* Gray.

Darwin, *Ag. latifolium* Cav., v. *galapageium* Robinson.

Deam, 330 = *Ag. conyzoides* L.; 6169 = *Ag. rugosum* Coult.; 6208 = *Ag. rugosum* Coult.?

Drill, *Ag. conyzoides* L.

Dugès, 24, 426 = *Ag. corymbosum* Zuccag.

Duss, 934, 2520, 4681 partim = *Ag. conyzoides* L.; 4681 partim = *Ag. Houstonianum* Mill.

Eggers, 303 = *Ag. conyzoides* L.; 3445 = *Ag. Houstonianum* Mill.

Ehrenberg, 463a = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman.

Ervendberg, 100 = *Ag. Houstonianum* Mill.

Everett, 16 = *Ag. conyzoides* L.

Fendler, 652 = *Ag. conyzoides* L.

Friedrichsthal, *Ag. scabriusculum* (Benth.) Hemsl.

Fuertes, 458 = *Ag. latifolium* Cav.

Galeotti, 2098 = *Al. microcephala* (Hemsl.) Robinson; 2159 = *Ox. arbutifolius* (HBK.) Gray; 2160 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 2200 = *Al. echioides* (Less.) Robinson; 2202 = *Ag. tomentosum* (Benth.) Hemsl.; 2380 = *Ox. arbutifolius* (HBK.) Gray; 2451 = *Ag. salicifolium* Hemsl.

Garber, *Ag. littorale* Gray.

Gardner, 3809 = *Al. angustata* Gardn.) Benth.; 3810 = *Al. cinerea* (Gardn.) Benth.; 4837 = *Al. fastigiata* (Gardn.) Benth.; 4838 = *Al. foliosa* (Gardn.) Benth. & Hook. f.; 4863 = *Al. longifolia* (Gardn.) Robinson.

Gaudichaud, 106 = *Ag. latifolium* Cav.

Gaumer, 1 = *Ag. littorale* Gray, v. *hondurense* Robinson; 20 = *Ag. maritimum* HBK., f. *calvum* Robinson; 93 = *Ag. maritimum* HBK., v. *intermedium* (Hemsl.) Robinson; 395 = *Ag. Gaumeri* Robinson; sine num. = partim *Ag. littorale* Gray, v. *hondurense* Robinson, partim *Ag. littorale* Gray, v. *hondurense* Robinson, f. *setigerum* Robinson.

Ghiesbreght, 111, 547 = *Ag. tomentosum* (Benth.) Hemsl.

Glaziou, 10,980 = *Al. myriadenia* (Sch. Bip.) Bak.; 21,579 = *Al. dubia* Robinson.

Goll, 114 = *Ag. rugosum* Coult.? 238 = *Ag. Houstonianum* Mill.

Gregg, 372 = *Ag. corymbosum* Zuccag.

Guillemin-Tarayre, *Ag. corymbosum* Zuccag.

Haenke, *Al. isocarphoides* (DC.) Robinson.

Hahn, 1190 = *Ag. conyzoides* L.

Hartweg, 142 = *Ag. corymbosum* Zuccag., v. *euryphyllum* Robinson.

Heller, 542, 1999, 6143; = *Ag. conyzoides* L.

Henry, 9094 = *Ag. conyzoides* L.

Heyde, 530 = *Ag. conyzoides* L.

Heyde & Lux (distrib. J. D. Sm.), 3781 = *Ag. conyzoides* L.; 4228 = *Ag. elachycarpum* Robinson; 4243 = *Ag. rugosum* Coult.; 6153 = *Al. guatemalensis* Robinson.

Hildebrandt, 3304a, 3500 = *Ag. conyzoides* L.

Holst, 8860 = *Ag. conyzoides* L.

Holway, 3509, 5235 = *Ag. corymbosum* Zuccag., v. *latifolium* (DC.) Robinson.

Hooker f., *Ag. conyzoides* L.

Hooker f. & Thompson, *Ag. conyzoides* L.

Humboldt & Bonpland, 1273 = *Ag. maritimum* HBK.; 3949 = *Al. ageratoides* HBK.; sine num. *Ag. corymbosum* Zuccag., v. *latifolium* (DC.) Robinson, *Ox. arbutifolius* (HBK.) Gray.

Joad, *Ag. conyzoides* L.

Jenman, 5398 = *Ag. conyzoides* L.

King (legulus cl. Kingii), 11 = *Ag. conyzoides* L.

Kotschy, 327 = *Ag. conyzoides* L.

Kralik, *Ag. conyzoides* L.

Kuntze, *Al. fastigiata* (Gardn.) Benth.

Langlassé, 310 = *Ag. salicifolium* Hemsl.

Lehmann, 3600, 3601, 4666 = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.

Liebmann, 76 = *Ox. arbutifolius* (HBK.) Gray; 82, 83, 84 = *Al. ageratoides* HBK.; 143, 144 = *Al. echioides* (Less.) Robinson; 147 = *Al. Wendlandii* (Sch. Bip.) Robinson; 212, 213 = *Ox. arbutifolius* (HBK.) Gray; 214 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 238 = *Ox. glanduliferus* (Sch. Bip.) Gray.

Linden, 349 = *Al. microcarpa* (Benth.) Robinson? 489 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 1121 = *Ox. arbutifolius* (HBK.) Gray; 1155 = *Ox. glanduliferus* (Sch. Bip.) Gray; 1156 = *Al. echioides* (Less.) Robinson; 1185 = *Ag. tomentosum* (Benth.) Hemsl.

Lund, *Al. fastigiata* (Gardn.) Benth.

Luschnath, *Ag. conyzoides* L.

Malme, 1678 = *Al. Regnellii* Malme.

March, *Ag. latifolium* Cav.

Martius, 672 = *Ag. conyzoides* L.

Maxon & Hay, 3487 = *Ag. conyzoides* L.

Merrill, 35, 563, 1955 = *Ag. conyzoides* L.

Moore, 469 = *Ag. conyzoides* L.

Müller, 1129 = *Al. echioides* (Less.) Robinson.

Née, *Ag. latifolium* Cav.

Nelson, 286 = *Ox. arbutifolius* (HBK.) Gray; 858 = *Al. microcephala* (Hemsl.) Robinson; 991 = *Ag. stachyofolium* Robinson; 1112 = *Ox. glanduliferus* (Sch. Bip.) Gray; 1208 = *Ag. albidum* (DC.) Hemsl.; 1213 = *Ag. tomentosum* (Benth.) Hemsl.; 1446 = *Ag. paleaceum* (Gay) Hemsl.; 2822a = *Ag. albidum* (DC.) Hemsl., v. *Nelsonii* Robinson; 3528 = *Al. platylepis* Robinson; 3542 = *Ag. conyzoides* L.; 5006 = *Ag. corymbosum* Zuccag.

Nichols, 33 = *Ag. Houstonianum* Mill.

Oersted, 241, 247, 248 = *Al. microcarpa* (Benth.) Robinson; 251 = *Ag. Oerstedii* Robinson; sine num. = partim *Ag. scabriusculum* (Benth.) Hemsl., partim *Ag. tomentosum* (Benth.) Hemsl.

Palmer (anno 1874) 192 = *Ag. littorale* Gray; (anno 1880) 427 partim = *Ag. corymbosum* Zuccag., v. *europhyllum* Robinson; 427 partim, 428 = *Ag. corymbosum* Zuccag.; (anno 1885) 31 = *Ag. corymbosum* Zuccag., f. *album* Robinson; 110 = *Ag. corymbosum* Zuccag., v. *longipetiolatum* Robinson; (anno 1886) 289 = partim *Ag. corymbosum* Zuccag., partim *Ag. salicifolium* Hemsl.; 290, 351 = *Ag. corymbosum* Zuccag., v. *jaliscense* Robinson, f. *lactiflorum* Robinson; 437 partim = *Ag. platypodum* Robinson; 715 = *Ag. corymbosum* Zuccag., v. *jaliscense* Robinson; (anno 1891) 1587 = *Ag. corymbosum*, v. *latifolium* (DC.) Robinson; (anno 1892) 1834 = *Ag. latifolium* Cav.; 1850, 1890 = *Ag. conyzoides* L.; 2066 = *Ag. Houstonianum* Mill.; (anno 1896) 506 = *Ag. corymbosum* Zuccag.;

(anno 1898) 307 = *Ag. corymbosum* Zuccag.; (anno 1902) 101 = *Ag. corymbosum* Zuccag., v. *euryphyllum* Robinson; 382 = *Ag. corymbosum* Zuccag., v. *subsetiferum* Robinson; (anno 1904) 154½, 387 = *Ag. corymbosum* Zuccag.; (anno 1906) 486 = *Ag. corymbosum* Zuccag.

Palmer & Riley, 65 = *Ag. Houstonianum* Mill.; 712, 739 = *Ag. maritimum* HBK.; 1068 = *Ag. conyzoides* L.

Parry & Palmer, 315 = *Ag. corymbosum* Zuccag., v. *euryphyllum* Robinson; 317 = *Ag. corymbosum* Zuccag.; 318 = *Ag. corymbosum* Zuccag., v. *euryphyllum* Robinson.

Peck, 80 = *Ag. Peckii* Robinson; 99 = *Ag. radicans* Robinson.

Pittier, 1891 = *Ag. conyzoides* L.; 2390, 3415, 3533, 4139 = *Al. microcarpa* (Benth.) Robinson; 4914 = *Ag. riparium* Robinson; 6995 = *Ag. Houstonianum* Mill.

Pohl, 358 = *Al. Pohlii* (Sch. Bip.) Bak.; 371, 535 = *Al. fastigiata* (Gardn.) Benth.; 669 = *Al. foliosa* (Gardn.) Benth & Hook. f.

Pollard, Collins & Morris, 12 = *Ag. littorale* Gray.

Pringle, 669 = partim *Ag. corymbosum* Zuccag., partim *Ag. corymbosum* Zuccag., f. *album* Robinson; 2166 = *Al. callosa* (Wats.) Robinson; 3599 = *Ag. corymbosum* Zuccag., v. *latifolium* (DC.) Robinson; 4290 = *Ox. arbutifolius* (HBK.) Gray; 4739 = *Al. callosa* (Wats.) Robinson; 4816 = *Ag. albidum* (DC.) Hemsl.; 4833 = *Ox. glanduliferus* (Sch. Bip.) Gray; 5675 = *Ag. paleaceum* (Gay) Hemsl.; 5786 = *Ag. tomentosum* (Benth.) Hemsl.; 6177 = *Ag. paleaceum* (Gay) Hemsl.; 6229 = *Al. alata* Hemsl.; 6234 = *Ag. salicifolium* Hemsl.; 6267 = *Ag. tomentosum* (Benth.) Hemsl.; 6612 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 6754 = *Ag. tomentosum* (Benth.) Hemsl.; 7851 = *Ag. lucidum* Robinson; 8065 = *Ag. Houstonianum* Mill.; 8362 = *Ag. lucidum* Robinson; 8555 = *Ox. arbutifolius* (HBK.) Gray; 9045 = *Ag. salicifolium* Hemsl.; 9047 = *Ag. corymbosum* Zuccag.; 9353 = *Al. callosa* (Wats.) Robinson; 9522 = *Ag. tomentosum* (Benth.) Hemsl.; 9542 = *Ag. corymbosum* Zuccag., f. *album* Robinson; 9841 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 9842 = *Ag. corymbosum* Zuccag., v. *latifolium* (DC.) Robinson, f. *albiflorum* Robinson; 9843 = *Ag. corymbosum* Zuccag., v. *latifolium* (DC.) Robinson; 9844 = *Ag. lucidum* Robinson; 9845 = *Ag. salicifolium* Hemsl.; 9846 = *Al. alata* Hemsl.; 11,819 = *Al. Wendlandii* (Sch. Bip.) Robinson; 11,898 = *Ag. salicifolium* Hemsl.; 13,022 = *Ag. lucidum* Robinson.

Purpus, 187 = *Ox. arbutifolius* (HBK.) Gray; 1179 = *Ag. tomentosum* (Benth.) Hemsl.; 1497 = *Ox. adscendens* (Sch. Bip.) Robinson & Greenman; 1565 = *Ag. corymbosum* Zuccag.; 1567 = *Ox. arbutifolius* (HBK.) Gray; 1870 = *Ag. tomentosum* (Benth.) Hemsl.; 2199 = *Al. echioides* (Less.) Robinson; 2547 = *Ag. tomentosum* (Benth.) Hemsl.; 2767 = *Ox. arbutifolius* (HBK.) Gray; 4561 = *Ag. corymbosum* Zuccag.; 4814 = *Ag. tomentosum* (Benth.) Hemsl.

Ramos, 2086 = *Ag. conyzoides* L.

Regnell, III 675 = *Ag. conyzoides* L.

Remy, 229 = *Ag. conyzoides* L.

Renson, 190 = *Ag. conyzoides* L.

Ricksecker, 250 b, 430 = *Ag. conyzoides* L.

Riedel, 711, 959 = *Al. fastigiata* (Gardn.) Benth., 1346 = *Ag. conyzoides* L.

Rose, 2538 = *Ag. corymbosum* Zuccag., f. *album* Robinson; 2585 = *Ag. corymbosum* Zuccag., v. *jaliscense* Robinson, f. *lactiflorum* Robinson; 2739 = *Ag. corymbosum* Zuccag., v. *euryphyllum* Robinson.

Rose & Hay, 5738, 5760 = *Ox. arbutifolius* (HBK.) Gray.

Rose, Painter & Rose, 9631 = *Ag. corymbosum* Zuccag.

Rudio, *Ag. conyzoides* L.; *Ag. Houstonianum* Mill.

Rusby, 1642, 1643 = *Ag. conyzoides* L.

Sallé, *Al. echioides* (Less.) Robinson.

Salvin & Godman, 48 = *Ag. rugosum* Coult.

Sartorius, *Al. echioides* (Less.) Robinson; *Ag. Houstonianum* Mill.
Schaffner, 202, 293 = *Ag. corymbosum* Zuccag.
Schiede, 304 = *Al. echioides* (Less.) Robinson; sine num. *Ag. Houstonianum* Mill.
Schomburgk, Rich., 488 = *Ag. scorpioideum* Bak.
Schomburgk, Rob., 353 = *Ag. scorpioïdeum* Bak.
Schumann, 40b = *Ag. corymbosum* Zuccag.
Seaton, 55 = *Ag. Houstonianum* Mill.; 237 = *Ox. arbutifolius* (HBK.) Gray; 346, 358 = *Ag. tomentosum* (Benth.) Hemsl.
Seemann, 267 = *Ag. conyzoides* L.
Sello, 119 = *Al. myriadenia* (Sch. Bip.) Bak.; 120, 122 = *Al. fastigiata* (Gardn.) Benth.
Shafer, 207 = *Ag. conyzoides* L.; 1099 = partim *Ag. maritimum* HBK., partim *Ag. maritimum*, f. *calvum* Robinson.
Shannon (distrib. J. D. Sm.), 600 = *Ag. conyzoides* L.
Shufeldt, 101 = *Ag. conyzoides* L.
Simpson, 246 = *Ag. littorale* Gray.
Sinclair, *Ag. petiolatum* (Hook. & Arn.) Hemsl.; *Ag. salicifolium* Hemsl.; *Ag. scabriusculum* (Benth.) Hemsl.
Sintenis, 59, 3661, 5816 = *Ag. conyzoides* L.; 5874 = *Ag. conyzoides* L., f. *album* (Willd.) Robinson.
Smith, C. L., 145 = *Ag. Houstonianum* Mill.; 365 = *Ag. albidum* (DC.) Hemsl.; 594 = *Ag. paleaceum* (Gay) Hemsl. 979 = *Ag. Houstonianum* Mill.
Smith, H. H., 523 = *Ag. conyzoides* L.
Smith, H. H. & G. W., 543 = *Ag. conyzoides* L.
Smith, L. C., 277 = *Ag. paleaceum* (Gay) Hemsl.; 285 = *Ag. tomentosum* (Benth.) Hemsl.; 424 = *Ag. albidum* (DC.) Hemsl.
Snodgrass & Heller, 423 = *Ag. latifolium* Cav., v. *galapageium* Robinson.
Stuhlmann, *Ag. conyzoides* L.
Taylor, 191 = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.
Thieme (distrib. J. D. Sm.), 5323 = partim *Ag. conyzoides* L.
Thomas, *Al. echioides* (Less.) Robinson.
Tonduz, 4139 = *Ag. latifolium* Cav.; 7281, 8479, 10,816 = *Al. microcarpa* (Benth.) Robinson.
Triana, 1157 = *Ag. latifolium* Cav.
Tuerckheim, H. von (distrib. J. D. Sm.), 488 = *Ag. conyzoides* L.; II 999 = *Ag. conyzoides* L.; II 2051 = *Ag. rugosum* Coult.
Tweedy, 312 = *Ag. littorale* Gray.
Ule, 3904 = *Al. fastigiata* (Gardn.) Benth.
Underwood & Griggs, 871, 872 = *Ag. conyzoides* L.
Valdez, 13 = *Ag. Gaumeri* Robinson.
Velasco, 8967 = *Ag. rugosum* Coult?
Wagener, *Ag. latifolium* Cav.
Ward, *Ag. corymbosum* Zuccag.
Warming, 402 = *Al. myriadenia* (Sch. Bip.) Bak.; sine num. partim *Al. angustata* (Gardn.) Benth., partim *Al. Pohlii* (Sch. Bip.) Bak.
Watt, 10,431 = *Ag. conyzoides* L.
Whyte, 252 = *Ag. polyphyllum* Bak.
Wight, 26 = *Ag. latifolium* Cav.
Wilkes, *Ag. conyzoides* L.; *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.
Williams, *Ag. conyzoides* L.
Wislizenus, 181 = *Ag. corymbosum* Zuccag.
Wright, 1129 = *Ag. corymbosum* Zuccag.; 1310 = *Ag. conyzoides* L.; 1631 (anno 1859–1860) = *Ag. latifolium* Cav.; 1631 (annis 1860–1864) = *Ag. maritimum* HBK.; 2798 = *Ag. domingense* Spreng.; sine num. *Ag. conyzoides* L., *Ag. domingense* Spreng., *Ag. scabriusculum* (Benth.) Hemsl.
Wright, Parry & Brummel, 250 = *Ag. conyzoides* L., v. *inaequipaleaceum* Hieron.
Zimmermann, 70 = *Ag. conyzoides* L.
Zollinger, 23 = *Ag. conyzoides* L.

III. SOME NEW COMBINATIONS REQUIRED BY THE INTERNATIONAL RULES.

By C. A. Weatherby.

While employed at the Gray Herbarium during the summer of 1911 I was engaged in determinative work and in the re-arrangement and incidental revision of certain groups, notably the *Nyctaginaceae*. In the course of these activities it was found impossible to name certain species in accordance with the provisions of the International Rules of Botanical Nomenclature without employing new combinations. I have been requested to put these names on published record and they may be stated as follows:

Nyctaginaceae.

Oxybaphus ciliatifolius, n. nom. *Oxybaphus aggregatus* Torr. Bot. Mex. Bound. 168 (1859), not Vahl, Enum. ii. 41 (1805). *Allionia ciliata* Standley, Contr. U. S. Nat. Herb. xii. 345 (1909), not *Oxybaphus ciliatus* Phil. ex Meigen in Engl. Bot. Jahrb. xvii. 231 (1893).

Oxybaphus comatus (Small), n. comb. *Oxybaphus nyctagineus,* var. *pilosus* Gray in Torr. Bot. Mex. Bound. 174 (1859). *Allionia comata* Small, Fl. Southeast. U. S. 407 (1903).

Oxybaphus Brandegei (Standley), n. comb. *Allionia Brandegei* Standley, Contr. U. S. Nat. Herb. xii. 346 (1909).

Oxybaphus rotatus (Standley), n. comb. *Allionia rotata* Standley, Contr. U. S. Nat. Herb. xii. 347 (1909).

Oxybaphus giganteus (Standley), n. comb. *Allionia gigantea* Standley, Contr. U. S. Nat. Herb. xii. 348 (1909)

Oxybaphus pratensis (Standley), n. comb. *Allionia pratensis* Standley, Contr. U. S. Nat. Herb. xii. 351 (1909).

Oxybaphus Carletoni (Standley), n. comb. *Allionia Carletoni* Standley, Contr. U. S. Nat. Herb. xii. 355 (1909).

Oxybaphus exaltatus (Standley), n. comb. *Allionia exaltata* Standley, Contr. U. S. Nat. Herb. xii. 355 (1909).

Oxybaphus cardiophyllus (Standley), n. comb. *Oxybaphus Cervantesii* of authors, in part, not Sweet, Brit. Fl. Gard. ser. 1, t. 84. (1823). *Allionia cardiophylla* Standley, Contr. U. S. Nat. Herb. xiii. 405 (1911).

IV. ON THE GRAMINEAE COLLECTED BY PROF. MORTON E. PECK IN BRITISH HONDURAS, 1905-07.

By F. Tracy Hubbard.

Prof. Peck's collection of grasses from British Honduras includes four new species and a number of plants that are little known, several of them not previously reported from Central America. The grasses indicate that the flora of British Honduras is composed of some North Mexican, but chiefly of West Indian and South American elements; a fact which Prof. Robinson tells me is true of the other families.

A list of the species collected is as follows —

Tripsacum dactyloides L. Sp. Pl. ed. 2, **2**: 1378 (1763).— Pine ridge, Ycacos Lagoon, March 8, 1907, *M. E. Peck*, no. 703.

Manisuris granularis (L.) Sw. Prodr. Veg. Ind. Occ. 25 (1788).— Open ground, Toledo, February 7, 1907, *M. E. Peck*, no. 653.

Ischaemum latifolium Kunth, Rev. Gram. **1**: 371, t. 99 (1830).— Swampy ground near Manatee Lagoon, December 30, 1905, *M. E. Peck*, no. 254.

Trachypogon plumosus (Humb. & Bonpl.) Nees, Agrost. Bras. 344 (1829).— Pine ridge near Manatee Lagoon, September 8, 1905, *M. E. Peck*, no. 139.

Andropogon condensatus HBK. Nov. Gen. et Spec. **1** : 188 (1816). — Pine ridge near Manatee Lagoon, June 21, 1905, *M. E. Peck*, no. 59.

Andropogon domingensis (Spreng.), comb. nov. *A. hirtiflorus* Kunth, Rev. Gram. **2** : 569, t. 198 (1832). *Streptachne domingensis* Spreng. ex Schultes, Mant. **2** : 188 (1824). *Schizachyrium domingense* (Spreng.) Nash in No. Am. Fl. **17** : 103 (1912).— Pine ridge near Manatee Lagoon, January 8, 1906, *M. E. Peck*, no. 282. There are three older uses of the combination *A. domingensis* viz. Steud. Nom. ed. 1, 45 (1821); Spreng. ex Steud. Nom. ed. 2, **1**: 91 (1841) in syn; and Fourn. Mex. Pl. **2**: 61 (1886). The second use being in synonymy is invalid and as the first and third uses are both based on *Anatherum domingense* R. & S. Syst. Veg. **2**: 809 (1817) which is given as a synonym of *Andropogon leucostachyus* HBK. (1816) [teste specim.] cf. Kunth, Rev. Gram. **1**: 164 (1824) and generally so considered,—they do not interfere with the present combination.

Andropogon leucostachyus HBK. Nov. Gen. et Spec. **1**: 187 (1816).— Pine ridge near Manatee Lagoon, August 10, 1905, *M. E. Peck*, no. 116.

ANDROPOGON SPATHIFLORUS (Nees) Kunth, Enum. Pl. **1**: 496 (1833).— Low pine ridge near Manatee Lagoon, June 21, 1905, *M. E. Peck,* no. 54.

ANDROPOGON VIRGINICUS L. Sp. Pl. **2**: 1046 (1753).— Pine ridge near Manatee Lagoon, September 27, 1905, *M. E. Peck,* no. 150.

ARUNDINELLA DEPPEANA Nees ex Steud. Syn. Pl. Gram. 115 (1854). — Thicket near Manatee Lagoon, February 28, 1906, *M. E. Peck,* no. 359; river bank, Moho R., March 16, 1907, *M. E. Peck,* no. 735. This species may not be distinct from *A. peruviana* (Presl) Steud. l. c. as affirmed by Nash in No. Am. Fl. **17**: 143 (1912), but from the material in the Gray Herbarium it seems to be distinct.

LEPTOCORYPHIUM LANATUM (HBK.) Nees, Agrost. Bras. 84 (1829). — Pine ridge near Manatee Lagoon, August 30, 1905, *M. E. Peck,* no. 137.

DIGITARIA SETIGERA Roth ex R. & S. Syst. Veg. **2**: 474 (1817). *D. setosa* Desv. ex Hamilt. Prodr. Pl. Ind. Occ. 6 (1825). *Syntherisma digitatum* (Sw.) Hitchc. Contr. U. S. Nat. Herb. **12**: 142 (1908) based on *Milium digitatum* Sw. Prodr. Veg. Ind. Occ. 24 (1788) not *D. digitata* Buese in Miq. Pl. Jungh. 381 (1855).— Cultivated ground near Manatee Lagoon, June 18, 1905, *M. E. Peck,* no. 44.

THRASYA CAMPYLOSTACHYA (Hack.) Chase in Proc. Biol. Soc. Wash. **24**: 115 (1911). *Panicum campylostachyum* Hack. in Oesterr. Bot. Zeitschr. **51**: 367 (1901).— Pine ridge near Manatee Lagoon, June 18, 1905, *M. E. Peck,* no. 70. This interesting species originally described from Costa Rica seems to be limited in its distribution to the southern portion of Central America. The determination was kindly verified by Mrs. Chase, Assistant Agrostologist of the Bureau of Plant Industry.

Mesosetum filifolium, sp. nov. Culmi caespitosi, basin versus a vaginis multis arcte imbricatis obtecti, erecti, tenues, 35–70 cm. altitudine, inflorescentiam versus puberuli aliter nodis breviter hispido-barbatis exceptis glabri. Vaginae radicales squamiformes, tumidiusculae, hispidae, abrupte ad folii junctionem contractae babulataeque; eae culmarum elongatae sed internodio breviores, glabrae, in laminas gradatim transeuntes. Ligula barbulata, circa 0.5 mm. longa. Folia erecta, setaceo-filiformis, basin versus conduplicata, 7–28 cm. longa, 0.5 mm. lata, glabra, ea culmarum reducta, supremum saepe vix 1 cm. longum. Inflorescentia erecta, 4–7 cm. longa, 5 mm. diametro; axis flexuosus fere capillaris, excavationi non ostendentius, puberulus vel scaber; pedicellum breve, hispidum. Spiculae patenti-erectae, lineari-oblongae, apicem versus tenebrae, albido-villosae, 5 mm. longae, 1.5 mm. latae; gluma prima quam secunda brevior vel eam subaequans, 3-nervata, acuta, dense hirsuta villis adpressis apicem glumae superantibus, ad basin penicillatum collectis; gluma secunda 5-nervata, involuto-acuminata, quam lemma sterile paullum longior, hirsuta villosa; lemma sterile dorso-concavum, 2-carinatum, 5-nervatum, abrupte acuminatum, carinis apicem versus longe villoso-penicillatis; lemma fertile 5-nervatum, ad apicem carinatum, apice hispidulo et acuto, quam palea 2-nervata paullum longius.— Type (in the Gray Herb.) and only specimen seen, pine ridge near Manatee Lagoon, August 30, 1905, *M. E. Peck*, no. 136. This species is closely allied to *M. exaratum* (Trin.) Chase, but is taller, much longer leaved, the inflorescence longer and the spikelets are larger, more villous and with more pointed parts.

AXONOPUS LAXIFLORUS (Trin.) Chase in Proc. Biol. Soc. Wash. **24**: 133 (1911). *Anastrophus laxiflorus* (Trin.) Nash in No. Am. Fl. **17**: 163 (1912).— Pine ridge near Manatee Lagoon, July 8, 1906, *M. E. Peck*, no. 464. I have not been able to compare this, but ex char. I believe it to be the above.

PASPALUM CAESPITOSUM Flügge, Gram. Monogr. 161 (1810).— Open ground, Toledo, March 26, 1907, *M. E. Peck*, no. 769. Not previously reported from Central America, former distribution southern Florida, Bahamas, Jamaica, Cuba, Porto Rico and South America.

PASPALUM ORBICULATUM Poir. in Lam. Encycl **5**: 32 (1804). *P. pusillum* Vent. ex Flügge, Gram. Monogr. 100 (1810).— Open ground, Toledo, October 1, 1906, *M. E. Peck*, no. 541.

PASPALUM PANICULATUM L. Syst. Nat. ed. 10, **2**: 855 (1759).— Open ground, Toledo, October 1, 1906, *M. E. Peck*, no. 542; clearing near Manatee Lagoon, February 27, 1906, *M. E. Peck*, no. 970. A very variable species, the Peck specimens show extremes of pilosity.

Paspalum Peckii, sp. nov., perenne, dense caespitosum, 80 cm. altitudine, radicibus fibrosis. Culmi erecti, crassi, glabri, nodis fuscis paullum breviter pilosis. Vaginae infimae saepe squamiformes, adpresse breviter pilosae; superiores imbricatae, supra aliquid conduplicatae, internodia aequantes vel ea superantes, glabrae sed per majorem vel minorem partem longitudinis suae cum marginibus papilloso-pilosis et ad folii junctionem cum annulo piloso. Ligula membranacea, 2 mm. longa. Folia erecta, rigidiuscula, linearia, plana vel conduplicata, longe acuminata, 10–34 cm. longa, 5–10 mm. lata, marginibus scabris; lamina subtus glabra vel sparse pilosa, supra breve pilosa basin versus densius induta, longe villosa ad basin penicillata. Inflorescentia terminalis, subdigitata, 10–15 cm. longa, in pedunculis rigidis, canaliculatis, glabris vel sparse hispidis longe exserta; racemi 2–3, saepius 3, 10–13 cm. longi, rhachi membranaceo-alato 1 mm. lato, spiculas binas in ordinibus duobus ferente, in margine basin versus sparse longeque ciliato, ad axis principalis junctionem piloso, aliter glabro marginibus et costa media et pedicellis scabris exceptis. Spiculae turgidae, elliptici-obovatae, acutae, glabrae, albo-virides saepe purpureo-coloratae, circa 2.7 mm. longae, 1.5 mm. latae, aliquando 2-florae,— inferior staminifera, superior hermaphrodita; gluma prima plerumque adest, quintam partem vel etiam dimidiam partem spiculae longitudine aequans, truncata vel acuminata, 1-nervata cum nervo scabro; gluma secunda 5-nervata, quam lemma sterile brevior; lemma sterile 5-nervatum, fructus superans, ad apicem incrassatum puberulumque; palea sterilis aliquando adest, indurata; fructus ellipsoidalis, circa 2.2 mm. longus, 1.4 mm. latus.— Type (in the Gray Herb.) and only specimen seen, pine ridge near Manatee Lagoon, July 18, 1905, *M. E. Peck,* no. 71. *P. Peckii* belongs to the section *Monostachya* and is most nearly allied to *P. pilosum* Lam. (*Panicum monostachyum* HBK. not *Paspalum monostachyum* Vasey), but differs from it in the greater number of racemes [*P. pilosum* occasionally has 2, but usually only 1], the longer less pubescent leaf-blades and the thinner sterile lemma. I wish to thank Mrs. Chase for her kindness in comparing the Peck specimen with the material at Washington and for her notes relating to its difference from other species of this group.

PASPALUM PECTINATUM Nees, Agrost. Bras. 34 (1829).— Pine ridge, Ycacos Lagoon, March 1, 1907, *M. E. Peck,* no. 680. Determined by Mrs. Chase who states that they have (in Washington) only one North American collection of this species, Costa Rica, *Biolley,* no. 2651. *P. pectinatum* is a South American species.

PASPALUM PEDUNCULATUM Poir. in Lam. Encycl. Suppl. **4**: 315 (1816).— Pine ridge near Manatee Lagoon, August 8, 1905, *M. E. Peck,* no. 110.

PASPALUM PULCHELLUM HBK. Nov. Gen. et Spec. **1**: 90, t. 26 (1816).— Pine ridge near Manatee Lagoon, July 31, 1905, *M. E. Peck,* no. 86. Previously reported from Cuba, Hispaniola, Barbados and South America; new for Central America.

PASPALUM SCHREBERIANUM (Flügge) Nash in No. Am. Fl. **17**: 190 (1912).— Wet pine ridge near Manatee Lagoon, July 18, 1905, *M. E. Peck,* no. 69 a. Determined by Mrs. Chase. The specimen (in the Gray Herb.) consists of two inflorescences which were tied with no. 69, *Paspalum Underwoodii* Nash, and extends the range to Central America as the former distribution was the West Indies and South America.

Paspalum Underwoodii Nash in Bull. Torr. Bot. Club, **30**: 375 (1903).— Wet pine ridge near Manatee Lagoon, July 18, 1905, *M. E. Peck*, no. 69. Determined by Mrs. Chase. A West Indian species not previously reported from Central America.

Panicum chrysopsidifolium Nash in Small, Fl. Southeast U. S. 100 (1903).— Forest near Sibune R., May 8, 1906, *M. E. Peck*, no. 425. Previously reported from Florida, Louisiana, Cuba and Porto Rico; new for Central America.

Panicum cyanescens Nees Agrost. Bras. 220 (1829).— Low pine ridge near Manatee Lagoon, January 5, 1906, *M. E. Peck*, no. 271, A variable species resembling *P. parvifolium* Lam., but more upright, with longer leaves and larger panicle. It is a South American species apparently new to North America. The references to *P. cyanescens* in Hemsl. Biol. Cent.-Am. Bot. **3**: 487 (1885) and Fourn. Mex. Pl. **2**: 20 (1886) are probably misapplications of the name and referable to *P. parvifolium*. *Wright*, no. 3459 cited by Fournier is according to Prof. A. S. Hitchcock's notes on this number in the Gray Herbarium a mixture of *P. parvifolium* Lam. and *P. nitidum* Lam.

Panicum fasciculatum Sw. Prodr. Veg. Ind. Occ. 22 (1788).— Cultivated ground near Manatee Lagoon, January 27, 1906, *M. E. Peck*, no. 317.

Panicum fusiforme Hitchc. Contr. U. S. Nat. Herb. **12**: 222 (1909). — Pine ridge near Manatee Lagoon, July 7, 1906, *M. E. Peck*, no. 453a. Distribution previously limited to the extreme southeastern portion of the United States and Cuba. New for Central America.

Panicum laxum Sw. Prodr. Veg. Ind. Occ. 23 (1788).— Pine ridge near Manatee Lagoon, June 21, 1905, *M. E. Peck*, no. 60.

Panicum maximum Jacq. Coll. Bot. **1**: 76 (1786).— Thickets near Manatee Lagoon, November 10, 1905, *M. E. Peck*, no. 195.

Panicum pilosum Sw. Prodr. Veg. Ind. Occ. 22 (1788).— Pine ridge near Manatee Lagoon, June 10, 1905, *M. E. Peck*, no. 28.

Panicum pulchellum Raddi, Agrost. Bras. 42 (1823).— Pine ridge near Manatee Lagoon, January 8, 1906, *M. E. Peck*, no. 279.

Panicum Rudgei R. & S .Syst. Veg. **2**: 444 (1817).— Pine ridge, Monkey R., December 26, 1906, *M. E. Peck*, no. 588.

Panicum sphaerocarpon Ell. Sketch **1**: 125 (1816).— Pine ridge near Manatee Lagoon, June 21, 1905, *M. E. Peck*, no. 61.

Panicum stenodoides, sp. nov., perenne, virgulta e caudice noduloso formans, circa 30 cm. altitudine, olivaceum, radicibus paullum carnosis. Culmi erecti, tenues, firmiusculi, basin versus toto vel fere glabri, supra papilloso-pilosi, nodis glabris. Vaginae infimes squami-

formes, glabrae, superiores breves, circa 2 cm. longae, patente papilloso-pilosae, abrupte ad folii junctionem contractae. Ligula membranacea, brevissima, 0.25 mm. longa. Folia erecta, rigidiuscula linearia, subtus plana apicem versus involuto-setacea, 5–14 cm. longa, 1–2 mm. lata, lamina utrinque papilloso-pilosa. Paniculae terminales, solitariae vel per occasionem cum secunda breviori ex axe eodem producta, breve exsertae vel ad basin inclusae, plerumque bractea parva setiforme suffultae, circa 1 cm. longae, 2–4 mm. diametro, 5–7-spiculatae, axe pedicelloque scabris. Spiculae obtusae, turgidae, paullum ad basin attenuatae, 2 mm. longae, circa 1.5 mm. latae, valde nervatae; gluma prima quam spicula duplo brevior, 3-nervata, acuta; gluma secunda 9-nervata, lemma sterile 9-nervatum aequans; fructus ellipsoidalis, acutus, circa 1.7 mm. longus, 1 mm. latus, paullum glumam secundam et lemma sterile superans.— Type (in the Gray Herb.) and only specimen seen, low pine ridge, Ycacos Lagoon, March 5, 1907, *M. E. Peck*, no. 681. This species belongs to the small section *Tenera* as characterized in Hitchc. & Chase, Contr. U. S. Nat. Herb. **15**: 97 (1910). It is very similar to *P. stenodes* Griseb. and *P. caricoides* Nees, but differs from the former in having pilose culms, sheath and leaves and larger spikelets; from the latter it differs very noticeably in the absence of long stiff hairs on the pedicel.

PANICUM TRICHANTHUM Nees, Agrost. Bras. 210 (1829).— Open ground, Toledo, March 27, 1907, *M. E. Peck*, no. 775. The Peck specimen is rather stouter and the spikelets slightly larger than usual.

PANICUM TRICHOIDES Sw. Prodr. Veg. Ind. Occ. 24 (1788).— Clearing near Manatee Lagoon, January 27, 1906, *M. E. Peck*, no. 314; open ground, Toledo, February 2, 1907, *M. E. Peck*, no. 637.

PANICUM VIRGATUM L. Sp. Pl. ed. 1, **1**: 59 (1753).— Swamp near Manatee Lagoon, July 18, 1905, *M. E. Peck*, no. 73; swamp near Manatee Lagoon, August 13, 1905, *M. E. Peck*, no. 123.

PANICUM (§ PTYCHOPHYLLUM) CRUS-ARDEAE Willd. ex Nees, Agrost. Bras. 253 (1829).— Forest, upper Moho R., October 18, 1906, *M. E. Peck*, no. 565. The proper position of the section *Ptychophyllum* seems to be very uncertain. By some it is considered a section of *Setaria* while by others it is retained in *Panicum*. As this species has never been transferred to *Setaria*, I am maintaining the old name though presumably its affinity is more with *Setaria* than *Panicum*.

ICHNANTHUS PALLENS (Sw.) Doell in Mart. Fl. Bras. **2**^{2}: 290 (1877). — Forest near Manatee Lagoon, October 18, 1905, *M. E. Peck*, no. 172; forest near Manatee Lagoon, January 27, 1906, *M. E. Peck*, no. 312.

Lasiacis divaricata (L.) Hitchc. Contr. U. S. Nat. Herb. **15**: 16 (1910). *Panicum divaricatum* L. Syst. Nat. ed. 10, **2**: 871 (1759).— Thicket near Manatee Lagoon August 2, 1905, *M. E. Peck,* no. 93.

Lasiacis Grisebachii (Nash) Hitchc. in Bot. Gaz. **51**: 302 (1911). *Panicum Grisebachii* Nash in Bull. Torr. Bot. Club **35**: 301 (1908).— Forest near Manatee Lagoon, November 10, 1905, *M. E. Peck,* no. 197. A species apparently hitherto restricted to Cuba.

Lasiacis procerrima (Hack.) Hitchc. ex Chase in Proc. Biol. Soc. Wash. **24**: 145 (1911). *Panicum procerrimum* Hack. in Oesterr. Bot. Zeitschr. **51**: 431 (1901).— Forest near Manatee Lagoon, August 6, 1905, *M. E. Peck,* no. 107.

Lasiacis sp.— Pine ridge, Monkey R., December 28, 1906 *M. E. Peck,* no. 593. This may be *Panicum martinicense* Griseb. Fl. Brit. W. Ind. 552 (1864), as it answers the description and is reported by Grisebach as coming from Jamaica [Martinique!, Panama!, Guiana!]. The identity of Grisebach's species is, however, at present uncertain and consequently it is unwise to make a new combination. Prof. Hitchcock writes me that a specimen of *Sieber,* no. 29 from Martinique (cited by Grisebach) is *Lasiacis Swartziana* Hitchc., though this particular specimen may not be the one from which Grisebach drew his description.

Sacciolepis vilvoides (Trin.) Chase in Proc. Biol. Soc. Wash. **21**: 7 (1908).— Wet pine ridge, Ycacos Lagoon, May 15, 1905, *M. E. Peck,* no. 901. This species has been previously reported from southern Mexico and Brazil.

Homolepis aturensis (HBK.) Chase in Proc. Biol. Soc. Wash. **24**: 146 (1911).— Pine ridge near Manatee Lagoon, June 18, 1905, *M. E. Peck,* no. 45.

Oplismenus Burmannii (Retz.) Beauv. Agrost. 54 (1812).— Forest near Manatee Lagoon, February 22, 1906, *M. E. Peck,* no. 349.

Setaria imberbis (Poir.) R. & S. Syst. Veg. **2**: 891 (1817).— Cultivated ground, Toledo, September 9, 1906, *M. E. Peck,* no. 491.

Olyra latifolia L. Amoen. Acad. (Pugill. Jam.) **5**: 408 (1759).— Forest near Manatee Lagoon, August 8, 1905, *M. E. Peck,* no. 109; in the same collection but without more precise data, *M. E. Peck,* no. 609.

Olyra latifolia L. forma.— Thicket, Toledo, September 21, 1906, *M. E. Peck,* no. 533. A narrow-leaved form.

Lithachne pauciflora (Sw.) Beauv. Agrost. 135, 168 (1812) by implication only, no combination made,— cf. Ind. Kew. **2**^{1}: 98 (1895). *Olyra pauciflora* Sw. Prodr. Veg. Ind. Occ. 21 (1788). *L.*

axillaris Beauv. Agrost. 166, Atlas 15, t. 24 f. 2 (1812).— Forest, Toledo, September 12, 1906, *M. E. Peck,* no. 507.

ARISTIDA DIVARICATA Humb. & Bonpl. ex Willd. Enum. Hort. Berol. 99 (1809).— Pine ridge near Manatee Lagoon, July 18, 1905, *M. E. Peck,* no. 72.

Aristida pseudospadicea, sp. nov., perennis, dense caespitosa, circa 80 cm. altitudine, radicibus fibrosis. Culmi erecti, tenues sed rigidi, basin versus aliquid ramosi, supra simplices, albo-virides, ad nodos purpurascentes, omnino glabri. Vaginae e basi imbricata tumidiuscula paullum angustatae, in parte superiori culmos laxissime includdentae vel patentes, quam internodia multo breviores, glabrae, ad folii junctionem angulo recto abruptissime contractae et cum annulo atro-brunneo cinctae. Ligula annularis brevis hispida in auriculas vaginae procurrens, circa. 0.2 mm. longa. Folia baseos superioribus similia, erecta, plana vel conduplicata, longe setaceo-acuminata, 8–30 cm. longa, 1–2 mm. lata; lamina subtus glabra, supra sparse longe tenuiterque pilosa basin versus pilis crebrioribus instructa. Inflorescentia panicula simplex, gracilis; 20–26 cm. longa, 2–4 cm. diametro; radiis in axillis solitariis (per occasionem secundo breviore), adpresso-ascendentibus vel paullo patentibus, inferioribus remotis; axe radiisque glabris. Spiculae glabrae, albo-virides vel paullum albo-violaceae, 9–11 mm. longae, circa 1 mm. diametro, callo obconico in summa parte barbato, circa 1 mm. longo; glumae carinatae, 1-nervatae, gluma prima in nervo scabra, acuminata, quam secunda aliquanto breviori, gluma secunda aristato-acuminata vel paullum bifida cum arista brevi; lemma quam gluma secunda longius, ad apicem aliquid tortum, scabrum, partibus tribus aristae divaricatis, subaequantes vel lateralibus multo brevioribus, parti media ad 35 mm. longa; palea circa 1 mm. longa.— Type (in the Gray Herb.) and only specimen seen, pine ridge near Manatee Lagoon, June 11, 1905, *M. E. Peck,* no. 31. *A. pseudospadicea* is most nearly allied to *A. spadicea* HBK. from which it differs in its more slender habit, absence of flat curled basal leaves, long-pilose upper leaf-surface and smaller spikelets. Similar differences separate it from *A. arizonica* Vasey. The comparatively long callus separates it at once from *A. tincta* Trin. & Rupr. which has a very short one. I wish to thank Mrs. Chase for comparing the material with that at Washington.

SPOROBOLUS CUBENSIS Hitchc. Contr. U. S. Nat. Herb. **12**: 237 (1909).— Pine ridge, Ycacos Lagoon, March 5, 1907, *M. E. Peck,* no. 694. A Cuban and Porto Rican species apparently new for Central America.

SPOROBOLUS VIRGINICUS (L.) Kunth, Rev. Gram. **1**: 67 (1829).— Low ground near Manatee Lagoon August 10, 1905, *M. E. Peck*, no. 120.

SPARTINA GOUINI[2] Fourn. ex Hemsl. Biol. Cent.-Am. Bot. **3**: 509 (1885) nomen; Fourn. Mex. Pl. **2**: 135 (1886).— Swampy shore of Manatee Lagoon, August 14, 1905, *M. E. Peck*, no. 130. Previously collected in Mexico only. The Peck material agrees perfectly with a specimen, in the Gray Herbarium, labelled *Spartina densiflora* Brongn. (*S. Gouini* Fourn.!) [alkaline meadows, Hacienda de Angostura, San Luis Potosi, Mexico, July 10, 1891, *C. G. Pringle*, no. 3760]. Judging from Brongniart's description of *S. densiflora* (a Chilean species) in which he calls attention to its affinity to *S. glabra* Muhl. I doubt very much that the Mexican *S. Gouini* is the same and a Chilean specimen in the Gray Herbarium, labelled *S. densiflora* Brongn. [Valdivia: Ensenada bei Corral, in Gräben 5/1 1905, *Dr. Otto Buchtien*, no. 1285] certainly is not the same as *Pringle*, no. 3760, but shows a strong resemblance to *S. glabra* Muhl.

LEPTOCHLOA FILIFORMIS (Lam.) Beauv. Agrost. 71, 166 (1812). *L. mucronata* (Michx.) Kunth. Rev. Gram. **1**: 91 (1829).— Clearing near Manatee Lagoon, January 21, 1906, *M. E. Peck*, no. 296.

ELEUSINE INDICA (L.) Gaertn. Fruct. **1**: 8 (1788).— Cultivated ground near Manatee Lagoon, June 8, 1905, *M. E. Peck*, no. 43.

ERAGROSTIS ELLIOTTII Wats. in Proc. Am. Acad. **25**: 140 (1890).— Pine ridge near Manatee Lagoon, November 25, 1905, *M. E. Peck*, no. 222a. Not previously reported from Central America, former distribution southeastern United States, Cuba and Porto Rico.

ERAGROSTIS HYPNOIDES (Lam.) BSP. Prelim. Cat. N. Y. Pl. 69 (1888).— Swamp near Sibune R., May 1, 1906, *M. E. Peck*, no. 143.

ERAGROSTIS MEXICANA (Lag.) Link, Hort. Berol. **1**: 190 (1827).— Wet pine ridge near Manatee Lagoon, July 18, 1905, *M. E. Peck*, no. 69b.

ERAGROSTIS sp.— affinis *E. acutiflorae* (HBK.) Nees.— Pine ridge near Manatee Lagoon, January 8, 1906, *M. E. Peck*, no. 281a. The Peck material is too immature to determine more definitely than above. Mrs. Chase has kindly compared this with the material at Washington and has been unable to identify it.

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[2] Since the preparation of this article Prof. A. S. Hitchcock's paper on Mexican Grasses [Contr. U. S. Nat. Herb. **17** (1913)] has appeared. In this,— pages 329, 330,— he refers *Pringle*, no. 3760 to *S. spartinae* (Trin.) Merr. U. S. Bur. Pl. Ind. Bull. **9** : 11 (1902). I have no doubt that he is correct in so doing; consequently the Peck material should be named *S. spartinae* (Trin.) Merr.

Arundinaria? — Lower Moho R., October 16, 1906, *M. E. Peck*, no. 560. Without inflorescence, probably an *Arundinaria*, possibly *A. longifolia* Fourn.

V. DIAGNOSES AND TRANSFERS AMONG THE SPERMATOPHYTES.

By B. L. Robinson.

In the course of routine work at the Gray Herbarium during the last few months the writer has found it necessary to employ a number of new names, resulting either through transfers of the plants in question or rendered needful by the provisions of the International Rules of Botanical Nomenclature. That these names may have proper status they are here given published record, together with a few diagnoses of new species. Two plants characterized at the Gray Herbarium by Mr. Sidney F. Blake and a transfer made by Mr. Sumner C. Brooks are included by request.

Inga (§ **Diadema**) **Peckii**, spec. nov., pedunculis exceptis glabra; ramis teretibus cortice griseo a lenticellis numerosis scabrato tectis plus minusve geniculatis; foliolis 2–3-jugis oblongis petiolulatis subcoriaceis supra subnitidis subtus opacis vel subglaucescentibus apice basique acuminatis 9–17 cm. longis 3.5–7.5 cm. latis penninerviis; petiolulis 4–6 mm. longis; rhachi commune ca. 1.5 dm. longo striato-angulato exalato; pedunculis brevibus gracilibus ex eodem axillo plurimis; receptaculo ovoideo, capitibus globosis saepius 12–18-floris; floribus sessilibus; calyce ca. 2 mm. longo subcylindrato margine brevissime dentato obsolete hispidulo-ciliato aliter glabro; corolla glabra graciliter tubulata sursum leviter gradatimque ampliata, dentibus limbi brevissimis deltoideis; tubo staminum exserto; legumine glabro falcato-oblongo ca. 13 cm. longo ca. 2.2 cm. lato 11-seminato.— British Honduras, *Prof. Morton E. Peck*, no. 673 (type, in Gray Herb.). A species nearly related to *I. jinicuil* Schlecht. but differing in its much shorter peduncles, thicker duller and less reticulated leaflets, shorter corolla-teeth, exserted stamen-tube, etc. From the imperfectly described and wholly obscure *I. coriacea* (Moc. & Sess.) G. Don it differs in having leaflets symmetrical not oblique at the base.

Acacia bucerophora, spec. nov., fruticosa vel arborea; ramis tereti-

bus, cortice griseo pustulis suberosis fulvis maculato; aculeis infrastipularibus geminis rigidis patenti-ascendentibus arcuatis cavis 4–6 cm. longis basi 5 mm. diametro apicem attenuatam versum cum foramine verisimiliter a formicis facto munitis; foliis ca. 2 dm. longis ca. 8 cm. latis; petiolo 1 cm. longo subtereti supra canaliculato; pinnae 14–16-jugis lineari-oblongis contiguis vel etiam imbricatis falcatis basi obliquis apice rotundatis ca. 6 mm. longis 1 mm. latis glabris subtus pallidioribus; inflorescentiis globosis usque ad 15 ex eodem axillo; pedunculis brevibus 1–2.5 cm. solum longis ca. 2–4 mm. supra basin cum involucro annulari squamuliformi saepius 3-partito instructis nec non media in parte cum involucello simili munitis; calyce subcylindrico parum superne ampliato breviter 4-dentato; corolla breviter exserta glabra; legumine sessili falcato-lineari ca. 14 cm. longo 6 mm. solum lato margine valde incrassato faciebus concavis glabris apice basique attenuato.— British Honduras, *Prof. Morton E. Peck*, no. 632 (type, in Gray Herb.). Another species of the highly interesting ant-inhabited Acacias like *A. sphaerocephala* Cham. & Schlecht., *A. spadicigera* Cham. & Schlecht., *A. Hindsii* Benth., etc., but differing from all these markedly in the double involucre of the peduncles, as well as in the long narrow pod, and various other features.

SESBANIA VESICARIA (Jacq.) Ell., var. **atro-rubra** (Nash) S. C. Brooks, comb. nov. *Glottidium floridanum* (Willd.) DC., var. *atro-rubrum* Nash, Bull. Torr. Bot. Club, xxiii. 101 (1896). *G. vesicarium atrorubrum* (Nash) Small, Fl. S. E. U. S. 615 (1903).

Aeschynomene tenerrima, spec. nov., herbacea gracilis palustris ca. 5 dm. alta ubique glaberrima; caule tereti pallide brunneo deorsum leviter spongiose incrassato supra gracillimo paucirameo, ramis ascendentibus subfiliformibus; foliis 2–3.5 cm. longis brevissime petiolatis; rhachi filiformi; foliolis ca. 18-jugis minutis oblongis basi obliquis apice rotundatis 2–2.3 mm. longis 1 mm. latis utrinque pallide viridibus concoloribus; racemis axillaribus 2–3-floris gracillimis; bracteis lanceolatis subscarioso-brunneis vel -purpureis vix 1 mm. longis; floribus anthesi ca. 2.8 mm. longis; calyce herbaceo pallide viridi plus minusve bilabiato, fructifero persistenti brunnescenti; petalis violascentibus ca. 2–2.5 mm. longis; legumine longe (ca. 5 mm.) graciliterque stipitato valde ex calice exserto, articulo unico semiorbiculari 4 mm. longo glabro margine incrassato apice cum styli basi torta vel curvata appendiculato faciebus leviter reticulato-venosis.— Swamp near Ycaco Lagoon, British Honduras, 15 May, 1907, *Prof. Morton E. Peck*, no. 900 (type, in Gray Herb.). A very delicate palustrine species with exceedingly small leaflets and inconspicuous

flowers, not closely related to any other Mexican or Central American species, though clearly of the genus.

WISSADULA SPICATA (HBK.) Presl, Rel. Haenk. ii. 117 (1830). *Abutilon spicatum* HBK. Nov. Gen. et Spec. v. 271 (1821). To the synonymy of this species may be added *Wissadula elongata* Brandegee, Zoe, v. 210 (1905), from Cofradia in the vicinity of Culiacan, Sinaloa.

Linociera oblanceolata, spec. nov., fruticosa vel arborea; ramis gracilibus teretiusculis cortice griseo tectis; ramulis fulvido-puberulis; foliis vix coriaceis oblanceolatis utrinque glabris 11–13 cm. longis 3.5–4.2 cm. latis apice caudato-acuminatis basi sensim angustatis, petiolo proprio teretiusculo puberulo 3–4 mm. solum longo; paniculis laxis 6–8 cm. longis gracillimis ex axillis superioribus foliorum verorum oriuntibus flavido-puberulis; floribus graciliter 3–5 mm. longe pedicellatis; calycis lobis ovatis acuminatis patentibus 1.2 mm. longis flavido-puberulis; petalis margine valde involutis caudiformibus 7–10 mm. longis; antheris 1 mm. longis obtusis, filamentis brevissimis, connectivo apice non productis; fructu crassiuscule clavato 1.9 cm. longo 8 mm. diametro in specimine sicco nigrescente.— Forest, upper Moho River, British Honduras, 16 March, 1907, *Prof. Morton E. Peck,* no. 719 (type, in Gray Herb.). This species differs from *L. caribaea* (Jacq.) Knobl. by its unappendaged anthers. From *L. Bakeri* Urb. of Cuba it may be distinguished by its larger oblanceolate acuminate and even more shortly petioled leaves.

Strychnos (§ **Longiflorae**) **Peckii,** spec. nov., fruticosa cirrhifera; ramis subteretibus a cortice griseo laevi tectis; ramulis gracilibus rectis patentissimis plus minusve sulcatis fulvo-puberulis apicem versus saepissime bifoliatis; cirrhis spiraliter aduncis 2–3 cm. longis usque ad 4 mm. crassis lignescentibus; foliis ovato-ellipticis acute acuminatis 7–11 cm. longis 3.4–5 cm. latis firme membranaceis glabris supra viridibus laevibus subtus distincte pallidioribus obscure puncticulatis 5-nervatis, nerviis saltim basim versus minutissime strigillosis; inflorescentiis axillaribus brevibus multifloris subglobosis glomeruliformibus; bracteis lineari-lanceolatis 3–4 mm. longis, bracteolis minutis obscurisque; pedicellis brevibus fulvo-puberulis; calyce rotato 5-lobo, lobis ovatis acuminatis patentibus ciliolatis; corolla 1 cm. longa, tubo cylindrato 7 mm. longo extus patente fulvo-tomentello, lobis patentibus acutatis supra ut faucibus dense flavo-barbatis; antheris anguste oblongis vix exsertis; stylo nunc modice nunc vix exserto.— Forest, Sittee River, British Honduras, 15 April, 1907, *Prof. Morton E. Peck,* no. 856 (type, in Gray Herb.). This species appears to approach *S. Erichsonii* Rich. Schomb. of British Guiana,

but the latter (known to the writer only from Progel's description in the Flora Brasiliensis, vi. pt. 1, 274) is said to have coriaceous leaves about three times as long as broad, and a corolla which is granulose-pulverulent on the outside. Furthermore, the figure of the corolla (Progel, l. c., t. 82, f. II.) shows the bearding only at the throat, while in the species here characterized it extends well out upon the lobes.

Gymnolomia acuminata Blake, spec. nov., herbacea ramosa strigoso-hispida caule tenui; foliis inferioribus oppositis superioribus alternis lanceolatis longe acuminatis crenulato-serrulatis basi cuneatis utrinque scabro-hispidis infra sparse glandulosis 5–8 cm. longis 1.2–1.8 cm. latis petiolis submarginatis 1 cm. longis dense hispido-strigosis basi ampliatis; ramis floriferis tenuibus ad apicem 3–5-capitulatis cum bracteis paucis linearibus subsessilibus munitis; capitulis longe (2.5–7.5 cm.) pedunculatis subglobosis 8–9 mm. diametro (radiis exclusis); disco convexo; involucri 6–7 mm. alti squamis triseriatis dense strigillosis interioribus ovalibus 3.5 mm. latis exterioribus duplo brevioribus late oblongis 2 mm. latis; radiis plerumque 8 flavis late oblongis subintegris 8 mm. longis; corollis disci flavis 3.5 mm. longis basi non ampliatis 10-nervatis glabris; styli ramis brevibus clavatis apice incrassatis breviappendiculatis; paleis subtruncate tridentatis apice pilosis; acheniis glabris atris oblongis incrassatis epapposis multistriatis 2.1 mm. longis apice obliquis.— Prope Gómez Fárias, Tamaulipas, Mexico, 13–21 April 1907, *Palmer* 582 (spec. typ. in herb. Grayano).—Species ad *G. latibracteatam* Hemsl. arcte affinis, foliis infra hispido-strigosis capitulis minoribus squamis strigosis non minute strigillosis bene distincta.

Flourensia retinophylla Blake, spec. nov. frutex ramosissima cortice brunneo-canescente ramulis junioribus viscido-glutinosis; foliis anguste lanceolatis integerrimis utroque acuminatis mucronulatis glutinosis in petiolum brevisissimum marginatum angustatis 2.5–3.5 cm. longis 4–7.5 mm. latis punctatis venis reticulatis costa valida; capitulis in apicibus ramulorum dense racemosis (3–6) discoideis ca. 12-floris glutinosis turbinatis 10–12 mm. altis; pedunculis brevibus 1–3-bracteolatis; involucri 8 mm. alti squamis laxis 2–3-seriatis oblongo-lanceolatis subobtusis luteo-viridibus; paleis firmis obtusis mucronatis ca. 3-nervatis 8.5–9.8 mm. longis; corollis luteis 5 mm. longis (tubulo 1.1 mm.) dentibus apice resinosis; acheniis cuneatis 6 mm. longis leviter incrassatis dense villosis, aristis 2 serrulatis basi subampliatis 3 mm. longis, squamellis nullis.— Sierra de la Paila, Coahuila, Mexico, Nov. 1910, *Purpus* 4728 (spec. typ. in herb. Grayano conservatum).— Planta ut *F. laurifolia* DC. distributa, a qua

foliis parvis anguste lanceolatis et capitulis minoribus brevipedunculatis et floribus paucioribus satis differt.

JAUMEA TENUIFOLIA (Sch. Bip.) Klatt, Leopoldina, xxiii. 146 (1887), p. 6 of reprint. *Neurolaena tenuifolia* Sch. Bip. acc. to Klatt, l. c. This species, imperfectly characterized by Dr. Klatt, is shown by his fragments of type material to have rested upon a very immature specimen of some plant with pappus of capillary bristles and with the young corollas (still closed) purplish-tomentose. Though from the fragments at hand it is impossible to place the species in any genus, it may be said with perfect definiteness that the plant, with its fine capillary pappus, is not a *Jaumea.* Its habit is rather that of a *Eupatorium* than of a *Neurolaena*, but the florets are too immature to furnish distinctive generic or even tribal characters. The type number (*Liebmann*, no. 202 from Chinantla, Mexico) is doubtless in other herbaria and may well be in some of them represented in sufficiently mature condition to permit more precise identification. Information on this subject would be welcomed.

OXYPAPPUS SCABER Benth. Bot. Sulph. 118, t. 42 (1844). There seems to be no doubt that Dr. Gray was entirely right in reducing to the synonymy of this species *Pentachaeta gracilis* Benth. in Hook. Ic. xii. 1, t. 1101 (1872), though the latter is maintained as a valid species by Hooker f. & Jackson in the Index Kewensis.

Schkuhria schkuhrioides (Link & Otto), comb. nov. *Achyropappus schkuhrioides* Link & Otto, Ic. Pl. Rar. 59, t. 30 (1828). *Schkuhria senecioides* Nees, Del. Sem. Hort. Bonn. 1831; Hemsl. Biol. Cent.-Am. Bot. ii. 212 (1881).

Actinea Palmeri (Gray), comb. nov. *Actinella Palmeri* Gray, Proc. Am. Acad. xix. 31 (1883). *Plateilema Palmeri* (Gray) Cockerell, Bull. Torr. Bot. Club, xxxi. 462 (1904).

ACTINEA SCAPOSA (DC.) Ktze., var. **linearis** (Nutt.), comb. nov. *Actinella scaposa, β linearis* Nutt. Trans. Am. Phil. Soc. vii. 379 (1841). *Tetraneuris linearis* (Nutt.) Greene, Pittonia, iii. 267 (1898).

DYSSODIA Cav. The various efforts which have been made to separate *Hymenatherum* Cass. from *Dyssodia* Cav. have proved so unsatisfactory that it seems best to follow Hoffmann in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 5, 265 (1890), and regard the distinctions as being at best of subgeneric or sectional value. In accordance with this view, however, it is necessary to transfer several species, as follows:

Dyssodia anomala (Canby & Rose), comb. nov. *Hymenatherum anomalum* Canby & Rose, Contrib. U. S. Nat. Herb. i. 105, t. 7 (1891).

D. aurantia (L.), comb. nov. *Aster aurantius* L. Sp. Pl. ii. 877 (1753). *Aster americanus, foliis pinnatis,* etc., Rel. Houst. 7, t. 18 (1781). *Dysodia appendiculata* Lag. Nov. Gen. et Spec. 29 (1816). *Clomenocoma aurantia* (L.) Cass. Dict. Sci. Nat. ix. 416 (1817), lix. 66 (1829). *Clappia aurantiaca* Benth. in Hook. Ic. xii. t. 1104 (1872).

D. aurantiaca (Brandegee), comb. nov. *Hymenatherum aurantiacum* Brandegee, Zoe, v. 258 (1908).

D. pentachaeta (DC.), comb. nov. *Hymenatherum Berlandieri* DC. Prod. v. 642 (1836). *H. pentachaetum* DC. l. c.; Gray, Proc. Am. Acad. xix. 42 (1883), where the two species of DeCandolle are united under the name of the latter from the standpoint of priority of position. *Thymophylla pentachaeta* (DC.) Small, Fl. S. E. U. S. 1295, 1341 (1903). In making the transfer to *Dyssodia* the author would have preferred to reinstate the first name employed by DeCandolle, but this is forbidden by Art. 46 of the International Rules, which reads, "When two or more groups of the same nature are united, the name of the oldest is retained. If the names are of the same date, the author chooses, and his choice cannot be modified by subsequent authors." In the case in hand, the two names *Berlandieri* and *pentachaetum* were of the same date. The species seem first to have been united by Gray, who chose the second of the two names, and this, of course, is decisive according to the article just quoted.

D. concinna (Gray), comb. nov. *Hymenatherum concinnum* Gray, Syn. Fl. i. pt. 2, 446 (1884).

D. diffusa (Gray), comb. nov. *Hymenatherum diffusum* Gray, Pl. Wright. i. 116 (1852).

D. Greggii (Gray), comb. nov. *Thymophylla Greggii* Gray, Pl. Fendl. 92 (1849). *Thymophyllum Greggii* (Gray) Hemsl. Biol. Cent.-Am. Bot. ii. 221 (1881). *Hymenatherum Greggii* Gray, Proc. Am. Acad. xix. 42 (1883).

D. Hartwegi (Gray), comb. nov. *Hymenatherum Berlandieri* Benth. Pl. Hartw. 18 (1839), by error of determination, not DC. *H. Hartwegi* Gray, Pl. Wright. i. 117 (1852).

D. Neaei (DC.), comb. nov. *Hymenatherum* ? *Neaei* DC. Prod. v. 642 (1836). *H. boeberoides* Gray, Pl. Wright. i. 115 (1852). *H. Naei* Hemsl. Biol. Cent.-Am. Bot. ii. 220 (1881); Gray, Proc. Am. Acad. xix. 41 (1883).

D. neo-mexicana (Gray), comb. nov. *Adenophyllum Wrightii* Gray, Pl. Wright. ii. 92 (1853). *Hymenatherum neo-mexicanum* Gray, Proc. Am. Acad. xix. 40 (1883), not *H. Wrightii* Gray, Pl. Fendl. 89 (1849).

D. PAPPOSA (Vent.) Hitchc. Trans. St. Louis Acad. Sci. v. 503 (4 February, 1891). This combination is referred by Durand & Jackson, Ind. Kew. Suppl. 1, p. 147 (1902), to O. Kuntze, Rev. Gen. i. 334 (September, 1891). There can, however, be no question as to the priority of Prof. Hitchcock's publication, which was reviewed in the Bulletin of the Torrey Botanical Club, xviii. 91 (issued 10 March, 1891).

D. pinnata (Cav.), comb. nov. *Aster pinnatus* Cav. Ic. iii. 6, t. 212 (1795). *Dysodia pubescens* Lag. Nov. Gen. et Spec. 29 (1816). *D. subintegerrima* Lag. l. c. *Boebera incana* Lindl. Bot. Reg. t. 1602 (1833). *Dysodia incana* (Lindl.) DC. Prod. v. 640 (1836). *Boebera subintegerrima* (Lag.) Spreng. Syst. iii. 545 (1826). *Clomenocoma ? pinnata* DC. Prod. v. 641 (1836). *Dysodia integerrima* Hemsl. Biol. Cent.-Am. Bot. ii. 219 (1881), by error for *D. subintegerrima.*

D. polychaeta (Gray), comb. nov. *Hymenatherum polychaetum* Gray, Pl. Wright. i. 116 (1852). *Thymophylla polychaeta* (Gray) Small, Fl. S. E. U. S. 1295, 1341 (1903).

D. setifolia (Lag.), comb. nov. *Thymophylla setifolia* Lag. Nov. Gen. et Spec. 25 (1816). *Thymophyllum setifolium* Hemsl. Biol. Cent.-Am. Bot. ii. 221 (1881). *Hymenatherum setifolium* (Lag.) Gray, Proc. Am. Acad. xix. 42 (1883).

D. Thurberi (Gray), comb. nov. *Hymenatherum tenuifolium* var.? Gray, Pl. Wright. ii. 93 (1853). *H. Thurberi* Gray, Proc. Am. Acad. xix. 41 (1883).

D. tenuiloba (DC.), comb. nov. *Hymenatherum tenuilobum* DC. Prod. v. 642 (1836). *H. tenuifolium* Gray, Pl. Wright. i. 118 (1852), by error of determination, not Cass. *Thymophylla tenuiloba* (DC.) Small, Fl. S. E. U. S. 1295, 1341 (1903).

D. Treculii (Gray), comb. nov. *Hymenatherum* n. sp.? no. 13 Gray, Pl. Wright. i. 116 (1852). *Hymenatherum Treculii* Gray, Proc. Am. Acad. xix. 42 (1883). *Thymophylla Treculii* (Gray) Small, Fl. S. E. U. S. 1295, 1341 (1903).

D. Wrightii (Gray), comb. nov. *Hymenatherum Wrightii* Gray, Pl. Fendl. 89 (1849). *Thymophylla Wrightii* (Gray) Small, Fl. S. E. U. S. 1295, 1341 (1903).

POROPHYLLUM DECUMBENS DC. Prod. v. 650 (1836). *Kleinia suffruticosa* Lodd. Bot. Cab. xvi. t. 1561 (1829), not Willd. This is one of many plants, which have been described solely from cultivated specimens. The stock was thought to have originally come from Mexico, but though first brought into cultivation as early as the twenties, it has never been rediscovered in Mexico or Central America.

Furthermore the species seems to have passed out of cultivation long ago and is now obscure. The object of the present note is merely to call attention to the fact that, judged from character and Loddiges' colored plate, the species would seem to be exceedingly close to if not entirely identical with *P. linifolium* (L.) DC., var. *brevifolium* (Hook. & Arn.) Bak. Fl. Bras. vi. pt. 3, 283, t. 80, f. 2 (1884), a maritime plant, native of Uruguay.

POROPHYLLUM RUDERALE and P. ELLIPTICUM. Urban, Symb. Ant. i. 467 (1900), unites these species under the name *P. ellipticum* Cass. Dict. Sci. Nat. xliii. 56 (1826). This course may have been suggested by the fact that *P. ellipticum* Cass., a mere renaming of the original *Cacalia Porophyllum* L. Sp. Pl. ii. 834 (1753), in a certain sense perpetuates the earlier element in the combined species. However, the course is contrary to the International Rules of Botanical Nomenclature, which in Art. 46 are as follows: " Dans le cas de réunion de deux ou plusieurs groupes de même nature, le nom le plus ancien subsiste."[3] In this case the oldest available name (the Linnaean specific name being rejected on account of its identity with the generic name) is *ruderale*, which going back to *Kleinia ruderalis* Jacq. Enum. 28 (1760), much antedates *ellipticum* of Cassini.

The writer fully concurs with Prof. Urban that the two plants should be treated as mere varieties of the same species. The following naming will correspond to the requirements of the International Rules.

P. RUDERALE (Jacq.) Cass. Dict. Sci. Nat. xliii. 56 (1826), at least as to name-bringing synonym. *Kleinia ruderalis* Jacq. Enum. 28 (1760). *Cacalia ruderalis* (Jacq.) Sw. Prod. 110 (1788). *Porophyllum ellipticum*, var. *β ruderale* (Cass.) Urb. Symb. Ant. i. 468 (1900).

P. ruderale, var. **ellipticum** (Cass.) Gray in herb. *P. ellipticum* Cass. Dict. Sci. Nat. xliii. 56 (1826).

FAUJASIA FLEXUOSA (Lam.) Benth. & Hook. f. Gen. Pl. ii. 443 (1873) acc. to Hook. f. & Jacks. Ind. Kew. i. 948 (1893). Identical with this species appears to be *Cacalia cuspidata* Klatt, Ann. Sci. Nat.

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[3] It may be noted here that the English translation is inaccurate and misleading. It runs "When two or more groups of the same nature are united, the name of the oldest is maintained." In this very case it may be seen that the name of the older species is *ellipticum* for that species (under another designation, it is true) dates back to 1753, indeed even into pre-Linnaean times, while on the other hand the oldest (available) specific name is *ruderale*, dating from 1760. Thus it is clear that the English translation by the use of the expression "the name of the oldest" instead of "the oldest name" is capable of quite another interpretation from the official French version, "le nom le plus ancien," which happily, as well as the German "der älteste Name," is unequivocal.

ser. 5, xviii. 374 (1873), the type of which, collected on the Isle of Bourbon by Richard, is now in the Gray Herbarium. Other specimens of *F. flexuosa* at hand show the species to have a fairly wide range of variation as to leaf-contour, in some cases reaching the short, subcordate, conspicuously cuspidate-caudate form of Klatt's type, in others passing to elongate lance-oblong shapes more or less cuneate at the base.

CELMISIA Cass. Dict. Sci. Nat. vii. 356 (1817). This genus of the *Senecioneae* was founded by Cassini upon a single South African species, which he called *C. rotundifolia* and which later proved to have been the plant previously named *Arnica tabularis* by Thunberg, Prod. Fl. Cap. 154 (1800). The genus *Celmisia,* thus having been originally based upon a single species, there can be no question as to its type. Furthermore, the name has not been mentioned in the lists of nomina conservanda or nomina rejicienda, so it must take its course under the rules of priority. Cassini subsequently, Dict. Sci. Nat. xxxvii. 259 (1825), included in his genus a plant collected by Gaudichaud in Australia, namely *C. longifolia* Cass., a plant belonging to the *Astereae.* When the *Compositae* were treated by DeCandolle he unfortunately took this latter, Australian plant as the type of the genus, and referred the original South African species to a newly named genus, *Alciope* DC. Prod. v. 210 (1836). This was, of course, contrary to the clause of Article 45 of the International Rules, which reads: "If the genus contains a section or some other division which, judging by its name or its species, is the type or the origin of the group, the name is reserved for that part of it." DeCandolle's treatment has been generally accepted and perpetuated until the present day. It is true, Dr. Otto Kuntze noted the inconsistency and restored *Celmisia* to its original application, but he referred the Australian and New Zealand element of the complex to the genus *Aster,* a reduction not likely to be generally followed upon taxonomic grounds. The current International Rules appear to call for the following revision of the nomenclature in the genera concerned.

Celmisia tabularis (Thunb.), comb. nov. *Arnica tabularis* Thunb. Prod. Fl. Cap. 154 (1800). *Celmisia rotundifolia* Cass. Dict. Sci. Nat. vii. 357 (1817). *Ligularia tabularis* (Thunb.) Less. Syn. Comp. 390 (1832) by implication. *Alciope Tabularis* (Thunb.) DC. Prod. v. 210 (1836).

C. tomentosa (Burm. f.), comb. nov. *Conyza tomentosa* Burm. f. Prod. 26 (1768). *Arnica lanata* Thunb. Prod. Fl. Cap. 154 (1800). *Ligularia lanata* (Thunb.) Less. Syn. Comp. 390 (1832) by implication. *Alciope lanata* (Thunb.) DC. Prod. v. 210 (1836).

C. tomentosa, var. **grandis** (Thunb.), comb. nov. *Arnica grandis* Thunb. Prod. Fl. Cap. 154 (1800). *Alciope lanata β grandis* (Thunb.) DC. Prod. v. 210 (1836).

Elcismia, nom. nov. (Anogram.) *Celmisia* Cass. Dict. Sci. Nat. xxxvii. 259 (1825), and of most subsequent authors, not Cass. Dict. Sci. Nat. vii. 356 (1817). *Aster* § *Celmisiana* Ktze. in Post & Ktze. Lex. Gen. Phan. 49 (1904) — a name by its adjectival form inappropriate for use in generic rank.

E. Adamsii (Kirk), comb. nov. *Celmisia Adamsii* Kirk, Trans. N. Z. Inst. xxvii. 329 (1894).

E. Adamsii, var. **rugulosa** (Cheesem.), comb. nov. *Celmisia Adamsii,* var. *rugulosa* Cheesem. Man. N. Z. Fl. 313 (1906).

E. argentea (Kirk), comb. nov. *Celmisia sessiliflora,* var. *minor* Petrie, Trans. N. Z. Inst. xv. 359 (1882). *C. argentea* Kirk, Stud. Fl. N. Z. 292 (1899).

E. Armstrongii (Petrie), comb. nov. *Celmisia Armstrongii* Petrie, Trans. N. Z. Inst. xxvi. 269 (1894).

E. bellidioides (Hook. f.), comb. nov. *Celmisia bellidioides* Hook. f. Handb. N. Z. Fl. 135 (1864).

E. Brownii (F. R. Chapm.), comb. nov. *Celmisia Brownii* F. R. Chapm. Trans. N. Z. Inst. xxii. 444 (1890).

E. Campbellensis (F. R. Chapm.), comb. nov. *Celmisia Campbellensis* F. R. Chapm. Trans. N. Z. Inst. xxiii. 407 (1891). *C. Chapmani* Kirk, Gard. Chron. ix. 731, f. 146 (1891).

E. cordatifolia (J. Buchanan), comb. nov. *Celmisia cordatifolia* J. Buchanan, Trans. N. Z. Inst. xi. 427 (1879). *C. petiolata,* var. *cordatifolia* (J. Buchanan) Kirk, Stud. Fl. N. Z. 286 (1899).

E. coriacea (Forst. f.), comb. nov. *Aster coriaceus* Forst. f. Prod. 56 (1786). *Celmisia coriacea* (Forst. f.) Hook. f. Fl. Antarc. i. 36 (4 July, 1844), & Fl. N. Z. i. 121, t. 32 (1853); Raoul, Ann. Sci. Nat. ser. 3, ii. 119 (August, 1844).

E. Dallii (J. Buchanan), comb. nov. *Celmisia Dallii* J. Buchanan, Trans. N. Z. Inst. xiv. 355, t. 35 (1882).

E. densiflora (Hook. f.), comb. nov. *Celmisia densiflora* Hook. f. Handb. N. Z. Fl. i. 130 (1864).

E. discolor (Hook. f.), comb. nov. *Celmisia discolor* Hook. f. Fl. N. Z. i. 123 (1853).

E. dubia (Cheesem.), comb. nov. *Celmisia dubia* Cheesem. Man. N. Z. Fl. 308 (1906).

E. Gibbsii (Cheesem.), comb. nov. *Celmisia Gibbsii* Cheesem. Man. N. Z. Fl. 300 (1906).

E. glandulosa (Hook. f.) comb. nov. *Celmisia glandulosa* Hook. f. Fl. N. Z. i. 124 (1853).

E. Haastii (Hook. f.), comb. nov. *Celmisia Haastii* Hook. f. Handb. N. Z. Fl. 131 (1864).

E. Hectori (Hook. f.), comb. nov. *Celmisia Hectori* Hook. f. Handb. N. Z. Fl. 135 (1864).

E. hieraciifolia (Hook. f.), comb. nov. *Celmisia hieraciifolia* Hook. f. Fl. N. Z. i. 124, t. 34B (1853).

E. hieraciifolia, var. **oblonga** (Kirk), comb. nov. *Celmisia hieracifolia*, var. *oblonga* Kirk, Trans. N. Z. Inst. xxvii. 328 (1894).

E. holosericea (Forst. f.), comb. nov. *Aster holosericeus* Forst. f. Prod. 56 (1786). *Celmisia holosericea* (Forst. f.) Hook. f. Fl. Antarc. i. 36, (4 July, 1844); Raoul, Ann. Sci. Nat. ser. 3, ii. 119 (August, 1844).

E. incana (Hook. f.), comb. nov. *Celmisia incana* Hook. f. Fl. N. Z. i. 123 t. 34A (1853).

E. incana, var. **petiolata** (Kirk), comb. nov. *Celmisia incana*, var. *petiolata* Kirk, Stud. Fl. N. Z. 284 (1899).

E. laricifolia (Hook. f.), comb. nov. *Celmisia laricifolia* Hook. f. Fl. N. Z. ii. 331 (1855).

E. lateralis (J. Buchanan), comb. nov. *Celmisia lateralis* J. Buchanan, Trans. N. Z. Inst. iv. 226, t. 15 (1872).

E. lateralis, var. **villosa** (Cheesem.), comb. nov. *Celmisia lateralis*, var. *villosa* Cheesem. Man. N. Z. Fl. 302 (1906).

E. Lindsayi (Hook. f.), comb. nov. *Celmisia Lindsayi* Hook. f. Handb. N. Z. Fl. 132 (1864).

E. linearis (Armstr.), comb. nov. *Celmisia linearis* Armstr. Trans. N. Z. Inst. xiii. 337 (1881).

E. longifolia (Cass.), comb. nov. *Celmisia longifolia* Cass. Dict. Sci. Nat. xxxvii. 259 (1825).

E. longifolia, var. **alpina** (Kirk), comb. nov. *Celmisia longifolia*, var. *alpina* Kirk, Stud. Fl. N. Z. 289 (1899).

E. longifolia, var. **gracilenta** (Hook. f.), comb. nov. *Celmisia gracilenta* Hook. f. Fl. Antarc. i. 35 (1844). *C. longifolia*, f. *gracilenta* (Hook. f.) Kirk, Stud. Fl. N. Z. 289 (1899). *C. longifolia*, var. *gracilenta* (Hook. f.) Cheesem. Man. N. Z. Fl. 314 (1906).

E. longifolia, var. **major** (Kirk), comb. nov. *Celmisia gracilenta*, var. *β*. Hook. f. Fl. N. Z. i. 123 (1853). *C. longifolia*, ff. *major* & *asteliaefolia* Kirk, Stud. Fl. N. Z. 289 (1899).

E. longifolia, var. **graminifolia** (Hook. f.), comb. nov. *Celmisia graminifolia* Hook. f. Fl. Antarc. i. 35 (1844). *C. longifolia*,

f. *graminifolia* (Hook. f.) Kirk, Stud. Fl. N. Z. 289 (1899). *C. longifolia,* var. *graminifolia* (Hook. f.) Cheesem. Man. Fl. N. Z. 314 (1906).

E. Lyallii (Hook. f.), comb. nov. *Celmisia Lyallii* Hook. f. Handb. N. Z. Fl. 133 (1864).

E. Lyallii, var. **pseudo-Lyallii** (Cheesem.), comb. nov. *Celmisia Lyallii,* var. *pseudo-Lyallii* Cheesem. Man. N. Z. Fl. 312 (1906).

E. Mackaui (Raoul), comb. nov. *Celmisia Mackaui* Raoul, Choix Pl. Nouv. Zel. 19, t. 14 (1846).

E. Macmahoni (Kirk), comb. nov. *Celmisia Macmahoni* Kirk, Trans. N. Z. Inst. xxvii. 327 (1894).

E. Monroi (Hook. f.), comb. nov. *Celmisia Monroi* Hook. f. Handb. N. Z. Fl. 133 (1864).

E. parva (Kirk), comb. nov. *Celmisia parva* Kirk, Trans. N. Z. Inst. xxvii. 328 (1894).

E. petiolata (Hook. f.), comb. nov. *Celmisia petiolata* Hook. f. Handb. N. Z. Fl. 134 (1864).

E. petiolata, var. **membranacea** (Kirk), comb. nov. *Celmisia petiolata,* var. *membranacea* Kirk, Stud. Fl. N. Z. 286 (1899).

E. petiolata, var. **rigida** (Kirk), comb. nov. *Celmisia petiolata,* var. *rigida* Kirk, Stud. Fl. N. Z. 286 (1899).

E. Petriei (Cheesem.), comb. nov. *Celmisia Petriei* Cheesem. Man. N. Z. Fl. 311 (1906).

E. prorepens (Petrie), comb. nov. *Celmisia prorepens* Petrie, Trans. N. Z. Inst. xix. 326 (1887).

E. ramulosa (Hook. f.), comb. nov. *Celmisia ramulosa* Hook. f. Handb. N. Z. Fl. 733 (1864).

E. rupestris (Cheesem.), comb. nov. *Celmisia rupestris* Cheesem. Trans. N. Z. Inst. xvi. 409 (1884).

E. Rutlandii (Kirk), comb. nov. *Celmisia Rutlandii* Kirk, Trans. N. Z. Inst. xxvii. 329 (1894).

E. sessiliflora (Hook. f.), comb. nov. *Celmisia sessiliflora* Hook. f. Handb. N. Z. Fl. 135 (1864).

E. Sinclairii (Hook. f.), comb. nov. *Celmisia Sinclairii* Hook. f. Handb. N. Z. Fl. 132 (1864).

E. spectabilis (Hook. f.), comb. nov. *Celmisia spectabilis* Hook. f. Fl. N. Z. i. 122, t. 33 (1853).

E. Traversii (Hook. f.), comb. nov. *Celmisia Traversii* Hook. f. Handb. N. Z. Fl. 134 (1864).

E. verbascifolia (Hook. f.), comb. nov. *Celmisia verbascifolia* Hook. f. Fl. N. Z. i. 121 (1853).

E. vernicosa (Hook. f.), comb. nov. *Celmisia vernicosa* Hook. f. Fl. Antarc. i. 34, tt. 26, 27 (1844).

E. viscosa (Hook. f.), comb. nov. *Celmisia viscosa* Hook. f. Handb. N. Z. Fl. 133 (1864).

E. Walkeri (Kirk), comb. nov. *Celmisia Walkeri* Kirk, Trans. N. Z. Inst. ix. 549 (1877).

Luina stricta (Greene), comb. nov. *Prenanthes stricta* Greene, Pittonia, ii. 21 (1889). *Luina Piperi* Robinson, Bot. Gaz. xvi. 43, t. 6 (1891). *Psacalium strictum* Greene, Pittonia, ii. 228 (1892). *Rainiera stricta* Greene, Pittonia, iii. 291 (1898). Some years ago the writer submitted material of this species to Dr. O. Hoffmann, author of the revision of the *Compositae* in Engler & Prantl's Natürliche Pflanzenfamilien. Replying to a request for his opinion as to the generic affinities of the plant, he wrote, under date of 20 September, 1897, "Nach Untersuchung des mir übersandten Materials halte ich Ihre Ansicht, dass die fragliche Pflanze in die Gattung Luina gehört für richtig, vorausgesetzt dass man *Luina* von *Cacalia* trennt. * * * * Doch würde ich für eine Vereinigung von *Luina* und *Cacalia* nicht stimmen." Having from my first investigation of the plant in question felt that it was a *Luina,* and having been confirmed in this view by the high authority of Dr. Hoffmann, I here associate with the generic name the earlier specific designation, according to the provisions of the International Rules.

Serratula deltoides (Ait.), comb. nov. *Onopordon deltoides* Ait. Hort. Kew. iii. 146 (1789). *Carduus atriplicifolius* Trev. Hort. Vratisl. 1820. *Silybum atriplicifolium* (Trev.) Fisch. Ind. Sem. Hort. Petrop. 1824. *Rhaponticum atriplicifolium* (Trev.) DC. Prod. vi. 663 (1837).

Onoseris onoseroides (HBK.), comb. nov. *Isotypus onoseroides* HBK. Nov. Gen. et Spec. iv. 12, t. 307 (1820). *Onoseris Isotypus* Benth. & Hook. f. Gen. Pl. ii. 487 (1873).

Chaetanthera cochlearifolia (Gray), comb. nov. *Oriastrum cochlearifolium* Gray, Proc. Am. Acad. v. 144 (1861).

Chaetanthera dioica (Remy), comb. nov. *Egania dioica* Remy in Gay, Fl. Chil. iii. 327, t. 36, f. 1 (1847); Wedd. Chlor. And. i. 32 (1855). *Oriastrum dioicum* (Remy) Reiche, Fl. Chil. iv. 357 (1904).

Chaetanthera Philippii, nom. nov. *Chondrochilus involucratus* Phil. Fl. Atac. 27, t. 3B (1860), not *Chaetanthera involucrata* Phil. Anal. Univ. Chil. xlvii. 6 (1894).

Chaetanthera splendens (Remy), comb. nov. *Elachia splendens* Remy in Gay, Fl. Chil. iii. 315 (1847). *Tylloma splendens* (Remy), Wedd. Chlor. And. i. 27, t. 8A (1855).

Trichocline reptans (Wedd.), comb. nov. *Bichenia reptans* Wedd. Chlor. And. 25, t. 8B (1855).

Gerbera gossypina (Royle), comb. nov. *Chaptalia gossypina* Royle, Ill. 18, 247, 251 (mere mentions, without characterization) & t. 57, f. 2, with floral details ("analyses") rendering the plate valid publication according to Article 37 of the International Rules. *Onoseris lanuginosa* Wall. Cat. no. 2929 (1828), nomen nudum. *Oreoseris lanuginosa* (Wall.) DC. Prod. vii. 17 ("1838"), citing Royle both by page and plate number; Deless. Ic. iv. 34 (citing both Royle and DC.), t. 76 ("1839"). It is impossible to see how the name *lanuginosa* can stand under the International Rules. Wallich's original publication of the name in 1828 is accompanied by no description. The publications of DeCandolle and Delessert must have been prepared at the same time and in collaboration, since they each cite the other by reference to page or plate, but it is significant that they both cite Royle's publication, a seemingly conclusive evidence that it must have been already in print. A manuscript note in the copy of the fourth volume of Delessert's Icones in the library of the Gray Herbarium states that it was received in December, 1840.

Gerbera maxima (D. Don), comb. nov. *Chaptalia maxima* D. Don, Prod. Fl. Nepal. 166 (1825). *Perdicium semiflosculare?* Ham. ex D. Don, l. c., not L. *Tussilago macrophylla* Wall. Cat. no. 2989 (1828). *Berniera nepalensis* DC. Prod. vii. 18 ("1838"). *Gerbera macrophylla* (Wall.) Benth. in Benth. & Hook. f. Gen. Pl. ii. 497 (1873), according to Hook. f. Fl. Brit. Ind. iii. 391 (1882). *Gerbera nepalensis* (DC.) Sch. Bip. Flora, xxvii. 780 (1844).

LEUCHERIA INTEGRIFOLIA (Phil.) Reiche, Fl. Chil. iv. 420 (1905), as *Leuceria.* This binomial was published by its author in a foot-note with the statement that he did not regard it as valid. Why authors should wish to publish names which they do not believe to be valid is a psychological mystery, which need not be discussed here. The point of interest in the present case lies in the fact that the discredited binomial would appear after all to be the legitimate designation of the plant in question. Reiche founded his new combination upon *Chabraea integrifolia* Phil. Anal. Univ. Santiago, xli. 744 (1872), but he states that *Leuceria integrifolia* (Phil.) [Reiche] cannot be accepted because of the existence of an earlier *Leuceria integrifolia* Phil. However, this earlier homonym does not appear to have been published and it seems probable that Reiche had in mind *Chabraea integrifolia* Phil. Linnaea, xxviii. 716 (1856), which on a later page of his work (430) Reiche includes in the synonymy of *Leuceria lithospermifolia*

(DC.) Reiche. It is obvious that the existence of the binomial *Chabraea integrifolia* Phil. (1856), especially if invalid, does not prevent the validity of a *Leucheria integrifolia* founded upon Philippi's quite different *Chabraea integrifolia* of 1872. In the synonymy of this species may be placed *Leuceria Hahnii* Franch. Miss. Sci. Cap. Horn, v. 349, t. 3 (1889). *L. fuegina* Phil. Anal. Univ. Santiago, lxxxvii. 98 (1894). It is not improbable that this may also have been the *Chabraea suaveolens, β integrifolia* Sch. Bip. Flora, xxxviii. 121 (1855), nomen nudum.

Leucheria suaveolens (Urv.), comb. nov. *Perdicium suaveolens* Urv. Fl. des Iles Malouines, 43 (1825); Mém. Soc. Linn. Paris, iv. 611 (1826). *Lasiorhiza ceterachifolia* & *L. viscosa* Cass. Dict. Sci. Nat. xliii. 80, 81 (1826). *Leuchaeria gossypina* Hook. & Arn. Comp. Bot. Mag. ii. 43 (1836). *Chabraea suaveolens* (Urv.) DC. Prod. vii. 59 (1838).

PEREZIA VIRENS (D. Don) Hook. & Arn. Comp. Bot. Mag. i. 34 (1835). This species is referred by Reiche, Fl. Chil. iv. 445 (1905) to *P. Poeppigii* Less. Syn. Comp. 411 (1832). It is clear that the validity of *P. virens* must depend upon the date of its name-bringing synonym *Clarionea virens* D. Don, Trans. Linn. Soc. xvi. 208. It is true that the volume in which it was published bears the date 1833, but it is stated that Don's paper was read 1829 and there is every reason to suppose that the different papers of which the volume is composed were issued as printed and at quite different dates. Fortunately there is conclusive proof that Don's paper had reached print and issue before Lessing's publication, for Lessing cites Don's paper on pages 407, 408, 412, and elsewhere in his Synopsis, which should settle the matter of priority and lead to the re-instating of *P. virens* (D. Don) Hook. & Arn.

Trixis calcicola, spec. nov., fruticosa; ramis lignescentibus medullosis a cortice flavido-brunneo tectis foliosis costato-angulatis juventate latiuscule alatis; foliis alternis oblongis attenuato-acuminatis mucronato-denticulatis 1–1.2 dm. longis 2.3–3 cm. latis utrinque viridibus supra rugulosis puberulis subtus venosissimis tenuiter pubescentibus dense glanduloso-atomiferis basi angustatis et in alas latiusculas denticulatas decurrentibus; alis internodia aequantibus vel superantibus usque ad 5 mm. latis deorsum cuneatim angustatis; corymbis ovoideis subthyrsiformibus densis ca. 1 dm. diametro et minus altis; bracteolis involucri exterioris 5 elliptico-oblanceolatis 1.7 cm. longis 5 mm. latis tenuibus vix acutis basi angustatis extus glanduloso-puberulis; squamis involucri interioris 8 oblongo-linearibus

1.2 cm. longis ca. 2 mm. latis obtusiusculis; achaeniis 7.3 mm. longis subfusiformi-columnaribus griseis glanduloso-hispidulis; pappi setis laete albis 1.2 cm. longis tenuissimis levissime scabratis numerosis.— Limestone ledges, Iguala Cañon, near Iguala, Guerrero, Mexico, altitude 760 m., 28 December, 1906, *C. G. Pringle*, no. 13, 921 (type, in Gray Herb.). The broadly winged stems would seem to place this species near *T. pterocaulis* Robinson & Greenman and *T. alata* Don. The former, however, has the bractlets of the outer involucre shorter than the scales of the inner involucre, leaves subentire, etc., while in *T. alata* Don the bractlets of the outer involucre are ovate-lanceolate and acuminate. The long obscure *T. michuacana* LaLav. & Lex., which from the original description would appear to have some points in common and might well be found in the same region, is said to have a simple herbaceous stem and a thyrsiform inflorescence nearly a foot long.

Launaea picridioides (Webb.), l. c., comb. nov. *Rhabdotheca picridioides* Webb. in Hook. Nig. Fl. 146 (1849).

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THE GENERALIZED RIEMANN PROBLEM FOR LINEAR DIFFERENTIAL EQUATIONS AND THE ALLIED PROBLEMS FOR LINEAR DIFFERENCE AND q-DIFFERENCE EQUATIONS.

By George D. Birkhoff.

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The program of obtaining a characterization of a function in simple descriptive terms which are independent of the equations of definition of the function is a familiar one. To Riemann is due the formulation of this characterization for the algebraic functions and for the functions defined by ordinary linear differential equations without irregular singular points. In both of these instances the characterization involves a certain number of characteristic constants — the monodromic group constants in the last mentioned instance. Riemann also proposed the associated problem of assigning these constants at pleasure.[1]

During the last few years I have discovered that the program admits of extension in a number of directions. The aim of the present paper is to solve the generalized problem of Riemann for ordinary linear differential equations with irregular singular points, and the analogous problem for linear difference equations and for linear q-difference equations. The formulation of the first and second of these problems has been given by me earlier.[2] At about the same time as myself, Nörlund, in his fundamental work on linear difference equations, was led to formulate essentially the second problem.[3] The third is stated in the present paper.

The problem of Riemann for linear differential equations in its classic form was first solved by Hilbert.[4] His treatment and Plemelj's elegant completion thereof[5] reposed alike upon a certain theorem whose proof was made by means of the Fredholm theory. Owing to the deep-seated analogy between linear differential and difference and q-difference equations, I have been able to apply a convenient extension of the same theorem in all cases; my proof is based on a method of successive approximations.

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[1] Werke, (zweite Auflage) pp. 37–39, 67–69.

[2] Trans. Am. Math. Soc., **10**, 436–470 (1909), and **12**, 243–284 (1911). These two papers will be referred to as I and II respectively.

[3] Mémoires de l'Académie Royale des Sciences et des Lettres de Danemark, series 7, **6**, 309–326 (1911); C. R. vol. 156, pp. 200–202 (1913).

In the second of these papers, Nörlund gives a formulation and explicit solution of the hypergeometric difference equation problem.

Inasmuch as I have been able to simplify Hilbert's and Plemelj's treatment of the classic Riemann problem, I have ventured to include my treatment of it also.

PART I. THE PRELIMINARY THEOREM.

§ 1. *Some Definitions.*

Let C be a simply closed analytic curve in the complex x-plane. If the arc length along this curve from a fixed to a variable point is measured by s, and if l be the length of C, it is clear that x is a single-valued analytic function of s with period l for s real, and that dx/ds is not zero. Consequently if we introduce a new variable τ defined by

$$\tau = e^{\frac{2\pi\sqrt{-1}s}{l}},$$

a one-to-one analytic correspondence is set up between the points of the unit circle $|\tau| = 1$ in the τ-plane and the points of C. It will therefore be possible to choose $\rho > 1$ so that the circular ring in the τ-plane,

$$\frac{1}{\rho} \leqq |\tau| \leqq \rho,$$

is transformed in a one-to-one and conformal manner into a ring in the x-plane bounded by simply closed analytic curves C_1 and C_2, within and without C respectively, while at the same time the circle $|\tau| = 1$ is transformed into C. Let $\tau = \tau(x)$ be the function which effects this transformation.

Also let $a(x)$ be any function continuous together with its derivatives of all orders along C,[6] and analytic save at a finite number of

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[4] Gött. Nachr. (1905), pp. 307–338.

[5] Monatsh. f. Math. u. Phys., **19**, 205–246 (1908).

[6] By definition we take $df(x)/dx$ along a curve L as follows:

$$\frac{df(x)}{dx} = \lim_{x' \doteq x} \frac{f(x') - f(x)}{x' - x} \qquad (x', x \text{ on } L).$$

points of C. These restrictions on $a(x)$ ensure that we can choose regular curves D_1 and D_2, within and without C respectively, and osculating C at the points where $a(x)$ is not analytic (Fig. 1) in such a way that on the continua limited by D_1, D_2, we have

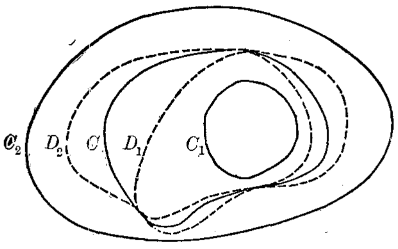

FIG. 1.

$$(1) \qquad |a(x)| \leqq K, \qquad \left|\frac{a(x)-a(x')}{x-x'}\right| \leqq K;$$

in these continua $a(x)$ is defined as the analytic extension of $a(x)$ on C. It is possible to extend further the definition of $a(x)$ throughout the ring formed by C_1, C_2 in such wise that inequalities of the type (1) hold; for this purpose it is clearly sufficient to choose real and imaginary components that join on continuously to the like components of $a(x)$ along D_1 and D_2, and to make each component satisfy inequalities of the same nature as (1). Such a choice can always be made.

§ 2. *On a First Type of Integral.*

Let us turn now to consider the integral

$$(2) \qquad \frac{1}{2\pi\sqrt{-1}}\int_C \frac{\tau^p(t)\,g^+(t)}{t-x}\,a(t)\,dt,$$

where $g^+(x)$ is a function analytic within C and continuous along C, and p is zero or a positive integer. Following Plemelj (loc. cit.) we shall term a function $g^+(x)$ of this description a *regular inner function* and affix to it a superscript $+$; likewise a superscript $-$ will indicate that a function is a *regular outer function,* i. e. is analytic in the extended plane without C, and continuous along C.

We can demonstrate at once that the integral (2) represents a regular inner function $f^+(x)$, or a regular outer function $f^-(x)$, according as x is within or without C. In the first place these functions are analytic within and without C respectively, as appears from (2).

In the second place, by Cauchy's integral theorem we have

$$(3) \qquad 0 = \frac{1}{2\pi\sqrt{-1}}\int_C \frac{\tau^p(t)\,g^+(t)}{t-x}\,dt - \frac{1}{2\pi\sqrt{-1}}\int_{C_1} \frac{\tau^p(t)\,g^+(t)}{t-x}\,dt,$$

provided that x lies between C and C_2, since the function $\tau^p(x)\ g^+(x)$ is analytic in the ring C, C_1 and continuous along its boundary. Thus we may write

$$(4)\quad f^-(x) = \frac{1}{2\pi\sqrt{-1}}\int_C \tau^p(t)\, g^+(t)\,\frac{a(t)-a(x)}{t-x}\,dt + \frac{a(x)}{2\pi\sqrt{-1}}\int_{C_1}\frac{\tau^p(t)\, g^+(t)}{t-x}\,dt.$$

The first integrand on the right-hand side is continuous in x and t, for t on C and x in the ring C, C_2 unless $x = t$, when the integrand is not defined; in the neighborhood of a pair of values $x = t$, the integrand remains finite by (1). Hence the first integral approaches a continuous limit as x approaches the boundary. Inasmuch as t is restricted to lie on C_1 in the second integrand, the same statement is certainly true of the second integral. Hence $f^-(x)$ may be so defined as to be continuous along C.

Likewise by means of the relation

$$(5)\quad \tau^p(x)\, g^+(x) = \frac{1}{2\pi\sqrt{-1}}\int_C \frac{\tau^p(t)\, g^+(t)}{t-x}\,dt - \frac{1}{2\pi\sqrt{-1}}\int_{C_1}\frac{\tau^p(t)\, g^+(t)}{t-x}\,dt,$$

valid for x between C and C_1 by Cauchy's integral formula, we obtain

$$(6)\quad f^+(x) - \tau^p(x) g^+(x)\, a(x) = \frac{1}{2\pi\sqrt{-1}}\int_C \tau^p(t)\, g^+(t)\,\frac{a(t)-a(x)}{t-x}\,dt + \frac{a(x)}{2\pi\sqrt{-1}}\int_{C_1}\frac{\tau^p(t)\, g^+(t)}{t-x}\,dt.$$

From this equation we can at once infer that $f^+(x)$ may be so defined as to be continuous along C.

Thus $f^+(x)$ and $f^-(x)$ are respectively regular inner and outer functions.

A comparison of the relations (4) and (6) which are both valid along C gives us the fundamental equation

$$(7)\qquad f^+(x) - f^-(x) = \tau^p(x)\, g^+(x)\, a(x) \quad \text{along } C.$$

Let us now consider the maximum modulus of $f^-(x)$ outside of or along C. This maximum modulus, and likewise that for $g^+(x)$ within or along C, are attained on C of course. Suppose that we have along C

$$(8)\qquad |\, g^+(x)\,| \leqq L.$$

Now modify the contour C of integration in (4) to D_1. The integrand is analytic in t over the continua enclosed by C and D_1, so that the value of the integral will not thereby be altered. (It must be remembered that x lies without C in (4).) From this modified form of (4) we obtain

$$(9) \qquad |f^-(x)| \leqq \frac{KL}{2\pi}\left\{\int_{D_1} |\tau^p(t)\,dt| + \int_{C_1}\left|\frac{\tau^p(t)}{t-x}\,dt\right|\right\},$$

upon applying (8) and (1).

But the two integrals which appear in the right hand member of this inequality tend to zero as the unspecified integer p increases; in fact we have $|\tau(x)| < 1$ within C so that $\tau^p(x)$ tends uniformly to zero in any closed continuum within C, as p becomes infinite. It is to be observed that the quantity $|t-x|$ which appears in the second integrand is never less than the minimum distance from C to C_1, since x lies without C, and t is a point of C_1.

These considerations demonstrate that for a given positive ϵ, however small, the integer p may be chosen so large that *for every regular inner function* $g^+(x)$, we have

$$(10) \qquad \text{maximum of } |f^-(x)| \leqq \epsilon\,\{\text{maximum of } g^+(x)\} \quad \text{along } C.$$

§ 2. *On a Second Analogous Type of Integral.*

In the same way we may treat an integral

$$(2') \qquad \frac{1}{2\pi\sqrt{-1}}\int_C \frac{\tau^{-p}(t)\,g^-(t)}{t-x}\,a(t)\,dt,$$

where $g^-(x)$ is a regular outer function, and p is zero or a positive integer. As before we denote the value of the integral for x within C by $f^+(x)$, and for x without C by $f^-(x)$. A discussion parallel to the earlier one in § 1 shows that $f^+(x)$ and $f^-(x)$ as thus defined are regular inner and outer functions respectively; in this case equation (4) is replaced by

$$(4') \quad f^+(x) = \frac{1}{2\pi\sqrt{-1}}\int_C \tau^{-p}(t)\,g^-(t)\,\frac{a(t)-a(x)}{t-x}\,dt + \frac{a(x)}{2\pi\sqrt{-1}}\int_{C_2}\frac{\tau^{-p}(t)\,g^-(t)}{t-x}\,dt,$$

and (6) likewise by

$$(6') \quad f^-(x)+\tau^{-p}(x)\, g^-(x) a(x) = \frac{1}{2\pi\sqrt{-1}} \int_C \tau^{-p}(t)\, g^-(t) \frac{a(t)-a(x)}{t-x}\, dt + \frac{a(x)}{2\pi\sqrt{-1}} \int_{C_2} \frac{\tau^{-p}(t)\, g^-(t)}{t-x}\, dt.$$

From these two equations there results at once

$$(7') \qquad f^+(x) - f^-(x) = \tau^{-p}(x)\, g^-(x)\, a(x) \quad \text{along } C.$$

In order to develop an inequality for the modulus of $f^+(x)$ in this case, we note that the contour C in (4′) may be modified to D_2. The modification yields

$$(9') \qquad |f^+(x)| \leqq \frac{KL}{2\pi} \left\{ \int_{D_2} |\tau^{-p}(t)\, dt| + \int_{C_2} \left| \frac{\tau^{-p}(x)}{t-x} dt \right| \right\}$$

where L is the maximum of $|g^-(x)|$ along C. But $|\tau^{-p}(x)|$ tends to zero for x outside of C as p becomes infinite, since for such an x we have $|\tau(x)| > 1$. We conclude therefore that for any positive ϵ however small, the integer p may be taken so large that *for every regular outer function* $g^-(x)$ we have

$$(10') \quad \text{maximum of } |f^+(x)| \leqq \epsilon \{\text{maximum of } |g^-(x)|\} \quad \text{along } C.$$

A further property of the function $f^-(x)$, which is apparent from its definition, is that this function vanishes at $x = \infty$.

§ 3. *Solution of a Pair of Matrix Equations.*

Throughout the present paper we shall be concerned with linear equations in n unknown functions, whose complete solution may be expressed in terms of n particular solutions. On this account we shall employ the matrix notation.

We consider first a pair of *matrix equations*

$$(11) \qquad \left\{ \begin{aligned} F^+(x) - F^-(x) &= \tau^p(x)\, G^+(x)\, A(x), \\ G^-(x) - G^+(x) &= \tau^{-p}(x)\, F^-(x)\, A^{-1}(x) - I, \end{aligned} \right. \qquad \text{along } C.$$

Here $\tau(x)$ is the function defined in § 1; the matrix $A(x)$ is a given matrix $(a_{ij}(x))$ $(i, j = 1, \ldots, n)$ of which each element is defined along C and has the properties specified in § 1 for the function $a(x)$ (namely, it is continuous together with its derivatives of all orders along C, and analytic save at a finite number of points); furthermore the determinant $|A(x)|$ is not to be zero along C. The symbol I stands for the unit matrix (δ_{ij}) in which $\delta_{ii} = 1$, $\delta_{ij} = 0$ for $i \neq j$, and $A^{-1}(x)$ stands for the matrix inverse to $A(x)$. The matrices $F^+(x)$, $G^+(x)$, $F^-(x)$, $G^-(x)$ are to be determined to satisfy (11), the first two as matrices of regular inner functions and the last two as matrices of regular outer functions. The matrix products on the right hand side are the customary matrix products, and the factors $\tau^p(x)$, $\tau^{-p}(x)$ stand for the matrices $(\tau^p(x)\,\delta_{ij})$ and $(\tau^{-p}(x)\,\delta_{ij})$ respectively.

It may be proved without difficulty that for p taken large enough a solution of these equations exists. To effect this proof we apply a method of successive approximations based on the sequence of equations

$$
\begin{aligned}
&F_0^+(x) = F_0^-(x) = G_0^-(x) = O, \quad G_0^+(x) = I, \\
(12)\quad &\begin{cases} F_1^+(x) - F_1^-(x) = \tau^p(x)\, G_0^+(x)\, A(x), \\ G_1^-(x) - G_1^+(x) = \tau^{-p}(x)\, F_0^-(x)\, A^{-1}(x) - I, \end{cases} \qquad \text{along } C, \\
&\begin{cases} F_2^+(x) - F_2^-(x) = \tau^p(x)\, G_1^+(x)\, A(x), \\ G_2^-(x) - G_2^+(x) = \tau^{-p}(x)\, F_1^-(x)\, A^{-1}(x) - I, \end{cases} \qquad \text{along } C, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

The symbol O is used to denote a matrix of zero elements, and the superscripts $+$ and $-$ are used to designate matrices of regular inner and outer functions respectively.

If we write $P_0^-(x) = F_0^-(x) = O$, $Q_0^+(x) = G_0^+(x) = I$, and furthermore

$$
(13)\quad \begin{cases} P_m^+(x) = F_m^+(x) - F_{m-1}^+(x), \quad P_m^-(x) = F_m^-(x) - F_{m-1}^-(x), \\ Q_m^+(x) = G_m^+(x) - G_{m-1}^+(x), \quad Q_m^-(x) = G_m^-(x) - G_{m-1}^-(x), \end{cases}
$$

it is clear that the sequence of equations (12) is equivalent to

$$
(14)\quad \begin{cases} P_m^+(x) - P_m^-(x) = \tau^p(x)\, Q_{m-1}^+(x)\, A(x), \\ Q_m^-(x) - Q_m^+(x) = \tau^{-p}(x)\, P_{m-1}^-(x)\, A^{-1}(x), \end{cases} \qquad \begin{matrix} \text{along } C, \\ (m = 1, 2, \ldots). \end{matrix}
$$

Here the superscripts are employed as before.

The form of equations (14) is such that we can determine $P_m^+(x)$, $P_m^-(x)$, $Q_m^+(x)$, $Q_m^-(x)$ in terms of $P_{m-1}^-(x)$, $Q_{m-1}^+(x)$ so that the mth pair of equations (14) is satisfied. In fact the first one of the mth pair of matrix equations may be broken up into n^2 ordinary equations

$$
p_{i,j,m}^+(x) - p_{i,j,m}^-(x) = \tau^p(x) \sum_{\lambda=1}^{n} q_{i,\lambda,m-1}^+(x)\, a_{\lambda j}(x) \quad \text{along } C, \qquad (i, j = 1, \ldots, n),
$$

where the third subscript on the functions corresponds to the subscript on the matrix. We have already obtained a solution of an equation of the form

$$f^+(x) - f^-(x) = \tau^p(x)\, q_{i,\lambda,m-1}{}^+(x)\, a_{\lambda j}(x) \quad \text{along } C$$

(compare with (7)) in the form of a definite integral. By forming the sum of $f^+(x)$ and $f^-(x)$ for $\lambda = 1, \ldots, n$ we obtain for every i and j, elements $p_{i,j,m}{}^+(x)$ and $p_{i,j,m}{}^-(x)$ which form the elements of $P_m{}^+(x)$ and $P_m{}^-(x)$ with the desired property (14).

Likewise we can break up the second one of the mth pair of matrix equations (14) into n^2 equations. A solution may here be built up in a similar way (compare (7′). It must be observed that since the determinant of $A(x)$ does not vanish along C, the elements of $A^{-1}(x)$ satisfy the conditions imposed on $a(x)$ at the outset.

Now if we recall the method of solution of (14), it is clear from (10), (10′) that along C the maximum modulus of any element of $P_m{}^-(x)$ or $Q_m{}^+(x)$ does not exceed $n\epsilon L_{m-1}$ where L_{m-1} denotes the maximum modulus of any element of $P_{m-1}{}^-(x)$ or $Q_{m-1}{}^+(x)$ along C, and ϵ is arbitrarily small uniformly for all values of m. This relation may be expressed in the simpler form

$$L_m \leqq n\epsilon L_{m-1}. \tag{15}$$

The series formed by the elements in any ith row and jth column of the series of matrices

$$P_0{}^-(x) + P_1{}^-(x) + \ldots, \qquad Q_0{}^+(x) + Q_1{}^+(x) + \ldots,$$

will therefore converge absolutely and uniformly provided that ϵ is taken so small that $n\epsilon < 1$.

But the sums of $m + 1$ terms of these two series of matrices are $F_m{}^-(x)$ and $G_m{}^+(x)$ respectively, whose elements therefore converge uniformly to the elements of matrices $F^-(x)$ and $G^+(x)$ of regular outer and inner functions respectively. If we recall that $P_0{}^-(x) \equiv O$, and that the integral form of representation of each element of $P_m{}^-(x)$ (see (2)) makes each element of this matrix reduce to zero at $x = \infty$, it is plain that at infinity $F^-(x)$ reduces to the matrix O.

If we turn now to consider $F_m{}^+(x)$ and $G_m{}^-(x)$ along C, we see from what precedes and from equations (12), that these matrices also converge uniformly along C, and therefore respectively within C and without C, to matrices $F^+(x)$ and $G^-(x)$ of regular outer and inner functions. Since $G_0{}^-(x) = O$, it is clear that $G^-(x)$, as well as $F^-(x)$, reduces to O at $x = \infty$.

The matrices $F^-(x)$, $F^+(x)$, $G^-(x)$, $G^+(x)$ thus obtained will satisfy (11), as appears from (12) by letting n become infinite.

§ 4. *Application to the Solution of a Single Matrix Equation.*

Multiply the second matrix equation (11) on the right by $\tau^p(x)A(x)$ and subtract it, member for member, from the first equation (11). There results

$$F^+(x) = \tau^p(x)\,[I + G^-(x)]\,A(x) \quad \text{along } C. \tag{16}$$

Inasmuch as $G^-(x)$ reduces to a matrix of zero elements at $x = \infty$, the determinant of $I + G^-(x)$ and also of $F^+(x)$ is not identically zero.

The matrix equation (16) admits of further simplification. In fact the function $\log \tau(x)$ is analytic along C and increases by $2\pi\sqrt{-1}$ as x makes a positive circuit of C. If c lies within C the function

$$\phi(x) = \log \tau(x) - \log(x-c)$$

is accordingly single-valued as well as analytic along C. But $\phi(x)$ is of the form

$$\tau^p(x)\,g^+(x)\,a(x) \qquad (p = 0,\ g(x) = 1,\ a(x) = \phi(x)),$$

so that by (7) we can find $\theta^+(x)$ and $\theta^-(x)$ such that

$$\theta^+(x) - \theta^-(x) = \phi(x) \quad \text{along } C;$$

this gives us

$$\tau^p(x) = (x-c)^p\, e^{p\theta^+(x)}\, e^{p\theta-}(x).$$

Now let us write

$$\Phi(x) = e^{-p\theta^+(x)}\,F^+(x), \qquad \Psi(x) = (x-c)^p\,e^{p\theta^-(x)}\,[I + G^-(x)].$$

By these equations we define $\Phi(x)$ as a matrix of regular inner functions, and $\Psi(x)$ as a matrix of functions analytic outside of C except for a pole of order p at $x = \infty$, and continuous along C; the determinant of neither $\Phi(x)$ nor $\Psi(x)$ vanishes identically. Between $\Phi(x)$ and $\Psi(x)$, by (16), we have the matrix relation

$$\Phi(x) = \Psi(x)\,A(x) \quad \text{along } C. \tag{17}$$

It is this type of matrix equation which is important for the present paper.

§ 5. *Further Properties of* $\Phi(x)$ *and* $\Psi(x)$.

It is necessary for us to investigate further the nature of any solution $\Phi(x)$, $\Psi(x)$ of (17) in the neighborhood of the curve C. In the first place, it is to be observed that at points of C where all of the elements of $A(x)$ are analytic, the same is true of the elements of $\Phi(x)$ and $\Psi(x)$; in truth, the equation (17) shows us that analytic extension is possible across the curve at such points.

We shall prove that the elements of these matrices possess derivatives of all orders, continuous at all points of C.

Since the elements of $A(x)$ have line derivatives of all orders along C we may write, for t and y upon C,

$$(18)\quad A(t) = A(y) + (t-y)\frac{d}{dy}A(y) + \dots + \frac{(t-y)^k}{k!}\frac{d^k}{dy^k}A(y) + (t-y)^k B(t, y),$$

where the elements of $B(t, y)$ are continuous functions of t and y along C.[7] Also by Cauchy's integral formula in matrix form we have from (17) for x within C

$$\frac{d^{k-1}\Phi(x)}{dx^{k-1}} = \frac{(-1)^{k-1}(k-1)!}{2\pi\sqrt{-1}}\int_C \frac{\Psi(t)\,A(t)}{(t-x)^k}\,dt \qquad (k = 2, 3, \dots).$$

If we substitute the above expression for $A(t)$ in this last equation, we obtain a number of terms of the form (save for a constant multiplier)

$$(19)\qquad \int_C \frac{(t-y)^l\,\Psi(t)}{(t-x)^k}\,dt\,\frac{d^lA(y)}{dy^l} \qquad (l < k).$$

The integral is not altered in value if C is replaced by C_2 which lies outside of C. Therefore each of these terms represents a function analytic in x and continuous in y along C. There remains a single term not of the form (19), namely

$$\frac{(-1)^{k-1}(k-1)!}{2\pi\sqrt{-1}}\int_C\left(\frac{t-y}{t-x}\right)^k\Psi(t)\,B(t, y)\,dt.$$

---

[7] When a differentiation or integration sign appears before a matrix, it is understood to apply to each separate element of the matrix. The stated property of $B(t, y)$ comes at once from the explicit formula

$$B(t, y) = \frac{1}{k!}\int_y^t\left(\frac{z-y}{t-y}\right)^k\frac{d^{k+1}A(z)}{dz^{k+1}}\,dz.$$

If now x be made to approach a point x_0 of C, and if y be taken as the foot of the normal from x to C, this term approaches a limit which is continuous along C. In fact the factor $(t-y)^k/(t-x)^k$ remains finite for t along C, and approaches the limit 1 uniformly save in the vicinity of x_0. Thus if x (and y also of course) approaches x_0, $d^{k-1}\Phi(x)/dx^{k-1}$ approaches a limit along C continuous for an arbitrary x_0.

A similar proof shows that $\Psi(x)$ has derivatives of all orders, continuous outside of and along C. This proof is based on the fact that the elements of $A^{-1}(x)$ satisfy the restrictions placed on the function $a(x)$ in § 1.

The above results also lead to the conclusion that at any point γ of C at which one or more elements of $A(x)$ fails to be analytic, the elements of $\Phi(x)$ and $\Psi(x)$ admit of asymptotic expansion in a series in positive integral powers of $x-\gamma$. This is an immediate consequence of an expansion like (18) for $\Phi(x)$ or $\Psi(x)$ in which now t and y can be any points within or without C respectively, and $B(t, y)$ is continuous in t and y.[8]

§ 6. *A Normal Form for* $\Phi(x)$ *and* $\Psi(x)$.

By a series of simple normalizations it is always possible to obtain a solution $\Phi(x)$, $\Psi(x)$ of (17) such that $\Phi(x)$ is (as before) a matrix of regular inner functions, and $\Psi(x)$ is a matrix of function analytic, without C in the extended plane except for a possible pole at $x=\infty$, *and furthermore such that* $|\Phi(x)|$ *does not vanish within or on* C, *and* $|\Psi(x)|$ *does not vanish without* C.

This solution may be directly obtained from that found in § 4. If $|\Phi(x)|$ vanishes at $x=c$ within C say, we can determine a matrix M of constants such that all the elements of the first row of $M\Phi(x)$ vanish at $x=c$, while $|M|\neq 0$. Now $M\Phi(x)$, $M\Psi(x)$ yield a new solution of (17), which has the properties given for $\Phi(x)$, $\Psi(x)$ in § § 4, 5. If we divide the elements of the first row of $M\Phi(x)$ by $x-c$, we obtain a matrix $\overline{\Phi}(x)$ of functions analytic within C and continuous along C; if the same operation be applied to $\Psi(x)$, we obtain $\overline{\Psi}(x)$, a matrix of functions analytic without C in the extended plane save for a possible pole at $x=\infty$. Moreover $\overline{\Phi}(x)$ and $\overline{\Psi}(x)$ yield a solution of (17), since the effect of this operation is to alter the matrix on either side only in the removal of a factor $x-c$ from the first row. By this device we have diminished the multiplicity of the zero of $|\Phi(x)|$ at

---

[8] For the relation between the existence of derivatives and of asymptotic series see W. B. Ford, Bull. Soc. Math. France, **39**, 347–352 (1911).

$x = c$ by one unit, without the introduction of further zeros of $|\Phi(x)|$ or $|\Psi(x)|$ in the finite plane.

An entirely similar process eliminates a zero of $|\Psi(x)|$ without C, or a zero of $|\Phi(x)|$ and $|\Psi(x)|$ along C. In consequence of the results of § 5, if either of these functions vanishes along C, the other does also, both at least to the first order.

This process may be continued so long as there remain zeros of $\Phi(x)$ or $\Psi(x)$. It must however finally come to an end. If this were not the fact it would follow at once that both $|\Phi(x)|$ and $|\Psi(x)|$ have a zero of infinite multiplicity at some one point of C where an element of $A(x)$ fails to be analytic. But this cannot be the case, for let $\tilde{\Phi}(x)$ and $\tilde{\Psi}(x)$ be a solution of the following matrix equation

$$(20) \qquad \tilde{\Phi}(x) = A^{-1}(x)\,\tilde{\Psi}(x) \quad \text{along } C,$$

where the elements of $\tilde{\Phi}(x)$ and $\tilde{\Psi}(x)$ are restricted like $\Phi(x)$ and $\Psi(x)$ were found to be in § 4. The existence of such a solution becomes manifest by a mere interchange of the rôle of rows and columns in what precedes. Now from (17) and (20) we conclude that

$$|\Phi(x)|\cdot|\tilde{\Phi}(x)| = |\Psi(x)|\cdot|\tilde{\Psi}(x)| \quad \text{along } C.$$

The function represented by either side of this equality is not identically zero; and it appears from the two representations that it is analytic in the finite plane, and analytic or with a pole at infinity. Hence this function is a polynomial, and the multiplicity of the zeros of either $|\Phi(x)|$ or $|\Psi(x)|$ at any point of C is finite.

When the process comes to an end the following result has been obtained: if the elements of $A(x)$ are unlimitedly differentiable along C, analytic save at a finite number of points of C, and if $|A(x)|$ is not zero along C, there exists a solution $\Phi(x)$, $\Psi(x)$ of the equation (17)

$$\Phi(x) = \Psi(x)\,A(x) \quad \text{along } C,$$

in which the elements of $\Phi(x)$ are analytic within C, unlimitedly differentiable along C, and $\Phi(x)$ is of determinant not zero within or on C; and in which the elements of $\Psi(x)$ are analytic without C in the extended plane save for a possible pole at $x = \infty$, unlimitedly differentiable along C, and $\Psi(x)$ is of determinant not zero without C.

Here the point $x = \infty$ appears as an exceptional point. It is evident that an arbitrary point $x = a$ not on C may be used to take the rôle of the point at infinity. In fact a may also be taken to be a point of C. When this is the case, the elements of $\Phi(x)$ and $\Psi(x)$ are finite, or become infinite to finite order at $x = a$. To obtain such a solution $\Phi(x)$, $\Psi(x)$ it is only necessary to divide each element of $\Phi(x)$, $\Psi(x)$ by a suitable power of $x - a$, so chosen as to make each element of these matrices analytic at $x = \infty$; and then to apply the normalization above indicated, letting $1/x - a$ replace x. The curve C can also be taken to be a simply closed analytic curve which passes through $x = \infty$.

The main part of the conclusion that has been deduced above was obtained by Hilbert and Plemelj (loc. cit.) with the aid of the Fredholm theory of linear integral equations. Independently of their work, I treated a special case (see I, § 1) which arose in a different form in connection with my study of the irregular singular points of ordinary linear differential equations.

My proof in this special case suggested to me the above treatment of the general case by the method of successive approximations. The restrictions here placed on the elements of $A(x)$ and on the curve C are not essential to this treatment, and I have very little doubt that these may be replaced by the weaker restrictions of Hilbert and Plemelj. Nevertheless I have been content to use the simplest restrictions consistent with the applications in view.

A second proof in the special case has recently been given by me, Math. Ann., vol. 74 (1913) pp. 122 (see also Bull. Am. Math. Soc., vol. 18, 1911, p. 64). This second proof, which suggested itself to me at about the same time as the first, is practically the same as that given by Hilbert and Plemelj. To my considerable regret this relationship escaped my observation until it was too late for me to make suitable reference.

§ 7. *The Preliminary Theorem.*

The following is an extension of the preceding results which is convenient for the applications:

PRELIMINARY THEOREM. *Let $C_1, \ldots, C_r$ be r simply closed analytic curves in the extended complex plane. Let $A_1(x), \ldots, A_r(x)$ be matrices of functions defined and unlimitedly differentiable along $C_1, \ldots, C_r$ respectively, analytic save at a finite number of points of these curves and of determinant not zero. If furthermore at any point of intersection of C_α, C_β, the matrices $A_\alpha(x)$, $A_\beta(x)$ are such that the formal derivatives of all orders of the matrix*

$$A_\alpha(x)\,A_\beta(x) - A_\beta(x)\,A_\alpha(x) \tag{21}$$

vanish, there exists a matrix $\Phi(x)$ *with the following properties:*

(1) *each element of* $\Phi(x)$ *is analytic except along* $C_1, \ldots, C_r$ *and at an arbitrary point* $x = \alpha$ *where the elements may become infinite to finite order;* $|\Phi(x)|$ *nowhere vanishes save possibly at* $x = \alpha$*;*

(2) *the elements of* $\Phi(x)$ *are continuous and unlimitedly differentiable along each curve* C_i *from either side, analytic from either side save at points of intersection of the curves, or at those points where an element of* $A_i(x)$ *fails to be analytic, or at* $x = \alpha$*; if* α *lies on a curve* C_i*, the matrix* $(x - \alpha)^l A_i(x)$ [*or* $x^{-l}A_i(x)$ *if* $\alpha = \infty$] *is unlimitedly differentiable along* C_i *for a suitable* l.[9]

(3) *if a* + *and* − *side of each curve* C_i *are chosen, then*

$$\lim_{x \doteq x_i^+} \Phi(x) = [\lim_{x \doteq x_i^-} \Phi(x)]\, A_i(x_i) \qquad (i = 1, \ldots, r),$$

where the approach to the arbitrary point x_i *of* C_i *is along the* + *and* − *side respectively.*

Let us begin by establishing the theorem in the case when α is not a point of $C_1, \ldots, C_r$. It has already been established for $r = 1$ (see § 6), with the single notational difference that $\Phi(x)$ was replaced by either of two matrices $\Phi(x)$ and $\Psi(x)$, according as x was within or without C. To establish the theorem then, we need only show that if it is true for $r \leqq k$ it is also true for $r = k + 1$, when the theorem follows by induction.

Assume that $\Phi_k(x)$ is the solution for $r = k$, and for the matrices $C_1, \ldots, C_k, A_1(x), \ldots, A_k(x)$, where $C_1, \ldots, C_{k+1}$, and $A_1(x), \ldots, A_{k+1}(x)$ satisfy the requirements of the theorem for $r = k + 1$. Let us suppose for the moment that a solution $\Phi_{k+1}(x)$ exists with the desired properties. If we write

$$\Phi_{k+1}(x) = U(x)\, \Phi_k(x), \tag{22}$$

the following facts are clear from (1), (2), (3) of the theorem: (1′) each element $U(x)$ is analytic except along $C_1, \ldots, C_{k+1}$ and at the specified point α, where its elements may become infinite to finite order; $|U(x)|$ nowhere vanishes save possibly at $x = \alpha$; (2′) the elements of $U(x)$ are continuous and unlimitedly differentiable along each curve C_i from either side, analytic save at points of intersection of the curves or at those points where an element of $A_i(x)$ fails to be analytic;

---

[9] A function will be termed unlimitedly differentiable at $x = \infty$ if when we write $x = 1/x'$, the function of x' obtained by the substitution is unlimitedly differentiable at $x' = 0$.

$$\begin{aligned} &\lim_{x \doteq x_i+} U(x) = \lim_{x \doteq x_i-} U(x) \qquad (i = 1, \ldots, k), \\ (3') \quad &\lim_{x \doteq x_{k+1}+} U(x)\,\Phi_k(x) = [\lim_{x \doteq x_{k+1}-} U(x)\,\Phi_k(x)]\,A_{k+1}(x_{k+1}). \end{aligned}$$

The condition (3′) necessitates that $U(x)$ is analytic at any point of C_i $(i = 1, \ldots, k)$ and throughout the extended plane save at a and at points of C_{k+1}. The condition (3′) gives us in addition

$$\lim_{x \doteq x_{k+1}+} U(x) = [\lim_{x \doteq x_{k+1}-} U(x)]\,\Phi_k(x_{k+1})\,A_{k+1}(x_{k+1})\,\Phi_k^{-1}(x_{k+1}).$$

Conversely if $U(x)$ satisfies these conditions (1′), (2′), (3′) it is apparent that $\Phi_{k+1}(x)$, given by (22), will satisfy the conditions prescribed for $\Phi(x)$ in the theorem.

But in view of the above simplification of (3′), the conditions (1′), (2′), (3′) on $U(x)$ are precisely those of the theorem on $\Phi(x)$ if we take $r = 1$, $C = C_{k+1}$, $A_1(x) = \Phi_k(x)\,A_{k+1}(x)\,\Phi_k^{-1}(x)$. To complete a proof we need only show that this matrix fulfils the conditions prescribed in the theorem along C_{k+1}; for then a matrix $U(x)$ will exist which satisfies these conditions.

The elements of the matrix

$$\Phi_k(x)\,A_{k+1}(x)\,\Phi_k^{-1}(x) \tag{23}$$

are analytic at points of C_{k+1} which are not points of intersection or contact with $C_1, \ldots, C_k$, or points at which an element of $A_{k+1}(x)$ fails to be analytic; at these latter points the elements of $\Phi_k(x)$ are analytic so that the above matrix is unlimitedly differentiable in the neighborhood of such a point. The determinant of this matrix is equal to $|\,A_{k+1}(x)\,|$ and nowhere vanishes along C_{k+1}.

It remains only to examine the elements of the matrix near points of intersection or contact of C_{k+1} with one of the curves $C_1, \ldots, C_k$. Suppose first that C_{k+1} intersects a single curve C_i at such a point. As x moves along the curve C_{k+1} and passes from the positive to the negative side of C_i, the matrix $\Phi_k(x)$ changes to $\Phi_k(x)\,A_i(x)$, so that the matrix (23) changes to

$$\Phi_k(x)\,A_i(x)\,A_{k+1}(x)\,A_i^{-1}(x)\,\Phi_k^{-1}(x).$$

Bearing in mind the condition of permutability imposed on $A_1(x), \ldots, A_r(x)$ at such a point of intersection (see (21)), it becomes apparent that the elements of the matrix (23) are continuous at this point, and have equal backward and forward derivatives of all orders at the point. Thus the elements of (23) are unlimitedly differentiable in the neighborhood of such a point along C_{k+1}. The same is true at more complicated points of intersection or points of contact, as a similar argument shows.

The theorem is now demonstrated, at least for the case when a does not lie on a curve $C_1, \ldots, C_r$. If a does lie on such a curve, we first choose an a' which is not on these curves to replace a, and then resort to the simple device used at the close of § 6 to replace a' by a.

PART II: THE PROBLEM OF RIEMANN AND ITS GENERALIZATION.

§ 8. *On Cauchy Matrices.*

Let T be a matrix of constants of determinant not zero, corresponding to the coefficients of a linear transformation. According to the well-known theory of classification of such matrices, based on the Cayley-Weierstrass elementary divisor theory, we may write

$$T = C^{-1}I'C,$$

where C is a matrix of constants, and *in general* I' is of the form $(\rho_j\delta_{ij})$.

Consider along with the matrix I', the matrix of functions $I'(x) = (x^{k_j}\delta_{ij})$ where $2\pi k_j\sqrt{-1} = \log \rho_j$ $(j = 1, \ldots, n)$. When x makes a positive circuit of $x = 0$, $I'(x)$ becomes $I'(x)\,I'$. The matrix

$$T(x) = I'(x)\,C$$

will accordingly alter to

$$I'(x)\,I'C = I(x)\,C\cdot C^{-1}I'C = T(x)\,T,$$

when x makes such a circuit, i.e. $T(x)$ will be affected by the prescribed linear transformation.

Such a matrix $T(x)$ or a simple modification thereof exists for every transformation T and is called a Cauchy matrix.[10] A certain measure of arbitrariness enters into the determination of $T(x)$ when T is given; in general $k_1, \ldots, k_n$ are undetermined up to an additive integer. We note that the determinant of $T(x)$ is not zero for $x \neq 0, \infty$.

A final property of the Cauchy matrices which is important for us is that they form the matrix solution of a linear differential system

$$x\frac{dT(x)}{dx} = LT(x),$$

---

[10] Cf. Schlesinger, Vorlesungen über lineare Differentialgleichungen, pp. 122–140, where a complete treatment is given.

where L is a matrix of constants. In fact we have

$$x\frac{dT(x)}{dx} = x\frac{dI'(x)}{dx}C.$$

But the matrix $x\,dI'(x)\,/\,dx$ may be written as

$$KI'(x), \qquad K = (k_j\delta_{ij}).$$

Thus we obtain

$$x\frac{dT(x)}{dx} = KI'(x)C = KC\cdot C^{-1}I'(x)C = LT(x).$$

§ 9. *The Monodromic Group Problem.*

The most elementary existence theorems for ordinary linear differential equations show that the linear system

$$\text{(24)} \qquad \frac{dY(x)}{dx} = R(x)\,Y(x)$$

admits a matrix solution $Y(x)$ whose elements are analytic in the finite plane at every *ordinary point* where the elements of $R(x)$ are analytic, and analytic at infinity if the elements of $R(x)$ vanish to at least the second order at the ordinary point infinity; furthermore $|\,Y(x)\,|$ is not zero at such points. All other points of the plane are called *singular points* in contradistinction to the ordinary points. A finite singular point at which the elements of $R(x)$ are analytic or have a pole of the first order, or the point $x = \infty$ if each element of $R(x)$ vanishes to the first but not always to the second order at that point, is termed a *regular singular point.*

Suppose now that the elements of $R(x)$ are rational, and that all of the singular points $a_1, \ldots, a_m$ are regular. These will be taken to lie in the finite plane. It is easy to show that the elements of $Y(x)$ become infinite to only a finite order at $x = a_1, \ldots, a_m$.[11] When x makes a positive circuit of one of these points, $Y(x)$ changes to $Y(x)\,T_i$, where T_i is a matrix of constants and $|T_i| \neq 0$; in fact the most general solution is of the form $Y(x)\,T$ where T is an arbitrary matrix of constants for which $|\,T\,| \neq 0$, and after the circuit is made in the x-plane, $Y(x)$ is still a solution of (24).

If we start from a point $x = c$ and make a circuit of $a_1, \ldots, a_m$ in such wise that the combined circuit is reducible to a point, it is clear that

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[11] Cf. Trans. Am. Math. Soc., **11**, 199–202 (1910).

$$T_m T_{m-1} \ldots T_1 = I, \tag{25}$$

a necessary relation between the matrices $T_1, \ldots, T_m$.

The problem of Riemann is the following: For assigned points $a_1, \ldots, a_m$ and assigned matrices $T_1, \ldots, T_m$ for which (25) holds, to construct a matrix $Y(x)$ of functions of determinant not identically zero, with elements analytic save at $a_1, \ldots, a_m$ where these elements are finite or become infinite to finite order, and undergoing a transformation to $Y(x)\ T_i$ as x makes a positive circuit of $a_i\ (i = 1, \ldots, m)$.

A solution of this problem has been given by Hilbert and completed by Plemelj (loc. cit.). It is possible to obtain a more simple solution on the basis of the preliminary theorem.

Let us surround $a_1, \ldots, a_m$ by small non-overlapping simply closed analytic curves $C_1, \ldots, C_m$ and let us pass through $a_1, \ldots, a_m$ in cyclical order another closed analytic curve D which meets each curve C_i only twice (Fig. 2), in l_i and m_i say.

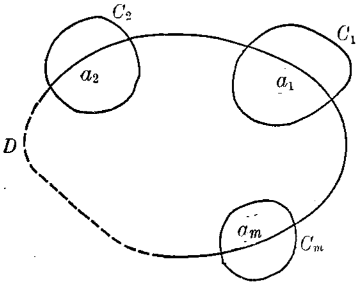

FIG. 2.

Choose matrices $A_1 \ldots, A_m$, $A_{m+1} = A_1$ of constants such that

$$A_{i+1}^{-1} A_i = T_i \qquad (i = 1, \ldots, m). \tag{26}$$

Here A_1, for example, may be taken at pleasure and $A_2, \ldots, A_m$ are then determined. Define a matrix $\bar{A}(x)$ along D to be equal to A_i on that part of the curve which lies between C_i and C_{i+1} $[C_{m+1} = C_1]$ and equal to

$$\frac{x - m_i}{l_i - m_i} A_{i-1} + \frac{x - l_i}{m_i - l_i} A_i \qquad (i = 1, \ldots, m) \tag{27}$$

for x on the part of D within C_i. We will suppose that l_i, m_i and D were so chosen that D does not pass through one of the finite number of point for which the determinant of any matrix (27) vanishes.

The matrix $\bar{A}(x)$ as here defined is continuous along D, analytic save at the points where D intersects $C_1, \ldots, C_m$, and of determinant not zero along D.

By slightly modifying each element of $\bar{A}(x)$ along a small segment near either end of the arc of D within C_i for $i = 1, \ldots, m$, it is possible to obtain a matrix $A(x)$ whose elements are analytic save at the end points of these segments, and unlimitedly differentiable along D. Such a matrix $A(x)$ satisfies all of the conditions necessary for the application of the preliminary theorem in the case $r = 1$.

Taking D as the curve and $A(x)$ as the matrix, we can affirm the existence of a certain matrix $\Phi(x)$, possessing in particular the property

$$\lim_{x \doteq x_1^+} \Phi(x) = [\lim_{x \doteq x_1^-} \Phi(x)]\, A(x_1), \tag{28}$$

the approach to the point x_1 of D being from within and without D respectively. Let us choose $a = a_m$.

If we extend $\Phi(x)$ analytically across D between C_i and C_{i+1}, it becomes $\Phi(x)A_i$ by (28). If we extend this matrix analytically back across D between C_{i+1} and C_{i+2} $[C_{m+2} = C_2]$, it becomes $[\Phi(x)\, A_{i+1}^{-1}]\, A_i^{-1}$, or $\Phi(x)\, T_i$. That is, the matrix $U(x)$ obtained from $\Phi(x)$ by analytic extension is analytic outside of $C_1, \ldots, C_m$ and undergoes a transformation to $U(x)T_i$ when x makes a positive circuit of a_i. Furthermore the determinant of $U(x)$ is not zero outside of $C_1, \ldots, C_m$.

Denote by $Z_i(x-a_i)$ the Cauchy matrix belonging to the transformation T_i, so that $Z_i(x-a_i)$ undergoes a transformation to $Z_i(x-a_i)\, T_i$ as x makes a positive circuit of a_i. Write

$$Y(x) = Z(x)\, U(x), \tag{29}$$

where $Y(x)$ is the solution of the Riemann problem to be constructed.

The elements of $Z(x)$ must in the first place be single-valued and analytic outside of $C_1, \ldots, C_m$, since $U(x)$ undergoes the same transformation as that prescribed for $Y(x)$ about the points $a_1, \ldots, a_m$, and $|U(x)| \neq 0$.

Furthermore within C_i, the elements of $Y(x)$ are to be analytic except at a_i where they may become infinite to finite order. Hence the elements of the matrix $Y(x)Z_i^{-1}(x-a_i)$ must be single-valued and analytic within C_i, by the definition of $Z_i(x-a_i)$, except for a possible pole at $x = a_i$. Along C_i this matrix may be written

$$\bar{Z}_i(x) = Z(x)\, [U(x)\, Z_i^{-1}(x - a_i)] \qquad (i = 1, 2, \ldots m). \tag{30}$$

If we write $\Phi(x) = Z(x)$ outside of $C_1, \ldots, C_m$ and also $\Phi(x) = \bar{Z}_i(x)$ within C for $i = 1, \ldots, m$, the equations (30) may be written

$$(31) \quad \lim_{x=x_i^+} \Phi(x) = [\lim_{x=x_i^-} \Phi(x)]\, A_i(x_i), \qquad A_i(x) = U(x)\, Z_i^{-1}(x - a_i). \quad (i = 1, \ldots, m).$$

This suggests another application of the preliminary theorem, since the curves $C_1, \ldots, C_m$ and the matrices $A_1(x), \ldots, A_m(x)$ of known functions satisfy the necessary restrictions.

Let $\Phi(x)$ be the solution given by the theorem for $a = a_m$, and let $\overline{Z}_1(x), \ldots, \overline{Z}_m(x)$, $Z(x)$, be defined as equal to $\Phi(x)$ within $C_1, \ldots, C_m$ and outside of these curves respectively. These functions will then satisfy (30). Let $Y(x)$ be defined by (29). This matrix is clearly composed of elements analytic without and along $C_1, \ldots, C_m$, as are those of $U(x)$; within C_i, $Y(x)$ continues analytically into $\overline{Z}_i(x)\, Z_i(x - a_i)$ by (30) for $i = 1, \ldots, m$, and consequently its elements are analytic throughout the plane except possibly at $a_1, \ldots, a_m$; its elements become infinite only to a finite order at a_i since the elements of $Z_i(x - a_i)$ become infinite only to finite order at a_i. Furthermore by (29) $Y(x)$ undergoes a linear transformation to $Y(x)\, T_i$ as x makes a positive circuit of a_i. Thus the Riemann problem has been solved.

It is worthy of note that $|\, Y(x) \,|$ does not vanish for $x \neq a_i$ (i $= 1, \ldots, m$). This is an immediate consequence of the fact that $|\, \overline{Z}_1(x) \,|, \ldots, |\, \overline{Z}_m(x) \,|$, $|\, Z(x) \,|$ do not vanish in their regions of definitions save at these points, and of the fact that the Cauchy matrix $Z_i(x - a_i)$ has a determinant which does not vanish save possibly at $x = a_i$ and $x = \infty$.

§ 10. *A Generalization. Equivalence.*

A more general result can be deduced exactly as the results of § 9 were obtained. Let us say that two matrices of functions $Y_1(x)$ and $Y_2(x)$ whose elements are analytic in the vicinity of $x = a$, but not in general single-valued or analytic at $x = a$, are *properly equivalent at* $x = a$ if we have

$$Y_1(x) = A(x)\, Y_2(x).$$

where $A(x)$ is composed of elements single-valued and analytic at $x = a$, of determinant not zero there; if this condition is not satisfied, but if the elements of $A(x)$ have a pole or are analytic at $x = a$, let us say that $Y_1(x)$ and $Y_2(x)$ are *improperly equivalent at* $x = a$.

This definition is convenient for the statement of the following result: *Let* $a_1, \ldots, a_m$ *be m given points; let* $T_1, \ldots, T_m$ *be matrices of constants such that* $T_m\, T_{m-1} \ldots T_1 = I$; *let* $Z_1(x), \ldots, Z_m(x)$ *be matrices of* *functions analytic of determinant not zero in the vicinity of* a_i *and undergoing a transformation to* $Z_1(x)\ T_1, \ldots, Z_m(x)\ T_m$ *as* x *makes a positive circuit of* $a_1, \ldots, a_m$ *respectively. There exists then a matrix* $Y(x)$ *of functions of determinant not zero for* $x \neq a_1, \ldots, a_m$ *and analytic save at these points, which undergoes a transformation to* $Y(x)T_i$ as x *makes a positive circuit of* a_i $(i = 1, \ldots m)$; *furthermore* $Y(x)$ *is properly equivalent to* $Z_1(x), \ldots, Z_{m-1}(x)$ at $a_1, \ldots, a_{m-1}$ *and properly or improperly equivalent to* $Z_m(x)$ *at* a_m.

The result above stated is obtained when we replace the Cauchy matrices $Z_1(x-a_1), \ldots, Z_m(x-a_m)$ of § 9 by matrices $Z_1(x), \ldots, Z_m(x)$ having the properties specified. The line of attack is identical with that given in § 9. The facts concerning equivalence follow at once from the relations analogous to (30):

$$Y(x) = \overline{Z}_i(x)\, Z_i(x) \qquad (i = 1, \ldots, m). \tag{32}$$

Here $\overline{Z}_i(x)$ is composed of elements analytic at $x = a_i$ $(i = 1, \ldots, m-1)$, and of determinant not zero there; also $\overline{Z}_m(x)$ is composed of elements analytic at $x = a_m$ or with a pole at that point.

§ 11. *Final Form of Solution.*

There is a certain lack of symmetry between the rôle of $a_1, \ldots, a_m$ in the solution of the Riemann problem obtained in § 9, provided the given Cauchy matrices $Z_1(x - a_1), \ldots, Z_m(x - a_m)$ were taken in the most general possible form. We shall now proceed to show that *if* $Z_m(x - a_m)$ *be a properly chosen Cauchy matrix associated with the transformation* T_m, *the equivalence of* $Y(x)$ *and* $Z_m(x)$ *can be made proper at* $x = a_m$ *also.*

A form of reduction that is well-known suffices to establish this fact.[12] Nevertheless, inasmuch as a similar type of reduction is necessary later in the present paper, I give this reduction herewith.

Let us begin with the matrix $Y(x)$, obtained in § 9, which we may assume to be improperly equivalent at a_m to the Cauchy matrix $Z_m(x - a_m)$. Now we have (§ 8)

$$Z_m(x - a_m) = I'(x - a_m)\, C$$

so that from (30)

$$Y(x)\, C^{-1} = \overline{Z}_m(x)\, I'(x - a_m),$$

---

[12] Plemelj, loc. cit., pp. 237–240.

The matrix of elements on the right can be written in the form

$$
\begin{array}{l}
(x-a_m)^{k_1}(a_{11}+b_{11}(x-a_m)+\ldots),\ldots,(x-a_m)^{k_n}(a_{1n}+b_{1n}(x-a_m)+\ldots) \\
(33)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
(x-a_m)^{k_1}(a_{n1}+b_{n1}(x-a_m)+\ldots),\ldots,(x-a_m)^{k_n}(a_{nn}+b_{nn}(x-a_m)+\ldots)
\end{array}
$$

Here it is supposed that the exponents of the Cauchy matrix do not differ by integers; but a similar form can be found in all cases. In each column of $Y(x)$ the highest possible power of $x - a_m$ is exhibited which leaves the coefficients of this power analytic in character at $x = a_m$.

If $|a_{ij}| \neq 0$ we can write this matrix in the form $A(x)I'(x - a_m)$, where $I'(x)$ is the matrix $(x^{k_j}\delta_{ij})$, and where $A(x)$ is the matrix obtained from (33) by striking out the exhibited powers of $x - a$. From the equation

$$Y(x) = A(x)\,I'(x - a_m)\,C^{-1}$$

we see that $Y(x)$ is properly equivalent to a Cauchy matrix belonging to T at $x = a_m$.

On the other hand if $|a_{ij}| = 0$ we proceed as follows: It is readily verified that if $Y(x)$ be one matrix solution of the Riemann problem satisfying the relation of equivalence given in § 10, then $DY(x)$ $[|D| \neq 0]$ is also a solution. Consequently it is no restriction in the consideration of (33) to assume that

$$a_{11} = a_{12} \ldots = a_{1n} = 0,$$

since when $|a_{ij}| = 0$ this relation may always be made to hold by multiplying on the left by a suitable matrix D. Denote by $y_1(x), \ldots, y_n(x)$ the elements of the first row after a factor $(x - a_m)^l$ has been removed, where l is the exponent of the highest power of $x - a_m$ that may be taken out and leave the elements $y_1(x), \ldots, y_n(x)$ of the respective forms

$$(x-a_m)^{k_1}(f_1+g_1(x-a_m)+\ldots),\ldots,(x-a_m)^{k_n}(f_n+g_n(x-a_m)+\ldots)$$

It follows that $f_1, \ldots, f_n$ are not all zero.

We may now add constant multiples of $y_1(x), \ldots, y_n(x)$ to the successive rows of $Y(x)$ and obtain a new matrix $Y(x)$. In fact this is equivalent to multiplying $Y(x)$ on the left by a matrix

$$P(x) = \left| \begin{array}{cccc} 1, & 0, & \ldots\ldots\ldots\ldots & 0 \\ \dfrac{c_2}{(x - a_m)^l}, & 1, & \ldots\ldots\ldots\ldots & 0 \\ \ldots & \ldots & \ldots\ldots\ldots\ldots & \ldots \\ \dfrac{c_n}{(x - a_m)^l}, & 0, & \ldots\ldots\ldots\ldots & 1 \end{array} \right|$$

Inasmuch as $| P(x) | = 1$, and the elements of $P(x)$ are analytic save at $x = a_m$, this modification cannot affect any of the properties already secured for $Y(x)$. However by choosing $c_2, \ldots, c_m$ properly we can clearly make all the coefficients $a_{2i}, \ldots a_{ni}$ vanish at $x = a_m$ provided $f_i \neq 0$. In this manner we can increase an exponent k_i by 1 without altering the determinant of $Y(x)$ except by a constant factor.

Inasmuch as $| Y(x) |$ does not vanish identically, a succession of steps of this type will finally bring to light a solution $Y(x)$ for which $Y(x)\, C$ has the form (33) and in addition $| a_{ij} | \neq 0$. When this stage is reached $Y(x)$ will be properly equivalent to a Cauchy matrix belonging to T_m at the point a_m.

When $Y(x)$ has thus been given a normal form, it is the solution of a linear differential system (24) with regular singular points at $x = a_1, \ldots, a_m$ and having no other singular points, as may be at once proved.[13] The elements of $R(x)$ therefore have the form of rational functions whose numerators are polynomials in x of degree at most $m - 2$, and whose denominators are the product of $(x - a_1), \ldots, (x - a_m)$.

§ 12. *Irregular Singular Points and Canonical Systems.*

The Cauchy matrix is the simplest possible matrix of functions to which a matrix solution of a given linear differential system is properly equivalent at a regular singular point. Let us determine the simplest possible matrix $Z(x)$ to which the matrix solution $Y(x)$ of a given linear differential system (24), in which the elements of $R(x)$ need not be rational, is properly equivalent at a prescribed *irregular* singular point. It is convenient to take this point to lie at infinity. If the highest order of any pole of an element of $xR(x)$ at $x = \infty$ is $p + 1$ $(p \geqq 0)$, then $p + 1$ is said to be the *rank* of the singular point $x = \infty$.

According to the results of § 10 we can find a matrix $Z(x)$ which at $x = \infty$ is properly equivalent to $Y(x)$ and at another point $x = 0$

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[13] Schlesinger, loc. cit. pp. 215–221.

is improperly equivalent to a Cauchy matrix which at $x = 0$ undergoes a transformation inverse to that which $Y(x)$ undergoes at $x = \infty$. Here we take $m = 2$, $a_1 = 0$, $a_2 = \infty$. The condition $T_2 T_1 = I$ is satisfied.

By means of a modification precisely like that of § 11 we can make $Z(x)$ *properly* equivalent to a suitable Cauchy matrix at $x = 0$, and yet preserve the other properties listed in § 10.

Now consider

$$R_1(x) = \frac{dZ(x)}{dx} Z^{-1}(x).$$

Since $|Z|(x)| \neq 0$ for $x \neq 0, \infty$, and since $Z(x)$ and $dZ(x)/dx$ undergo the same substitution about $x = 0$, the elements of $R_1(x)$ are single-valued, and analytic for $x \neq 0, \infty$. Since $Z(x)$ is properly equivalent to a Cauchy matrix at $x = 0$, the elements of $R_1(x)$ have poles of at most the first order at $x = 0$.[14] Moreover since $Z(x)$ is properly equivalent to $Y(x)$ at $x = \infty$ we have

$$Z(x) = A(x)\, Y(x),$$

where the elements of $A(x)$ are analytic at $x = \infty$ and also $|A(x)| \neq 0$ at $x = \infty$. Therefore we obtain

$$\frac{dZ(x)}{dx} = A(x)\frac{dY(x)}{dx} + \frac{dA}{dx} Y(x) = \left[A(x)\, R(x) + \frac{dA}{dx}\right] Y(x),$$

and

$$R_1(x) = \frac{dZ(x)}{dx} Z^{-1}(x) = \left[A(x)\, R(x) + \frac{dA(x)}{dx}\right] A^{-1}(x).$$

Hence the elements of $R_1(x)$ are analytic or have a pole of order not greater than p at $x = \infty$.

From this analysis it follows that $xR_1(x)$ is a matrix of polynomials of degree at most $p + 1$, so that $Z(x)$ is itself the solution of a linear differential system

$$x\frac{dZ}{dx} = P(x)\, Z, \tag{34}$$

where $P(x)$ is a matrix of polynomials of degree at most $p + 1$. This is the *canonical* form of equation with an irregular singular point of rank $p + 1$ at $x = \infty$.

At a finite singular point $x = a$, the canonical system is of a type obtained from (34) by a transformation $x' - a = 1/x$.

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[14] The detailed proof is entirely similar to that given herewith to determine the nature of the elements of $R(x)$ at $x = \infty$. Cf. Schlesinger, loc. cit., pp. 143–144.

In order not to introduce artificial difficulties we shall consider the regular singular point to be of rank zero. To this case the above argument applies also, and the canonical system is that satisfied by a Cauchy matrix.

If $Y(x)$ is the matrix solution of a differential system (24) in which the elements of $R(x)$ are rational, the above argument shows that at each of its singular points $a_1, \ldots, a_m$, $Y(x)$ is properly equivalent to the matrix solutions $Z_1(x), \ldots, Z_m(x - a_m)$ of canonical differential systems.

Conversely the results of § 10 lead to the conclusion that given $T_1, \ldots, T_m$ such that $T_m T_{m-1} \ldots T_1 = I$, and canonical differential systems belonging to the singular points $a_1, \ldots, a_m$ with matrix solutions $Z_1(x), \ldots, Z_m(x)$ undergoing a transformation to $Z_1(x) T_1, \ldots, Z_m(x) T_m$ at these points respectively, there will exist a matrix solution $Y(x)$ of a rational differential system (24) which undergoes a transformation to $Y(x) T_i$ as x makes a positive circuit of a_i and which is properly equivalent to $Z_i(x)$ at this point, for $i = 1, \ldots, m-1$, and properly or improperly equivalent to $Z_m(x)$ at $x = a_m$.

It is therefore essential to obtain a characterization of the matrix solution of a canonical system (34), and further to solve the associated inverse problem, before solving the general problem of characterization for a system (24) with irregular singular points.

§ 13. *The Problem of the Irregular Singular Point.*

In my paper referred to[15] I characterized the nature of the matrix solution of a canonical linear differential system (34), at least in the case that the roots of a certain characteristic equation were distinct; the case of equal roots introduces complications of an algebraical nature, and is put to one side in the present paper.

The results which I obtained may be recapitulated as follows: If the singular point is taken at $x = \infty$, there exists a formal matrix solution of (34)

$$S(x) = (e^{p_j(x)} x^{k_j} s_{ij}(x)),$$

$$(35) \qquad p_j(x) = \alpha_j \frac{x^{p+1}}{p+1} + \beta_j \frac{x^p}{p} + \ldots + \lambda_j x, \qquad (j = 1, \ldots, n),$$

$$s_{ij}(x) = s_{ij} + s_{ij}^{(1)} \frac{1}{x} + \ldots, \qquad (i, j = 1, \ldots, n),$$

---

[15] I, §§ 6, 7.

where $| s_{ij} | \neq 0$. The quantities $\alpha_1, \ldots, \alpha_n$ are the roots of the characteristic equation alluded to, and the series $s_{ij}(x)$ are in general divergent. We shall assume for the time being that no three of the points $\alpha_1, \ldots, \alpha_n$ lie on the same straight line in the complex plane. Let now $\tau_1, \ldots, \tau_N$ denote the $N = n(n-1)(p+1)$ arguments in order of increasing angular magnitude such that for some j and k $(j \neq k)$

$$(36) \qquad \Re\{(\alpha_j - \alpha_k) x^{p+1}\} = 0, \qquad \arg x = \tau_m.$$

Here "$\Re$" denotes "the real part of". Let us write $\tau_{N+1} = \tau_1 + 2\pi$, and let j_m and k_m denote the value of j and k corresponding to m, so ordered that the real part (36) changes from positive to negative as $\arg x$ increases through $\arg x = \tau_m$. There exist then N matrix solutions $Z_1(x), \ldots, Z_N(x)$ such that for $i = 1, \ldots, N$

$$(37) \qquad Z_m(x) \backsim S(x), \qquad \tau_m \leqq \arg x < \tau_{m+1},$$[16]

and such that along $\arg x = \tau_{m+1}$, $Z_{m+1}(x)$ and $Z_m(x)$ differ only in their j_mth column, the j_mth column of $Z_{m+1}(x)$ being obtained from that of $Z_m(x)$ by the addition of the k_mth column of $Z_m(x)$, affected with a suitable constant multiplier c_m, to the j_mth column. As a matter of definition we take

$$(38) \qquad Z_{N+1}(x) = Z_1(x) I', \qquad I' = (e^{2\pi k_j \sqrt{-1}} \delta_{ij}).$$

The proof of the existence of $Z_1(x), \ldots, Z_N(x)$ having these properties can be directly based on the existence of a solution $Z(x)$ asymptotically represented by $S(x)$ along every particular ray.[17]

The properties so far stated are characteristic of the behavior of the matrix solution not alone of a canonical system but of any system with singular point of rank p at $x = \infty$ in the neighborhood of the singular point,[18] and are invariant under a transformation $\bar{Y}(x) = A(x) Y(x)$ where $A(x)$ is a matrix of elements analytic at the singular point in question and of determinant not zero there.

When $Z_1(x), \ldots, Z_{N+1}(x)$ are in addition the solution of a canonical system, the matrices $Z_m(x)$ are analytic in the finite plane for $x \neq 0$

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[16] The relation "$z(x) \backsim s(x)$, $\arg x = \sigma$," means in the present paper that $z(x)$ is asymptotically represented by $s(x)$ *in some sector (however small) that includes arg $x = \sigma$ as an interior ray.* This slight modification of the conventional meaning of the symbol "$\backsim$" and its natural extension to matrices is convenient for the present paper.

[17] I, § 6.

[18] I, § 6.

and of determinant not zero. At $x = 0$ these matrices are properly equivalent to a Cauchy matrix. These further facts come at once from the form of (34).

Conversely let us prove that if $Z_1(x), \ldots, Z_N(x)$ exist possessing such properties, they are all matrix solutions of one and the same canonical system (34).[19]

In the first place it is clear that the matrix

$$P(x) = x \frac{dZ_m(x)}{dx} Z_m^{-1}(x) \qquad (m = 1, \ldots, N)$$

is defined in each sector $\tau_m \leqq \arg x < \tau_{m+1}$ $(i = 1, \ldots, N)$ and hence is defined in the entire plane. Now $Z_{m+1}(x)$ is obtained from $Z_m(x)$ by multiplication on the right by the matrix

$$I + C_m,$$

where C_m is a matrix of zero elements except for the single element c_m in the k_mth row and j_mth column. It appears therefore that $P(x)$ is single-valued and analytic along each ray $\arg x = \tau_m$; for we have along this ray

$$\frac{dZ_{m+1}(x)}{dx} Z_{m+1}^{-1}(x) = \frac{dZ_m(x)}{dx}[I + C_m]\,[I + C_m]^{-1} Z_m^{-1}(x) = \frac{dZ_m(x)}{dx} Z_m^{-1}(x).$$

Moreover, on account of the equivalence of $Z_1(x), \ldots, Z_m(x)$ to a Cauchy matrix at $x = 0$, the elements of $xP(x)$ are analytic at $x = 0$.

Secondly, in the neighborhood of $x = \infty$, the matrix $P(x)$ is asymptotic to

$$x \frac{dS(x)}{dx} S^{-1}(x), \qquad \tau_m \leqq \arg x < \tau_{m+1}, \tag{39}$$

where the meaning of the notation is manifest. It is to be recalled that the relation (37) holds (by definition) in a small sector including $\arg x = \tau_m$ as an interior ray. This enables us to write

$$\frac{dZ_m(x)}{dx} \backsim \frac{dS(x)}{dx}, \qquad \tau_m \leqq \arg x < \tau_{m+1}.^{20}$$

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[19] I stated this fact without proof earlier; see I, p. 468.
[20] Ford, loc. cit.

Thus $P(x)$ is represented asymptotically in the *complete* vicinity of $x = \infty$ by (39). But we have formally

$$\frac{dS(x)}{dx} = \left(e^{p_j(x)} x^{r_j} \left\{ \left(\frac{dp_j(x)}{dx} + \frac{r_j}{x} \right) s_{ij}(x) + \frac{ds_{ij}(x)}{dx} \right\} \right). \tag{40}$$

If (35) and (40) be used in evaluating the expression (39), it is seen that each element of $P(x)$ is given asymptotically by a power series in descending integral powers of x, with leading term in x^{p+1} or lower power of x. It follows that the elements of $P(x)$ are analytic or have a pole of at most the $(p+1)$th order at $x = \infty$. Hence the elements of $P(x)$ are polynomials of degree at most $p + 1$.

The central problem of the irregular singular point is, for a given choice of $p_1(x), \ldots, p_n(x)$, $r_1, \ldots, r_n$ and of $c_1, \ldots, c_N$ above described, to construct a matrix $Y(x)$ with the above specified properties.[21]

§ 14. *Solution of the Problem of* § 13.

In order to solve the problem just stated we make an application of the preliminary theorem of Part I, taking $r = N/2$ and for the curves $C_1, \ldots, C_{\frac{1}{2}N}$ *not the $N/2$ straight lines formed by the N rays* $\arg x = \tau_m$ $(m = 1, \ldots, N)$ but by the N rays $\arg x = \tau'_m$ $(m = 1, \ldots, N)$, which are obtained from them by a slight rotation ϵ in a clockwise direction in the complex x-plane. It is evident that if ϵ be taken small enough we shall have

$$Z_m(x) \backsim S(x), \qquad \tau'_m \leqq \arg x \leqq \tau'_{m+1},$$

for $m = 1, \ldots, N$. Furthermore we shall have for any m

$$\Re\,(a_{jm} - a_{km})\,x^{p+1} > 0, \qquad \arg x = \tau'_m,$$

for $m = 1, \ldots, N$. This fact is essential to the solution.

The matrices $A_1(x), \ldots, A_{\frac{1}{2}N}(x)$ which are to be used in the application of the theorem are defined as follows. Write

$$T(x) = (x^{p_j(x)} x^{r_j} \delta_{ij}),$$

and then put

$$\bar{A}_m(x) = T(x)\{I + C_m\}T^{-1}(x) \qquad (m = 1, \ldots, N), \tag{41}$$

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[21] Cf. I, § 7.

where x in $\bar{A}_m(x)$ is taken along arg $x = \tau'_m$, and the determinations of $T(x)$ chosen are obtained from one another by allowing arg x to increase from τ_1 to τ_N. The matrix $\bar{A}_m(x)$ thus defined is analytic along its line save at $x = 0$. Also $\bar{A}_m(x) - I$ is a matrix whose only non-zero element is

$$c_m e^{p_{km}(x)-p_{jm}(x)} x^{r_{km}-r_{jm}}.$$

Now we have

$$p_{km}(x) - p_{jm}(x) = (\alpha_{km} - \alpha_{jm}) \frac{x^{p+1}}{p+1} + (\beta_{km} - \beta_{jm}) \frac{x^p}{p} + \ldots + (\lambda_{km} - \lambda_{jm}) x,$$

which quantity by definition of j_m, k_m has *a negative real part* for x along the ray arg $x = \tau'_m$, at least when $|x|$ is sufficiently large. In consequence of the form of this non-zero element it is certain that $\bar{A}_m(x)$ is unlimitedly differentiable along the ray at $x = \infty$ and behaves there like the matrix I. As this is true along each of the rays, the matrices $\bar{A}_m(x)$ clearly satisfy the conditions of the theorem in the vicinity of $x = \infty$. Choose $A_1(x), \ldots, A_{\frac{1}{2}N}(x)$ along the straight lines $C_1, \ldots, C_{N/2}$ as equal to the corresponding matrix $\bar{A}_m(x)$ along either of its component rays outside of some circle $|x| = r$, within which they are chosen so as to satisfy the conditions of the preliminary theorem. Since $\bar{A}_1(x), \ldots, \bar{A}_N(x)$ are of the nature above described at $x = \infty$, the matrices $A_1(x), \ldots, A_{N/2}(x)$ are unlimitedly differentiable, and satisfy the permutability condition (see (21)) of the preliminary theorem. It is therefore possible to make such a choice of $A_1(x), \ldots A_{N/2}(x)$.[22]

It follows by the theorem that there exists a matrix $\Phi(x)$ of determinant nowhere zero save possibly at $x = a = 0$, analytic except along the rays arg $x = \tau_m'$ and such that

$$\text{(42)} \qquad \lim_{x=x_m^+} \Phi(x) = [\lim_{x=x_m^-} \Phi(x)] \bar{A}_m(x_m),$$

where x_m is a point of arg $x = \tau_m'$ for which $|x_m| \geqq r$. It also follows from the theorem that each element of $\Phi(x)$ is represented asymptotically by a series in negative powers of x in each sector (τ'_m, τ'_{m+1}); since $A_m(x) \backsim I$, these series are the same in all the sectors and we may write

$$\text{(43)} \qquad \Phi(x) \backsim (s_{ij}(x)),$$

where $s_{ij}(x)$ is a series of negative powers of x in which the determinant of the constant terms is not zero.

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[22] Cf. § 9.

Now write for $m = 1, \ldots, N + 1$

$$Z_m(x) = \Phi(x)\, T(x) \qquad (\tau'_m \leqq \arg x \leqq \tau'_{m+1}).$$

From (41) and (42) there results along $\arg x = \tau'_{m+1}$

$$Z_{m+1}(x) = Z_m(x)\,[I + C_m], \qquad |\, x \,| \geqq r.$$

The functions $Z_m(x)$ may accordingly be continued analytically across each ray $\arg x = T_m$ and represent matrices analytic for $|\, x \,| \geqq r$, of determinant not zero. The relation (43) leads to the conclusion that

$$Z_m(x) \backsim S(x) \qquad (\tau'_m \leqq \arg x \leqq \tau'_{m+1}).$$

where $S(x)$ is of the desired form (35), and thence to the conclusion that this asymptotic representation is valid for $\tau_m \leqq \arg x < \tau_{m+1}$. The relation between $Z_{N+1}(x)$ and $Z_1(x)$ is that stated in (38).

In § 13 it was shown that, if matrices $Z_1(x), \ldots, Z_N(x)$ had the above properties and the further properties that they were analytic in the finite plane of determinant not zero for $x \neq 0$, and at $x = 0$ were properly equivalent to a Cauchy matrix, then these matrices were solutions of a canonical system (34) with irregular singular point at $x = \infty$. The same arguments can be used to establish that the $Z_1(x), \ldots, Z_N(x)$ before us are solutions of a differential system (24) having coefficients rational in character at $x = \infty$, with poles of order not more than p. But we have proved in § 12 that the matrix solution of such an equation is properly equivalent to that of a canonical linear differential system at $x = \infty$. Consequently a transformation $\overline{Z}(x) = A(x) Z(x)$, where $A(x)$ is a matrix of functions analytic in character at $x = \infty$ of determinant not zero there, makes $\overline{Z}(x)$ the solution of such a canonical system. In particular the matrices

$$\overline{Z}_1(x) = A(x)\, Z_1(x), \qquad \ldots\ldots, \qquad \overline{Z}_N(x) = A(x)\, Z_N(x)$$

form the solution of our problem, a fact which is apparent if we note that $\overline{S}(x) = A(x)\, S(x)$ has the same form as $S(x)$.

It has therefore been completely established under the stated restrictions that the characteristic constants which occur in the characterization of the matrix solutions of a canonical linear differential system can be chosen at pleasure.

The restriction that no three of the quantities $\alpha_1, \ldots, \alpha_n$ shall lie on a straight line is not essential, for if it is not satisfied and *if no two of the polynomials* $p_1(x), \ldots, p_n(x)$ *are identical* it will be possible to replace the rays $\tau_1, \ldots, \tau_N$ which have coalesced by an equal number of curved rays so chosen that the real part of only one of the differences $p_i(x) - p_j(x)$ changes sign along the ray, and thence to apply the preliminary theorem in much the same fashion as before. If two or more polynomials $p_1(x), \ldots, p_n(x)$ are identical it is not necessary to modify the nature of the rays.

Furthermore it would be possible to construct an analogous existence proof when $S(x)$ has for its elements so-called *anormal series.*

In other words, the results here obtained are of an entirely general nature.

§ 15. *The Generalized Riemann Problem.*

The problem which I proposed in my paper on singular points (I) was the following: " *To construct a system of linear differential equations of the first order with prescribed singular points*

$$x_1, x_2, \ldots x_m, x_{m+1} = \infty$$

of respective ranks

$$q_1, \ldots, q_{m+1},$$

and with a given monodromic group, the characteristic constants being assigned for each singular point." This problem is virtually solved by what precedes.

It is of course understood that certain obvious conditions of compatibility are satisfied, the first being the one already noted,

$$T_{m+1}T_m \ldots T_1 = I.$$

However a second necessary condition must also be imposed. Take any assigned point $x = x_i$. The solution $Z_1(x)$ appertaining to this point (see § 14) is transformed successively into

$$Z_2(x)(1 + C_1)^{-1}, \quad Z_3(x)(I + C_2)^{-1}(I + C_1)^{-1},$$
$$\ldots, Z_{N+1}(x)(I + C_N)^{-1} \ldots (I + C_1)^{-1},$$

as x passes over the rays $\tau_1, \ldots \tau_N$ respectively. Hence after a complete circuit of a_i, $Z_i(x)$ alters to

$$Z_1(x) I' (I + C_N)^{-1} \ldots (I + C_1),$$

where the matrix $\overline{T}_i$ of transformation is explicitly determined in terms of the characteristic constants. But in order for this set of characteristic constants to be possible, some solution $Z(x) = Z_1(x) C$ must undergo precisely the transformation by T_i, i. e.,

$$\overline{T}_i = C T_i C^{-1} \quad (i = 1, \ldots, m).$$

This second condition is satisfied if the elementary divisors associated with T_i and $\overline{T}_i$ are the same.

With the understanding that the two conditions of compatibility are satisfied, we can assert that there exist $m+1$ canonical systems with the prescribed characteristic constants at $x_1, \ldots, x_{m+1}$ and with solutions $V_1(x), \ldots, V_{m+1}(x)$ undergoing a transformation to $V_1(x)T_1, \ldots, V_{m+1}T_{m+1}$ as x makes a positive circuit of $x_1, \ldots x_{m+1}$ respectively. Hence there will exist a matrix $Y(x)$ of elements analytic and of determinant not zero save at $x_1, \ldots, x_{m+1}$, properly equivalent to $V_i(x)$ at $x = a_i$ $(i = 1, \ldots, m)$, and properly or improperly equivalent to $V_{m+1}(x)$ at $x_{m+1} = \infty$. This follows upon application of the results of § 10. The matrix $Y(x)$ thus obtained is a matrix solution of a differential system of the form required and has the monodromic group and characteristic constants required, if the equivalence at $x_{m+1} = \infty$ be proper. This is an immediate consequence of the results of §§ 13, 14.

If this is not the case it is easy to show that if one merely increases the characteristic constants $r_1, \ldots, r_n$ for the singular point $x_{m+1} = \infty$ by suitable integers this equivalence at infinity becomes proper. For consider the matrix $S(x)$ corresponding to the matrix solution of the equivalent canonical system at $x = \infty$. The corresponding matrix $\overline{S}(x)$ for $Y(x)$ is of the form $A(x)\ S(x)$ where the elements of $A(x)$ are rational in character at $x = \infty$. Hence if a proper choice of C be made, the matrix $\overline{S}(x)\,C$ may be written (compare (33), § 11).

$$\begin{vmatrix} e^{p_1(x)}x^{\bar{r}_1}\left(a_{11} + \dfrac{b_{11}}{x} + \ldots\right), \ldots, e^{p_n(x)}x^{\bar{r}_n}\left(a_{1n} + \dfrac{b_{1n}}{x} + \ldots\right) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e^{p_1(x)}x^{\bar{r}_1}\left(a_{n1} + \dfrac{b_{n1}}{x} + \ldots\right), \ldots, e^{p_n(x)}x^{\bar{r}_n}\left(a_{nn} + \dfrac{b_{nn}}{x} + \ldots\right) \end{vmatrix}$$

where $\bar{r}_1, \ldots, \bar{r}_n$ differ from $r_1, \ldots, r_n$ by integers. By a process of reduction exactly like that given in § 11 this matrix may be replaced by a series of matrices of the same form in which the exponents $\bar{r}_1, \ldots, \bar{r}_n$ are increased so long as $|a_{ij}| \neq 0$. We conceive of $Y(x)$ as affected by the same series of operations, which have no effect on the properties above specified. Furthermore the transformed $Y(x)$ and $\overline{S}(x)$ stand always in the same relation to each other after as before transformation. If the process comes to an end so that $|a_{ij}| \neq 0$, the linear differential system with matrix solution $Y(x)$ will have a singular point of rank q_{m+1} at $x = \infty$ (compare § 13, pp. 547–548) and the associated properly equivalent canonical system has characteristic constants only modified as stated.

Now we have

$$| Y(x) | \sim | S(x) |$$

in the *complete* vicinity of $x = \infty$, which is made up of the sectors $(\tau_1, \tau_2), \ldots (\tau_N, \tau_{N+1})$. It follows that the series for $| S(x) |$ converges and hence must be of the form

$$e^{p_1(x) + \ldots + p_n(x)} \cdot x^p \left(a + \frac{b}{x} + \ldots \right), \quad a \neq 0,$$

so that always

$$r_1 + \ldots + r_n \leqq p.$$

But the above reductions increase $r_1 + \ldots + r_n$ and do not affect $| S(x) |$ or the value of p, and so must terminate.

We can state then that *either a solution of the stated problem, or of a modified problem in which the constants* $r_1, \ldots, r_n$ *of one of the singular points are altered to* $r_1 + l_1, \ldots, r_n + l_n$ *respectively, where* $l.$, $\ldots$, l_n *are integers, will exist.*

The matrix $Y(x)$ thus obtained is not always unique. The most general determination is however of the form $P(x)\,Y(x)$ where $P(x)$ is a matrix of polynomials of constant determinant which fulfills other conditions. Thus the notion of "primitive systems" admits of extension to the case of irregular singular points.[23]

PART III; THE LINEAR DIFFERENCE EQUATION PROBLEM.

§ 16. *Formulation of the Problem.*

Let

$$Y(x + 1) = Q(x)\,Y(x) \tag{44}$$

be a linear difference system in which the elements of $Q(x)$ are polynomials of degree μ in x.[24] In my earlier paper on linear difference equations I demonstrated that, at least if the above equation admits a formal matrix solution

$$S(x) = [x^{\mu x} (\rho_j e^{-\mu})^x x^{r_j} s_{ij}(x)], \tag{45}$$

in which $s_{ij}(x)$ is a power series proceeding according to negative powers of x with the determinant of the leading coefficients not zero, there exist two matrix solutions $Y^{-}(x)$ and $Y^{+}(x)$, with elements analytic in the finite plane save for poles, such that $Y^{-}(x) \backsim S(x)$ in any left half plane and $Y^{+}(x) \backsim S(x)$ in any right half plane.[25] The existence of such a solution was proved by Nörlund and Galbrun by methods based on the Laplace transformation somewhat earlier.[26]

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[23] Cf. Plemelj, loc. cit. pp. 240–245. Like theorems may be proved in a similar manner here.

[24] Essentially the most general linear difference system with rational coefficients may be reduced to this form; see II, § 5.

These matrices $Y^{-}(x)$ and $Y^{+}(x)$ are connected by a relation

$$Y^{-}(x) = Y^{+}(x)\,P(x) \tag{46}$$

where $P(x)$ is evidently a matrix of periodic functions of period 1.

From the form of (44) it appears that $Y^{-}(x)$ is a matrix of entire functions, while $Y^{+}(x)$ is analytic save for poles.

In my paper I determined explicitly the nature of the elements $p_{ij}(x)$ of $P(x)$ to be the following:

$$\tag{47} \left\{ \begin{aligned} p_{ii}(x) &= 1 + c_{ii}^{(1)} e^{2\pi\sqrt{-1}x} + \ldots + c_{ii} e^{2\pi(\mu-1)\sqrt{-1}x} + e^{2\pi r i\sqrt{-1}} e^{2\pi\mu\sqrt{-1}x} \\ &\qquad\qquad (i = 1, \ldots, n) \\ p_{ij}(x) &= e^{2\pi\lambda_{ij}\sqrt{-1}x}\left[c_{ij}^{(0)} + \ldots + c_{ij}^{2\pi(\mu-1)\sqrt{-1}x}\right] \\ &\qquad\qquad (i \neq j;\ i, j = 1, \ldots, n), \end{aligned} \right.$$

Here λ_{ij} stands for the least integer as great as

$$\Re\left(\frac{1}{2\pi\sqrt{-1}}\left[\log\rho_j - \log\rho_i\right]\right)\cdot \tag{48}$$

An analogous determination in certain cases at about the same time was made by Nörlund (loc. cit.).

It is not difficult to show that, if $Y^{-}(x)$ and $Y^{+}(x)$ have the properties above outlined, then conversely they are solutions of a linear difference system (44) in which the elements of $Q(x)$ are rational if not polynomial.[27] For this reason the constants ρ_j, r_j, $c_{ij}^{(k)}$ may be called the characteristic constants of $Y^{-}(x)$ and $Y^{+}(x)$.

This characterization suggested to me the following problem: To construct a linear difference system (44) with assigned characteristic constants in which the elements of $Q(x)$ are polynomials in x of degree not greater than μ.

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[25] These matrices $Y^{-}(x)$ and $Y^{+}(x)$ correspond to $G(x)$ and $H(x)$ of II. In certain cases it may be necessary to consider half planes not bounded by a vertical line. I refer to this possibility later.

[26] Nörlund, Dissertation, Copenhagen (1911); Galbrun, Dissertation, Paris (1910).

[27] See II, § 7.

§ 17. *Solution of the Problem of* § 16.

In order to treat the problem of § 16 we apply the preliminary theorem. We shall take $r = 1$, and take C_1 to be the axis of imaginaries in the complex plane unless $|P(x)| = 0$ at a point of that axis. The matrix $A_1(x)$ is taken equal to

$$T(x)\,P(x)\,T^{-1}(x), \qquad T(x) = (x^{\mu x}(\rho_j e^{-\mu})^x x^{r_j}\delta_{ij}),$$

except near to $x = 0$, where it is chosen in any way so as to satisfy the restrictions of the preliminary theorem there (compare § 9). Since the elements of $T(x)$ are in general multiple-valued functions of x, it is necessary to specify which branch of $T(x)$ to select. We shall choose a continuous branch of $T(x)$ in the right half plane, and a continuous branch in the left half plane in such a way that these branches coincide along the upper half of the axis of imaginaries. The first factor $T(x)$ in the expression for $A_1(x)$, will be identified with the first of these branches, and the last factor $T^{-1}(x)$ will be the inverse of the second of these branches.

It is therefore clear that, along the upper half of the axis of imaginaries, the element in the ith row and jth column of $A_1(x)$ is, for $i \neq j$,

$$e^{2\pi\lambda_{ij}\sqrt{-1}x}\rho_i{}^x\rho_j{}^{-x}x^{r_i-r_j}\,[c_{ij}{}^{(0)} + \ldots + c_{ij}{}^{(\mu-1)}e^{2\pi(\mu-1)\sqrt{-1}x}]$$

while the diagonal elements are the same as for $P(x)$. But by definition of λ_{ij},

$$(49) \qquad 1 > \Re\left(\lambda_{ij} - \frac{1}{2\pi\sqrt{-1}}(\log\rho_j - \log\rho_i)\right) \geqq 0.$$

Let us exclude at present the case of the equality sign; the element of $A_1(x)$ in the ith row and jth column $(i \neq j)$ will therefore vanish to infinite order together with its derivatives as x goes to infinity along the upper half of the axis of imaginaries. The diagonal elements diminished by 1 have the same properties.

Hence we have $A_1(x) \backsim I$ along the upper half of the axis of imaginaries, while all the derivative matrices of $A_1(x)$ tend to matrices of zero elements as x becomes infinite. If $A_1(x)$ has this character along the lower half of the axis also, it is clear that this matrix satisfies all the restrictions imposed in the theorem.

Let us demonstrate that such is actually the case. The determination of $T(x)$ on the left-hand side of the lower half of the axis of imaginaries is obtained from that on the right-hand side by a complete positive circuit of $x = 0$, during which $T(x)$ changes from

$$(x^{\mu x}(\rho_j e^{-\mu})^x x^{r_j}\delta_{ij}) \quad \text{to} \quad (e^{2\pi\mu\sqrt{-1}x} e^{2\pi\sqrt{-1}r_j} x^{\mu x}(\rho_j e^{-\mu})^x x^{r_j}\delta_{ij}).$$

The ith diagonal element of $A_1(x)$ may now be written

$$c_{ii}^{(0)} e^{-2\pi\sqrt{-1}r_j} e^{-2\pi\mu\sqrt{-1}\,x} + \ldots + 1,$$

while the element in the ith row and jth column of $A(x)$ $(i \neq j)$ may be written

$$e^{2\pi(\lambda_{ij}-1)\sqrt{-1}x} \rho_i^x \rho_j^{-x} x^{r_i - r_j} [c_{ij}^{(0)} e^{-2\pi(\mu-1)\sqrt{-1}x} + \ldots + c_{ij}^{(\mu-1)}].$$

Bearing (49) in mind we readily perceive that $A_1(x)$ does have the indicated properties along the lower half of the axis of imaginaries.

According to the preliminary theorem we can then determine a matrix $\Phi(x)$ such that

$$\lim_{x \doteq x_1^-} \Phi(x) = [\lim_{x \doteq x_1^+} \Phi(x)] A_1(x), \tag{50}$$

where x_1 is a point of the axis of imaginaries and the approach is from the left-hand and right-hand side of that axis respectively. If we take $x = a = 0$, the determinant of $\Phi(x)$ is not zero in the finite plane except at $x = 0$ possibly, and the elements of this matrix are analytic at any point not on the axis. Along the axis as defined from either side these elements have continuous derivatives of all order, and will be analytic at more than certain distance d from the origin. In the vicinity of $x = \infty$, $\Phi(x)$ is represented asymptotically by a matrix of series in $1/x$ with determinant of leading coefficients not zero. This matrix is the same on either side of the axis, since $A(x) \backsim I$.

Let us denote $\Phi(x)$ by $U^+(x)$ for x in the right half plane and by $U^-(x)$ for x in the left half plane, and write

$$\bar{Y}^+(x) = U^+(x)\,T(x), \qquad \bar{Y}^-(x) = U^-(x)\,T(x).$$

From equation (50) we see then that

$$\bar{Y}^-(x) = \bar{Y}^+(x)\,P(x) \tag{51}$$

for $|x| > r$ along the axis of imaginaries. From the asymptotic form of $U^+(x)$ and $U^-(x)$ at $x = \infty$ we obtain

$$\bar{Y}^-(x) \backsim \bar{S}(x), \qquad \bar{Y}^+(x) \backsim \bar{S}(x),$$

in the left and right half plane, where $\bar{S}(x)$ is of the same form as $S(x)$ above. The relation (51) shows that $\bar{Y}^-(x)$ is composed of elements analytic in the right half plane.

Let us now apply the preliminary theorem a second time, taking $r = 1$, and for C_1 a circle with center at the origin and radius so large as to include within it all those points of the axis of imaginaries at which an element of $A_1(x)$ as chosen above is not analytic, and also so as not to pass through a zero of $|\,\overline{Y}^-(x)\,|$.

In this second application of the theorem we choose $A_1(x)$ to be $[\overline{Y}^-(x)]^{-1}$, and in this way satisfy the restrictions of the theorem. Furthermore let us take $a = \infty$.

Along the circle C_1 we have for the solution $\Phi(x)$

$$(52) \qquad \lim_{x=x_1+} \Phi(x) = [\lim_{x=x_1-} \Phi(x)]\,[\overline{Y}^-(x_1)]^{-1},$$

where the approach to the point x_1 of C is from without and within C respectively. Now write

$$Y^-(x) = \Phi(x)\,\overline{Y}^-(x), \qquad Y^+(x) = \Phi(x)\,\overline{Y}^+(x)$$

for x outside of C_1. It follows that $Y^-(x)$ is composed of elements analytic in this region; also along C_1, $Y^-(x)$ coincides with the inner determination of $\Phi(x)$ by (52), so that the elements of $Y^-(x)$ are also analytic within and on C_1. Hence $Y^-(x)$ is a matrix of entire functions. Similar considerations show that the elements of $Y^+(x)$ are analytic in the right half plane like the elements of $\overline{Y}^+(x)$.

At $x = \infty$, $Y^-(x)$ and $Y^+(x)$ are asymptotically represented by a matrix $S(x)$ in which however $r_1, \ldots, r_n$ are not necessarily the same as in $\overline{S}(x)$, and in which the determinant of the leading coefficients may be zero. This results from the fact that the elements of $\Phi(x)$ are rational in character at $x = \infty$ and in consequence can be expanded in convergent series in descending integral powers of x. Inasmuch as we have $|\,S(x)\,| = |\,\Phi(x)\,|\cdot|\,\overline{S}(x)\,|$, it is also true that $|\,S(x)\,|$ cannot reduce formally to zero.

Finally from (51) and (52) we infer that

$$(53) \qquad Y^-(x) = Y^+(x)\,P(x).$$

A first conclusion to be derived is that $Y^-(x) \backsim S(x)$ in any left half plane, and that also $Y^+(x) \backsim S(x)$ in any right half plane. In fact we have already determined the asymptotic form of $P(x)$, and this known form combined with the known asymptotic form of $Y^+(x)$ in the right half plane gives us the form of $Y^-(x)$ in the part of the plane to the left of any line parallel to the axis of imaginaries; a similar remark applies to the asymptotic form of $Y^+(x)$ in any right half plane.

One further remark comes in appropriately at this point. The definition of $\overline{Y}(x)$ and $\overline{Y}^{-}(x)$ ensured that $|\overline{Y}^{-}(x)|$ and $|\overline{Y}^{+}(x)|$ do not vanish to the left or right of the axis of imaginaries respectively, save possibly for $x = \infty$. Hence $|Y^{-}(x)|$ and $|Y^{+}(x)|$ do not vanish in these left or right half planes respectively.

We may now enter upon a series of modifications of $Y(x)$ which will preserve the above stated properties and secure in addition that the determinant of the leading coefficients of $S(x)$ is not zero. To this end we write the matrix $S(x)$ in the form

$$\begin{vmatrix} x^{\mu x}(\rho_1 e^{-\mu})^x x^{r_1}\left(a_{11}+\frac{b_{11}}{x}+\dots\right), & \dots, & x^{\mu x}(\rho_n e^{-\mu})^x x^{r_n}\left(a_{1n}+\frac{b_{1n}}{x}+\dots\right) \\ \dots & \dots & \dots \\ x^{\mu x}(\rho_1 e^{-\mu})^x x^{r_1}\left(a_{n1}+\frac{b_{n1}}{x}+\dots\right), & \dots, & x^{\mu x}(\rho_n e^{-\mu})^x x^{r_n}\left(a_{nn}+\frac{b_{nn}}{x}+\dots\right) \end{vmatrix}$$

and carry out reductions parallel to those given in § 11. The same reductions are supposed to be simultaneously effected upon $Y^{-}(x)$ and $Y^{+}(x)$ (compare § 15). This set of reductions will terminate, since $|S(x)|$ does not vanish identically, and, when it does, $Y^{-}(x)$ and $Y^{+}(x)$ will have the desired additional property.

Consider now the matrix

$$Q(x) = Y^{-}(x+1)\,[Y^{-}(x)]^{-1} = Y^{+}(x+1)\,[Y^{+}(x)]^{-1}.$$

These two forms for $Q(x)$ are equal in virtue of (53) and yield us at once the asymptotic form of the elements of $Q(x)$ in the complete vicinity of $x = \infty$ as descending power series in x with leading term of the μth degree at most.[28] Hence the elements of $Q(x)$ are rational in character at $x = \infty$, with a pole of at most order μ there.

In the finite plane to the left or along the axis of imaginaries the first expression for $Q(x)$ shows that the elements of $Q(x)$ are analytic without exception. It will be recalled that $Y^{-}(x)$ is a matrix of entire function of determinant not zero in the left half plane inclusive of the axis of imaginaries. On the other hand the elements of $Q(x)$ are analytic to the right of the axis of imaginaries, as the second form shows.

Accordingly $Q(x)$ is a matrix of polynomials of degree at most μ and $Y^{-}(x)$, $Y^{+}(x)$ are solutions of the rational difference system (44).

Finally it may be observed that in case $|P(x)| = 0$ along the axis of imaginaries, a parallel line may be used to take the same rôle; or indeed any simple analytic curve without a horizontal tangent and with vertical asymptote, provided that $|P(x)| \neq 0$ along the curve.

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[28] Cf. I, § 7.

If the equality sign obtains in (49) it will be necessary to employ a curve with asymptote not quite in the vertical and to employ half planes not bounded by a vertical line.

It is also possible to replace $S(x)$ by certain *anormal forms,*[29] and thus extend the above results to the most general case.

Our conclusion may be summed up as follows: *There exists a linear difference system* (44) *with matrix solutions* $Y^-(x)$, $Y^+(x)$ *which either possesses prescribed characteristic constants* $\rho_j, r_j, c_{ij}^{(k)}$, *or else constants* ρ_j, $r_j + l_j$, $c_{ij}^{(k)}$ *where* $l_1, \ldots, l_n$ *are integers. For an arbitrary curve which meets each line parallel to the real axis only once, having a vertical asymptote, and which does not pass through a point* $|P(x)| = 0$, *there exist such matrices* $Y^-(x)$, $Y^+(x)$ *with the further property that* $|Y^-(x)| \neq 0$ *to the left of the curve while the elements of* $Y^+(x)$ *are analytic and* $|Y^+(x)| \neq 0$ *to the right of the curve.*

It is worthy of note that this last property determines the location of the poles of the elements of $Y^+(x)$ completely: namely, they occur to the left of the curve and at the points for which $|P(x)| = 0$. This appears from the formula

$$Y^+(x) = Y^-(x)\, P^{-1}(x),$$

which also permits us to affirm that the precise maximum order of pole of any element of $Y^+(x)$ is the order of the zero of $|P(x)|$.

PART IV: THE LINEAR q-DIFFERENCE EQUATION PROBLEM.

§ 18. *On Linear q-Difference Equations.*

A linear q-difference system may be written

$$Y(qx) = Q(x)\, Y(x) \qquad |q| > 1, \tag{54}$$

where $Q(x)$ is a matrix of polynomials of degree μ or less, in analogy with the normal form (46) of linear difference systems. The apparently more general case in which the elements of $Q(x)$ are rational in x may be reduced to this form readily. Let the least common denominator of the elements of $Q(x)$, written as quotients of relatively prime polynomials, be

$$(x - a_1) \ldots (x - a_l)$$

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[29] Analogous to the anormal series for linear differential equations. These forms have recently been obtained by Mr. P. M. Batchelder.

and let $g_i(x)$ be a solution of the q-difference equation of the type (54)

$$g(qx) = (x - m)\, g(x) \tag{55}$$

for $m = a_i$. If one takes for new variable

$$\overline{Y}(x) = g_1(x) \ldots g_l(x)\, Y(x),$$

a new matrix equation (54) in $\overline{Y}(x)$ is obtained with $\overline{Q}(x)$ polynomial in x.

Let us write

$$t = \frac{\log x}{\log q}.$$

In terms of this new variable a solution of (55) for $m = 0$ is

$$q^{\frac{1}{2}(t^2 - t)}.$$

For $m \neq 0$, the transformation

$$x = m\bar{x}, \qquad y(x) = e^{\pi\sqrt{-1}\frac{\log \bar{x}}{\log q}}\, m^{\frac{\log \bar{x}}{\log q}}\, \bar{y}(\bar{x})$$

takes (55) to the normal form

$$y(qx) = (1 - x)\, y(x). \tag{56}$$

Two solutions of this equation are

$$\left\{\begin{aligned} y_0(x) &= \left(1 - \frac{x}{q}\right)\left(1 - \frac{x}{q^2}\right)\cdots, \\ y_\infty(x) &= q^{\frac{1}{2}(t^2-t)} e^{-\pi\sqrt{-1}\,t}\, \frac{1}{1 - \frac{1}{x}} \cdot \frac{1}{1 - \frac{1}{qx}} \cdots, \end{aligned}\right. \tag{57}$$

as one may verify by direct substitution. The function $y_\infty(x)$ plays the same rôle for the linear q-difference equations as the gamma function does in the theory of linear difference equations. I have mentioned these functions in order to supply an example later.

The fundamental existence theorems for linear q-difference equations are essentially a consequence of the work of Grévy[30] and Leau.[31] The first complete treatment has been given by Carmichael,[32] and the result may be expressed as follows: There exist in general two matrix solutions

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[30] Paris thesis, 1894.

[31] Paris thesis, 1897.

[32] Am. Jour. Math., **34**, 147–168 (1912).

$$\left\{\begin{aligned} Y_0(x) &= (x^{\rho_j}\, a_{ij}(x)) \\ Y_\infty(x) &= q^{\frac{\mu}{2}(t^2-t)}\, (x^{-\sigma_j} b_{ij}(x)), \end{aligned}\right. \tag{58}$$

where each function $a_{ij}(x)$ is analytic at $x = 0$, and each function $b_{ij}(x)$ is analytic at $x = \infty$; and where furthermore the determinants of the leading coefficients of $a_{ij}(x)$ and $b_{ij}(x)$ at $x = 0$ and $x = \infty$ respectively are not zero. It is only the case when such series exist that will be here considered.

It follows at once from (54) that $Y_0(x)$ is a matrix of functions analytic for $x \neq 0, \infty$, and that $Y^-(x)$ is a matrix of functions analytic in the finite plane except for poles when $x \neq 0$. Further, if we write

$$Y_0(x) = Y_\infty(x)\, P(x),$$

then $P(x)$ is a matrix of functions analytic for $x \neq 0, \infty$, and possessing the property that $P(qx) = P(x)$.[33] These properties are in close analogy with the properties for a linear difference system, to which indeed (54) reduces formally by the substitution $x = q^t$.

I propose now to determine completely the nature of $P(x)$, as I have done for the analogous functions $P(x)$ associated with the linear difference system; it is the doubly periodic functions which enter here instead of the simply periodic functions. By means of this determination it will be possible for us to state the problem which, for this field, is analogous to the problems above treated for linear differential and linear difference equations.

§ 19. *On the Matrix $P(x)$.*

Let us make the transformation $x = q^t$ and write

$$P(x) = \overline{P}(t).$$

The function $\overline{P}(t)$ is a single-valued function of t analytic save for poles; for, this transformation takes the Riemann surface of infinitely many leaves, with logarithmic branch points at $x = 0$ and $x = \infty$ in a one-to-one and conformal manner into the t-plane.

Let us conceive of the t-plane as divided into parallelograms which belong to the periods $\omega = 1$, $\omega' = 2\pi\sqrt{-1}/\log q$, and let $ABCD$ be a parallelogram with vertices

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[33] Cf. Carmichael, loc. cit., p. 159.

$$x_0, \qquad x_0 + 1, \qquad x_0 + 1 + \frac{2\pi\sqrt{-1}}{\log q}, \qquad x_0 + \frac{2\pi\sqrt{-1}}{\log q},$$

respectively. At homologous points of BC and AD, $P(t)$ has the same value, since $\overline{P}(t+1) = \overline{P}(t)$.

To obtain the relation between $P(t)$ at homologous points of AB and DC we consider first the matrix

$$(59) \qquad P(x) = Y_\infty^{-1}(x)\, Y_0(x).$$

It is apparent from the form of the elements of $Y_0(x)$ near $x = 0$ as given by (58) that, if a positive circuit of $x = 0$ be made, $Y_0(x)$ will change to $Y_0(x)\,K$, where

$$K = (e^{2\pi\rho_j\sqrt{-1}}\,\delta_{ij});$$

likewise, upon a similar circuit, $Y_\infty(x)$ will change to

$$(-1)^{\mu} e^{2\pi\mu\sqrt{-1}\,t}\, e^{-\frac{2\pi^2\mu}{\log q}}\, Y_\infty(x)\, L,$$

where

$$L = (e^{-2\pi\sigma_j\sqrt{-1}}\delta_{ij}).$$

If these modified matrices be substituted in (59) we obtain the form which $P(x)$ assumes after x has made a positive circuit of the origin. This is

$$(-1)^{\mu} e^{-2\pi\mu\sqrt{-1}\,t}\, e^{\frac{2\pi^2\mu}{\log q}} L^{-1} P(x)\, K.$$

If therefore $p_{ij}(x)$ denotes the element in the ith row and jth column of $P(x)$, such a circuit modifies $p_{ij}(x)$ to

$$(-1)^{\mu} e^{-2\pi\mu\sqrt{-1}\,t}\, e^{\frac{2\pi^2\mu}{\log q}}\, e^{2\pi(\sigma_i+\rho_j)\sqrt{-1}}\, p_{ij}(x).$$

But this circuit in the x-plane corresponds to a passage from a point of AB in the t-plane to the homologous point of DC; in this way, letting $\overline{p}_{ij}(t)$ stand for the element of $\overline{P}(t)$ in the ith row and jth column, we find

$$(60) \quad \overline{p}_{ij}\left(t + \frac{2\pi\sqrt{-1}}{\log q}\right) = (-1)^{\mu} e^{-2\pi\mu\sqrt{-1}\,t}\, e^{\frac{2\pi^2\mu}{\log q}}\, e^{2\pi(\sigma_i+\rho_j)\sqrt{-1}}\ \overline{p}_{ij}(t)$$

We have also seen that

$$(61) \qquad \bar{p}_{ij}(t+1) = \bar{p}_{ij}(t).$$

Now let us attempt to satisfy (60) and (61) by writing

$$(62) \qquad \bar{p}_{ij}(t) = ce^{at^2+bt}\sigma(t-\alpha_1)\ldots\sigma(t-\alpha_\mu),$$

where $\sigma(t)$ is the Weierstrass sigma function belonging to the periods

$$\omega = 1, \qquad \omega' = \frac{2\pi\sqrt{-1}}{\log q},$$

which satisfies the relations

$$(63) \quad \begin{cases} \sigma\left(t+\dfrac{2\pi\sqrt{-1}}{\log q}\right) = -e^{\eta'\left(t+\frac{\pi\sqrt{-1}}{\log q}\right)}\sigma(t), \\ \sigma(t+1) = -e^{\eta(t+\frac{1}{2})}\sigma(t), \qquad \eta\dfrac{2\pi\sqrt{-1}}{\log q} - \eta' = 2\pi\sqrt{-1}. \end{cases}$$

The above choice of ω and ω' meets the requirement $\Re(\omega'/\omega) > 0$, since $|q| > 1$.

A direct substitution into (61) determines

$$(64) \qquad a = -\eta\frac{\mu}{2}, \qquad b = \eta\sum_{i=1}^{\mu}\alpha_i + \mu\pi\sqrt{-1} + 2k\pi\sqrt{-1},$$

in which k denotes any integer. If these values of a and b are taken and a direct substitution is made in (60), there is obtained the further condition

$$(65) \qquad \sum_{i=1}^{\mu}\alpha_i + \frac{\mu\pi\sqrt{-1}}{\log q} = \sigma_i + \rho_j + l - \frac{2k\pi\sqrt{-1}}{\log q},$$

in which l denotes an integer. These conditions are necessary and sufficient that an expression of the form (62) shall satisfy both (60) and (61). If the value for $\sum_{i=1}^{\mu}\alpha_i$ deduced from (65) be used in (64) the expression for b simplifies to

$$\eta(\sigma_i + \rho_j + l) - \eta'\left(\frac{\mu}{2} + k\right).$$

But if one adds or subtracts a period to α_i the precise effect is to alter k or l by an integer. It is therefore always permissible to take $k = l = 0$ and to write

$$(66) \quad \bar{p}_{ij}(t) = c_{ij}e^{\frac{-\eta\mu}{2}t^2+\left[\eta(\sigma_i+\rho_j)-\frac{\eta'\mu}{2}\right]t}\sigma\left(t-\alpha_1^{(i,j)}\right)\ldots\sigma\left(t-\alpha_\mu^{(i,j)}\right),$$

provided that

$$\sum_{\lambda=1}^{\mu} a_\lambda^{(i,j)} = \sigma_i + \rho_j - \frac{\mu\pi\sqrt{-1}}{\log q}. \tag{67}$$

This is of course under the assumption that $\bar{p}_{ij}(t)$ may be represented in the form (62). But this fact may be proved at once. For let $\psi(t)$ be any function which satisfies (60) and (61), and $\phi(t)$ the particular one above obtained. The function $\psi(t)/\phi(t)$ is doubly periodic, analytic save for poles, and can therefore be expressed as a quotient of products of sigma functions

$$C\frac{\sigma(t-\gamma_1)\ldots\sigma(t-\gamma_k)}{\sigma(t-\beta_1)\ldots\sigma(t-\beta_k)}, \qquad \Sigma\gamma_k = \Sigma\beta_k.$$

But this quotient when multiplied by $\phi(t)$, which is expressed as a product of sigma functions, must yield $\psi(t)$, an entire function. This necessitates that for each zero of $\sigma(t-\gamma_j)$ in the numerator $(i = 1, \ldots, \mu)$ there must be a congruent zero $x = \beta_j$ of $\sigma(x-\beta_j)$ in the denominator. Such pairs of corresponding factors may be combined leaving only an exponential factor e^{ct+d}. Thus $\psi(t)$ appears in the same form as $\phi(t)$.

Our result is therefore that the element $p_{ij}(x)$ of $P(x)$ is of the form (66) $(t = \log x/\log q)$, where $\rho_1, \ldots, \rho_n$, $\sigma_1, \ldots, \sigma_n$ are the constants that appear in the series representations (58), and where the conditions (67) are fulfilled.

It is interesting to apply these results to the equation (56) in which $n = 1$, $\mu = 1$, $\rho_1 = 0$, $\sigma_1 = \pi\sqrt{-1}/\log q$, and $Y_0(x)$ and $Y_\infty(x)$ reduce respectively to $y_0(x)$ and $y_\infty(x)$ defined in (57). In this case we have therefore

$$y_0(x) = y_\infty(x)\,p(x) \tag{68}$$

where

$$p(x) = c\,e^{\frac{-\eta}{2}\left(\frac{\log x}{\log q}\right)^2 - \pi\sqrt{-1}\frac{\log x}{\log q}}\,\sigma\left(\frac{\log x}{\log q}\right).$$

The constant c may be determined by writing (68) in the form

$$y_0(x) = [(x-1)\,y_\infty(x)]\left[\frac{p(x)}{x-1}\right]$$

and allowing x to approach 1. Since $\sigma(0) = 0, \sigma'(0) = 1$, this gives (see (57))

$$c = \left(1-\frac{1}{q}\right)^2\left(1-\frac{1}{q^2}\right)^2\ldots \tag{69}$$

The relation (68) with the explicit values of $y_0(x)$, $y_\infty(x)$ and $p(x)$ substituted in is essentially one of the fundamental product formulas for the sigma function.

§ 20. *The q-Difference Equation Problem.*

It is now easy to show that conversely if $Y_0(x)$, $Y_\infty(x)$ are matrices of functions of the form (58) in the vicinity of $x = 0$ and $x = \infty$ respectively, analytic for $x \neq 0, \infty$, save for poles, and if the matrix $P(x)$, defined by the relation $Y_0(x) = Y_\infty(x)P(x)$, is composed of elements $p_{ij}(x)$ which are left unchanged when x is replaced by qx, then $Y_0(x)$ and $Y_\infty(x)$ are matrix solutions of a linear q-difference system (54) with rational coefficients. In fact, if we write

$$Q(x) = Y_0(qx)\,Y_0^{-1}(x) = Y_\infty(qx)\,Y_\infty^{-1}(x),$$

it is seen at once that for $x \neq 0, \infty$, the only singularities of $Q(x)$ are poles, while the first and second of these forms for $Q(x)$ ensure that $Q(x)$ is composed of elements analytic at $x = 0$ and with a pole of at most order μ at $x = \infty$ if not analytic there. This suffices to establish the fact that the elements of $Q(x)$ are rational.[34] In order to conclude further that the elements of $Q(x)$ are polynomials of degree μ it is sufficient to know that the plane may be divided into two parts by a loop about $x = 0$ meeting each equiangular spiral or radial line

$$(70) \qquad \theta = c + \frac{\arg q}{\log|q|}\log r \qquad (r,\ \theta,\ \text{polar coördinates})$$

only once and not passing through a point $|P(x)| = 0$, such that the elements of $Y_0(x)$ are analytic and $|Y_0(x)|$ is not zero within or along the loop, while the elements of $Y_\infty(x)$ are analytic and $|Y_\infty(x)|$ is not zero outside the loop. Under these conditions the first expression for $Q(x)$ makes it evident that its elements are analytic within or along the loop, and the second expression makes it clear that the same is true without the loop, since if x is a point without the loop so is qx.

It is natural to term the $2n$ constants ρ_j, σ_j and the $n^2(\mu+1)$ constants c_{ij}, $a_1^{(i,j)}, \ldots, a_\mu^{(i,j)}$ *the characteristic constants.* These constants are not all independent, since there are n^2 relations between the constants $a_k^{(i,j)}$ of the type (67). Furthermore the constants c_{ij} are not uniquely determined by the given q-difference system. For, any ith column of $Y_0(x)$ is only determined up to a constant factor f_i and likewise any ith column of $Y_\infty(x)$ is only determined up to a constant factor h_i. This fact appears from (58). The effect is to allow one to replace c_{ij} in $p_{ij}(x)$ by $f_j c_{ij}/h_i$, and thus vary at will $2n - 1$ of the characteristic constants c_{ij}. There remain then essentially only

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[34] Compare II, § 7.

$$2n + n^2(\mu + 1) - n^2 - (2n - 1) \quad \text{or} \quad n^2\mu + 1$$

characteristic constants. It is readily verified that these constants are all invariant under a transformation $\overline{Y}(x) = C\overline{Y}(x)$, where C is an arbitrary matrix of constants.

But the equation (54) involves $n^2(\mu + 1)$ arbitrary coefficients in $Q(x)$ of which there are $n^2(\mu + 1) - (n^2 - 1)$ or $n^2\mu + 1$ invariants under the same linear transformation. Hence we have found as many invariants for the q-difference system as invariant characteristic numbers for the solutions.

We are thus led to formulate the following problem: To construct a q-difference system (54), with coefficients of degree not greater than μ in x, having any assigned set of characteristic constants.

§ 21. *Solution of the Problem of* § 20.

Here also we shall employ the preliminary theorem, but the application of it is even simpler than in the earlier cases.

The conclusion that we shall derive is the following: *There exists a linear q-difference system* (54) *with the matrix solutions* $Y_0(x)$, $Y_\infty(x)$ *either possessing prescribed characteristic constants* $\rho_i, \sigma_i, c_{ij}, \alpha_1^{(i,j)}, \ldots, \alpha_\mu^{(i,j)}$ *or else constants* $\rho_j, c_j + l_j, c_{ij}, a_1^{(i,j)}, \ldots, \alpha_\mu^{(i,j)}$, *where* $l_1, \ldots, l_n$ *are integers. For an arbitrary loop about* $x = 0$ *which cuts each spiral* (70) *only once and does not pass through a point* $|P(x)| = 0$, *there exist matrices* $Y_0(x)$, $Y_\infty(x)$ *with the further property that* $|Y_0(x)| \neq 0$ *within or along the loop while the elements of* $Y_\infty(x)$ *are analytic and* $|Y_\infty(x)|$ *is not zero without the loop.*

Let C_1 be a specified loop of this description which may be taken to be analytic.[36] We may take the matrix $A_1(x)$ of the preliminary theorem to be

$$T_\infty(x)\, P(x)\, T_0^{-1}(x),$$

where

$$T_0(x) = (x^{\rho_j}\delta_{ij}), \qquad T_\infty(x) = q^{\frac{\mu}{2}(t^2-t)}(x^{-\sigma_j}\delta_{ij}),$$

since the elements of $A_1(x)$ are single-valued and analytic along C_1 by (60).

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[36] It is only a question as to how the loop weaves among the zeros of $|P(x)|$.

According to the theorem there exists a matrix $\Phi(x)$ such that

$$\lim_{x=x_1+} \Phi(x) = [\lim_{x=x_1-} \Phi(x)] A_1(x_1), \tag{71}$$

where x_1 is an arbitrary point of C_1 and the $+$ and $-$ signs denote approach to x_1 from within and without C_1 respectively; the matrix $\Phi(x)$ possesses certain other properties: it is composed of elements analytic for x not on C_1, save at a point a which we shall take to be at infinity; furthermore its determinant does not vanish in the finite plane.

Let us denote by $U_0(x)$ the matrix $\Phi(x)$ within C_1, and its analytic extension across C_1; similarly by $U_\infty(x)$ let us denote the matrix $\Phi(x)$ without C_1 and its analytic extension across C_1. Consider then the matrices

$$Y_0^-(x) = U_0(x)\, T_0(x); \qquad Y_\infty(x) = U_\infty(x)\, T_\infty(x).$$

From (71) we obtain at once

$$Y_0^-(x) = Y_\infty(x)\, P(x), \tag{72}$$

and prove without difficulty that $Y_0^-(x)$ and $Y_\infty^+(x)$ as thus defined have the characteristics demanded save possibly that $Y_\infty(x)$ may not be precisely of the form (58) at $x = \infty$, as it would be if $\Phi(x)$ were composed of elements analytic at $x = \infty$ and if also $|\Phi(x)|$ were different from zero at $x = \infty$. Nevertheless one can always write $Y_\infty(x)$ in the form

$$q^{\frac{1}{2}(t^2-t)} \begin{vmatrix} x^{-\bar{\sigma}_1}\left(a_{11} + \dfrac{b_{11}}{x} + \ldots\right), \ldots & x^{-\bar{\sigma}_n}\left(a_{1n} + \dfrac{b_{1n}}{x} + \ldots\right) \\ x^{-\bar{\sigma}_1}\left(a_{n1} + \dfrac{b_{n1}}{x} + \ldots\right), \ldots & x^{-\bar{\sigma}_n}\left(a_{nn} + \dfrac{b_{nn}}{x} + \ldots\right) \end{vmatrix}$$

where $\bar{\sigma}_1, \ldots, \bar{\sigma}_n$ differ from $\sigma_1, \ldots, \sigma_n$ by integers. By a process of reduction precisely like that employed earlier (§ 11) one may further modify $Y_0(x)$ and simultaneously $Y_\infty(x)$ so as to preserve all of the properties already noted and to finally obtain $|a_{ij}| \neq 0$. It is to be recalled that $|Y_0(x)|$ is not identically zero and can at most vanish to a finite order at $x = \infty$; for it is this fact that enables us to conclude that the process of reduction terminates.

The argument of the preceding paragraph shows that $Y_0(x)$ and $Y_\infty(x)$ will be matrix solutions of a system (54) with coefficients polynomials in x of degree μ at most.

It is deserving of notice that the properties stated above determine the location of the poles of the elements of $Y_\infty(x)$, namely at the zeros of $|P(x)|$ within the loop, the maximum order of the pole of any element being precisely the order of the corresponding zero of $|P(x)|$. This is an immediate consequence of the relation.

$$Y_\infty(x) = Y_0(x)\,P^{-1}(x).$$

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STUDIES ON THE PERIPHERAL NERVOUS SYSTEM OF AMPHIOXUS.

By Harriet Lehmann Kutchin.

With Eight Plates.

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By Harriet Lehmann Kutchin.

Presented by E. L. Mark. Received June 2, 1913.

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Introduction.

The literature on the nervous system of Amphioxus presents such divergence of opinion that it is probable many facts with regard to the structure, distribution and functions of this system still remain to be determined and illustrated. It will be impossible to more than theorize as to the primitiveness or the degeneracy of the nervous system of Amphioxus until the main features of its organization are more clearly understood. The present research aims to contribute toward a knowledge of the structure and distribution of the peripheral nerves, and it is hoped may aid in furnishing a basis for comparison with other vertebrates.

The following studies were undertaken in 1904 by the writer while holding the Alice Freeman Palmer Fellowship of Wellesley College.

Contributions from the Bermuda Biological Station for Research. No. 28.

I wish to express my sincere thanks to that institution for the exceptional opportunities afforded by the fellowship. Professor E. L. Mark of Harvard University suggested this field for investigation, and I am also indebted to him for kindly advice and criticism. The work was carried on in 1904–1905 at the Bermuda Biological Station and in the Harvard Zoölogical Laboratories while I was registered as a Graduate Student in Radcliffe College. In the spring of 1905 the problem was continued at the Naples Zoölogical Station, through the kindness of the "Association for Maintaining the American Women's Table at the Zoölogical Station at Naples and for Promoting Scientific Research by Women." I am under obligation to those in charge of the Naples Station for many courtesies extended throughout my stay. During the year 1905–1906 laboratory privileges were kindly afforded me for further work in the Zoölogical Laboratories of the University of Chicago.

Two species have been used as a basis for study, the Caribbean Amphioxus, Branchiostoma caribaeum, found in Bermuda waters, and the form so abundant at Naples, Branchiostoma lanceolatum.

LITERATURE.

Retzius (**'91**) and Dogiel (**:02**) have reviewed the literature on the peripheral nervous system of Amphioxus in considerable detail, presenting particularly points where lack of agrreement exists. Other authors, as Rohde (**'88**) and Heymans et van der Stricht (**'98**), give historical accounts in connection with their personal observations. It is therefore unnecessary to give here a connected discussion of the literature; the work of the several authors will be taken up in connection with each question in its bearing upon the descriptive part of this paper. An extensive bibliography of Leptocardii is given by Lönnberg (**:01**—, pp. 206–214).

MATERIAL AND METHODS.

At the Bermuda Biological Station the tissues of a large number of Branchiostoma caribaeum were impregnated with methylene blue by the *intra-vitam* method. Gold chloride and the methods of Golgi were also extensively used. Many variations were employed in the use of all these methods, with the view of bringing out details of structure, and these methods were found to give excellent results. Material was also hardened in various fixing fluids, 10% formol proving to be particularly valuable. The same methods were employed at Naples upon Branchiostoma lanceolatum, but a larger number of preparations was made, to provide against the uncertainty of impregnation. Further discussion of these methods will be taken up in the body of this paper, but a few general remarks are in place here.

The best impregnations of both superficial and visceral nerves were obtained by immersion of the living animals in sea-water colored a moderately dark blue with a stock mixture of $\frac{1}{2}$% to 1% of methylene blue in normal salt solution. The sea-water mixture should never be opaque. It is probable that the small quantity of salt in the mixture causes the epithelium to loosen, thus permitting more direct action of the methylene blue. Such preparations are not, of course, suitable for study of sensory endings in the skin. Specimens immersed in this mixture do not appear impaired in vigor at the end of two or three hours. The subsequent exposure to air recommended by Dogiel (**: 02**) is of great importance; the length of time required for different nerves can be determined only by continued observation under the microscope. The specimen must be kept moist with the methylene-blue mixture during such exposure. Fixation was usually effected by the ammonium-picrate method, and it was found that the addition of a few drops of 1% osmic acid to each 100 cc. of the ammonium picrate used for fixing these preparations greatly aided in their preservation. Material treated in this way, preserved in the usual ammonium-picrate and glycerine mixture, and carefully guarded against unnecessary exposure to light, was found in excellent condition at the end of two years, while other preparations fixed without osmic acid were practically useless after one year.

The gold-chloride method recommended by Hardesty (**: 02**) for use after fixation with 10% formol proved useful in demonstrating motor fibers and their endings. I regard this method as worthy of wider use because of its accurate fixation and the comparative absence of artifacts. The action of the gold chloride is, however, no more certain in this method than in others.

Mallory's (**: 00**) method for study of the central nervous system proved useful in the case of Amphioxus for the peripheral nervous system. Vom Rath's fluid (strong) was employed, but with indifferent results, except for the central nervous system. The methods of Golgi for impregnation with silver, and their various modifications, are all useful in obtaining impregnations of different parts of the nervous system. The rapid method is usually most certain and reliable. Picro-carmine employed after treatment with very dilute osmic acid is suitable for only the thinnest tissues. Structures much beneath the external surface receive absolutely no fixation, and are therefore quite unreliable for study.

A Nernst electric lamp, or carefully adjusted direct sunlight, was found to bring into view nerves which it was quite impossible to observe with the ordinary adjustments of light. This is particularly true of methylene-blue preparations of the whole animal, which show plexuses between dorsal nerves, and of thick Golgi preparations. With such strong light the nerves in thick parts of the body may be seen *in situ* with surprising clearness. This method has the disadvantage of soon tiring the eyes. Various methods of dissection were employed on impregnated and stained material for the study of the visceral nerves.

DORSAL NERVES.

A. Nerves of the Rostrum.

Nerve I.— The usually accepted first-nerve pair was first noted by Goodsir (**'41**), and has been repeatedly described. Owsjannikow (**'68**) interpreted it as the *trigeminus*. Schneider (**'79**) says that from its position the first nerve should be the *opticus*. Rohon (**'82**, p. 60) regards the first two pairs of nerves as analogous to the sensory elements of the *trigeminus*, the third pair to the corresponding (gleichnamigen) *facialis* of higher vertebrates. Later authors hesitate to compare the first nerve with any nerve of the higher vertebrates until some facts with regard to its function are established. Dogiel (**: 02**) designates the first nerve as purely sensory, since its branches ramify exclusively in the skin. The latter author has described in detail the manner of branching of this nerve in Branchiostoma lanceolatum, and defines the territory which it innervates as the end of the rostrum, with an occasional extension into the adjoining dorsal and ventral skin regions. Dogiel finds some variation in the size and manner of branching of the first nerve.

Edinger (**: 06**) describes a pair of nerves arising, like an olfactory nerve, from the base of the brain anterior to the nerve pair which is commonly designated as the first. My observations have not brought to light such a nerve, but I do not deny its existence, since I have not used the silver-impregnation method of Bielschowsky, by means of which Edinger demonstrated the nerve fibers which he describes.

My observations with regard to the commonly accepted nerve I agree in the main with those of Dogiel. This nerve and its branches, as well as nerve II, exhibit peculiarities during the process of *intra-vitam* impregnation with methylene blue. Nerves I and II usually become colored in the course of 20 to 30 minutes, before the sensory nerves of the thicker parts of the body do. If the immersion in methylene blue is continued, these anterior nerves shortly lose their color, but if examined after a considerably longer immersion (1½ to 3 hours), nerves I and II again appear impregnated, frequently with great brilliancy. This indicates that there is more than one epoch in the course of immersion in methylene blue when a successful impregnation of the anterior sensory nerves may be obtained. Another possibly significant fact with regard to the physiological character of the first two sensory nerves is that the methods of Golgi repeatedly produced no impregnation in these nerves in specimens whose sensory nerves were otherwise well impregnated.

Figures 1 to 5 are drawn to show the distribution of nerve I in Branchiostoma caribaeum. These furnish a basis for comparison with the previously published figures of Branchiostoma lanceolatum. Figure 6 (Pl. **2**) shows the exit of nerve I from the neural tube in B. caribaeum, and Figure 11 (Pl. **3**) presents its exit in B. lanceolatum. A comparison of the two species shows no striking differences in the manner of branching of this nerve, nor in the territory which it innervates. I find, as does Dogiel, that some variations occur in the size of this nerve, and in its area of distribution. In the same specimens complementary variations are to be noted in the manner of distribution of the branches of nerve II.

In Figures 11 and 12 particular care has been taken to illustrate the relation of the first two nerves to the most anterior myomere. These figures represent the rostrum without its epithelial covering, this having sloughed off in the course of treatment following immersion in methylene blue. The fact that the main trunk of the first nerve usually gives off no branches for a considerable distance after its exit from the neural tube is of interest as distinguishing it from all other dorsal nerves. The cells of Quatrefages have no definite arrangement with relation to the nerves of the rostrum, as may be readily noted by comparison of Figures 1 to 5.

The main stem of nerve I, as well as the anterior end of the nerve cord, does not bear a constant relation to the chorda dorsalis nor to the anterior portion of the first myomere. Occasionally little of the main stem of nerve I is visible; in other cases it may be observed for almost its entire length, while in still other specimens its union with the neural tube, and even a portion of the tube itself, are exposed to view. The main stem usually lies close to the dorsal surface of the chorda.

Nerve II.— In 1841 Johannes Müller described the second nerve, enumerating it as the first (J. Müller **'44**, p. 95, 96). Quatrefages (**'45**) numbered the branches of nerve II as nerves II, III, IV, and V, and figured the terminal branches of nerve I in the rostrum as arising from nerve II. This conception was based upon his idea that the first was an *optic* nerve, a short stalk connecting brain and "eye spot." Schneider (**'79**) figures (Taf. 15, Fig. 1) two roots for nerve II of the right side, stating (p. 14) that this nerve usually possesses two roots upon one side, and upon the other divides immediately after its origin.

Dogiel (**:02**) gives an extended and accurate account of the distribution of the branches of nerve II, as well as the variations which occur in the form of the nerve itself, and the territory which it innervates. He finds from one to three nerves in the region of the exit of nerve II, conditions not being symmetrical in this respect on the two sides of the body. He enumerates these irregularly occurring nerves as distinct nerves, without considering their relation to the myomeres. Although his figures do not present the muscle segments with clearness, they convey an inaccurate idea of the number and position of these segments. Dogiel describes several forms of variation from the usual course to the rostrum of nerves I, II, and III (as he enumerates them). In one case nerve II is poorly developed, and a branch of his nerve III innervates the ventral part of the rostrum (Dogiel's Fig. 6*b*, which illustrates this condition, shows the roots of nerves II and III in close proximity). In another case, of frequent occurrence, nerve II is strongly developed and only the posterior ventral part, if any, of the rostrum receives branches of nerve III. (In Dogiel's Fig. 6*c*, and 6*d*, illustrating this case, the nerve designated as III is the one commonly accepted as III.) Occasionally his nerve III (shown in his Fig. 7, which is not the nerve usually enumerated as III) ramifies exclusively in the skin of the dorsal fin, a ventral branch being absent. In such cases the skin of the posterior ventral portion of the rostrum is supplied by branches from his nerve IV. (From Dogiel's Fig. 7 this is clearly the commonly accepted nerve III.) In another form of variation (Dogiel's Fig. 8*a*, and 8*b*) his nerve III consists of two or three dorsal branches and a small ventral ramus, which does not reach the ventral border of the rostrum. According to his figures this nerve (III) makes its exit near the root of nerve II. In a few cases (illustrated by his Fig. 9) Dogiel observed directly posterior to the large root of nerve II two small nerves, which he designated as nerves III and IV. Neither of these possesses a ventral ramus which reaches to the ventral border of the rostrum; in such cases the skin of the rostrum is in part supplied by his nerve V. It is clear from his figure that the nerve numbered V is the one usually designated as III.

From the foregoing description it will be seen that Dogiel does not regard the anterior sensory nerves as occupying definite relations to the myomeres. Changes in the numbering of the anterior sensory nerves must of course affect the numbering of all the other sensory nerves. Thus one, two or three roots in the region of the exit of nerve II, if designated in accordance with Dogiel's view as separate nerves, would correspondingly change the numbering of all the succeeding dorsal nerves, which show such a constant relation to the muscle segments. Since the two sides are frequently not symmetrical with regard to the number of roots in the region of nerve II, one might have, according to Dogiel's system of enumeration, the correspondingly numbered nerves of the two sides belonging to myomeres *not* correspondingly numbered.

I have examined a large number of specimens, frequently with the aid of artificial light, and find that roots accessory to the main root of nerve II are often present, as is shown in my Figures 11 and 12 (Pl. **3**). These two figures present the two sides of the same individual and illustrate the manner of exit of the roots of nerves II and III, and their relation to the myomeres. They also show the lack of symmetry in the roots of nerve II on the two sides of the body. Figures 6 and 7 (Pl. **2**) show other conditions in the roots of nerve II. These do not differ in their form and position from the nerves described by Dogiel for this region. The main root of nerve II, and those roots that make their exit in close proximity to it, always lie anterior to the first myomere of the adult. Small roots may lie considerably posterior to the main root of nerve II and still occupy a position anterior to the first myomere. This is possible owing to the form of the muscle segment. Examination of a large number of individuals leaves in my mind no doubt of this condition. A distinct myomere intervenes between the root (or roots) of nerve II and that of the commonly accepted nerve III (Figs. 11 and 12). Moreover, the territories innervated by these nerves, while subject to some variation, are comparatively well defined. The dorsal nerves make their exit in definite relation to the myomeres, as is illustrated in Figure 6, and a logical enumeration of the nerves could not allow the association of the same myomere with different nerves in different individuals. Nerve III has a definite character and territory of distribution; its number should not depend upon the number of roots possessed by nerve II, nor on their being variously designated as nerves III, IV or V. It is not an unknown condition for a dorsal nerve in the more posterior parts of the body to have two distinct roots making their exit through the same myoseptum. The possible cranial character of the nerves anterior to the first myomere in the adult can scarcely authorize the enumeration of each separate root in the region of the place of exit of nerve II as a distinct nerve root; because the lack of symmetry on the two sides of the body, and the irregularity of the occurrence of these roots, would leave no basis for rational enumeration. The fact is that nerve II supplies a fairly definite territory, consisting usually of a large part of the rostrum and an adjoining portion of the dorsal fin.

I have not observed the small muscle segments which Dogiel figures (his Fig. 6*a*, 6*b*, 8*b*) anterior to the origin of nerve II. The anterior boundary of the first muscle segment of the adult is often difficult to distinguish, and in my own observations strong artificial light was frequently employed to make this clear. Careful camera drawings were made of the dorsal portions of the anterior myomeres, as shown in Figures 6, 11 and 12. In specimens of both species the anterior projection of each muscle segment forms a much more acute angle than Dogiel's figures indicate. In this connection Hatschek's (**'92**) views regarding the first myomere may be mentioned. He states that in the larvae of Amphioxus the muscle fibrillae of the rostral process are well formed even out to the tip of the rostrum; but in the fully developed animal they become rudimentary, only a remnant persisting. In transverse sections of young Amphioxus (not larvae) treated with Mallory's differential stain, I have observed the cut ends of muscle fibers, lying in front of the anterior border of the first adult myomere. These fibers were short and few in number, but clearly differentiated by the stain from the surrounding connective tissue.

Nerve II was studied to a considerable extent in Branchiostoma caribaeum, and its distribution in the rostrum of this species is shown in Figures 1–5. As this species has been little illustrated, it is thought best to reproduce the conditions rather fully, for such figures form an interesting basis of comparison with Branchiostoma lanceolatum. No very essential differences, however, occur between the two species. Variations in the number and arrangement of the roots of nerve II are probably as frequent in one form as in the other. Hatschek (**'92**, Fig. 6) figures a branch of nerve II as passing to the deeper nerve plexus of the mouth. I have never observed such a branch, but am aware that variations occur. However, such a condition can scarcely be the typical one. Nerves I and II usually send no branches to deep lying structures.

No new observations concerning the olfactory pit were made in the present study, and this structure has not been included in the enumeration of the dorsal nerves.

With regard to the spinal or cerebral character of the more anterior dorsal nerves, and especially those having no ventral roots, there has been much variation of opinion. Owsjannikow (**'68**) and Stieda (**'73**, p. 48) designate the first two nerve pairs as cranial. Stieda, however, denies the existence of nerves of special sense. Langerhans (**'76**, p. 279) says that the first and second nerves are distinguished from the others by the possession of peripheral ganglion cells, as well as by their origin. Schneider (**'79**, p. 14) regards the first and second nerves, and the bulbus olfactorius as cranial, but considers it uncertain whether they differ in function from the remaining sensory nerves. Rohde (**'88**) calls the first five sensory nerve pairs cerebral. It does not fall within a discussion of the distribution and structure of the peripheral nerves, to treat of the question as to the cranial or spinal character of the more anterior dorsal nerves. The structure of the neural tube, and the functions of these nerves must enter largely into such a determination. If, as Hesse (**'98**) states, the whole spinal cord reacts to light stimulation, we may well make careful investigation of the physiological character of the more anterior nerves, before ascribing any special degree of cephalization to Amphioxus. Heymans et van der Stricht (**'98**, p. 68) say, in regard to this question, that the anterior extremity of the spinal cord of Amphioxus might perhaps be considered anatomically as a head (in its first stage of development), but that physiologically the head perhaps extends over the whole nerve axis, though especially developed in the segment corresponding to the buccal and branchial region.

B. Nerves of the Buccal Region.

Nerve III.— The third dorsal nerve usually sends a small branch, or branches, to the posterior part of the rostrum, but by far the greater part of its branches are confined to the buccal region. It may, therefore, be properly included with the nerves of this group.

Certain earlier authors, as Schneider ('**79**) and Rohon ('**82**), regarded the third pair of dorsal nerves as the first true sensory spinal nerves, this interpretation being based partly on the fact that the first pair of motor roots is associated with this pair of sensory roots. These authors enumerated the "spinal nerves" separately, designating the third sensory pair as the first spinal sensory roots. At present it seems safest to enumerate this pair of dorsal nerves as the third, without attempting to distinguish between so-called cranial and spinal nerves.

Heymans et van der Stricht ('**98**) have given a careful account of the distribution of the branches of this pair of nerves in Branchiostoma lanceolatum, and Dogiel (**: 02**) discusses nerve "III" in the same species at considerable length. As has been noted, however, Dogiel ascribes no definite territory of distribution to the third pair of nerves, its number depending upon the number of roots present near the place of exit of nerve III from the neural tube. This uncertainty makes comparison difficult and much involved. It may be stated, however, that in his discussion of the innervation of the border of the mouth Dogiel designates the commonly accepted third nerve as III, as is shown in his Figur 1.

I have studied this nerve in both species, chiefly upon specimens impregnated with methylene blue, although gold chloride and picrocarmine bring out excellently many details in the nerves of the oral hood. The finer branches of the nerves of the buccal region are usually impregnated in from 15 to 30 minutes in the methylene-blue mixture previously described, while the main trunks of these nerves require a longer immersion. A rather long subsequent exposure to air (20 to 30 minutes) gave peculiarly brilliant results for the finer branches. Fixation with ammonium molybdate and osmic acid, after Dogiel's (**: 02**) method, is well adapted to this region. The thin tissues fix rapidly, and may be dehydrated quickly without loss of impregnation. It is advisable before fixation to detach the anterior portion of the animal, cutting a little behind the velum. By dissecting along the mid-dorsal line, a flat, fairly thin preparation may be obtained for mounting in balsam. Thick pieces of tissue fixed in ammonium molybdate are usually unsatisfactory for sections or other methods of study.

In the two species studied surprisingly little difference was found in the distribution of the branches of the third pair of nerves. Variations occur as frequently in Branchiostoma caribaeum as in B. lanceolatum, and are similar in character. In both species nerve III usually possesses a dorsal and a ventral ramus, although in rare cases either may be absent, with the possible exception of the ventral ramus of the left side. I was unable to find any specimens in which the latter branch was entirely lacking. Occasionally the root subdivides at or near its place of exit from the neural tube, and either ramus is represented by two or more branches. The primary division in nerve III usually occurs near the neural tube, but considerable variation exists in this regard. Different forms of division in the two species are illustrated in Figures 6, 7, 11 and 12. In these figures the comparative uniformity in the place of exit of the nerve through the myoseptum may be noted.

The curious post embryonic history of the buccal region brings about the well known asymmetry in certain of the buccal nerve pairs. The dorsal ramus of the third nerve of either side usually innervates the neighboring portion of the dorsal fin, and a small area overlying the adjoining trunk muscles. A small branch of the ventral ramus of nerve III on either side of the body supplies the posterior ventral portion of the rostrum. Twigs from this branch may anastomose with branchlets of nerve II. In its course over the side muscles, this more anterior ventral branch of nerve III gives off numerous subdividing side branches. The main branch of the ventral ramus of either side takes a course ventrad and noticeably anteriad over the side muscles, and also gives off a number of side branches destined for the skin of this region. In this part of its course, or at the ventral border of the trunk muscles, this main branch of the ventral ramus of the third nerve, on either side of the body, gives off a branch (or branches) which passes to the anterior portion of the border of the mouth of the corresponding side and breaks up into the outer mouth plexus of this region. This branch usually leaves the more ventral side of the main branch, but may vary considerably in size and position. The main ventral branch of left nerve III is usually of greater size than the corresponding branch of the right side, and supplies a larger territory. This main branch divides at or near the ventral border of the trunk muscles of the left side into two branches, one of which passes beneath the ventral border of these muscles and emerges on the right side of the body, where it is continued across the oral hood to form the anterior portion of the inner mouth plexus of this side. The other branch traverses the oral hood on the left side, and breaks up into the left anterior portion of the inner mouth plexus. Thus the most anterior portion of this inner plexus on either side of the hood, is supplied by branches from left nerve III, while the outer mouth plexus of the same region is formed symmetrically from branches of nerve III, one the right nerve, the other the left. Heymans et van der Stricht (**'98**) also note this condition in Branchiostoma lanceolatum. Dogiel (**: 02**) describes in detail the structure of the plexuses of the mouth border. These plexuses will be briefly considered later in this paper. I found no branches of left nerve II joining the inner mouth plexus, as described by Hatschek; on the contrary, the specimens observed indicate that the second nerve pair is not concerned in the innervation of the deep-lying structures of the mouth border.

Interesting variations occur in the distribution of the branches of nerve III in both species. In one case (B. lanceolatum) a branch of right nerve III anastomosed with the branch of left nerve III on the right side, thus actually joining the inner mouth plexus. Ordinarily the ramifications of left nerve III in the inner mouth plexus on either side join each other near the median line, but in one specimen (B. caribaeum) no anastomosis occurred, thus presenting the curious condition of an incomplete "nerve ring" in the inner mouth plexus.

Nerve IV.— This nerve usually impregnates with methylene blue coincidentally with nerve III, but the portion extending over the trunk muscles often shows a better impregnation after longer immersion. Bilateral asymmetry exists in the fourth nerve pair, as in the third pair, and many variations occur in its manner of division, and the distribution of its branches. As shown in Plate **2**, Figure 6 (B. caribaeum), nerve IV arches slightly more than nerve III immediately after its exit from the neural tube. Near its place of exit, this nerve divides into a dorsal and a ventral ramus. More than one dorsal ramus may be present, and the ventral ramus may subdivide near its origin; but I have never noted the entire absence of the ventral ramus, or its equivalent, on either side of the body. Occasionally the dorsal ramus of a sensory nerve is absent, or represented by a twig of practically no importance. In such cases its territory is supplied by branches of an unusually well developed dorsal ramus of one of the two adjoining nerves. The place of division of nerve IV into a dorsal and a ventral ramus may be readily noted in Figure 6. This place of division is often concealed by the muscles, but frequently may be ascertained by the aid of strong light, especially in methylene-blue preparations preserved in glycerine and ammonium picrate. Such preparations often become very transparent. Heymans et van der Stricht (**'98**) say that the bifurcation of the dorsal roots into dorsal and ventral rami is always made before exit from the myoseptum, except in the case of the last caudal root. According to my observations, the place of division in the first 6–8 anterior nerves may occur at, or slightly after, the exit of the nerve from the myoseptum, although it frequently occurs nearer the neural tube.

The stem of the dorsal ramus of nerve IV is usually deflected more or less anteriorly (Figs. 6 and 7, Pl. **2**). This ramus divides repeatedly, distributing its branchlets over the surface of the neighboring trunk muscles, and in that portion of the dorsal fin lying approximately dorsal to the nerve root. The ventral ramus of nerve IV is usually of greater size than the corresponding ramus of nerve III, especially upon the left side. This ramus of nerve IV takes a general ventrad course over the trunk muscles; but in its course over these muscles it forms an arch the convexity of which is directed anteriorly. This arch is frequently more prominent on the left side, and a like one may also be noted in the ventral ramus of nerve V (Pl. **2**, Fig. 7). Dogiel (**: 02**) states that an arch is also formed in the course of the ventral ramus of nerve III, and occasionally in nerve VI. I, however, have observed a *marked* arch only in the course of nerves IV and V. On the left side this arch is accentuated by the fact that the branch of this ramus which innervates internal substructures bends toward the interior of the body. The ventral ramus of nerve IV gives off numerous subdividing branches in its course over the side muscles. On the right side it divides into two or more descending branches, which innervate a portion of the outer wall of the oral hood, and finally break up into a part of the outer mouth plexus. These descending branches may be formed either in the course of the ventral ramus over the trunk muscles, or at the ventral border of these muscles, or even in the oral hood. The branches which cross the oral hood usually give off only small side branchlets. The ventral ramus of left nerve IV innervates a much larger territory, and shows considerable variation in the number of its branches, and their manner of distribution. This ramus usually divides into its main branches near the ventral border of the side muscles, but often varies in this regard. A branch, or branches, to the outer plexus of the mouth may be given off from the main branch at any point in its course over the trunk muscles (Fig. 7). Other branches to this plexus may be given off at or near the place of main division mentioned above.

The remaining branches formed at this place of division are destined for less superficial structures. Of these, one or more take a course ventrad across the oral hood and form that part of the inner plexus of the mouth lying posterior to the portion formed by branches of left nerve III. The branches of left nerve IV which aid in the formation of the inner mouth plexus lose their slender, thread-like appearance as they approach the plexus, and become flattened and band-like (Pl. **2**, Fig. 7, *i;* Pl. **4**, Fig. 18, *i.*). These branches lie interior to those (*o.*) forming the outer mouth plexus, the difference in focus increasing in their distal portions. The branches forming the inner plexus also lose their fibrous appearance in methylene-blue preparations as they approach the plexus, and appear granular; this change may be noted in all branches of dorsal nerves innervating deep-lying structures. At the place of main division of the ventral ramus of left nerve IV a very large branch is also given off, which bends at once beneath the ventral border of the trunk muscles, and passes to the right side of the oral hood. Here it may emerge at varying places, usually, however, near the ventral border of the trunk muscles of the right side, and often anterior to the ventral branches of right nerve IV (Fig. 18). This branch of left nerve IV subdivides in various ways in the right wall of the oral hood, and the branches thus formed break up into that portion of the inner mouth plexus of the right side which lies posterior to the part formed by the branch of left nerve III, previously described. No other nerve branches from nerves on either side of the body appear to be concerned in the formation of the inner plexus of the mouth border on the right side, in either species.

Another large, more posterior branch of left nerve IV leaves the ventral ramus at its main place of division. This branch bends beneath the trunk muscles toward the interior, and after reaching the median region ventral to the notochord, runs posteriad as a thick, straight trunk. In methylene-blue preparations in which epithelial structures are not retained, this nerve branch appears to end abruptly a short distance in front of the velum, and slightly to the right of the median line (Fig. 8, IV*a*). This nerve may correspond to the branch of nerve V described by Hatschek (**'92**, p. 144) as innervating the sensory groove of Hatschek. Much variation certainly occurs in the distribution of the branches of the dorsal nerves in the buccal region. In this connection it may be noted that Heymans et van der Stricht (**'98**, Pl. VI, Fig. 22) figure a branch of left nerve IV passing directly to the right side of the velum. I, however, have been unable to find any connection between this nerve branch and the nerves of the velum; but, in some instances another large branch of the ventral ramus of left nerve IV clearly did join in the innervation of the velum. This branch is usually given off from the ventral ramus of the latter nerve at or near the main place of division, and most frequently enters the velum on the left side of the body. In one specimen (B. lanceolatum), however, after taking a posteriorly directed course for some distance along the inner border of the trunk muscles of the left side, this branch turned dorsad running toward and a little beyond the median plane, where it entered the velum slightly to the right of this plane. This is the only instance observed of a left nerve branch entering the velum on the right side of the body. In some cases a branch of left nerve IV is *indirectly* connected with the innervation of the velum by anastomosis with a large branch of left nerve V which passes directly to the velum. These variations occur in both species, and make it difficult to define the territory innervated by the fourth nerve pair. Such variations no doubt account, in large measure, for the apparent disagreements between the various descriptions of this nerve.

Nerve V.— Nerve V (Pl. **2**, Fig. 7) is in many respects similar to nerve IV. It is usually of about the same size, and the ventral ramus forms an arch similar to that described for nerve IV. The dorsal ramus of nerve V innervates a territory corresponding to that of the dorsal ramus of nerve IV, but more posterior, and the portion of the ventral ramus taking a course over the trunk muscles gives off branches similar to those leaving this portion of the ventral ramus of nerve IV. The place of division of nerve V into a dorsal and a ventral ramus in B. caribaeum is illustrated in Figure 6. The fifth nerve pair is also bilaterally asymmetrical.

The ventral ramus of right nerve V usually divides at or near the ventral border of the trunk muscles into two or more branches, which cross the oral hood and form that part of the outer mouth plexus lying posterior to the portion formed from branches of right nerve IV. Occasionally only a single small branch of right nerve V joins the outer plexus of the mouth border. The ventral ramus of left nerve V is larger than that of right nerve V, and exhibits considerable variation in its manner of division. The main branches are usually given off at or near the ventral border of the lateral muscle. In Figure 7 (B. caribaeum) a branch to the inner mouth plexus may be observed leaving the ventral ramus about half way in its course over the trunk muscles. One or more branches usually pass to the outer mouth plexus from the main place of division of this ventral ramus, and a like number leave this place for the inner plexus of the mouth border. A band-like nerve branch is also given off from left nerve V near the ventral border of the side muscles, which frequently anastomoses with a branch of left nerve VI (Figure 8). This nerve branch is usually concerned in the innervation of the velum. Another form of connection between nerves V and VI is illustrated in Figure 7. As has been stated, a branch of left nerve IV sometimes anastomoses with left nerve V in this region; left nerves VI and VII often show some form of connection with it in the neighborhood of the ventral border of the trunk muscles. Thus, with the slight connection sometimes occurring between nerves II and III, a basis is formed for the band-like nerve described by Hatschek (**'92**) as connecting nerves II–VII in this region. Hatschek's Figure 6 must be somewhat diagrammatic, however, since this condition is found only on the left side, and even here these connecting nerves are not constant in occurrence or size. In nearly all cases left nerve V sends one or more branches to the velum. These branches usually leave the ventral ramus near the ventral border of the side muscles (Figs. 7, 8), and enter the velum on the left side of the body. At least one of these branches forms an anastomosis with a branch or branches of left nerve VI, and frequently a similar anastomosis is formed with a branch or branches of left nerve VII. These anastomoses vary greatly in character; it is often difficult to determine the share of each nerve in the innervation of the velum. This is especially true when, as sometimes happens, branches are given off to the inner mouth plexus ventral to the anastomosis between branches of left nerves V and VI (Fig. 7).

Nerve VI.— This nerve requires only a brief description. A form of division of the trunk of this nerve in B. caribaeum may be noted in Figure 6 (Pl. **2**). The dorsal rami of this nerve pair are similar to those of the fifth pair of nerves; their main branches are illustrated in Figures 6 and 7. The ventral ramus of the sixth nerve, both the right and the left, is usually smaller than the corresponding ramus of nerve V, but the cutaneous branches given off in its course over the trunk muscles correspond to those leaving the similar portion of nerve V. In both species the remaining branches of right nerve VI are distributed in the oral hood and external portion of the mouth border of the right side. The ventral ramus of right nerve VI divides near the ventral border of the trunk muscles into two or more slender branches, which take a general course ventrad across the oral hood, and break up into a portion of the outer mouth plexus. The corresponding ramus of left nerve VI sends a similar branch, or branches, to the outer plexus of the mouth border on the left side of the body. Left nerve VI frequently sends a branch, or branches, to the inner plexus on the left side of the mouth, but this is not uniformly the case. In several specimens examined branches of left nerves III, IV, and V formed the entire left portion of the inner mouth plexus. Heymans et van der Stricht ('**98**, p. 36) state that the plexus of Fusari is formed from branches of left dorsal nerves III–VI, and perhaps VII. As has been stated, a branch of left nerve V usually anastomoses with a branch, or branches, of left nerve VI. From this anastomosis a branch passes to the velum, frequently uniting in its course with a branch of left nerve VII (Fig. 7). This branch to the velum may divide in various ways before entering the latter structure. Its branches, however, enter the velum on the left side of the body.

Nerve VII.— Figure 6 (Pl. **2**) shows this nerve leaving the neural tube slightly nearer its dorsal surface than nerve VI. The dorsal ramus of nerve VII on either side of the body is similar to that of nerve VI, except for a slight increase in size, due to the larger territory lying dorsal to the seventh nerve root. The ventral ramus of nerve VII on either side of the body does not differ from that of nerve VI in the region of the trunk muscles. Right nerve VII sends one or more branches to the posterior portion of the outer mouth plexus. These branches give off small side branches in their course across the oral hood. Nerve VII of the right side occupies a position considerably posterior to that of left nerve VII, and its ventral ramus frequently lies exterior to the velum. In such cases right nerve VII is usually the most posterior nerve of the right side sending branches to the outer plexus of the mouth border. In nearly all the specimens examined the branches of left nerve VII were found to be exclusively cutaneous in their distribution. This nerve sends branches to the outer mouth plexus of the left side, similar to the corresponding branches of right nerve VII. I have been unable to find a branch, or branches, of left nerve VII in either species directly connected with the inner plexus of the mouth border. As has been noted, Heymans et van der Stricht ('**98**) state that left nerve VII is perhaps concerned in the formation of the plexus of Fusari. This may be the case in rare instances, but is certainly not the usual condition. These authors also state that left nerve VII contributes toward the innervation of the velum. Dogiel (: **02**) also says that nerve VII not infrequently takes part in the innervation of the velum. The specimens examined by me revealed no large branches of left nerve VII passing directly to the velum. In a few cases a small branch, or branches, of this nerve anastomosed with branches of left nerve V or VI (Figure 7); therefore fibers from branch VII may take this course to the velum; but it is difficult to determine the course of nerve fibers after such an anastomosis.

Nerve VIII.— This nerve will be briefly considered here, although its branches, for the most part, supply the branchial region. It may be noted in Figure 6 that this nerve leaves the neural tube very near its dorsal surface, and arches noticeably immediately after its exit. The dorsal ramus of nerve VIII on either side of the body, and the branches of this nerve pair in the region of the trunk muscles, are so similar to the corresponding branches of the two next anterior nerves, already described, that a separate description is unnecessary. I have been unable to find a branch of the ventral ramus of right nerve VIII which unquestionably joins the outer plexus of the mouth border. However, since branches of this nerve sometimes lie considerably anterior to the velum, it seems probable that they may occasionally contribute toward the formation of that plexus. Left nerve VIII frequently sends a small branch to this plexus (Fig. 7), but such a branch is as frequently lacking. None of the specimens examined showed a branch of left nerve VIII passing to the velum, as described by Dogiel (**: 02**).

The variation in the territory innervated by the individual anterior nerves is of considerable interest, since the branches of a nerve usually supplying a certain area, may be supplanted by those of another nerve which leaves the neural tube at some distance from the place of exit of the nerve ordinarily innervating the given territory. It appears from this that the same nerve does not always carry fibers to precisely the same area. The plexuses and communicating nerve bands of this region may have a bearing on this point, and lead to doubt as to which nerves actually supply certain territories. The relation of the nerve components of the neural tube to each other in this region must also be of interest. It is certainly a fact worthy of consideration that a great nerve, like nerve IV, leaving the neural tube at a definite place, may or may not take a prominent part in the innervation of the velum.

The plexuses of the mouth border have been frequently described, especially by Fusari (**'89**) and Dogiel (**: 02**), and will not be taken up in detail in this paper. Besides the inner and outer plexuses of this region, Dogiel describes a "Zwischengeflecht," lying between them and taking its origin chiefly from the outer plexus. The nerve branches taking part in the formation of the plexuses of the mouth border in the specimens of both species observed by me have been already enumerated. Figure 18 (Pl. **4**) illustrates a portion of the outer plexus in Branchiostoma caribaeum. This plexus does not appear to be essentially different from the corresponding plexus in B. lanceolatum. The inner plexus is also apparently similar in the two species.

C. Nerves of the Velum.

The nerves of this structure have been so little described and figured that they will be considered under a separate heading in this paper. Rathke, as early as 1841, mentions the velum, giving the number of tentacles as sixteen. Rolph (**'76**) notes that the velum carries about ten cirri. Rohon (**'82**), van Wijhe (**'93**), and Hatschek (**'92**) mention nerve branches passing to the velum; but the most detailed description of the innervation of this structure is found in the work of Heymans et van der Stricht (**'98**). These authors state that the voluminous circular nerve of the velum is formed from branches of left dorsal nerves IV, V, VI and VII, and appears to occupy about the middle part of the sphincter muscle, at the interior of which it gives off large branches. With regard to nerves passing to the velar tentacles, these authors state (p. 36): "nous avons également observé à l'intérieur des tentacules des fibres nerveuses que nous avons pu poursuivre jusque entre des cellules épithéliales." Contrary to van Wijhe, they found in their preparations no impregnation of sense cells in the velum. As mentioned in the discussion of nerve IV, Heymans et van der Stricht describe a posterior branch of left dorsal nerve IV which passes under the body of Hatschek, and reaches the right portion of the velum. These authors illustrate (Pl. VI, Fig. 22) the large nerve branches passing to the velum, and also its circular nerve. Dogiel (**:02**) finds branches from left nerves IV, V, VI, VII, and even VIII and IX, taking part in the innervation of the velum. The branches of nerves IV and V are usually, however, most important. Dogiel's enumeration of the more anterior nerves must be considered in connection with this statement. This author shows (Taf. 20–21, Fig. 13) a portion of the plexus of the circular muscle of the velum. In this figure branches of left nerves V, VI and VIII form the velar plexus. In his Figur 1, branches from left nerves IV, V, VI and VIII pass to the velum. The branches of nerves VIII and IX which he finds occasionally passing to the velum are fine branchlets of the rami viscerali of these nerves.

My personal observations on the innervation of the velum were made largely from dissected specimens of Branchiostoma lanceolatum and B. caribaeum impregnated by the intra-vitam method with methylene blue. Most of these specimens were fixed in ammonium picrate, and preserved in a mixture of the ammonium picrate and glycerine. A few specimens were fixed in ammonium molybdate, dissected and mounted in balsam. In either case, a few drops of osmic acid added to the fixing fluid appeared to aid greatly in preserving the impregnation. It was usually most convenient to use only the anterior portion of the body in studying the velum, severing the body a short distance behind that structure. This anterior portion was prepared for study under a dissecting microscope by cutting with fine scissors along the longitudinal axis, usually on the right side, as close as possible to the dorsal fin. Certain specimens were cut in different regions, so as to make it possible to determine the course of the nerve branches on both sides of the body. These preparations were arranged on a slide in the preserving mixture, with the aid of camel's hair brushes. The velar tentacles were arranged in the position showing the nerves to best advantage. Thin supports beneath the cover-glass were usually necessary to prevent injury. Strong light was found useful in following the deep-lying nerve branches, but the velar plexus, when impregnated, is usually visible with ordinary light. The velar nerves require a long immersion in the methylene-blue mixture ($1\frac{1}{2}$ to $2\frac{1}{2}$ hours), with a subsequent exposure to air of about one hour. Unless osmic acid is added to the fixing fluid, the epithelial covering of the velum sloughs off to a large extent.

Although considerable variation exists in the innervation of the velum, Figure 8 (Pl. 2) presents a fairly typical picture of the branches of the dorsal nerves which pass to this structure, and the distribution of the nerves in the velum itself. Figure 9 shows the nerves of a large and a small velar tentacle in greater detail. These figures present the first published views of the entire plexus of the velar muscle, and of the nerves of the velar tentacles. These figures do not show the epithelial covering of the velum, except for occasional cells apparently remaining attached to nerve branches.

Figure 8 may be taken as a basis for the description. In this specimen branches of the ventral rami of left nerves V and VI supply the velum. A somewhat complicated alternation of anastomosis and division takes place between these branches as they pass to the velum. A large branch of left nerve V takes a course along the ventral muscle border to the ventral ramus of left nerve VI, and anastomoses with the latter. Two branches destined for the velum are given off near this place of union. The larger (α) takes a backward course along the ventral border of the trunk muscles for a short distance, and then bends around the muscle border to the inner surface of the side muscles, where it is continued posteriorly with considerable deflection dorsalward. As it bends toward the interior, this large branch gives off a much smaller one (γ), which passes posteriad and ventrad, dividing after a short distance into two branches, which reach the velum a short distance ventral to the muscle border. These two latter branches unite in the velar muscle, and the band-like nerve thus formed may be followed in almost the entire circumference of the ring muscle, although its size varies considerably in different portions.

The smaller branch (β), leaving the place of anastomosis between the branches of nerves V and VI, takes a general course backward over the inner surface of the trunk muscles, running for a short distance dorsal to the larger branch. Near the velum these two nerve branches (α, β) unite, forming a short, thick trunk. This trunk divides into two short, diverging branches of equal size. The more dorsal branch (*b*′) enters the velum about the width of two myomeres dorsal to the ventral muscle border of the left side, and is continued dorsad in the circular muscle of the velum as a slightly irregular, thick band. This band curves over to the right side of the velum, and breaks up into the velar plexus. The more ventral (δ) of these short, diverging branches continues ventrad in the ring muscle of the velum, losing its band-like character at the base of the fifth tentacle ventral to the border of the trunk muscles. These band-like nerves lie chiefly near the bases of the skeletal portions of the velar tentacles, but do not form a continuous "band nerve." The velar plexus lies, for the most part, between these thick, flat nerves and the free portions of the tentacles. A comparatively large branch passes to each tentacle obliquely from one of the band-like nerves; this oblique deflection is usually ventralward, but may vary. A single large tentacle in the mid-ventral region receives two main branches, one from the right side, and one from the left. The velar plexus is completed by numerous small branches, which anastomose to form an irregular, loose-meshed network between the band nerves of the circular muscle and their large branches which supply the tentacles. Toward the more external border of this plexus many apparently free-ending small nerve branches are visible. These may terminate in some form in the velar epithelium, or may possibly join a finer unimpregnated plexus.

The nerves of the velar plexus generally exhibit a flattened, granular appearance when impregnated with methylene blue, and their outlines do not present the clear-cut aspect noticeable in cutaneous branchlets of dorsal nerves. Their structure calls to mind that of the nerve branches forming the inner plexus of the mouth border, and the inner abdominal plexus. Apparent oval nuclei, or nucleated cells, may be frequently observed in the course of the finer branches of the velar plexus (Pl. 2, Fig. 9).

The tentacles of the velum are usually alternately large and small, but this arrangement does not obtain with absolute regularity. Their number does not appear to be constant, but is in the neighborhood of fifteen. The innervation of the tentacles presents a beautiful appearance when impregnated with methylene blue. Figures 8 and 9 illustrate these nerves with different degrees of detail. From these figures it may be observed that each large nerve passing to a tentacle breaks up suddenly, at the base of its tentacle, into a great number of small branches, which divide, anastomose and interlace throughout the entire free portion of the tentacle. Such meshes as are formed are usually much elongated parallel to the long axis of the tentacle. These nerves of the velar tentacles are also granular in appearance, and occasional small nuclei or cells occur in the course of the nerve threads. Free nerve ends may be noted, but these must always be considered in connection with the fact that in methylene-blue preparations little epithelium favorable for studying the general innervation of the velar tentacles persists. In some instances small, darkly-staining thickenings of the nerve ends may be seen projecting beyond the border of the tentacles, or exterior to the network of nerves (Fig. 9, *c*). These may represent some form of nerve ending in the epithelium of the velar tentacles. The elaborate innervation of the tentacles suggests that many nerve endings may exist in them.

The variation mentioned in the nerves supplying the velum has no doubt led to the apparently conflicting statements with regard to the numbers of these nerves. In the specimens observed, only branches of nerves of the left side of the body passed to the velum, and these usually enter the velar muscle on the left side. A branch of left nerve IV has been described which entered the velum slightly to the right of a mid-dorsal line, but this case appeared to be exceptional. Nerve IV may send a branch directly to the velum, or a branch may anastomose with that of nerve V on its way to the velum. In Figure 7 branches of nerves VI and VII anastomose with the branch of nerve V passing to the velum. It is difficult to explain just what these anastomoses mean. Nerve IV sometimes sends a large branch to the velum, nerves V and VI usually send branches of considerable size, but nerve VII seldom contributes largely toward the innervation of the velum. The branch of left nerve IV indicated at IV*a* in Figure 8 appears to be associated with the nerves of the velum, but is actually entirely free from the velar muscle. I have frequently observed a branch of left nerve IV ending in this manner in this region. Variation no doubt occurs in this nerve branch, and it may therefore correspond to the branch of nerve V described by Hatschek ('**92**, Figur 6, *N. S.*) as passing to the groove of Hatschek. Heymans et van der Stricht ('**98**) state that the N. recurrens of Hatschek ends at the velum, and has nothing in common with the branchial plexus. I have never observed a large branch from the buccal nerves reaching the branchial basket.

D. *Nerves of the Branchial Region.*

In this group are included the dorsal nerves lying between the velum and the region anterior to the atriopore. The term "branchial" is therefore unsatisfactory in some respects, but is perhaps the one most available for use in a description of the nerves. In B. lanceolatum this region is supplied by nerves VIII to XL, or XLI, inclusive, while in B. caribaeum it seems probable that nerves XXXVIII and XXXIX innervate the region of the atriopore. This indicates that the difference in the number of myotomes in the two species occurs, in part, in the branchial region. Dogiel (:**02**) states that in B. lanceolatum nerve XLII is the most posterior nerve supplying the region of the atriopore. He describes the fine-meshed plexus ramifying about this opening, and finds that branches of nerves XXXVIII–XLII may take part in its formation. Variations certainly occur in these nerves, as in the case of other dorsal nerves.

The nerves of this region impregnate particularly well with methylene blue, but require a longer immersion than those of the anterior part of the body, especially in the case of the visceral branches. Strong light was very useful in following the finer nerve branches in the thicker portions of the body. Various branches of these dorsal nerves have been repeatedly described; the accounts, which are somewhat fragmentary, will be taken up in connection with the description of my personal observations.

The dorsal nerves of this region differ little from each other in their main characteristics, and in certain respects resemble those of the buccal region. It may be noted that, as seen in Plate **2**, Figure 6, the arch formed by each dorsal nerve as it leaves the neural tube gradually increases in height toward the middle region of the body. Here the arch remains practically constant, but toward the posterior region it again diminishes in height, and at the extreme posterior end, as will be noted later, no arch is present. In the anterior portion of the branchial region the place of division of the dorsal nerves into dorsal and ventral rami is usually at, or just before, the exit of each nerve from the myoseptum (Fig. 6). In the remainder of this region the place of division lies deeper, due, in the main, to the greater thickness of the trunk muscles; for the majority of the nerves of this group divide at a comparatively uniform distance from the neural tube. Much variation exists with regard to the place of exit of the dorsal and ventral rami through the myoseptum (Pl. **6**, Fig. 34; Pl. **7**, Figs. 37, 39). Johnston (:**05**) states that "the typical place of division is about half way between the cord and the dermis," rather than close beneath the dermis as described by Hatschek. Johnston finds much variation in the place of division, and frequent cases of separate emergence of the dorsal and ventral rami.

The dorsal rami of the nerves of this group resemble those of the more anterior region. They are largest in the thickest portion of the body, and possess a great number of branches, and, like the anterior dorsal rami, innervate the cutaneous area over the neighboring trunk muscles, and the adjoining portions of the dorsal fin. These dorsal rami frequently emerge from the myoseptum not as single stems, but in several branches, division having taken place beneath the dermis (Figs. 34, 39).

The ventral rami of these nerves are usually large, but may vary, one or two rami being increased in size at the expense of an adjoining ramus (Fig. 39). A ventral ramus may even be entirely absent. Both dorsal and ventral rami give off a number of repeatedly dividing branches in their course over the trunk muscles. In many methylene-blue preparations which retain little epithelium, the finer branches appear to end in innumerable delicate branchlets, the distal ends of which are often noticeably directed outward. Frequently these slender branchlets are continued into fine threads, which anastomose with one another and with similar threads proceeding from adjoining nerves, forming a delicate, irregularly meshed plexus. Such plexuses are not limited to this region, but are also present in connection with nerves posterior to the atriopore. These plexuses have been so little observed and figured that a detailed account will be given of their distribution and character in connection with the discussion of sensory nerve endings. The ventral rami of the dorsal nerves of the branchial region (with a few possible exceptions) divide near the ventral border of the side muscles, to form three main branches (see sketch, Pl. **4**, Fig. 15a). This division is accomplished in various ways, and each resulting branch may be represented by one or more twigs. The most exterior branches formed by this division are the *rami cutanei ventrales*, noted by early authors, and well described by Fusari (**'89**), Heymans et van der Stricht ('98) and Dogiel (: 02). These branches ramify over the metapleural fold and ventral wall of the atrium, forming a cutaneous network, the meshes of which are generally elongated longitudinally. Nerve VIII is usually the most anterior nerve sharing in the formation of this plexus. Fusari ('89) describes nerve cells and ganglionic knots in connection with this network. It is of interest to note that these cutaneous abdominal branches may leave the ventral ramus at or near the ventral muscle border, or the ventral ramus may divide much earlier in its course, sending one or more twigs from the branches thus formed to the superficial abdominal region (Figs. 37, 39). In Figure 39 the absence of a ramus cutaneus ventralis in connection with nerve XXXVI is to be noted.

The remaining two divisions of the ventral ramus are frequently formed from one main short stem; this may bend toward the interior beneath the ventral border of the trunk muscles, or may penetrate a myoseptum at some distance (the width of one, two or even three myomeres) dorsal to this ventral muscle border. These two branches may, however, leave the ventral ramus independently, instead of by a common stem. On the inner surface of the trunk muscles the short stem mentioned above divides into two main branches. One of these, the ramus visceralis ascendens, ascends over the inner surface of the trunk muscles, while the other, the ramus visceralis descendens, descends to the inner surface of the transverse muscles. Nerves VIII or IX, and the succeeding dorsal nerves of this region usually possess a ramus visceralis descendens, but I was unable to find a ramus visceralis ascendens anterior to that of nerve XII. The most anterior ascending visceral ramus is small, but the succeeding one is larger, showing the characteristic fan-like branching. It is entirely possible that branches pass to the most anterior branchial bars from the descending visceral ramus, or the inner abdominal plexus. In this connection it may be noted that the fan-like branches of the most anterior ascending visceral rami observed, were directed obliquely forward. The diagrammatic sketch, Figure 15a (Pl. 4), made from several transverse sections, illustrates a fairly typical form of division of the ventral ramus. This shows a branch of a ventral ramus penetrating a myoseptum the width of one myomere above the ventral muscle border. In these sections, stained with Mallory's differential stain, the connective tissue may be readily discerned on either side of the penetrating branch. Upon its exit on the inner surface of the side muscles, the ascending visceral branch takes a course dorsad over the connective tissue covering of the muscles.

This branch runs for some distance inside a gonadial pouch. The descending branch passes downward over the inner surface of the most ventral myomere to the inner surface of the transverse muscles. Here it appears to lie between layers of connective tissue. Both the ascending and descending visceral rami show a faint bluish tinge in sections stained with Mallory's differential stain, due, perhaps, to the presence of a thin connective-tissue sheath.

The distribution of the branches of the *rami viscerales descendentes*, forming the inner abdominal plexus, has been well described by Fusari (**'89**), Heymans et van der Stricht (**'98**), and Dogiel (**: 02**). Fusari and Dogiel note a network of extremely fine threads connected with the coarser meshes of the inner abdominal plexus. Dogiel describes these threads as somewhat varicose. He also finds other fine branchlets given off from the coarser plexus, which run to the abdominal muscles, and break up into a great number of repeatedly dividing finer threadlets that possess small varicose thickenings. Dogiel is of the opinion that these threadlets weave about the transverse muscles in an exceptionally thick plexus, analogous to that which he has described for the "ring-muscle" of the mouth. He does not find the fibers described by Heymens et van der Stricht as passing directly from the coarser plexus to penetrate between the lamellae of the transverse muscle, where they terminate in swellings analogous to the endings of nerve fibers in smooth muscle. Dogiel notes small three-cornered nuclei in the angles formed by branchlets of the coarser abdominal plexus. Fusari finds no ganglion cells in connection with this plexus, but notes small nuclei at the knotted places of anastomosis. He designates the nerves of the inner abdominal plexus as sympathetic.

The finer plexus described by Fusari and Dogiel is clearly visible in certain of my methylene-blue preparations. In these preparations the threads of this finer plexus are less varicose than those figured by Dogiel (**: 02**, Fig. 14, 15). However, at rather infrequent intervals swellings resembling small nuclei may be seen. Many of these finer threads cross the interior faces of the coarser meshes, indicating that they lie closer to the epithelium covering the inner surface of the transverse muscles than does the main plexus. A few fine threads, however, cross the external surfaces of the larger meshes. The meshes of the finer plexus are generally elongated parallel to the long axis of the animal. A secondary plexus is also present in that portion of the abdominal region which lies between the transverse muscles of either side of the body, but its meshes do not appear to be as close here as in other regions. In transverse sections stained with Mallory's differential stain, portions of the larger plexus may be seen lying on the inner surface of the transverse muscles, in connective tissue. Since many of the threads of the finer plexus are nearer the surface of the atrial cavity than those of the coarser plexus, they must be closely associated with the bases of the cells lining that cavity. In portions of my preparations the finer plexus is extremely complicated, and the meshes are not decidedly elongated. The meshes of the main inner abdominal plexus sometimes appear knotted in methylene-blue preparations, even though the knots are not differentiated in any way; but apparent nuclei are frequently present in the course of the nerve threads and at places of junction. Certain large meshes of the inner abdominal plexus send branches to the gonadial pouches (Pl. **3**, Fig. 13, *i.*), although the main supply for these structures is derived from the ramus visceralis ascendens. In Golgi preparations of Branchiostoma caribaeum, nerve threads were found scattered in the transverse muscles. It was impossible to determine their origin, but closely associated threads evidently did not all lie in the same focus. The greater number of these threads are arranged more or less nearly parallel to the long axis of the animal (Pl. **5**, Fig. 23).

Impregnation with methylene blue does not always produce the same results in the structural appearance of these visceral nerves. In some preparations the threads of the inner abdominal plexus appear granular, while in others distinct fibers can be traced. But notwithstanding these variations, the visceral nerves can always be distinguished from the cutaneous nerves by their rougher outlines.

Rami viscerales ascendentes.— Johannes Müller (**'41**, **'44**, p. 96), and Leuckart und Pagenstecher (**'58**) observed nerves passing to the branchial basket. Schneider (**'79**, p. 15) observed and figured sensory nerve branches penetrating to the interior at the ventral border of the side muscles. These branches ramify as they ascend on the surface of the abdominal cavity. He ascribes the function of the *vagus* to these nerves. Rohon (**'82**, p. 24) agrees with this interpretation. He also describes in some detail, the distribution of nerves in the ligamentum denticulatum, and in the branchiae. Fusari (**'89**, p. 130) describes the branchial nerve branch as ascending obliquely upward and forward after penetrating to the interior, finally joining the branchial apparatus by way of the ligamentum denticulatum. He states that only the more anterior nerves send branches to the branchiae and that the nerves of the tail also lack the "sympathetic" branch. It will be remembered that Fusari designates the nerves of the inner abdominal plexus as sympathetic. According to Rohon, the branchial nerves end in tufts. Fusari (p. 133) differs from this, and states that they spread out in a membrane that covers and supports the branchial apparatus on its external surface. On this membrane the different nerve branches form a net of irregular meshes, which Fusari calls the nerve net of the branchiae (his Taf. VIII, Fig. 2). This figure probably illustrates a plexus formed by ascending visceral branches before they reach the branchial basket (compare Pl. **4**, Figs. 15, 16). Fusari probably saw a nerve net distributed over the gonadic pouches. Heymans et van der Stricht (**'98**, p. 41–44) describe in detail the branches of the rami viscerales ascendentes, and figure nerves in the ligamentum denticulatum, a small number of fibers on the branchial bars, and fibers on the cross bars, joining those running lengthwise of the primary and secondary bars (their Fig. 30). These authors also figure a plexus distributed along the median ventral portion of the branchial basket, and show a portion of a plexus covering the gonads. They never found nerve fibers in the interior of the branchiae. They note a plexus in the ligamentum denticulatum, parts of which resemble a longitudinal nerve trunk; but in their opinion an actual large longitudinal nerve does not usually exist, except as formed indistinctly by the anastomosis of different branches of the ascending visceral nerves. These authors are therefore inclined to deny the existence of a distinct vagus nerve. Other branches of the rami viscerales ascendentes were observed, which these writers (Pl. VIII, Fig. 26) regard as perhaps innervating the blood-vessels, digestive tube, and the parietal serous surface of the branchial cavity. Dogiel (**:02**) presents figures of Golgi preparations showing numerous fibers on the primary and secondary branchial bars, and on the cross bars; also nerve cells of spindle-like or angular shape, previously undescribed, possessing from three to six processes, which gradually divide into a great number of twigs. Dogiel finds such cells in all parts of the branchial basket, and regards them as analagous to the sympathetic cells in the intestinal plexus of the "Neunauge" (river-lamprey). He also finds ascending visceral branches posterior to the branchial basket, which he is ~~certain~~ supply the intestinal canal (Fig. 17*a*, 17*b*, 18*a*, 18*b*, 19*a*, and 19*b*).

My own observations on the ascending visceral nerves and their branches were made chiefly from specimens of B. lanceolatum, only a few individuals of B. caribaeum proving favorable for study of these nerves. Dissected specimens previously impregnated with methylene blue were particularly useful in following the ramifications of the rami viscerales ascendentes. A study of these nerve branches is attended with difficulties, since it is necessary to lift the pharynx wall to disclose the nerves on the internal surface of the side muscles, and in the ligamentum denticulatum, and this proceedure is more than likely to break the slender nerve threads passing to the ligamentum denticulatum and the branchial basket. The accompanying series of figures (Pl. **2**, Fig. 10; Pl. **3**, Figs. 13, 14; Pl. **4**, Figs. 15–21, Pl. **5**, Fig. 30) aims to illustrate the distribution of numerous branches of the rami viscerales ascendentes with as much clearness as these technical difficulties permit.

As has been noted, a visceral branch may bend around the ventral border of the side muscles, or penetrate a myoseptum further dorsalward. A dorsal nerve may possess two or more visceral branches, instead of one. Figures 13 and 15 (Pls. **3** and **4**) illustrate a condition of frequent occurrence, in which a branch of the ventral ramus penetrates to the interior through the myoseptum just dorsal to the most ventral myomere. In either case, division into a descending and an ascending visceral ramus takes place immediately after reaching the internal surface of the side muscles, the ascending branch running dorsalward, and the descending branch taking a general ventrad direction. In Figure 15 the descending ramus is small, but in Figure 13 its size is equal to that of the ascending ramus. Such variations in size are frequent, and when a branch is unusually small, compensation may ordinarily be noted in an adjoining nerve. The ramus visceralis ascendens gives off branches from either side as far as a main stem can be observed, but the origin of the more ventral of these branches is usually obscured by a gonadic pouch. When one of these pouches is empty (Fig. 13), the nerves which innervate its entire surface with an elaborate network may be readily seen arising either from the ascending visceral ramus, or its first branches, or from branches of the descending visceral ramus, or, finally, from meshes of the inner abdominal plexus. These nerve branches ramify to form a plexus which covers not only the more interior surface of the gonadial pouch, but also the surface adjoining the muscles. This plexus is extremely delicate and fine-meshed. The more exterior portion is continuous throughout the region of the gonadic pouches, the network arising from one nerve uniting with that which is in connection with an adjoining nerve. This network also joins the more dorsal fan-like branches of the rami viscerales ascendentes. This plexus is probably also continuous with that which is in connection with the intestinal canal, and it possibly extends beyond the anus. When the gonadic pouches are large and press against each other, the plexus on their more interior surfaces appears continuous and the origin of branches from the visceral rami is largely hidden. Figure 13 (Pl. **3**) shows only the larger meshes of the plexus on the internal surface of a gonadic pouch, but the origin of these branches is illustrated. In Figure 10 (Pl. **2**) the finer meshes of this plexus are shown under greater magnification. In methylene-blue preparations the threads of the gonadic plexus often appear faintly granular, indicating that they impregnate in a manner similar to the nerves of the inner abdominal plexus. Heymans et van der Stricht (**'98**) find the "peritoneal" plexus less rich under the sexual glands, but in this they probably refer only to the larger meshes. These authors do not figure the origin of branches running to the gonadic pouches, and give no detailed account of their innervation. Dogiel (**:02**) does not discuss the innervation of these organs.

The main stem of an ascending visceral ramus may usually be followed dorsad a distance equal to the width of one, two or even three myomeres from the ventral border of the side muscles, and then it breaks up into a large number of branches spreading out fan fashion over the inner surface of these muscles. In the anterior part of the body these fans are directed somewhat anteriad as well as dorsad (Pl. **4**, Fig. 15), but the branches of the more posterior ascending visceral rami take in general a dorsad direction. If the wall of the branchial basket is lifted to disclose these nerves, the numerous branches of a fan often appear broken off abruptly in the middle region of the side muscles, and the distal broken ends are most frequently bent away from the muscles as though lying in a membrane not closely attached to the side muscles. This is no doubt due to the entrance of these nerves into the ligamentum denticulatum. As may be noted in Figure 15, *c* and *d*, the branches of one fan anastomose with those of an adjoining fan, thus forming a continuous network along the inner surface of the side muscles, and in the ligamentum denticulatum. This plexus may or may not lie dorsal to the gonadic pouches, depending on the size of the pouches. As has been mentioned, these fans join nerve threads which innervate the sexual glands.

From the plexus formed by these fans, certain branches pass upward along the ligamentum denticulatum (Pl. **4**, Figs. 16, 17), while others ramify extensively in the "pocket" portions of this ligament. These nerves often present a fibrous appearance, as noted by Fusari; but this seems to depend on the time or manner of fixation. In other cases they appear dotted, with rough edges, the latter perhaps due to the deposit of a small quantity of coloring matter in the surrounding tissue. The nerves to the branchial basket pass upward along the ligamentum denticulatum in numerous small threads, and enter the basket not far from its dorsal region of attachment. The nerves in the "pocket" portions of the ligament run more or less longitudinally (Fig. 17), while those in the portions attached to the primary bars take a nearly vertical direction. As may be noted in Figure 17, nerves pass to the primary and secondary bars from various portions of the ligament. Figures 14 (Pl. **3**) and 19 (Pl. **4**) show the nerve threadlets of the primary and secondary branchial bars, and the connecting threads passing along the cross-bars. These figures, drawn from methylene-blue preparations, show an interesting likeness to Dogiel's Figur 17*a*, taken from a Golgi preparation. In the course of treatment after impregnation with methylene blue, the branchial epithelium sloughs off in large measure, exposing the nerve threads. The nerves which pass dorsad along the primary and secondary bars lie at first on the more exterior (lateral) surface of the branchial basket. As the threads ascend, they gradually shift toward the interior, and a greater number of threads are often found on the more posterior portion of each primary bar. Near the top of the primary bar these threads pass into a plexus lying between any given bar, and the next posterior secondary bar (Fig. 20). It is quite possible that as many fibers ascend on the anterior side of the bars to join this dorsal plexus, but the specimens observed did not show as great a number. Nerves passing along the secondary bars join this dorsal plexus with a smaller number of threads. In the middle region of the branchial basket, and in its ventral portion, the nerve threads lie on its more external surface. The nerves of the primary bars are loosely placed, and form an irregularly meshed network, the threads of which are not noticeably arranged lengthwise of the bar (Figs. 17, 19). On the secondary bars (Pl. **3**, Fig. 14), on the contrary, the slightly knotted nerve threads form a plexus consisting of numerous long, longitudinally arranged threads, connected by short branches. This plexus appears to be rather smoothly drawn over the surface of the bar, in contrast to that of the primary bars. The knotted appearance of threads on the secondary bars may be an artificial condition. Nerve threads crossing from one bar to another along the cross-bars form a plexus over the latter in their course. At least, in the region dorsal to the ligamentum denticulatum fibers pass from primary to secondary bars at other points than the cross-bars, thus adding to the elaborateness of the branchial plexus.

I find a large number of the previously mentioned special nerve cells, which were first described by Dogiel (:**02**) as connected with nerves of the branchial basket. Dogiel figures these cells as they appear in Golgi preparations, and states that in methylene-blue preparations he noted large round or oval nuclei on certain nerve branchlets of the branchial basket, which he identifies as the nuclei of the cells found in Golgi preparations of the branchial region. These "nuclei" are shown in his Figur 16, taken from a methylene-blue preparation. This figure shows a condition frequently present in my methylene-blue preparations. Comparatively smooth lines, darkly colored with methylene blue, run parallel to the branchial bars, but do not appear to be connected with the evident nerves of the pharynx. Possibly these may be blood vessels; at any rate the nerve cells (Figs. 19, 21) which are impregnated with methylene blue in my preparations are not connected with these lines, but with the branchial plexus. Figures 19, 21, and 30 show these cells in methylene-blue preparations, and in a Golgi preparation. They correspond in shape and number of processes to those described by Dogiel. The nuclei of the cells impregnated with methylene blue are often difficult to discern, as is frequently the case in other nerve cells impregnated more or less with methylene blue. This led to the suggestion that these bodies might themselves be nuclei. Upon measurement of these so-called cells, and those impregnated after the method of Golgi, it was found that the size in either preparation was practically the same, the bodies in methylene-blue preparations being if anything, a little rounder and fuller. In the Golgi preparation illustrated (Fig. 30) a lighter nucleus is plainly visible in the multipolar cell. These cells are present on both primary and secondary bars, and are present at least in the ventral and side regions of the pharynx, on its external surface. In the ventral region these cells are distributed in connection with the elaborate plexus which spreads over this portion of the branchial basket.

There are certain branches of the rami viscerales ascendentes which are connected with the fan-like branches of these rami, but apparently do not have the same ultimate destination (Fig. 15, *e*), since they are lost to view in the region between the muscles and the ligamentum denticulatum. Heymans et van der Stricht ('**98**) figure nerve branches of the ascending visceral rami (Pl. VIII, Fig. 26, *N D V*.), which they designate as digestive and vascular, but the destination of such branches has never been satisfactorily determined. It is certain, however, that further experiments with methylene-blue impregnation and other methods, will reveal many additional complications in the distribution of the visceral nerves of Amphioxus. The investigations up to the present time establish the fact that the visceral rami of the dorsal nerves are elaborately connected throughout the atrial region, and that multipolar nerve cells occur in connection with these visceral nerves.

E. Nerves Posterior to the Atriopore.

Dogiel (**:02**) has described in detail the main cutaneous branches of these nerves, and illustrates the larger branches in his Figures 2 and 3. He notes the presence of "rami communicantes" connecting two adjacent ventral rami of dorsal nerves in their course over the side muscles of this region, but does not limit the occurrence of such connecting branches to the posterior portion of the body.

In the specimens which I have observed, the cutaneous branches in this region are more numerous than represented by previous authors. Rich plexuses are frequently present both between the dorsal rami and between the ventral rami of these nerves on the same side of the body (Pl. **6**, Figs. 34–36). These plexuses will be described in connection with the sensory nerve endings. A "ramus communicans" was frequently found in this region, as well as in other parts of the body. The elaborate branching of the last dorsal nerve of Branchiostoma caribaeum is illustrated in Figures 31 and 33 for the right and left sides respectively. This pair of nerves is exclusively cutaneous, and its branches are more numerous than the figures by previous authors indicate. The most posterior dorsal nerve is posterior to the last ventral root, and, according to Hatschek's interpretation of the relation of the nerves to the myomeres, belongs to the preceding myomere. He states that each dorsal nerve belongs to the myomere anterior to the myoseptum through which it passes, and that the ventral root innervating this myomere appertains to this dorsal root.

Little attention has been previously given to visceral branches in this region. Heymans et van der Stricht (**'98**, p. 43) note in Golgi preparations apparent nerve fibers on both the parietal and visceral mucosa faces posterior to the abdominal pore. They were unable to find the origin of these nerves, or to trace their course, but suggest that they may come from the "abdominal" visceral nerves (Hatschek), or, more probably, as an extension of "thoracic" longitudinal nerves into the "abdominal" cavity. These authors seem to apply the term "abdomen" to the region containing the intestinal canal. They also note in Golgi preparations nerve fibers supplying the surface of the intestine, and (p. 44) branches from certain dorsal nerves (14th to 16th from the tail end) of the left side of the body supplying the anus. A branch from the right 13th (from the tail end) appears to have the same destination. Dogiel (**: 02**) states that the dorsal nerve pairs from XL, or XLI, to LIII, or LIV, inclusive, give off ascending visceral branches, which bend around the ventral border of the side muscles, or penetrate a myoseptum to reach the interior. Beginning with nerve XLI or XLII, these visceral branches divide into a great number of branchlets, which weave around the whole rectum reaching to the anus. A fine plexus is formed in the walls of the anus from exceedingly fine and slightly varicose branches of nerves LI, LII and LIII. Dogiel does not state that the nerves of one side are more concerned in the innervation of the anus than those of the other. It is uncertain whether or not he finds visceral branches from the nerves succeeding nerve LIV; his figures surely do not illustrate such branches. His Figures 19*a* and 19*b* show ramifications of the visceral branches of nerves XLII to XLVI inclusive. These extend on the inner surface of the side muscles from their ventral border a short distance dorsad. Both figures are drawn as seen from the exterior.

In the specimens I have examined, occasionally visceral branches were apparently lacking to certain dorsal nerves, but, as Dogiel has observed, the manner in which these penetrate to the interior may be such that a view from the exterior fails to give any evidence of their presence. Examination of a large number of specimens disclosed visceral branches to practically all the nerves of this region from XL to LXI, inclusive. However, visceral branches could never be discerned from the exterior for all of these nerves in a single specimen. In one case nearly all dorsal nerves as far as nerve LX showed visceral branches. I have never observed visceral branches in connection with nerves LXII, LXIII or LXIV (when 64 nerves are present). The innervation of the anus, which lies on the left side, is usually supplied by branches of *left* nerves LI–LIII, in both species. Variation occurs here, and it is probable that in Branchiostoma caribaeum more anterior nerves may occasionally innervate this structure. Frequently only two of the nerves mentioned above give off branches which weave about the anus. Right nerves LI, LII and LIII often show branches penetrating to the interior, but these could never be traced to the anus.

In methylene-blue preparations suitable for dissection, the intestine is rarely in favorable condition for study, although the main visceral nerve branches impregnate particularly well. Such preparations cut along the dorsal median line, and laid open with the internal surfaces uppermost, proved very interesting when studied with the aid of strong light. Figure 32 (Pl. 6) is drawn from a specimen dissected after this fashion. It is evident from this figure that many more nerve branches penetrate to the interior than can be observed from an external view. These branches appear smooth and darkly stained for a short distance after they reach the interior, and their finer branches ramifying over the base of the ventral fin have a similar appearance. As the visceral branches ascend and ramify on the internal surfaces of the muscles, they are more lightly colored, and often flatten out into comparatively broad bands, which appear loosely confined. On the right side may be noted branches which probably supply the posterior extension of the atrium. The nerves ramifying over the base of the ventral fin are not to be confused with the cutaneous supply. The latter may be seen in a specimen dissected and examined in the manner just described, but they lie at a much lower focus (i. e. nearer the surface). A few nerves were observed forming a network over the inner surface of the side muscles posterior to the anus, thus accounting for at least a part of the visceral nerves observed from the exterior in this region. The nerve supply for the intestine probably reaches the latter through the membrane (mesentery) which holds it in place beneath the notochord. This corresponds to the manner in which a nerve supply reaches the pharynx. The presence of the post-atrial extension on the right side of the body in this region, may bring about modifications in the arrangement of the visceral nerves of that side. Certain abruptly ending branches, indicated at *c*, *c*, in Figure 32, may be the result of imperfect impregnation, or have a significance not yet determined. It seems certain that a network of nerve threads similar to that noted in the branchial region spreads over the internal surfaces of the muscles posterior to the atriopore.

In certain transverse sections of the tail region, stained with Mallory's differential stain, a pathological condition was evident, involving nearly all the structures of one myomere, and present to a slight extent in an adjoining one.

Considerable variation was noted in the total number of dorsal nerve pairs in each species. In Branchiostoma lanceolatum the number counted most frequently was 63; while in B. caribaeum the number appeared to be between 56 and 60.

F. Spinal Ganglia.

Dogiel (: **02**) found special structures in connection with the dorsal nerves of Amphioxus which he interprets as spinal ganglia. These bodies appeared in specimens immersed from 3–6 hours in a dark blue or violet mixture either of a 1% methylene blue, or of a saturated solution of toluidin blue, in normal salt solution. They could also be seen in gold-chloride preparations when the lemon-juice and formic-acid method was used. These so-called ganglia consist of groups of from 3 to 7 round, oval or pear-shaped elements, situated at the place of exit of each dorsal nerve from the myoseptum, or near the proximal end of the dorsal or ventral ramus where it gives off small branches; in addition to these, similar, though smaller, bodies may be found even as distant as the ventral border of the trunk muscles. These structures are illustrated in Dogiel's Figures 20, 21, 22*a*, 22*b*, 22*c*, 22*d*, and 23.

By using Dogiel's methods I was able to observe these structures in methylene-blue and in gold-chloride preparations, and also found them in specimens fixed in weak osmic acid and stained with picro-carmine. They appeared not only at the places indicated by Dogiel, but by using strong light for study of the preparations, they were found at almost any point in the course of the dorsal nerves. In methylene-blue preparations the size and number increased with the length of immersion. These bodies were of varying size, in some cases quite large, while by using the higher magnifications on portions of tissue mounted under a cover-glass, similar structures of minute size could be seen in connection with the smaller nerve branches. Strong light revealed great numbers of these structures in connection with the nerves of the thicker parts of the body. The methods mentioned above always cause a marked enlargement of the ampulla-like dilation at the posterior end of the neural tube, and a general increase of size in the tube itself. These facts led to doubts as to the reliability of such methods, and numerous experiments were made in the spring of 1905 to determine the nature of these bodies. Dogiel's methylene-blue method, employing normal salt solution instead of sea-water, causes general swelling of the tissues, and in some cases death occurs, followed by certain post-mortem changes, before these bodies appear. In material "fixed" in weak osmic acid and stained with picro-carmine these structures were frequently present, or, if not, I was able at will to make them appear by pressing lightly on the cover-glass over the mounted specimen. In some cases the pigment of the neural tube had passed out into the swellings in the course of treatment with chemicals, or upon pressure, and contrasted brilliantly with the light stain. An excellent illusion was sometimes produced by the presence of well stained nuclei, which are either pushed out from the cord, or are sheath nuclei. The weak osmic acid has little effect on tissues beneath the surface, and the subsequent washing in water causes actual maceration. This would account for all manner of artifacts. The specimens treated according to the gold-chloride method mentioned above, show general distortion, and many structures are so displaced as to make such preparations quite unreliable for study. These bodies, if present in methylene-blue preparations, will persist when the preparations are preserved in the mixture of glycerine and ammonium picrate; but when fixed in ammonium molybdate and *dehydrated* they usually disappear. Similar structures were never found in well preserved and sectioned material. There seems, therefore, but little doubt that these so-called "ganglia" are artifacts. Since writing the above account, Johnston's (**:05**) paper was noted, in which he also concludes that these structures are of artificial formation. Johnston finds a small proportion of the ganglion cells of Amphioxus in the nerve cord, and the remainder in the dorsal roots, located as far as, and beyond, the place of division into dorsal and ventral rami.

G. Structure of the Dorsal Nerves.

In methylene-blue preparations of the dorsal nerves, darkly stained fibers can often be traced from the neural tube, or place of exit of the nerve, through the myoseptum to the ventral border of the lateral muscle. These fibers are more or less separated from one another, and are not of equal size. Usually a single fiber cannot be traced for this entire distance. Fibers can frequently be discerned in the rami cutanei ventrales which give these branches a darkly colored appearance. I have often traced one, two or three fibers into the common basal trunk of the descending and ascending visceral branches, but was unable to trace distinct fibers into either of those branches. As has been noted, however, these nerves sometimes present a fibrous appearance. Figures 40 (Pl. **7**) and 22 (Pl. **5**) show fibers of at least two sizes entering the dorsal nerve roots, the smaller fibers being by far the most numerous. This agrees with the observations of Johnston (**:05**). The different structural appearance of the visceral nerves has already been noted. This difference is particularly evident in methylene-blue preparations, and may be observed in all branches of dorsal nerves innervating deep-lying structures. It is entirely possible that structural distinctions will be established between the visceral nerves to different organs.

H. *Sensory Endings of Dorsal Nerves.*

This subject has long attracted investigators, and its study has brought forth a great variety of opinion and suggestion. Quatrefages (**'45**) described peripheral nerves ending in small ovoid bodies (the corpuscles of Quatrefages), which he suggested represent special mucous organs. Kowalewsky (**'67**) believed that the sensory nerves end in epithelial cells. Owsjannikow (**'68**) occasionally saw what appeared to be a nerve fiber connecting with a cylindrical epithelial cell. He also noted what he calls a nerve-net, lying in the deepest layer of the skin, a region well supplied with connective tissue. Reichert (**'70**) describes certain cells in the epithelial layer of the skin, which bear a spine-like external process. He designates these cells as "thorn" cells. Langerhans (**'76**) found no branches uniting two dorsal nerves, or any exchange of fibers. All dorsal nerves branch in simple tree fashion. He believes that nerve plexuses are lacking in all parts of the body, except at the mouth border. He found no vestige of a fine end plexus such as Marcusen describes, holding that Marcusen was deceived by connective-tissue fibrillae. Langerhans describes special small cells lying irregularly between the cylindrical epithelial cells of the skin. These special cells have a small body and large oval nucleus; they lack a cuticula (limiting or basement membrane), and often possess a thread-like process at the internal end. On the external surface each cell bears a long, stiff hair, which sometimes has a thickened base. These special cells are particularly numerous in the head region, but may be found on all parts of the body. The hairs may be seen on living animals, but never show motion. The finer dermal nerve branches pass through small canals in the so-called limiting membrane of the skin, each of which is located where two small fissures in the membrane cross each other, usually at right angles. After a short sub-epithelial course, these fibers unite with hair-bearing cells, which Langerhans declares to be the endings of the cutaneous nerves. These nerves are not in relation with the ordinary epithelial cells, in fact, there are not enough nerve branchlets to permit union with each such cell.

Rohon (**'82**) states that the greater number of nerve branchlets run out into the cutis and end in tufts. Fusari (**'89**) says that the cutaneous nerves divide into fine branches immediately beneath the cuticula (the limiting membrane of Langerhans), and that anastomoses are frequent in the skin of the ventral region, but rare on the sides and dorsum. The branches in the ventral region pass into fine free-ending fibrillae, or before ending show a spindle-shaped, nucleated enlargement. In all other regions (with the exception of the mouth) nerves, penetrating the cuticula, appear to terminate after a short course in free endings, though they can be traced to the bases of epithelial cells. Fusari is uncertain, however, whether there is any connection between these nerve fibers and the cells. He saw the sensory cells of Langerhans, and though inclined to his views, contends that a connection with the nerves is not demonstrated. He believes that the description of the nerve-net given by Marcusen is based on a peritoneal rather than a dermal plexus. Heymans et van der Stricht (**'98**, p. 33) regard the so-called sense-cells of Langerhans as merely compressed and flattened "cyclinder" cells, bearing no special relation to the nerves. These authors saw branches of a dorsal nerve form among themselves a plexus on the border of the fins; but they never found the plexuses mentioned by Fusari as frequent in the ventral region. According to these writers the nerve fibers ramify under the cutaneous epithelium, and terminate between, and perhaps within, ordinary epithelial cells. They found no special sense-organs in the skin, in connection with nerve fibers.

Retzius (**'98**) is convinced that cutaneous fibers end free at the bases of epidermal cells. He saw two kinds of cells in the epithelium, one smaller than the other, but neither bore a thread- or spine-like process. He found no direct connection between these cells and nerve fibers, nor could he discover any true peripheral sense-cells connected with nerves. Dogiel (**:02**) states that the "Nervi cutanei dorsales (laterales et ventrales)" are pure sensory branches without any motor fibers. These gradually divide in their course to the superficial layers of the skin, continually giving off branches of various lengths and thicknesses, which anastomose with similar neighboring branchlets to form a more or less elaborate plexus, his "Grundgeflecht." This plexus is especially plain, he says, in the skin of the ventral region. His further description is as follows: The branches from this plexus ultimately reach the homogeneous layer of the skin immediately beneath the epithelium, and can be observed in good preparations to pass through small canals in this layer. Each then divides into three or four or more fine branchlets, which spread out radially. These branchlets, which lie immediately beneath the epithelium, in turn divide repeatedly in this region, giving off fine, varicose threads, which form a close-meshed plexus. This is designated by Dogiel as the "sub-epithelial plexus." From this plexus fine threadlets penetrate between the epithelial cells, and there give off side branchlets, which twist about these cells and end between them; these threads were followed almost to the outer surface of the skin. Dogiel found such endings in the rostrum, in the middle region of the body, and in the tail. But in addition to these free nerve endings in the epithelium, Dogiel also saw in the epithelium what he regards as peripheral nerve cells; these occur in the rostrum, the tentacles, and the "head" region. He believes that they are scattered over the entire skin area. These cells have a spindle-shaped body, with a peripheral and a central process, and a large nucleus, nearly filling the thick part of the cell. The peripheral process is a short, thick cylinder or rod, which reaches close to the outer surface of the epithelium, and its end is either blunt, or somewhat pointed, but it never extends in the form of a thread beyond the free surface of the skin. The central process runs perpendicularly or obliquely toward the interior, and could occasionally be seen passing through the homogeneous layer of the skin and uniting with a nerve branchlet. Dogiel's Figur 28 presents a portion of the "sub-epithelial plexus" and its branches in the ventral region of the body, in connection with the well-known plexus formed from branches of the rami cutanei ventrales. His figures 29 and 30 show peripheral nerve cells in the skin of the rostrum and head. Dogiel also illustrates nerve-endings in the tentacles of the mouth.

In my own study of the sensory nerve terminations in Amphioxus a great number of methods were employed, including methylene-blue impregnation, the methods of Golgi, the various gold-chloride methods, picro-carmine staining, and sections of material fixed and stained in a variety of ways. As has been noted, the extensive arborescent appearance of the cutaneous branches of the dorsal nerves may be readily observed in methylene-blue preparations studied with the aid of strong light (Pl. **6**, Fig. 34; Pl. **7**, Figs. 37, 39). The arrangement of the sensory endings must depend largely on the distribution of the finer terminal nerve branches. The extreme readiness with which Amphioxus reacts to tactile stimuli applied to any part of the body, indicates a rich supply of cutaneous nerves. The sensory nerve branches reaching to the epithelial layer of the skin are most readily followed in methylene-blue preparations. The best impregnations of these nerves were obtained by immersion of the specimens from $1\frac{1}{4}$

to $1\frac{3}{4}$ hours. The subsequent exposure to air varied from 15 to 40 minutes. The specimens were fixed in ammonium picrate to which a little osmic acid had been added, or in ammonium molybdate and osmic acid. Direct sunlight, a Welsbach lamp, or a Nernst lamp were used in studying whole specimens.

The distal ends of the finer cutaneous nerve branchlets are often noticeably directed toward the exterior (Pl. **7**, Fig. 37). By focusing on the plane in which the exterior ends of these nerve branchlets lie, it may be observed that the surface of the cuticula is broken by numerous small clefts crossing each other either at right angles, or in the form of an X. By focusing downward upon a dark spot at the place of crossing, it can be determined that this spot is the end of a nerve branchlet, which can be followed to its place of union with one of the larger branches. These numerous small, exteriorly directed branchlets for the most part, therefore, penetrate the sub-epithelial so-called cuticula (better basement membrane) through the small canals first described by Langerhans. These small branchlets are more numerous than either Heymans et van der Stricht (**'98**), or Dogiel (: **02**) have figured. They were not always evident, and it was frequently impossible to demonstrate a basement membrane, which seems to indicate that this structure sometimes sloughs off with the epithelium. Figures 34 (Pl. **6**) and 39 (Pl. **7**), which show these exteriorly directed small branchlets in considerable numbers, also show other characteristic cutaneous nerve branches. I refer to large branches, lying close to the muscles, which may be noted connecting the branches of two or more dorsal nerves in their course over the side muscles (see also Pl. **6**, Fig. 36, p_1 and Pl. **7**, Fig. 38, p_1). Beside these connecting branches, there are, lying at a higher focus, finer plexuses, which connect the small, exteriorly directed branchlets. These finer plexuses may also be seen in Figures 35, p_2 and 38, p_2. In methylene-blue preparations this finer plexus is usually the most superficial nervous structure visible. The branches of the deeper plexus are usually of considerable size, and fewer in number than those of the more exterior plexus, and their meshes are larger than those of the outer plexus. As has been noted, the deeper plexus lies close to the muscles, and its branches may even run beneath other large branches of the dorsal nerves. This plexus between the ventral rami of dorsal nerves was found distributed over the side muscles in nearly all parts of the body. The finer plexus is composed of slender threads, forming close meshes; it was found in the regions where there is a coarser plexus, and also in the ventral fin, and between the dorsal rami of dorsal nerves. The nerve threads in either plexus always appear smooth, showing no swellings of any kind. Figure 34 (Pl. **6**) shows at *c* (near the upper margin of the figure) a cell which is apparently associated with a fine nerve plexus.

These plexuses were found in nearly all specimens, of both species, impregnated in the manner previously described. The specimens retaining considerable epithelium were quite unfavorable for study; on the other hand, with the sloughing off of the epithelium, portions of the superficial plexuses were undoubtedly lost. A record, given below, was made of the occurrence of these plexuses, indicating the species, the side of the body, and the number of the nerve with which they were associated. In nearly every case, portions of both the coarser and the finer plexus were present.

	Species.	Side of Body.	Number of Nerve.	
1.	B. lanceolatum	left	IX	
2.	" "	"	XI, XII	same individual
		right	" "	
3.	" "	left	XV, XVI, XVII	
4.	" "	right	XXI, XXII	same individual
		left	XXVI, XXVII	
5.	" "	right	XXVII	
6.	B. caribaeum	"	XXIX, XXX	
7.	B. lanceolatum	left	XXXII	
8.	" "	right	XXXV, XXXVI, XXXVII	
9.	B. caribaeum	left	" " "	
10.	B. lanceolatum	"	XXXVII, XXXVIII	
11.	" "	"	XLI, XLII, XLIII, XLIV	
12.	" "	"	XLII, XLIII, XLIV	
13.	" "	right	XLVI, XLVII, XLVIII	
14.	" "	"	LIV, LV	
15.	" "	"	LIII, LIV, LV, LVI, LVII	
16.	" "	"	LIV, LV, LVI	

It is evident from this record, that these plexuses cannot be looked upon as localized structures, or of infrequent occurrence. The record shows no plexuses (those of the mouth border are not included) in the rostrum, buccal region, or at the extreme posterior end of the animal. This may be due to lack of impregnation, or these regions may show differences in this respect. Figures 34, 35, 36 (Pl. **6**) and 38, 39, 41 (Pl. **7**) present the first published views of such cutaneous plexuses distributed over the side muscles, either *in situ* or otherwise.

The plexuses shown in Figures 34, 38, and 41 — formed by the breaking up, for a short distance, of the main stem of the ventral ramus of a dorsal nerve into a number of anastomosing branches,— are of frequent occurrence, but do not appear to have been noted by earlier authors. Such plexuses may have a morphological significance, or they may be pathological, the result of injury or disease. A simpler form of division of the ventral ramus is often present, in which the main stem divides into two branches of nearly equal size, which unite again after a short course. These modifications in the ventral ramus often serve to attract attention to fine plexuses in connection with them. "Rami communicantes" were often noted in both species.

In the figures showing plexuses between the dorsal nerves, branches may often be noted, in connection with at least the coarser plexus, which penetrate the myosepta toward the interior of the muscle (*a*, Figs. 34, 36, Pl. **6**). The destination of these branches is uncertain.

The absence, in methylene-blue preparations, of any great number of branches arising from the finer plexuses, is of interest. It may, perhaps, be accounted for by the fact that such thick preparations are not favorable for disclosing fine threadlets; higher magnifications of course cannot be used. Gold-chloride preparations of the skin were made to supplement those on which are based the foregoing account of terminal branches. The method of Ranvier was used, also that of Hardesty, in which impregnation with gold chloride follows fixation with 10% formol. The latter method gives accurate fixation as a rule, and the finer cutaneous nerve branches often adhere to the skin, when it is stripped off, instead of remaining attached to the muscles. Such preparations can be studied with the aid of the higher magnifications; but the epithelial cells always obscure the nerves more than is desirable. Figure 24 (Pl. **5**) is drawn from a gold-chloride preparation of material fixed in formol. This figure shows the internal surface of skin stripped from the dorsal fin toward the posterior end of the animal. A few epithelial cells are outlined (at *e*) to show their relative size, and special cells, to be described later, are indicated at *g* (upper part of figure). The fine threads apparently running out from the larger nerve branch are the most delicate threads so far noted in connection with the cutaneous branches of dorsal nerves; and the star-like places of anstomosis (*s*) of these threads remind one of those described by Dogiel. Apparent cells or nuclei (*c*) may be noted in connection with these threads, as the latter run to the special cells (indicated at *g*) situated between the epithelial cells of the skin. Unfortunately, however, connective-tissue threads stain very well with gold chloride, and therefore may lead to much confusion. In the ventral region, where the tissues are transparent, connective-tissue threads may be readily discerned in gold-chloride preparations, weaving about the nerves, and running out from them in various directions like guy-ropes. These threads thus furnish an excellent pitfall for the observer. Where exceedingly fine nerve branchlets and connective-tissue threads are associated with each other, one may well hesitate to decide between them. The bodies (*c*) connected with the fine threadlets shown in Figure 24, all lie in the same plane with the threadlets, whereas the epithelial cells and special cells (*g*) are more superficial, being immediately exterior to these threadlets. The latter may be terminal nerve branches, interweaving at the bases of the epithelial cells, and sending branches between them, but evidence furnished by some other method is needed for corroboration. If these threadlets are nerves, they are probably connected with the finer plexus noted in methylene-blue preparations.

The special cells (*g*) previously mentioned are shown in Figure 42 (Pl. **7**) under greater magnification. This figure illustrates the external surface of skin overlying the side muscles near the dorsal fin. This preparation was impregnated with gold chloride after Ranvier's method. In preparations of this kind the ordinary epithelial cells are light purple in color, while certain differentiated cells lying between them appear red, are somewhat smaller, and are oval or circular on their external faces instead of polygonal. One or two slightly modified epithelial cells (*s*) are usually associated with each special cell. These accompanying cells assume a darker purple color than the ordinary epithelial cells. The special cells (*g*) and the modified epithelial cells (*s*) were both present in certain methylene-blue preparations, distributed in the rostrum and anterior portion of the body as far as nerve XVI. Unfortunately, favorable preparations of the skin of more posterior portions of the body were lacking. In these methylene-blue preparations the special cells assume a dark blue color, while the modified epithelial cells are only slightly darker colored than the ordinary epithelial cells, and present a granular appearance.

Changes in focus show that the diameter of these special cells is slightly less at the surface than at a short distance toward the interior. The most striking feature, however, of these cells is a rather long, stiff hair-like structure attached to the outer surface of each cell. In gold-chloride preparations this hair-like structure is colored black, or a very dark purple. In methylene-blue impregnations it appears dark blue. As shown in Figure 42 (Pl. **7**), this "hair" is thicker at its distal than at its proximal end, and often bears a knob-like termination. Two apparent points of attachment to the cell are frequently visible; one of these occurs at almost any point on the outer surface of the cell, while the other is at one side of the cell, or perhaps between two cells. While these special cells may possibly correspond to the "thorn" cells (Stachelzellen) noted by Reichert (**'70**, p. 756), this stiff hair is much too long for such a comparison, nor does it resemble a spine, because of its thickened distal end; neither does it resemble any terminal nervous structure. One interesting feature is the varying length of such "hairs," and their absence from many of the special cells. The "hairs" were nearly always comparatively short in methylene-blue preparations. I suggest that these special cells may be gland cells, and that the "hairs" are hardened exudations adhering to the outer surface. Glands certainly exist in the skin of Amphioxus, and probably receive a nerve supply from the cutaneous plexuses. Attempts were made on hardened material to color the "hairs" with special stains for glands, but none of the material was properly fixed to make such tests of value. If these peculiar cells are gland cells, it is still uncertain whether sensory cells are present in the epithelium of all parts of the body.

Sensory cells are clearly present in the tentacles, and on the mouth border, as described by Dogiel (**: 02**, p. 192–195; Fig. 10, 31, 32). These cells impregnated successfully in certain of my methylene-blue preparations, and nerve fibers could be traced in the tentacles to the region of these cells. I was unable, however, to determine absolutely the connection between the two which in all probability exists in the form already shown by Dogiel. The long distal process of the end-cells in the papillae of the tentacles, figured by Dogiel was clearly evident in my preparations.

VENTRAL NERVES.

Owsjannikow (**'68**) describes nerve roots arising from the spinal cord at different levels. Although Stieda (**'73**) thought it probable that this author saw ventral nerve roots, he himself was the first to describe the ventral roots clearly; he notes that they do not lie in the cross-section plane of the dorsal roots, and he found no actual union between them and the dorsal roots. Langerhans (**'76**) saw only dorsal roots, and regarded them as carrying motor fibers. Schneider (**'79**) described each ventral nerve root as arising by several processes (each leaving the nerve cord by a separate orifice), which unite into a flat bundle; this soon spreads out, a small portion going dorsad, and a larger part ventrad. When the fibers reach the muscle layer they bend posteriorly and coalesce with the free margin of the muscle-plate. As the fibers approach the border of the muscle-plate, they become cross-striated. Schneider proposed the theory that the real motor nerves extend outward only as far as the sheath of the spinal cord, and that each muscle-plate sends a process to the spinal cord and receives its innervation there. Balfour (**'80**) denies the presence of motor roots. Rohon (**'82**, p. 14, 54) states that the ventral roots arise from multipolar ganglion cells, in part pigmented, which lie on both sides of, and in part ventral to, the central canal of the cord. He finds no morphological relation between dorsal and ventral roots outside the spinal cord and is entirely in the dark as to how the motor nerves end. Rohde (**'88**) thinks, with Schneider, that the motor fibers are probably in part direct processes of the muscle fibrillae; but he is uncertain whether there is a direct connection between the motor fibers and the nerve elements of the neural tube. Fibers in the ventral roots were followed centripetally to a delicate membrane on the inner side of the corda sheath, where they were believed to divide into very fine fibers, which, however, could not be traced in the spinal cord. Fusari (**'89**) finds, as did Schneider and Rohde, most of the ventral-root fibers cross-striated; but some of them are not striated, exhibit enlargements, and stain black by the Golgi method. Retzius (**'91**) notes in each ventral root an anterior division composed only of varicose fibers, and a posterior, somewhat smaller division carrying fibers mostly without varicosities. Each fiber of the posterior portion bears an oval body a short distance after its exit from the spinal cord. Retzius is unable to decide whether these are sheath nuclei, or of some other nature. Toward the spinal cord motor nerve fibers were followed singly into the outer layer of the cord, where they enter a low, granular mound; but beyond this they cannot be followed. In these mounds the fibers often bend in hook-fashion. Retzius thinks that the connection of motor fibers with central ganglion cells, which he does not doubt, cannot be very direct. Fibers occasionally divide dichotomously, but there are no special nerve endings. Especially characteristic of these fibers is their varicose-granular condition. Owing to the abundance and transverse elongation of the granules, they give a cross-striated appearance to the fibers, probably the cause of Schneider's erroneous views, for these are true nerve fibers, and not of muscular nature. He is inclined to accept Rohde's suggestion that the *transverse* muscles are innervated by fibers carried by the dorsal roots.

Heymans et van der Stricht (**'98**) believe that the presence of a pair of dorsal nerves behind the last ventral pair indicates the disappearance, or non-appearance, of a posterior myotome, and on this basis regard each dorsal root as corresponding to the following, not the preceding (as Hatschek states), ventral root. The division of a root into dorsal and ventral portions (Retzius) is based on an optical illusion, or on the effect produced by sections. The motor fibers, studied largely on Golgi preparations, penetrate between muscle plates and reach the peripheral zone of the myotome; they sometimes branch. Both ascending and descending fibers show varicosities, consisting of regular and of irregular swellings. The regular swellings are probably nuclei, but those of irregular occurrence are artifacts. Motor fibers show definite terminal bodies, which are flattened and conical, of spatulate form; they are perhaps to be considered as motor plates of the cylindrical muscle fiber, upon which the terminal nerve fiber inserts itself, not perpendicularly, but laterally. From the number of these terminal plates the authors believe that each muscle fiber is innervated. The nerve fibers of the ventral roots are non-medullated, and thicker than those of the dorsal roots. The striated fibers in ventral roots are not nerve fibers, but muscle fibers, deflected from their usual course.

Dogiel (**:02**) impregnated motor fibers and end-plates with methylene blue. He finds that the typical motor nerve ending is not a plate, but a flattened cone, the basal surface of which lies on the surface of a *muscle* plate. The cones appear larger in Golgi preparations than in those impregnated with methylene-blue, because in the former case silver is deposited to some extent in the muscle as well as in the nerve. Dogiel finds motor fibers penetrating (centripetally) the ventral part of the nerve cord in more or less thick bundles. These fibers arch backward, and can be followed for some distance. Certain fibers bend downward under the central canal, either from right to left, or vice versa, and so constitute a kind of commissural fiber. Dogiel could not follow fibers to cells, but states that they do not divide into fine threads, nor form a net on entering the nerve cord. Although he says that few varicosities are present in methylene-blue preparations, his Figures 43, 44*a*, and 44*b*, show frequent swellings in the motor nerve fibers. Repeated branchings of the motor fibers in their course toward the periphery are seen in his Figure 41.

In my own study the most successful impregnations of motor fibers were obtained by use of the rapid methods of Golgi, and by treatment with gold chloride after fixation with 10% formol. The methods of Golgi and their modifications have produced such varying results in the appearance of nerve fibers in vertebrates generally, that a great number of variations in length of time, temperature, etc., were employed in order to test, if possible, the reliability of the impregnations obtained. Several hundred specimens were subjected to these various modifications of the methods of Golgi, and enough specimens have been sectioned and examined to establish some definite facts. In general it was found that all animals must be cut into two or more pieces, depending on their size; since whole specimens, beside impregnating unsatisfactorily, usually show numerous artifacts. It is absolutely essential that the fluids employed be used in large quantities, and slight warmth was found advantageous in bringing about impregnation. A liberal amount of the silver nitrate should be used and changed at least once. The rapid methods were by far the most satisfactory, as shown by the greater number of fibers impregnated, and the freedom from artifacts. Dogiel states that varicosities in the motor fibers are constantly present in Golgi preparations. The accompanying figures (Pl. 5, Figs. 25–29; Pl. 8, Figs. 43–50) show that such is not the case. The difficulty in securing impregnations of motor fibers after they enter the neural tube suggests that their chemical composition may differ in that region. Modifications in the methods employed may hereafter bring results in this case, as in others.

The impregnations with gold chloride after fixation with 10% formol are comparatively free from distortion and artifacts, and are probably reliable as a basis for comparison. The impregnation of motor fibers and endings is somewhat uncertain with this method, but when successful the preparations are very satisfactory. The nerve fibers are not strikingly differentiated, but are favorable for study on account of their accurate fixation. These preparations show a tendency to fade after a considerable length of time. I did not succeed by this method in impregnating fibers distinctly as they enter the neural tube, but variations in the method may produce better results.

Strong light was particularly useful in studying the thick sections necessary for tracing the nerve fibers. The shrinkage in the surrounding tissues often causes wrinkling in fibers impregnated by the Golgi method (Pl. 8, Figs. 46, 47). These fibers also show a tendency to break, but the artificial nature of such breaks is usually evident.

The ventral nerves were studied in both species of Branchiostoma, and no differences were found which could not be readily explained by the uncertainty of the methods employed. The nerve fibers of the ventral nerves show such uniform smoothness in good preparations that it seems doubtful if any actual "varicosities" exist. The numerous smooth fibers illustrated in Figures 28, 29 (Pl. **5**), and 46 (Pl. **8**), and the uniform smoothness of fibers impregnated with gold chloride, certainly give grounds for such a conclusion. The small swellings sometimes present in my Golgi preparations are of irregular occurrence, and are usually found in whole specimens, or those impregnated according to the slow methods. It is true that occasionally structures resembling bipolar ganglion cells (Pl. **8**, Fig. 50) are to be found toward the proximal ends of fibers in the ventral nerves, but one hesitates to regard these structures as cells when their presence here would indicate a marked variation from the typical condition. In Figure 50 an apparent nucleus is present in the cell-like body attached to the nerve fiber; but every investigator who has used the methods of Golgi knows what bewildering artifacts can be produced.

According to my preparations the motor fibers of Amphioxus leave the inner border of the spinal cord sheath (Fig. 25, Pl. **5**) as rather slender, smooth fibers, apparently differing somewhat in size. These fibers are often curved or wrinkled in the sheath, and frequently bend sharply as they leave the covering of the neural tube. They are continued to the muscle border (Figs. 25, 26) as comparatively smooth threads, changing little in caliber. The spreading out of the ventral nerves has been described so often that it need not be taken up here. Upon reaching the side muscles certain motor fibers increase in size, while others do not change, depending apparently on their distance from the muscle fibers which they innervate. As a fiber approaches its distal end, it gradually thickens, and finally ends with the peculiar plate- or cone-like structure, first noted by Heymans et van der Stricht (**'98**). In contrast to the observations of Dogiel (**: 02**), little branching is evident in the course of these fibers. Figures 28, 29 (Pl. **5**) and 46 (Pl. **8**), which show a large number of motor fibers, do not reveal branching. In Figures 28 (Pl. **5**) and 43 (Pl. **8**) single fibers may be traced from near the internal border of the muscles to a short distance from its exterior border and in one case (Fig. 43) to the termination of the fiber. These long, single fibers are significantly frequent in various parts of the myotome. Branching certainly occurs (Pl. **8**, Figs. 48, 49), but my preparations do not reveal the extensive division in the region of the muscles described by Dogiel.

It may be noted in this connection that a greater number of nerve fibers are impregnated in the region of the muscles than in the neighborhood of the neural tube. This might be due to any one of several conditions; e. g., a fiber may change in character before it reaches the neural tube, thus presenting for impregnation different conditions in its two parts; or branching may take place more freely than my specimens indicate. Perhaps the mechanical tension on the nerves, between their exit from the neural tube and the nearest point of the muscle, due to the action of reagents, may destroy, or render unsuitable for impregnation, that part of the nerve.

The motor endings shown in Figures 27, 28 (Pl. 5) and 43–45 (Pl. 8) are probably fairly typical. They agree in size and form with those in gold-chloride preparations, and the surrounding muscle does not show the deposit of silver sometimes evident in material impregnated by the Golgi method. These endings were found in nearly all portions of the side muscles except the region adjoining the skin. They are present in considerable numbers a short distance from the exterior. Many endings were found dorsal to the neural tube, not far from the dorsal fin. I was unable to trace motor fibers into the neural tube beyond its sheath. No preparations of any kind showed motor fibers continuing in the neural tube in the manner figured by Dogiel in his Figur 45. In certain preparations, not cut in a true frontal plane, fibers appeared to end some distance inside the sheath of the neural tube, but on further study this proved to be an illusion, caused by the direction of the section. I am inclined to believe that the true course of motor fibers in the neural tube is as yet undiscovered.

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EXPLANATION OF FIGURES.

Each drawing was outlined with an Abbé camera lucida. The numbers of the dorsal nerves are indicated by Roman numerals. Unless otherwise stated, dorsal is up in all figures.

ABBREVIATIONS.

N. B.— Unfortunately an entirely consistent system of lettering figures could not be adopted owing to the incorporation of the provisional lettering in the drawings themselves. With the reduction in size accompanying the reproduction of the figures, many of these letters are extremely small. For explanation of all letters not embraced in the following list, the reader may consult the explanation of the Figure on which the letters occur.

ant.	Anterior.
b. n.	Band nerve of velum.
brs. gon.	Gonadial pouch.
ch.	Chorda dorsalis.
dx.	Right.
e. p.	End-plate.
e. s.	Eye-spot.
i.	Nerve branches to inner mouth plexus.
i. a. p.	Inner abdominal plexus.
l.	Lateral.
l. d.	Ligamentum denticulatum.
m.	Muscle.
m_1, m_2, m_3.	Muscle of myomeres 1, 2, 3 etc.
marg. l.	Lateral margin.
marg. m.	Median margin.
med.	Median.
m. v.	Ventral margin of muscle.
my'sep.	Myoseptum.
n.	Nucleus.
n. t.	Neural tube.
o.	Nerve branches to outer mouth plexus. In Fig. 2, *o* is olfactory pit.
par. go.	Wall of gonadium.
post.	Posterior.
Q.	Cells of Quatrefages.
r. c.	Ramus cutaneus.
r. c. v.	Ramus cutaneus ventralis.
r. d.	Ramus dorsalis.
r. v.	Ramus ventralis.
r. v. a.	Ramus visceralis ascendens.
r. v. d.	Ramus visceralis descendens.
sin.	Left.
t.^{1}	Velar tentacle, small.
t.^{2}	Velar tentacle, large, central.
v.	Velum.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE TECHNIQUE OF HIGH PRESSURE EXPERIMENTING.

By P. W. Bridgman.

Investigations in Light and Heat made and published with Aid from the Rumford Fund.

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THE TECHNIQUE OF HIGH PRESSURE EXPERIMENTING.

By P. W. Bridgman.

Presented, Jan. 14, 1914. Received, Dec. 29, 1913.

In this paper I propose to collect the results of several years' experience in designing apparatus and conducting experiments at high pressures. The rather unusual magnitude of the pressures, from 12000 to 30000 kgm / cm^2, has made necessary the development of methods different from those which have hitherto sufficed for more moderate pressures, up to perhaps 3000 kgm/cm^2. I shall endeavor to present enough of the details of manipulation so that any one may construct apparatus for experimenting in this as yet almost untouched field of higher pressures. It is not claimed that the methods presented furnish the only solution of the problems of high pressure technique; all that is claimed is that the methods given are possible solutions which have stood the test of constant use for a number of years.

The plan of presentation is to indicate the essential parts of a piece of high pressure apparatus, and then to describe in detail the peculiar features of construction of each of the parts.

The apparatus consists essentially of a chamber in which pressure is produced by a plunger, a mechanism for pushing the plunger into the chamber, a tube connecting the chamber in which pressure is produced with a second chamber adapted to the particular investigation, and a pressure gauge. The second chamber is the only part of the apparatus that need be varied for different experiments. The other parts will be described in detail here.

Packing.

Obviously an absolute essential to the success of any high pressure apparatus is some reliable method of packing. I shall describe here the broad principle of this packing, leaving for further description the numerous modifications for special uses. All the packing used in this work is so designed that at high pressures it is made tighter by the action of the pressure itself. Figure 1, showing the packing for the piston will make the principle clear. The piston P pushes the plug A through the medium of the hardened ring R, the cupped washer of soft steel, C, and the rubber packing B. The liquid compressed is below A at L. The plug A is provided with a stem which is long enough to reach into the ring R, but not long enough to reach to the piston P. If now we consider the equilibrium of A, we see that the fluid pressure over the lower end of A must be balanced by the pressure exerted by the packing B on an area less than that of A by the area of the unsupported stem. The result is that the hydrostatic pressure in the packing B per unit area is always a certain percentage higher than that in the liquid, so that the liquid can never leak past the piston. This principle is capable of manifold modification and adaptation, but it will always be found that there is somewhere an area unexposed to the action of pressure, so that the hydrostatic pressure in the packing itself is always higher than that in the liquid.

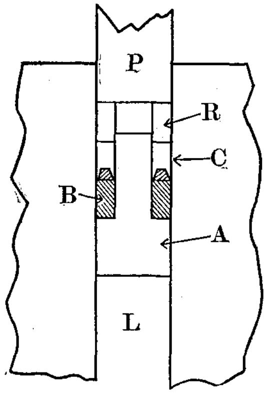

Fig. 1.

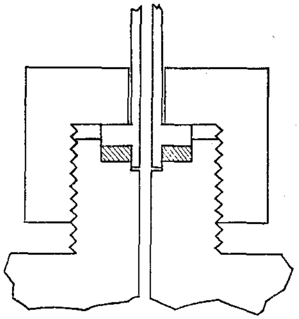

Fig. 2.

FIGURE 1. Shows the general principle of the packing by which the pressure in the packing B is always kept higher than that in the liquid at L. The scale of the diagram is $\frac{3}{4}$ actual size.

FIGURE 2. Shows a common previous type of packing for high pressures. The packing is compressed by a powerful screw into a confined space. This packing leaks when the pressure in the liquid rises as high as that initially applied with the screw.

It is instructive to compare this packing with that formerly used for high pressures, by Amagat, for instance, up to 3000 kgm. His packing is compressed initially by a powerful screw into a confined space. Reference to Figure 2 will show that such a packing must leak as soon as the pressure in the liquid reaches the pressure applied initially to the packing by the screw.

We proceed to the detailed consideration of the various parts of the apparatus.

The Mechanism for Moving the Piston.

This mechanism, of course, may be anything that will furnish a force of the required intensity and exert it over a long enough distance. It has been usual in previous high pressure work to use a screw to drive the piston. When the pressure to be produced becomes high, the screw becomes very inefficient, and it is highly desirable to replace it with a hydraulic press. In my early work up to 6000 kgm., the piston (½ inch in diameter) was driven by a screw. This screw had a pitch of 8 threads to the inch, and needed a six foot wrench to turn it. Its efficiency was less than 5%. Furthermore, even when made of nickel steel, the screws, which were one inch in diameter, finally broke down and had to be replaced. The hydraulic press which I have been using for the last six years has never given the slightest trouble. To avoid bulky apparatus, it is desirable to actuate the press with liquid at a fairly high pressure. For this purpose a pump of the Société Genevoise is convenient, giving 1000 kgm/cm^2 with a lever. The diameter of the piston of the hydraulic press is 2½ inches, which permits, therefore, of a pressure of 25000 kgm/cm^2 on a ½ inch piston, the size usually used. The barrel of the press is made of mild steel, 4 inches in outside diameter. The piston is threaded over the entire length and provided with a heavy nut by which the piston may be maintained in any desired position, even when the pressure on the low end is relieved. This arrangement has proved an indispensable convenience in operation. Opposed to the main hydraulic press is a smaller press with a ¾ inch piston, connected to the larger piston by tie rods and a yoke pressing against the nut. By this auxiliary press the piston of the large press can be rapidly returned to its initial position after completion of a stroke.

It is essential that the press be accurately constructed so that the thrust on the high pressure piston shall be exactly centered; otherwise the piston will buckle.

The packing on the low pressure piston of the press embodies the principle shown above. It might be possible to use a cup leather, but this would probably last for only a short time, for my experience has been that even so low a pressure as 1000 kgm. is sufficient to rapidly mechanically disintegrate the fibrous structure of the leather. Figure 3 shows the packing. The end piece A with stem B plays in the hole C in the end of the piston. The packing is a disc of $\frac{1}{4}$ inch soft rubber between two washers of $\frac{1}{8}$ inch red fibre. The rubber washers will wear away in time, but the fibre washers will have then become so adapted as to give the necessary tightness. In six years of constant use, this packing has been renewed only once, and then only as a matter of precaution, when the press was dismantled for another purpose. In order to make absolutely sure that the end of the stem B is always free from stress it is well to drill a small lateral hole at the bottom of C to allow any liquid to flow away that might percolate around the packing when it is relaxed, as it is when there is no pressure behind it.

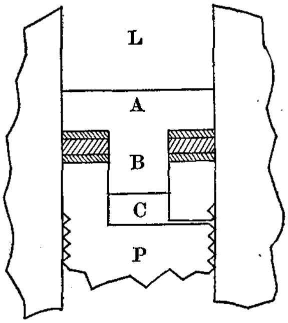

Figure 3. Shows a section of the cylinder of the hydraulic press with the packing on the end of the piston P. Pressure up to 1000 kgm. per sq. cm. is exerted on the piston by means of a liquid at L. The scale of the diagram is $\frac{1}{3}$ actual size.

The High Pressure Piston.

It has been found best to make this in two distinct parts; a piston proper by which pressure is transmitted from the press, and a packing plug driven by the piston into the cylinder. The piston proper is a cylindrical piece of steel with perfectly plane ends. It is made of tool steel, first turned between centers to the approximate dimensions, then hardened glass hard and left with the temper undrawn, ground to the final size between centers, and finally the center marks ground out, leaving the ends plane. If the center holes are not ground out, the piston is much more likely to crack. For pressures up to 15000 kgm. any high grade of carbon tool steel is good enough. For higher pressures it will be well to use one of the special tool steels that admit of being made especially hard. I have found chrome or silicon steels very suitable. One of these broke at 50,000 kgm/cm^2. It has been my experience that the new high speed steels are not so good for this purpose as the old fashion carbon steels. It is the hardness that counts — and brittleness is no disadvantage. After prolonged use, the pistons may be expected to show longitudinal cracks. I have never had a piston fail in actual use, although I have used pistons a number of times after the appearance of the cracks. The piston must not be too long, or it will buckle. About 4 inches is a good length for a diameter of $\frac{1}{2}$ inch. In use, of course, the piston is partially supported by the cylinder when high pressures are reached. A cylinder $\frac{1}{2}$ inch in diameter may easily support 25000 kgm. with an unsupported length of $1\frac{1}{2}$ inches, provided that it is properly centered.

Some sort of face plate must intervene between the hardened piston and the soft steel piston of the press. The form shown in Figure 4 is convenient. This is made of tool steel, hardened, and drawn to a blue. If not drawn, it is quite likely that it will crack, or even that the center will be sheared through. A thin (0.01 inch) copper washer at A ensures uniform seating of the piston. The piston, it may be mentioned, is most likely to crack longitudinally when the ends press against something soft that may flow out laterally, carrying the piston with it by friction. It is well, therefore, to make the center hole in the face plate a good fit for the piston.

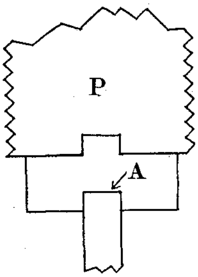

Fig. 4.

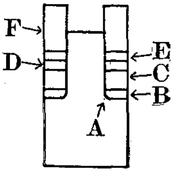

Fig. 5.

FIGURE 4. Shows the face plate of hardened steel between the half inch high pressure piston and the piston of soft steel, P, of the hydraulic press. The scale of the diagram is $\frac{2}{5}$ actual size.

FIGURE 5. The packing plug on the movable piston for use up to the highest pressures in the finally seasoned apparatus. The scale of the diagram is $\frac{4}{5}$ actual size.

The principle of the packing plug at the end of the piston has been described, but there are a number of details of construction. This plug may have two forms, one for use in the apparatus as finally set up ready for experiment, the other for use in giving the preliminary seasoning. I describe first the plug for regular use. Figure 5 shows the dimensions. It is particularly important that the corners at A should be left slightly rounded. The plug may be made an easy fit, say 0.001 of an inch too small, for the hole in the cylinder. The plug is further provided with a washer of copper, B, $\frac{1}{16}$ of an inch thick, a force fit for the hole. For convenience in handling, it is well to solder the washer B to the plug. Above B is the packing washer, C. $\frac{1}{8}$ of an inch is thick enough for this rubber washer if only a few strokes of the piston are to be made, but since the enormous friction rapidly wears away the rubber, the packing should be made thicker if longer use is contemplated. It is an advantage to keep the packing as thin as possible, for in this way the total friction is reduced. Above the packing is another copper washer, D, identical with the lower one, and above this a washer, E, of the same dimensions, of soft Chrome Nickel steel. Finally, above the soft steel washer, is a hardened steel ring, F, against which the piston bears. This ring is likely to crack. If pressures not over 12000 kgm. are to be used, the ring may be made of tool steel, hardened, and drawn to a black. But if pressures to 25000 are to be used, the ring must be left glass hard, and one may expect to renew this ring with each new setting up of the apparatus.

The packing plug just described is one of the few parts of the apparatus almost certain to give way after long use. Fortunately the results of failure are nothing more serious than the projection of the severed stem against the piston, and sometimes, though not usually, leak. Failure takes the form of separation of the stem from the head at the corner A by the "pinching-off"[1] effect. The grade of steel used for the plug is therefore, important; a peculiar sort of toughness is necessary. I have found best for this a Krupp Chrome Nickel steel, grade E. F. 60.0. The New York agents are Thos. Prosser and Son, 15 Gold St. It is strange that another grade of steel with a higher tensile strength, and better adapted for the construction of the cylinders, is not so good for the plug. The plug is to be hardened in oil, and the temper left undrawn. The degree of heating during the hardening is important, slight differences having a great effect on the resistance to the "pinching-off" effect. The precise temperature of heating can best be told by experiment.

The size of the rubber washers used for packing is also important; these should be as much larger than the hole as can conveniently be used. The reason is that under high pressure rubber becomes rigid and brittle like glass, and under the right conditions may even crack. If the rubber is initially so small that under the high pressure its natural volume compression would make it smaller than the hole, then it would certainly leak if it were not for the excess pressure in the packing over that in the liquid. But the rigidity of the rubber may become so great that the excess pressure is not sufficient to force it tightly against the walls of the cylinder, unless the rubber would of itself completely fill the hole at the high pressure.

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[1] P. W. Bridgman, Phil. Mag., **24**, 68 (1912).

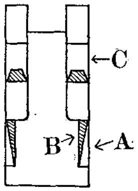

FIGURE 6. Piston packing for high pressures for use in seasoning the cylinders when provision must be made for stretch of the cylinder without leak. The scale of the diagram is $\frac{4}{5}$ actual size.

The form of plug to be used during the preliminary seasoning of the apparatus is essentially the form described above so modified as to allow it to follow the stretching of the cylinder. It could be used under all circumstances, except that the friction is considerably higher than that of the other form. It is shown in Figure 6. The possibility of distension of the head to follow the stretching of the cylinder is provided by a coned ring of copper, A, backed with solder, B, and the possibility of stretching at the upper end is provided by using a washer of soft steel, C, rather deeply cupped, the groove of the cup being filled with a solder. With such a plug, a pressure of 25000 kgm/cm.^{2} may be maintained in a cylinder which has stretched $\frac{1}{16}$ of an inch.

THE CYLINDERS.

The cylinders, or "bombs," need to be subjected to a seasoning process, because the pressures at which they are to be used are beyond the natural elastic limit of the steel. It is, therefore, necessary to raise the elastic limit by the application of a stress beyond the original elastic limit. But since the application of a stress great enough to permanently raise the elastic limit produces a permanent distortion of the cylinder, the cylinder must be machined again to the final size after the preliminary stretching.

The choice of steel for the cylinder is a matter of much importance. A high tensile strength, combined with a moderate elongation before rupture is essential. The Krupp Chrome Nickel steel mentioned for the piston is satisfactory for this purpose, or somewhat better is a

Chrome Vanadium steel, "Type D," made by the Halcombe Steel Co. of Syracuse, N. Y. This latter steel may show a tensile strength as high as 300,000 lb. per sq. in. Before the stretching process, the cylinder should be hardened by heating to from 870° to 1000°, and quenching in oil. The temper should not be drawn at all after this hardening. It is not possible to get either of these two steels glass hard like tool steel, unless by case hardening, so that it is possible to enlarge a hole by reaming in a piece of steel so hardened, or even to drill it.

The dimensions of the cylinders are capable of some variation. It has been my experience that the outside diameter of the cylinder should be from six to ten times that of the inside hole. Not much is gained by making it heavier than ten times, and it is likely to break if made much less than six times. Several cylinders of four times the diameter have eventually broken at 12000 kgm. although they withstood a seasoning pressure of 25000 kgm. Choice of dimensions must furthermore be influenced by the consideration that there is a distinct advantage in keeping the absolute dimensions of the apparatus small, because hardening by quenching cannot reach to the interior of a large piece of steel. I have found it convenient to use a diameter of about $\frac{1}{2}$ of an inch for the piston, and an outside diameter for the cylinder of from 4 to $4\frac{1}{2}$ inches. If the chrome vanadium steel is used, the hole may be made initially $\frac{1}{2}$ inch in diameter, and the stretch after seasoning to 25000 kgm. will be small enough so that the hole may be reamed to a final size of $\frac{17}{32}$ of an inch. If the Krupp steel is used, a slightly greater allowance for stretch must be made, but it will still be less than $\frac{1}{16}$ of an inch on a diameter of $\frac{1}{2}$ an inch.

The actual details of the seasoning require little comment. Pressure should be increased gradually, stopping after every increase long enough for the viscous yield to entirely disappear. The length of the steps by which the pressure is increased may be so chosen that the maximum is reached in from ten to fifteen steps. Pressure should be maintained at the maximum for several hours. One application of the seasoning pressure is sufficient, unless the stretch should be so great as to make a second stroke necessary to reach the maximum pressure. The seasoning pressure should be as high as can be reached without permanent deterioration of the steel. With the grades of steel mentioned above, this pressure may be safely as high as 25000 to 30000 kgm. A mixture of $\frac{2}{3}$ (by volume) glycerine and $\frac{1}{3}$ water is suitable for transmitting the pressure during the seasoning. The necessary length of stroke may be reduced by filling those parts of the interior of the cylinder which will not be reached by the piston with a core of brass or steel, thus reducing the volume and the compression of the liquid.

One must always be prepared for disappointment after constructing one of these cylinders, for in spite of the greatest care of the manufacturer one cannot at the present day be sure that the steel will be free from flaws. These flaws may develop during the preliminary seasoning, but are much more likely not to show themselves until the piece has been reamed to the final size. For example, one cylinder had been used for a year before finally a seam opened in a wall $2\frac{1}{2}$ inches thick, letting through a very fine stream of liquid. It is a matter of pure chance whether a flaw will be found or not; of two pieces from the same bar, one may have a flaw, and the other may be sound. It has been my experience that about three out of every four pieces are sound.

Connecting Pipes.

The proper construction of connections from one piece of apparatus to another has been until recently the most serious problem of all this high pressure work, and the cause of almost every explosion. The problem has at last, however, been satisfactorily solved. Diferent types of connection may be used, depending on the pressure to be carried. We begin with the connections for the lower pressures.

For low pressure transmission, such as for the low pressure end of the press up to 1000 kgm., the most convenient connection is copper tubing. The size I have used is $\frac{1}{4}$ of an inch outside diameter and $\frac{1}{16}$ of an inch inside diameter. This may be used either in its hard drawn state or else annealed. It will stand a single application of 1500 kgm. and may be used almost indefinitely to 1000 kgm. For coupling together two pieces of pipe, a cone coupling with right and left handed thread will be found very convenient. (See Figure 7.) The hollow cone may be best made of steel, since it is subjected to greater strain than the other, which may be made of brass. The copper tubing is attached to the cones by threading and soldering. The thread may well be as long as $\frac{1}{2}$ inch, and the unthreaded part another $\frac{1}{2}$ inch. If the soldered length at the end of the pipe is much less than 1 inch long, solder will be slowly extruded through the threads by the pressure, and there will eventually be leak.

For making connections to the cylinder the coupling shown in

Figure 8 is useful. It will be seen that this is an application of the packing principle described above. Here the projecting pipe itself is the area unsupported, so that the rubber packing, A, exerts its pressure on an area less than that acted on by the liquid by an amount equal to the area of the cross section of the pipe. Brass washers, B and C, are strong enough for the end of the tube. These brass washers should fit the hole within one or two thousandths of an inch, so that the rubber may not be blown through the cracks. Occasionally, after several years' use, a copper tube may fail by the "pinching-off" effect at the rubber packing.

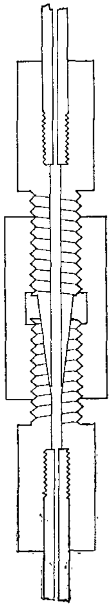

Fig. 7.

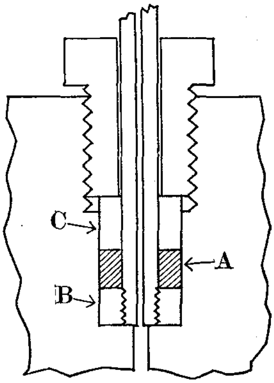

Fig. 8.

FIGURE 7. Coned coupling for copper tubing up to 1000 kgm. The scale of the diagram is $\frac{1}{2}$ actual size.

FIGURE 8. The packing for copper tubing, or, with slight modifications, for commercial steel tubing up to 4000 kgm. The scale of the diagram is $\frac{2}{3}$ actual size.

For pressures higher than 1000, up to 6000 or 7000 kgm., it is possible to get commercial steel tubing. The National Tubing Co., Pittsburg, Pa., draws a size of "Shelby" tubing $\frac{5}{16}$ of an inch outside diameter, and $\frac{1}{16}$ of an inch inside diameter, which I have found suitable. This may stand as much as 9000 kgm. on the first application of pressure. The tubing should not be softened, but left hard drawn. It is likely to have flaws in it, but these can be usually detected by a careful examination of the outside of the tubing. For pressures up to 4000 kgm. it is safe to use with this tubing a connection like that shown above (in Figure 8) for the copper tubing. But the right and left hand thread connection (of Figure 7) cannot be used, since for one thing a soldered joint will not stand the pressure, the solder being eventually pushed out. The washers, of course, should be of steel, and it pays to cup the upper steel washer, filling the groove of the cup with solder. At pressures in excess of 4000 kgm. the tube is likely to fail by the "pinching-off" effect, if the rubber type of packing is used. This "pinching-off" effect almost invariably takes place at the bottom of the thread, the weakest part of the tube.

For higher pressures than 4000, up to 6000 or 7000, a connection must be used which prevents the packing from coming into contact with the thread. The cone packing of Figure 9 answers this purpose. The cone, A, screwing over the end of the tube is of hardened nickel steel. The liquid is kept from coming into contact with the thread by a ring, B, of soft steel, protected by solder, C, above. A hollow cone of soft steel, D, cut at a slightly more acute angle than the solid cone is the packing. To ensure initial tightness, before pressure has been pushed high enough to make the cones conform to each other, a thin piece of rubber, or a ring of copper may be placed between the cones. This packing has never been quite satisfactory, since often rupture did ultimately occur at the base of the threads. At the Geophysical Laboratory in Washington a method has been developed by Dr. John Johnston for packing commercial steel tubing that will doubtless be found more convenient than the above.[2] He has tested the method to 8000 kgm., but whether the tubing will stand the continued application of 8000 I do not know.

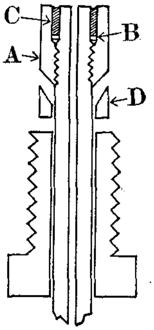

FIGURE 9. Coned packing for use with tubing up to 7000 kgm. The purpose is to keep the pressure from the threads on the outside of the tubing, where rupture is particularly likely to occur. The scale of the diagram is $\frac{2}{3}$ actual size.

For continued use to pressures higher than 7000 I have not been able to use any commercial drawn tubing, but have found it necessary to drill the tubing from the solid rod. The same grade of steel as that used for the cylinders should be used for the drilled tubing. The inside diameter of the tubing is $\frac{1}{16}$ of an inch, and it is quite possible with a little practise to drill pieces at least 17 inches long. The drill should be cut on the end of a long piece of drill rod; it does not pay to try to braze a long shank onto a short drill. Two essentials in successfully drilling a long piece of tube are to start with the drill accurately central, and to use as homogeneous stock as possible. The drill need not be expected to run more than $\frac{1}{2}$ of an inch out of center on a piece 17 inches long. After getting the drill accurately started for two or three inches it will be found convenient to put the drill in a hand tool holder and force it in by hand. This arrangement makes it much simpler to run the drill in and out of the hole to remove chips. Great care must be observed that the drill does not become clogged with chips. I have found that it pays to carefully clean out the hole with a swab after drilling not more than $\frac{1}{8}$ of an inch. It is easy, if all precautions are observed, to drill a hole $\frac{1}{16}$ of an inch in diameter 17 inches long in from seven to eight hours.

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[2] John Johnston and L. H. Adams, Amer. Jour. Sci., **31**, 505 (1911).

After drilling, the rod is to be turned off over the hole to the final size, so that the whole may be concentric, and then hardened in oil and left undrawn, exactly as are the cylinders.

The problem of the proper connections at the end of the pipe is one that has given great trouble. The coned connection described above was used for some time. The only change necessary from the form used for lower pressures is the interposition of a hardened steel washer between the soft cupped washer and the retaining screw. I have successfully reached 12000 kgm. a number of times with this packing, but ultimately rupture always occurred at the bottom of the threads. It may be that rupture at this point is not a pure pressure effect, but that the cylinders are too heavy for the tubing, so that there is some slight bending at the thread under the unavoidable hard handling of setting up the apparatus. Be that as it may, the cone packing is unsatisfactory for long continued service, particularly at temperatures much above that of the room.

The packing for the connecting tube finally adopted, and which has proved entirely satisfactory, is shown in Figure 10. The main improvement of this over the cone form is that it leaves the tube much heavier, the minimum outside diameter at a point exposed only to internal pressure being $\frac{3}{4}$ of an inch in the new form against about $\frac{1}{4}$ of an inch at the bottom of the thread in the cone form. The end of the tube has a heavy thread cut on it, and is provided with a milled head, H, for a wrench. The packing is composed of three rings; a thin ring of Bessemer, A, a ring of lead, B, $\frac{1}{16}$ of an inch square in cross section, and a ring of Bessemer, C, of $\frac{1}{16}$ inch square section. These rings fit over the end of the tube in an annular space $\frac{1}{16}$ of an inch wide between the tube and the walls of the cylinder. On screwing up the the tube, the rings are forced against a conical shoulder, D, on the tube. It will be noticed that the Bessemer ring C supports the hydrostatic pressure over the entire area of one face, but that at the rear face there is a vacant space, E, so that the pressure must be balanced by the force exerted by the cone on the *corner* of the ring. This means that the intensity of pressure in the steel at the corner is much greater than the pressure in the liquid, so that leak cannot occur. Here again, we have the principle of a packing with an unsupported area. Tightness at the outside of the Bessemer ring is secured by the stretching of the ring over the cone. The lead washer merely serves the purpose of giving initial tightness, and the thin Bessemer washer is to keep the lead washer in place. The projecting ledge against which the washers bear opposite the cone is shown in the figure as the rim of a nickel steel cup, F, resting loosely in the hole. This construction is convenient, because it provides a way of withdrawing the packing rings after use by inserting a threaded rod through the bottom of the cup and pulling the cup out. It is, however, quite possible to use part of the cylinder itself as the ledge, since the removal of the washers, even without the device of the cup, does not present serious difficulties. The lead ring and the heavy Bessemer ring cannot be used more than once, but the thin Bessemer ring for retaining the lead may be used several times.

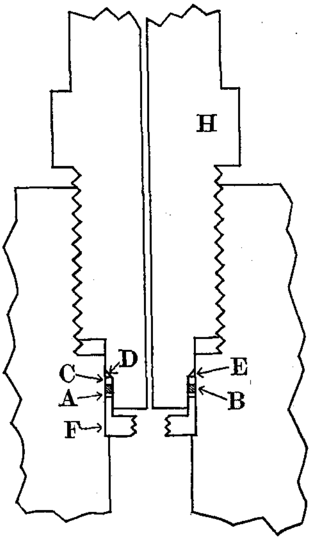

FIGURE 10. Detail of the final packing of rings of soft steel for the drilled tubing of nickel steel. The scale of the diagram is $\frac{2}{3}$ actual size.

This packing has, at the date of writing, been in continuous use for nearly a year, up to 13000 kgm. and 200°, without a single rupture or leak of any kind. It has not been tested higher, but there is no reason why it should not hold until the steel cylinder stretches enough to leak. It is a satisfaction in using this connection to feel that this part of the apparatus is at least no weaker than any other.

The Pressure Gauge.

Without doubt the most convenient means of measuring high pressure is to measure the change produced by the pressure in the electrical resistance of some such alloy as manganin. The details have been given in a previous paper.[3] The only structural difficulties *with the method are those incidental to getting* electrically insulated leads into the cylinder. The principle of the insulating plug has also been described in a previous paper, but not in much detail, and since several improvements have been introduced, it will pay to redescribe it. The plug is shown in Figure 11. It consists essentially of an outer shell, A, to which one electrode is silver soldered, and running through it and insulated from it, a thin steel stem, F, which forms the other electrode. The outer shell of the plug, A, is made of Krupp Chrome Nickel steel, hardened in oil. The packing of the shell used at present is soft rubber, after the same design as that used for the tubing at lower pressures. This packing is not shown in the diagram. There is no weakening of the plug here by a thread on the outside, so that the "pinching-off" effect is not nearly so troublesome as in the case of the packing for the tubing. Moreover, by making the shell with a taper shoulder, and placing over this a soft steel cone, against which the rubber bears, thus providing for the possibility of slip of one metal part over the other in the locality where the "pinching-off" effect would be most likely to occur if the metal were of one solid piece, and by using rubber washers only $\frac{1}{8}$ of an inch thick, the danger of "pinching-off" is reduced to a minimum. However, this design

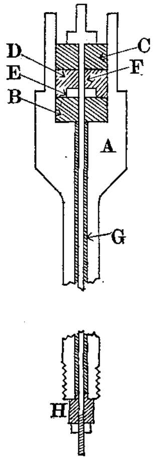

Figure 11. The insulating plug for leading electrical connections to the interior of the pressure chamber. The scale of the diagram is $\frac{2}{3}$ actual size.

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[3] P. W. Bridgman, These Proceedings, **47**, 321–343 (1911).

does not entirely do away with the effect; it is safe to count on a rupture about once a year if the apparatus is used continually to 12000 kgm. Failure of the plug need not result in complete rupture. Once or twice I have been troubled with an obscure leak which I finally traced to a crack at the base of the cone of A, where the "pinching-off" effect might be expected to occur. If ever occasion should arise to make new plugs, I intend to redesign them so as to use the ring packing of steel now used for the connecting tubes.

The insulating properties of the plug are provided by two layers of mica washers, B and C, turned (not punched) so as to be a force fit for the hole. Tightness against leak is provided by a layer of rubber, D, between the two layers of mica. The fine stem, F, is insulated from A for the rest of its length by a thin glass tube, G, slipping into the annular space between F and A. A small cylinder of hard rubber, H, at the outer end completes the insulation. The insulating properties of the plug are improved if all the parts are dipped into paraffine heated to from 120° to 140° immediately before assembling, and if they are kept hot enough during the assembling so that the paraffine remains melted. The insulation resistance is at least 100 megohms, the limit of the measuring instrument used.

It will be noticed that the insulating packing uses again the principle of the unsupported area, the central stem in this case being the unsupported part. But the area of the stem is so slight that the excess pressure provided by it may not be sufficient to overcome the friction in the mica washers against the side of the plug. It is therefore necessary to make the rubber washer initially considerably larger than the hole, or else the tightness will not be permanent. A washer initially $\frac{17}{32}$ of an inch to go into a hole $\frac{7}{16}$ of an inch in diameter is not too large.

The steel disc at E is necessary to prevent the rubber blowing out past the mica washers along the stem. This disc should be of hardened nickel steel. The stem F passing through the washers is also of hardened nickel steel. It, together with the head at its upper end, must be made from one piece. Attempts to braze or screw the head onto a piece of wire have uniformly failed. This thin stem is also likely to fail by the "pinching-off" effect, where it passes through the rubber, or if the friction of the washers is too great, it may be torn apart by the tension afforded by the expansion of the washers during decreasing pressure. Several times I have observed rupture from this cause at 6000 kgm. after decreasing pressure from a maximum of 12000.

The pressure of 6000 kgm. is still sufficient to project the fine stem with considerable violence after rupture takes place, so that one must be particularly careful not to yield to a false feeling of security in handling the plug during decreasing pressure. The insulating plug as thus constructed is at present the part of the apparatus most likely to fail. The mica washers eventually fail along a cone of shear reaching from the hardened disc to the hole in the stem. Slipping of the mica on the shear planes is likely to be accompanied by unsymmetrical yielding of the rubber and the hardened disc, so that the fine stem may be bent or even sheared off by the tilted disc.

It would probably pay in designing new apparatus to experiment with a form of insulating plug developed at the Geophysical Laboratory.[4] Soapstone instead of mica is the insulating material, and the rubber washer is dispensed with, so that it is simpler in construction. It has not been tested over so wide a range as the form described above, but has given perfect satisfaction up to 8000 kgm. The form described above has been used to 21000 kgm., although it would not stand this many times.

VALVES.

I have as yet not been able to construct satisfactory valves. However, it may be worth while to describe a substitute which has been found useful for one particular purpose. In the absence of valves the desired final pressure must be reached with one stroke of the piston. But if the apparatus connected to the cylinder is of more than two or three times the capacity of the cylinder itself, it will not be possible to do this because of the compressibility of the transmitting liquid. But since the greater part of the compression of a liquid occurs in the first few thousand kilograms, it would in many cases be sufficient if an initial pressure of a few thousand kilograms could be produced in the cylinder before the stroke begins. This is accomplished by the use of a small by-pass at the upper end of the cylinder, so situated that it is just uncovered when the piston is withdrawn to its extreme position. Initial pressure to the desired amount is produced in the apparatus by an auxiliary pump through the by-pass, which is then cut off by the descent of the piston. It is necessary to make this by-pass very minute, so that the rubber packing of the piston may not be blown out through it as the piston passes by. One convenient way of doing this is to drill and tap a convenient sized hole at the upper end of the cylinder, say for a 10–32 thread, $\frac{1}{4}$ of an inch long, and then to screw tightly into the tapped hole a carefully cut screw, across the threads of which (that is, parallel to the axis) a fine scratch has been made. This scratch is the channel by which the transmitting fluid gains access to the interior of the cylinder; its size may be regulated at pleasure by varying the size of the scratch. The screw may be made of soft tool steel, since the pressure it has to withstand is not high, thus permitting the end projecting into the cylinder to be reamed off.

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[4] John Johnston and L. H. Adams, Amer. Jour. Sci., **31,** 507 (1911).

The use of a by-pass necessitates the use of an auxiliary pump giving several thousand kilograms. I have found it convenient to use a second pump of the Geneva type,coupled to an intensifier of readily suggested design, stepping up the pressure four times.

PLUGS.

In adapting pieces of apparatus to new uses it will often be necessary to plug holes previously used for connections. This may be done with a plug packed according to the simple method of rubber washers used for the connecting pipes at low pressures (see Figure 12). If the stem and the head, A, are turned from one piece of Krupp Chrome Nickel steel and hardened, the "pinching-off" effect will seldom be troublesome. One should realize, however, that the "pinching-off" effect is treacherous, and likely to come at the most unexpected times. All parts liable to this effect should either be shielded, or else they should be pointed in such a direction that they can do no damage if projected by an explosion.

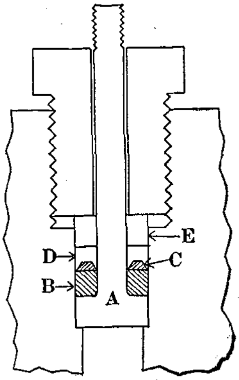

FIGURE 12. The plug for closing holes against high pressures. The plug A is packed with a rubber washer B, a soft steel washer D cupped and filled with solder at C, and the hardened washer E. The scale of the diagram is $\frac{5}{8}$ actual size.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.

Proceedings of the American Academy of Arts and Sciences.

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RECORDS OF MEETINGS, 1913–14.

OFFICERS AND COMMITTEES FOR 1914–15.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

BIOGRAPHICAL NOTICE.

Oliver Fairfield Wadsworth. By C. J. Blake.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

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(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

One thousand and twenty-sixth Meeting.

OCTOBER 8, 1913.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were forty-eight Fellows and many guests (including ladies) present:

The Corresponding Secretary presented the following correspondence:—G. D. Birkhoff, C. J. Bullock, G. W. Chadwick, J. L. Coolidge, Henry Crew, H. A. Christian, S. M. Crothers, D. R. Dewey, F. P. Fish, Arthur Foote, D. C. French, E. F. Gay, C. H. Grandgent, Robert Grant, C. B. Gulick, C. H. Haskins, E. V. Huntington, H. C. G. von Jagemann, J. R. Jewett, N. A. Kent, Wm. Lawrence, A. D. Little, F. B. Mallory, W. B. Munro, E. H. Nichols, W. A. Noyes, Harold Pender, B. L. Pratt, E. K. Rand, H. N. Sheldon, Moorfield Storey, and G. E. Woodberry, accepting Fellowship; from F. B. Dexter, J. T. Morse, Jr., and C. S. Hastings declining Fellowship; from Adam Politzer, accepting Foreign Honorary Membership; from Miss Sara Norton, presenting papers which passed into the possession of her father, the late Charles Eliot Norton, after the death of Chauncey Wright; from a Committee, proposing to commemorate in 1914 the seventh centenary of Roger Bacon's birth by erecting a statue in his honor, and asking subscriptions; from the Jardin Impériale Botanique de St. Pétersbourg, inviting the Academy to take part in the celebration of its bicentennial jubilee; from the World's Congress of International Associations, stating the object of the Congress; from the President, Trustees and Faculty of Princeton University requesting the presence of a delegate at the dedication of the Graduate College, on October 22nd.

The Chair announced the deaths of the following Fellows:—

Okakura Kakuzo, Fellow in Class III., Section 4; Francis Bartlett, Fellow in Class III., Section 4; Reginald Heber Fitz, Fellow in Class II., Section 4.

The President was authorized to appoint a delegate to attend the Princeton dedication.

The following papers were presented by title:—

"Contributions from the Gray Herbarium of Harvard University. New Series XLIII.— A Taxonomic Study of Setaria italica and its immediate Allies." By F. T. Hubbard. Presented by B. L. Robinson.

"Floating Islands." By Sidney Powers. Presented by S. F. Clarke.

"Phase Changes under Pressure. I. The Phase Diagram of Eleven Substances with especial reference to the Melting Curve. By P. W. Bridgman.

The following communications were given:—

"A Winter in Rome." By Professor E. K. Rand.

"Absence of Finality in Physical Science." By Professor John Trowbridge.

At the conclusion of Professor Trowbridge's Communication, a reception was held in the Reading-room, where refreshments were served.

One thousand and twenty-seventh Meeting.

NOVEMBER 12, 1913.

The Academy met at its House.

The PRESIDENT in the chair.

There were twenty-eight Fellows and one guest present.

The following letter was presented:— from Edward Channing, resigning Fellowship.

The following extracts from the will of Francis Amory, who died November 10, 1912, were read:—

EXTRACTS FROM WILL OF FRANCIS AMORY.

Section XXII. I give and bequeath unto the Academy of Arts and Sciences situated in the City of Boston, Mass. in memory of my late uncle, Francis Amory, of Beverly, Mass. the sum of $25,000, but In Trust nevertheless for the following uses and purposes, namely,

To be safely invested and allowed to accumulate till the expiration of 21 years from the date of my death, and then to devote the income alone thereof, to the award of a "Septennial prize" and a gold medal, or other token of honor and merit to be awarded by the President and Fellows of said Academy, at their discretion, to any individual or individuals who shall in their judgment and belief, during the said Septennial period next preceding any award thereof, through experiment, study or otherwise discover any notably useful remedy, or invent any cunning device or instrument for the treatment and cure of diseases and derangement of the human sexual generative organs in general, and more especially for the cure, prevention or relief of the retention of urine cystitis, prostatitis, etc., those distressing disorders so common to old men: moreover as a secondary consideration, I direct that should no invention or discovery as aforesaid be made and brought to the knowledge and deliberation of said President and Fellows during said Septennial period and be awarded as aforesaid, then and in that event, the said President and Fellows may award the said prize and medal or both, to any author or authors, during said period, who may have written any theoretical or practical treatise which in their judgment, may be of extraordinary or exceptional value and merit on the anatomy of said organs, or the treatment of said diseases. But in all cases of competition any aforesaid invention or discovery shall take precedence of any authors or theoretical, practical and approved treatise, any one such individual, however, being eligible to both said prize and medal: it being my intention, purpose and design not only to aid and reward as much as possible, those, the pursuit and practise of whose legitimate profession might naturally and spontaneously aid and lead them towards the realization and attainment of the objects and honors for which this bequest is founded, those, in a word, whose merit and reward are uniformly "the children of their voluntary efforts," but moreover also wishing and intending to render the same well worth the while and efforts of even the unprofessional, and to in every way possible awaken and stimulate the attention, zeal, ambition and sympathy therein, of the poorest, the most indifferent and unconcerned. Therefore should any such septennial period elapse without said prize or token or both for the next preceding period having been awarded, then and in that event the said income or any portion thereof, not having been then paid for said prize, medal or token or both for such period, shall be added to said capital sum of $25,000, and shall become part an parcel of said capital sum and said capital sum shall on no account be expended for any other purpose or object whatsoever.

Section XXV. I desire and suggest that my executors hereinafter named, or whoever shall be charged with the execution of this my will, shall, at the expense of my estate, during the period of three years after the probate of this will, and that thereafterwards, for the further period of twenty-five years, the respective institutions to whom I have devised and bequeathed the trust funds stated in Sections XXII, XXIII and XXIV of this my will, shall, on each and every anniversary of my death, publish in two or more of the principal newspapers printed in the City of Boston, a succinct but clear exhibit of the nature and design of every endowment or trust fund mentioned in this will, and expressly solicit the public to contribute thereto by subscription, donation or bequest.

Fully persuaded myself of the utility, importance and necessity of the several endowments or trust funds I have hereinbefore created, established and bequeathed, and desiring to promote the comfort, prosperity and happiness of my fellow men, I do hereby most earnestly recommend my fellow citizens, friends and relations generally, to contribute thereto by gift, donation and bequest, for the alleviation of misery, for the promotion and diffusion of knowledge and science, and for the honor and glory of the city.

Section XXVI. My executors, hereinafter named, or whoever shall be charged with the execution of this my will, shall render annually to the Probate Court a detailed and itemized account of their administration, and of the investment of the funds remaining in their hands; and the several institutions, mentioned in Sections XXII, XXIII and XXIV, shall annually, at the close of their fiscal years, publish in two or more of the principal newspapers, printed in Boston, a detailed and itemized account of their respective funds, held under the above trusts, and of the administration and application thereof during said year, so that it may at all times appear, on examination by the public, that my intentions regarding the same have been faithfully observed.

Section XXVII. If any legatee, devisee or annuitant hereinbefore named, shall not accept his, her or their legacy, devise or annuity, subject to the conditions and restrictions hereby imposed, within one year after it becomes due, then and in that event, I hereby give, devise and bequeath any and all legacies, divises or annuities so declined or any part or portion thereof, to the Boston Lying-in Hospital, 24 McLean Street, aforementioned, in the same manner and for the same intents and purposes as those mentioned in Section XXIV of this my will.

On motion of the Corresponding Secretary, it was

Voted, That the American Academy of Arts and Sciences agrees to accept the sum of twenty-four thousand dollars ($24,000.00) in payment of the legacy of twenty-five thousand dollars ($25,000.00) left to the Academy under the will of Francis Amory, deceased.

The above concession is made on account of the necessity of a compromise between the Executors of the Will and the Heirs of the Testator in order to prevent a contest.

It was also

Voted, That the Treasurer of the American Academy of Arts and Sciences is hereby authorized to receive and receipt for the above legacy on the terms above set forth.

The following Communication was given by Dr. W. S. Bigelow: —

"On the Method of Practising Concentration and Contemplation, by Chisho Daishi, a Monk of the Shuzenji Monastery of Tendai Mountain; translated from the Chinese by Okakura Kakuzu."

One thousand and twenty-eighth Meeting.

DECEMBER 10, 1913.

The Academy met at its House.

The President in the chair.

There were forty-one Fellows and three guests present.

A letter from Albert Bushnell Hart, declining Fellowship, was presented by the Corresponding Secretary.

Dr. Horatio R. Storer, a Resident Fellow of the Academy from 1858 to 1879, presented a small bronze bas-relief of himself, inscribed "To the Master in Surgery, Medical Numismatist and Lover of Man and Nature, Horatio R. Storer, M. D., LL.D., from his friend R. Tait McKenzie, M.D. 1913."

Mr. Baldwin Coolidge presented a small lithograph, from a drawing by George Rumford Baldwin, in 1830, of the house in which Count Rumford was born.

The above gifts were accepted by the Academy, and the Corresponding Secretary requested to acknowledge them.

Dr. Clarence J. Blake read a biographical notice of Dr. O. F. Wadsworth.

A communication was given by Professor G. W. Pierce on "Wireless Telephony," accompanied by lantern slides and apparatus, which was followed by discussion, and remarks by Professors Thomson, Webster and FitzGerald.

The meeting then adjourned to the Reading-room, where refreshments were served.

One thousand and twenty-ninth Meeting.

JANUARY 14, 1914.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were thirty-nine Fellows and two guests present.

The Corresponding Secretary presented the following letters: — from the Real Academia de Ciencias y Artes de Barcelona, inviting the Academy to send a delegate to the celebration of the one hundred and fiftieth anniversary of its foundation, January 18–20, 1914; from the Museo Nacional de Acqueologia Historia y Ethnologia, sending the felicitations of the New Year; from the Boston Rotary Club, enclosing tickets to its annual exhibition in Horticultural Hall, January 23–24, 1914.

The Chair announced the deaths of the following Fellows: — Seth Carlo Chandler, Class I., Section 1; Silas Weir Mitchell, Class II., Section 3; Benjamin Osgood Peirce, Class I., Section 2.

The Corresponding Secretary read the following Recommendation of the Council: —

The Council recommends that the following Standing Vote be adopted: —

There may be chosen by the Academy, under the same rules by which Fellows are now chosen, one hundred Resident Associates. Not more than forty Resident Associates shall be chosen in any one Class.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be one half that of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

It was

Voted, That this Recommendation be printed on the call for the next Meeting at which it can be considered.

The following gentlemen were elected Fellows of the Academy:— a printed list of nominees having been sent to all Voting Fellows with the notice of the December meeting, in accordance with Chapter III, Article 3 of Statutes:—

In Class I., Section 1 (Mathematics and Astronomy):—

Charles Leonard Bouton, of Cambridge; Clarence Lemuel Elisha Moore, of Boston.

In Class I., Section 2 (Physics):—

Henry Andrews Bumstead, of New Haven, Conn.; John Charles Hubbard, of Worcester; James Edmund Ives, of Worcester; Charles Herbert Williams, of Milton.

In Class I., Section 3 (Chemistry): —

Elmer Peter Kohler, of Cambridge; Arthur Becket Lamb, of Cambridge; Alexander Smith, of New York, N. Y.; Julius Oscar Stieglitz, of Chicago, Ill.

In Class I., Section 4 (Technology and Engineering): —

Francis Tiffany Bowles, of Boston; William Hubert Burr, of New York, N. Y.; Howard Adams Carson, of Malden; John Hays Hammond, of Gloucester; Rudolph Hering, of New York, N. Y.; Henry Marion Howe, of New York, N. Y.; Edward Furber Miller, of Newton; Frederick Haynes Newell, of Washington, D. C.; William Barclay Parsons, of New York, N. Y.; Charles Milton Spofford, of Boston.

In Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe): —

Herbert Percy Whitlock, of Albany, N. Y.

In Class II., Section 3 (Zoology and Physiology):—

Robert Payne Bigelow, of Brookline.

In Class II., Section 4 (Medicine and Surgery):—

Harvey Cushing, of Boston; Richard Pearson Strong, of Boston; Ernest Edward Tyzzer, of Wakefield.

In Class III., Section 1 (Theology, Philosophy and Jurisprudence):—

William Howard Taft, of New Haven, Conn.

In Class III., Section 2 (Philology and Archaeology):—

William Rosenzweig Arnold, of Cambridge; Jeremiah Denis Mathias Ford, of Cambridge; James Haughton Woods, of Cambridge.

In Class III., Section 3 (Political Economy and History):—

Henry Walcott Farnam, of New Haven, Conn.; Roger Bigelow Merriman, of Cambridge; George Grafton Wilson, of Cambridge; George Parker Winship, of Providence, R. I.

In Class III., Section 4 (Literature and the Fine Arts):—

George Pierce Baker, of Cambridge; William Allan Neilson, of Cambridge; Bliss Perry, of Cambridge.

The following gentlemen were elected Foreign Honorary Members of the Academy:—

In Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe):—

Waldemar Christofer Brøgger, of Christiania, Norway.

In Class III., Section 2 (Philology and Archaeology):—

Alfred Percival Maudslay, of London, England.

In Class III., Section 4 (Literature and Fine Arts):—

Sir James Augustus Henry Murray, of Oxford, England.

The following communications were given:—

Mr. Hammond Vinton Hayes. "Government Ownership of Telephones in England."

Dr. William Sturgis Bigelow. "Details of the Buddhist System."

The following paper was presented by title:—

"The Technique of High Pressure Experimenting." By P. W. Bridgman.

One thousand and thirtieth meeting.

February 11, 1914.— Special Meeting.

The Academy met at its House, called by the President for the purpose of making an appropriation from the income of the General Fund for the use of the Library.

The President in the chair.

There were thirty-seven Fellows present.

The following letters of acceptance of Fellowship were presented by the Corresponding Secretary:—from W. R. Arnold, G. P. Baker, R. P. Bigelow, C. L. Bouton, F. T. Bowles, W. H. Burr, Harvey Cushing, H. W. Farnam, J. D. M. Ford, Rudolph Hering, H. M. Howe, J. E. Ives, E. P. Kohler, A. B. Lamb, R. B. Merriman, E. F. Miller, C. L. E. Moore, W. A. Neilson, W. B. Parsons, Alexander Smith, C. M. Spofford, J. O. Stieglitz, W. H. Taft, E. E. Tyzzer, H. P. Whitlock, C. H. Williams, G. G. Wilson, G. P. Winship, J. H. Woods; from A. P. Maudslay, accepting Foreign Honorary Membership.

The death of Sir David Gill, Foreign Honorary Member in Class I., Section 1, was announced by the chair.

On motion of the Corresponding Secretary, it was

Voted, To appropriate one hundred and fifty ($150.) dollars for Expenses of the Library, and fifty ($50.) dollars for Books and Binding, from the income of the General Fund.

On the recommendation of the Council, it was

Voted, That the following Standing Vote be adopted:—

There may be chosen by the Academy, under the same rules by which Fellows are now chosen, one hundred Resident Associates.

Not more than forty Resident Associates shall be chosen in any one Class.

The election of Resident Associates shall be for a term of three years with eligibility for reëlection.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be one-half that of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

The following communications were given:—

"The Relation of Samaji to the Normal Waking Consciousness." By W. S. Bigelow.

"A Note on the Life of Victor Schumann, with some account of recent progress in the Extreme Ultra Violet." By Professor Theodore Lyman.

The following paper was presented by title:—

"Types of Abnormal Color Vision." By Dr. Louis Bell.

One thousand and thirty-first Meeting.

FEBRUARY 25, 1914.

The Academy met in conjunction with the Lawrence Scientific Association in the Testing-room of the Submarine Signal Company, 25 Atlantic Avenue, Boston.

The PRESIDENT in the chair.

There were sixteen Fellows and many members and guests of the Lawrence Scientific Association present.

The following communication was given: —

"Long Distance Submarine Signalling by Dynamo-Electric Machinery." By Mr. Reginald A. Fessenden.

It was

Voted, To express the thanks of the Academy to the Submarine Signal Co. for the great privilege extended to them of witnessing at their works the remarkable experiments of Mr. Fessenden.

One thousand and thirty-second Meeting.

MARCH 11, 1914.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were thirty-nine Fellows present.

The following letters were presented by the Corresponding Secretary:—from J. C. Hubbard, accepting Fellowship; from W. C. Brøgger and Sir James Murray, accepting Foreign Honorary Membership; from the Royal Society of Edinburgh, asking a subscription for the proposed celebration of the tercentenary of the publication of John Napier's "Logarithmorum Canonis Mirifici Descriptio"; to be held in Edinburgh, July 24, 1914; from the Circolo Matematico di Palermo, inviting the Academy to be represented at the thirtieth anniversary of its foundation, to be held April 14, 1914; from the Deutsche Shakespeare Gesellschaft, inviting the Academy to be represented at its fiftieth anniversary, April 22–24, 1914, in Weimar; from the Department of Fine Arts of the Panama-Pacific International Exposition, inviting the Academy to hold a meeting in San Francisco during the Expostion; from the Academia de la Historia, Madrid, requesting a delegate to the celebration of the 400th anniversary of the discovery of the Pacific, to be held at Seville, April 11–15, 1914.

The Chair appointed Professor W. T. Sedgwick to represent the Academy at Seville.

It was

Voted, That the Academy subscribe £2 for the Napier celebration.

On the recommendation of the Council, the following appropriations were made for the ensuing year: —

From the General Fund, $5900. to be used as follows: —

for House expenses	$1800.
for Library expenses	1950.
for Books, periodicals and binding	1000.
for Meeting expenses	150.
for Treasurer's office	600.
for General expenses	400.

From the income of the Publication Fund, $2500. to be used for publication.

From the income of the Rumford Fund, $2952.70 to be used as follows: —

for Research	$1000.
for Books, periodicals and binding	200.
for Publication	600.
to be used at the discretion of the Committee	1152.70

From the Warren Fund, $800. to be used at the discretion of the Committee.

It was

Voted, To transfer the sum of $50. from the Rumford Committee's appropriation for the current year for Publication, to the appropriation for Periodicals, books and binding.

The Chair appointed the following Councillors to act as Nominating Committee: —

Robert W. Willson, of Class I.,
Reginald L. Daly, of Class II.,
Joseph H. Beale, of Class III.

It was

Voted, That the President appoint a representative of the Academy, who, in conjunction with the Librarian, shall confer with representatives of the Boston Society of Natural History, on the possibility of cooperation between the two libraries.

The President appointed Archibald C. Coolidge.

The following communications were given: —

"The Antagonism of Emotional States and the Significance as suggested by Recent Physiological Researches." By Dr. W. B. Cannon.

"The Motion of a Radiating Oscillator." By Professor Edwin B. Wilson.

The following papers were presented by title: —

"The Demagnetizing Factors of Cylindrical Rods in High, Uniform Fields." By B. O. Peirce.

"On Electric Conduction and Thermo-electric Action in Metals." By Edwin H. Hall.

One thousand and thirty-third Meeting.

March 16, 1914.— Special Meeting.

The Academy met at its House, called by the President for the purpose of making an appropriation.

The President in the chair.

There were twenty-two Fellows present.

It was

Voted, To appropriate, from the income of the Publication Fund for publication during the current year, the sum of $609.72.

One thousand and thirty-fourth Meeting.

MARCH 25, 1914.— SPECIAL MEETING.

The Academy met at its House, in conjunction with the Departments of Geology of Harvard University and the Massachusetts Institute of Technology.

The PRESIDENT in the chair.

There were twenty-eight Fellows and many guests present.

The following communication was given by Dr. Arthur L. Day, Director of the Geophysical Laboratory at Washington.

"Observations of the Volcano Kilauea (Hawaii) in action."

At the conclusion of the Paper, adjournment was made to the third floor, where refreshments were served.

One thousand and thirty-fifth Meeting.

APRIL 8, 1914.

The Academy met at its House.

The PRESIDENT in the chair.

There were thirty-nine Fellows, with Guests present.

The Corresponding Secretary presented the following correspondence: from R. P. Strong and J. H. Hammond, accepting Fellowship; from H. A. Carson and Bliss Perry, declining Fellowship; from R. S. Peabody, resigning Fellowship; from the Naturwissenschaftliche Verein, Karlsruhe, announcing its fiftieth anniversary; from the Deutsche Shakespeare Gesellschaft, Weimar, giving a program of its fiftieth anniversary celebration.

The Chair announced the death of Edward S. Holden, Fellow in Class I., Section 1.

The following Communication was given:—

"Demonstration of a New X-ray Tube, devised by W. D. Coolidge, and a consideration of some Measurements relating to Quality as well as Quantity of X-light." By Dr. Francis H. Williams.

After an extended discussion, participated in by the President, Corresponding Secretary, Vice-President Thomson, Professor Webster, the author of the paper and others, adjournment was made to the third floor, where refreshments were served.

One thousand and thirty-sixth Meeting.

MAY 13, 1914.— ANNUAL MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were forty-five Fellows present.

The following letters were read:— from the University of Missouri, inviting the Academy to be represented at its seventy-fifth anniversary, June 3, 1914; from the President of the committee of the fourth centennial of the discovery of the Pacific, thanking the Academy for the designation of Professor W. T. Sedgwick as its delegate; from the editor of the Danish Weekly "Nordlyset" published in New York city, asking for an expression of welcome to Dr. Georg Brandes, a Foreign Honorary Member, who is expected in New York; from the Comité International de la Médaille et de la Fondation Henri Poincaré, requesting a list of members, with a view to soliciting subscriptions.

The following deaths were announced by the Chair:—

Sir John Murray, Foreign Honorary Member in Class II., Section 1; George William Hill, Fellow in Class I., Section 1; Alfred Noble, Fellow in Class I., Section 4; Charles Pickering Putnam, Fellow in Class II., Section 4; Charles Santiago Sanders Peirce, Fellow in Class III., Section 1.

The following report of the Council was presented:—

Since the last report of the Council, there have been reported the deaths of eleven Fellows:— Okakura-Kakuzo, Francis Bartlett, Reginald Heber Fitz, Seth Carlo Chandler, Silas Weir Mitchell, Benjamin Osgood Peirce, Edward Singleton Holden, Charles Pickering Putnam, George William Hill, Alfred Noble and Charles Santiago Sanders Peirce; and of two Foreign Honorary Members:— Sir David Gill and Sir John Murray.

Two Fellows have resigned:— Edward Channing and Robert Swain Peabody.

Seventy-one Fellows have been elected, of which number five have declined Fellowship, two have not yet accepted election.

Three Foreign Honorary Members have been elected.

The roll now includes 387 Fellows and 55 Foreign Honorary Members.

The annual report of the Treasurer was read, of which the following is an abstract:—

GENERAL FUND.

Receipts.

Balance, April 1, 1913	$517.16	
Investments	2,638.46	
Assessments	2,480.00	
Admissions	610.00	
Sundries	10.00	$6,255.62

Expenditures.

Expenses of Library	$2,986.47	
Expense of House	1,696.36	
Expense of Meetings	208.61	
Treasurer	186.57	
Insurance	110.80	
Fireproofing	241.03	
General Expenses of Society	378.56	
Interest on Bonds, bought	6.39	
Income transferred ot principal	221.73	$6,036.52
Balance, April 1, 1914		219.10
		$6,255.62

RUMFORD FUND.

Receipts.

Balance, April 1, 1913	$1,735.65	
Investments	3,190.36	
Sale of Publications	22.50	$4,948.51

Expenditures.

Research	$1,600.00	
Books, periodicals and binding	263.91	
Publication	669.38	
Medals	400.00	
Prof. Richards for Table of Constants	100.00	
Sundries	1.00	
Income transferred to principal	150.39	$3,184.68
Balance, April 1, 1914		1,763.83
		$4,948.51

C. M. WARREN FUND.

Receipts.

Balance, April 1, 1913	$737.04	$1,702.24
Investments	965.20	

Expenditures.

Research	$275.00	
Vault rent, part	4.00	
Income transferred to principal	32.22	$311.22
Balance, April 1, 1914		1,391.02
		$1,702.24

PUBLICATION FUND.

Receipts.

Balance, April 1, 1913	$1,067.04	
Appleton Fund investments	844.26	
Centennial Fund investments	2,388.49	
Sale of Publications	249.86	$4,549.65

Expenditures.

Publications	$4,375.82	
Vault rent, part	12.50	
Income transferred to principal	156.27	$4,544.59
Balance, April 1, 1914		5.06
		$4,549.65

May 13, 1914.

The following reports were also presented:—

REPORT OF THE LIBRARY COMMITTEE.

The general cataloguing of books in the Library has recently been brought up to date, and can probably be now maintained with moderate effort.

The large collection of pamphlets is filed alphabetically, and it has not yet seemed that the considerable cost of cataloguing it would be justified.

The most important work now awaiting attention is that of filling gaps in our serial publications, and as time permits, attention will be concentrated upon this. The labor and time which may be expended in this direction are naturally almost unlimited.

Supplementing previous action of the Council, the Librarian has endeavored, by correspondence and conferences to enlist interest in the publication by the Library of Congress or the Carnegie Institution of the Handbook of the Learned Societies of the Old World. It seems unlikely that any progress can be made, however, in the near future.

Mr. Thomas J. Homer of Boston, with the support of this and other libraries, is working on a new edition of the list of periodicals currently received by the principal libraries in this vicinity.

Plans for coöperation with the library of the Boston Society of Natural History are under consideration by a joint committee.

A valuable collection of manuscripts of the late Professor Benjamin Peirce has recently been received from his heirs for care and safekeeping, which will doubtless be permanent.

A copy of the eleventh edition of the Encyclopedia Britannica has been placed in the Reading Room.

The British Academy and the Cardiff (Wales) Naturalists' Society have been added to our exchange list.

During the year, 107 books have been borrowed from the Library by 26 persons, including 17 Fellows and 2 libraries. All but 7 books have been returned for examination, or satisfactorily accounted for.

The number of bound volumes on the shelves at the time of the last report was 32,715. 960 volumes have been added during the past year, making the number now on the shelves 33,675. This includes 91 purchased from the income of the General Fund, 23 from that of the Rumford Fund, and 846 received by gift or exchange.

265 of the last mentioned are brochures, placed on the shelves in pamphlet covers. The pamphlets added during the year number 453.

The expenses charged to the Library during the same period are: —

Salaries, including that of the Assistant Librarian, whose time is devoted largely to other services of the Academy,	$1493.88
Binding:	
General Fund	620.90
Rumford Fund	66.65
Purchase of periodicals and books:	
General Fund	673.80
Rumford Fund	197.26
Miscellaneous,	197.89
Total	$3250.38

Following is a list of periodicals purchased for the Library: —

Academy.
R Annalen der Physik und Chemie.
R " " " " " Beiblätter.
Annales de Chimie.
R Annales de Physique.
Annales des Mines.
Annales des Ponts et Chaussées.
Annales des Sciences Naturelles Botanique.
" " " " Zoologie.
Annals and Magazine of Natural History.
Année Scientifique et Industrielle.
Archiv für Anatomie und Physiologie.
Archiv für mikroskopische Anatomie.
Astronomical Journal.
Astronomische Nachrichten.
Berliner Astronomisches Jahrbuch.
R Central-Zeitung für Optik u. Mechanik.
Chemical News.
R Comptes Rendus.
Curtis's Botanical Magazine.
Deutsche Chemische Gesellschaft. Berichte.
R Dinglers Polytechnisches Journal.
Ecole Normale Sup. Annales Scientifique.
R Elektrotechnische Zeitschrift.
Engineering.
R Fortschritte der Elektrotechnik.
R Fortschritte der Physik.
Geological Magazine.
International Cat. of Scientific Lit.

Jahrbuch der Chemie (Meyer).
R Jahrbuch für Photographie.
Jahrbuch u. d. Fortschritte der Mathematik.
Jahresbericht der Gesammten Medicin.
Jahresbericht u. d. Fortschritte der Chemie.
Journal de l'Anatomie et de la Physiologie.
Journal de Mathématiques.
R Journal de Physique.
Journal für praktische Chemie.
Journal of Morphology.
Justus Liebig's Annalen der Chemie.
R London, Edin. & Dub. Philosophical Mag.
R Lumière Electrique.
Minerva.
Monatshefte für Mathematik u. Physik.
Mycologia.
Neues Jahrbuch für Mineralogie.
" " " " Beilage.
Nouvelles Annales de Mathématiques.
Nuovo Cimento.
Petermann's Mitteilungen.
R Physikalische Zeitschrift.
R Quarterly Journal of Microscopical Sci.
Revue Générale des Sciences.
Science.
Wer ist's.
Who's who.
Who's who in America.
R Wilson's Photographic Magazine.
Zeitschrift für Analytische Chemie.
Zeitschrift für Biologie.
Zeitschrift für Elektrochemie.
R Zeitschrift für Instrumentenkunde.

The use of the Library, while slightly larger than in previous years, is still far short of what it might be. It is hoped that members of the Academy may, as opportunity offers, bring the resources of the Library to the knowledge of scholars in need of them. The new parcel post regulations should facilitate a somewhat wider service.

H. W. Tyler, *Librarian.*

May 13, 1914.

R Rurchased from the income of the Rumford Fund.

REPORT OF THE RUMFORD COMMITTEE.

During the present year, grants have been made in aid of research as follows:—

October 8, 1913, to Professor Gilbert N. Lewis, of the University of California, in aid of his researches on the "Free Energy Changes in Chemical Reactions" (additional)	$300
To Professor William O. Sawtelle, of Haverford College, in aid of his research on the "Spectra of the Light from the Spark in an Oscillatory Discharge" (additional)	300
To Professor Harvey N. Davis, of Harvard University, in aid of various thermodynamical researches	200
November 12, 1913, to Professor Louis V. King, of McGill University, to defray expenses of computation for his research on the "Scattering and Absorption of Solar Radiations in the Earth's Atmosphere"	250
January 14, 1914, to Professor Alpheus W. Smith, of Ohio State University, in aid of his research on the "Hall and Nernst Effect in the Rare Metals."	100
To Professor Charles G. Abbott of the Smithsonian Astrophysical Observatory, in aid of his research on the "Application of Solar Heat to Domestic Purposes."	150
April 8, 1914, to Professor P. W. Bridgman, of Harvard University, in aid of his thermodynamical researches at high pressures	250

Reports of progress in their several researches have been received from the following persons: C. G. Abbott, P. W. Bridgman, W. W. Campbell, A. L. Clark, D. F. Comstock, H. N. Davis, E. B. Frost, F. E. Kester, F. G. Keyes, L. V. King, E. F. Nichols, E. L. Nichols, C. L. Norton, J. A. Parkhurst, T. W. Richards, G. W. Ritchey, F. A. Saunders, W. O. Sawtelle, A. W. Smith, F. W. Very, A. G. Webster, and R. W. Wood.

The following papers have been published in Volume 49 of the Proceedings of the Academy with aid from the Rumford Fund since the last annual meeting.

No. 1. "Thermodynamic Properties of Twelve Liquids between 20° and 80° and up to 12,000 Kgm. per Sq. Cm.," by P. W. Bridgman.

No. 4. "An improved Method for Determining Specific Heats of Liquids, with Data concerning Dilute Hydrochloric, Hydrobromic, Hydriodic, Nitric, and

Perchloric Acids and Lithium, Sodium and Potassium Hydroxides," by T. W. Richards and A. W. Rowe.

No. 11. "The Technique of High Pressure Experimenting," by P. W. Bridgman.

At the meeting of March 11, 1914, it was unanimously voted for the first time and at the meeting of April 8, 1914, for the second time to recommend to the Academy that the Rumford Premium be awarded to William David Coolidge for his invention of Ductile Tungsten and its application in the production of Radiation.

Charles R. Cross, *Chairman.*

May 13, 1914.

Report of the C. M. Warren Committee.

The C. M. Warren Committee begs to report that there was an unexpended balance of the appropriation for the use of the Committee of $360 at the beginning of the year. In addition to this, the sum of $500 was appropriated by the Academy.

During the year three grants have been made, one an additional grant of $50 to Professor E. W. Washburn, for use in connection with an adiabatic calorimeter; a grant of $225 to Professor R. F. Brunel for work on equilibria in organic reactions where optically active radicals are concerned; and $200 to Professor S. Lawrence Bigelow for study of osmotic membranes.

The Committee has received satisfactory reports of progress from most of those who have received grants from the Fund.

Respectfully submitted,

H. P. Talbot, *Chairman.*

May 13, 1914.

Report of the Publication Committee.

Between April 1, 1913, and April 1, 1914, there were published four numbers of Volume XLVIII (Nos. 18 to 21 inclusive), eleven numbers of Volume XLIX of the Proceedings, and one Memoir (Volume 14, No. 1) making in all 859 pages.

There was available for the use of the Publication Committee an unexpended balance from last year of $1067.04, an appropriation of $2500 and an additional appropriation of $609.72, and an amount of $249.86 from the sales of publications — in all, $4,425.62 from the

Publication Fund and sales. Bills against this appropriation to the amount of $4,422.44 have been approved by the Chairman. This leaves an unexpended balance of $4.18.

Bills aggregating $522.51, incurred in publishing papers on light and heat, have been referred to the Rumford Committee for payment from the Rumford Fund in accordance with authority of the Chairman of that Committee.

G. W. Pierce, *Chairman.*

May 13, 1914.

Report of the House Committee.

The House Committee submits the following report for the year 1913–1914:

The Committee had at its disposal at the beginning of the year a balance of $15.22. The appropriations by the Academy for the year amounted to $1700, making a total of $1715.22. Of this sum $1,696.36 has been expended, which may be summarized as follows:

Janitor	$725.00
Electricity, A. Light	96.78
B. Elevator	44.80
Gas	8.28
Water	12.60
Telephone	75.39
Coal, Furnace, 60 tons	413.98
Water heater, 4 tons	30.50
Ash tickets	14.35
Care of Elevator	33.33
Ice	14.40
Janitor's materials	10.34
Furniture	205.10
Up-keep	11.51
Total expenditure	$1,696.36
Appropriated	1,715.22
Unexpended balance	$98.86

During the year the Academy has held eight regular and two special meetings in the buildings, and the small rooms have been used by Councils and Committees. The Colonial Society has held its meetings in the building during the year, and the American Oriental Society held all day sessions on April 16th and 17th. The arrangement of the previous year with respect to janitorial service has been continued with satisfactory results. The building has been open during the same hours, namely, from 8 A. M. to 5 P. M., and 1 P. M. on Saturdays The Committee has received no suggestions regarding any change of hours from the members of the Academy.

The Committee again desires to express its sense of obligation to Mrs. Holden, the assistant librarian, for her coöperation and assistance.

Respectfully submitted,
H. P. TALBOT, *Chairman.*

May 13, 1914.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Dr. William David Coolidge, of Schenectady, N. Y., for his invention of Ductile Tungsten and its Application in the Production of Radiation.

On motion of the Treasurer, it was

Voted, That the Annual Assessment be ten (10) dollars.

The annual election resulted in the choice of the following officers and committees: —

JOHN TROWBRIDGE, *President.*
ELIHU THOMSON, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
A. LAWRENCE LOWELL, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
HARRY W. TYLER, *Librarian.*

Councillors for Four Years.

FREDERICK S. WOODS, of Class I.
ALFRED C. LANE, of Class II.
SAMUEL WILLISTON, of Class III.

Councillor for Three Years, in place of G. L. Kittredge, resigned.

WILLIAM S. BIGELOW, of Class III.

Finance Committee.

JOHN TROWBRIDGE, GARDINER M. LANE,
JOHN COLLINS WARREN.

Rumford Committee.

Charles R. Cross,	Erasmus D. Leavitt,
Edward C. Pickering,	Elihu Thomson,
Arthur G. Webster,	Louis Bell,

Arthur A. Noyes.

C. M. Warren Committee.

Henry P. Talbot,	Walter L. Jennings,
Charles L. Jackson,	Gregory P. Baxter,
Arthur A. Noyes,	James F. Norris,

William H. Walker.

Publication Committee.

George W. Pierce, of Class I.
Walter B. Cannon, of Class II.
Albert A. Howard, of Class III.

Library Committee.

Harry M. Goodwin, of Class I.
Samuel Henshaw, of Class II.
William C. Lane, of Class III.

House Committee.

Henry P. Talbot,	Louis Derr,

Hammond V. Hayes.

Committee on Meetings.

The President,	The Recording Secretary
William M. Davis,	Wallace C. Sabine

Arthur Fairbanks.

Auditing Committee.

Eliot C. Clarke,	Worthington C. Ford.

The following gentlemen were elected Fellows of the Academy,— a printed list of nominees having been sent to all Voting Fellows with the notice of the April meeting, in accordance with Chapter III., Article 3, of the Statutes: —

In Class I., Section 1 (Mathematics and Astronomy): —

Roland George Dwight Richardson, of Providence.

In Class I., Section 2 (Physics): —

William Johnson Drisko, of Boston; William Duane, of Boston; Charles Clifford Hutchins, of Brunswick, Me.; Ernest George Merritt, of Ithaca, N. Y.; Dayton Clarence Miller, of Cleveland, O.; Robert Andrews Millikan, of Chicago.

Class I., Section 3 (Chemistry): —

Marston Taylor Bogert, of New York; Harmon Northup Morse, of Baltimore; Thomas Burr Osborne, of New Haven; Samuel Cate Prescott, of Brookline; Martin André Rosanoff, of Worcester.

Class I., Section 4 (Technology and Engineering): —

Alexander Crombie Humphreys, of New York; Edward Dyer Peters, of Dorchester; George Chandler Whipple, of Cambridge.

Class II., Section 1 (Geology, Mineralogy and Physics of the Globe):

Louis Caryl Graton, of Cambridge.

Class II., Section 2 (Botany): —

George Perkins Clinton, of New Haven; Fred Dayton Lambert, of Tufts College; Burton Edward Livingston, of Baltimore; George Richard Lyman, of Hanover, N. H.; Alfred Rehder, of Jamaica Plain; Erwin Frink Smith, of Washington.

Class II., Section 3 (Zoölogy and Physiology): —

Edwin Grant Conklin, of Princeton; Herbert Spencer Jennings, of Baltimore; Ralph Stayner Lillie, of Worcester; Jacques Loeb, of New York; Herbert Vincent Neal, of Tufts College.

Class II., Section 4 (Medicine and Surgery): —

Alexis Carrel, of New York; Charles Value Chapin, of Providence; Frederick Cheever Shattuck, of Boston; Frederick Herman Verhoeff, of Boston.

Class III., Section 1 (Theology, Philosophy and Jurisprudence): —

James De Normandie, of Roxbury; George Angier Gordon, of Boston; John Wilkes Hammond, of Cambridge; Alfred Hemenway, of Boston; Nathan Matthews, of Boston; William Cushing Wait, of Medford; Eugene Wambaugh, of Cambridge; John Henry Wigmore, of Chicago.

Class III., Section 2 (Philology and Archaeology): —

Francis Greenleaf Allinson, of Providence; Maurice Bloomfield, of Baltimore; Bert Hodge Hill, of Athens, Greece; George Andrew Reisner, of Cambridge.

Class III., Section 4 (Literature and the Fine Arts): —

Ellery Sedgwick, of Boston; Owen Wister, of Philadelphia.

The following gentlemen were elected Foreign Honorary Members:—

In Class I., Section 1 (Mathematics and Astronomy): —

Johann Oskar Backlund, of St. Petersburg.

In Class I., Section 2 (Physics): —

Max Planck, of Berlin.

In Class I., Section 3 (Chemistry): —

Fritz Haber, of Berlin; Walter Nernst, of Berlin.

In Class II., Section 1 (Geology, Mineralogy and Physics of the Globe): —

Viktor Goldschmidt, of Heidelberg, Germany.

In Class II., Section 2 (Botany): —

John Briquet, of Geneva, Switzerland; Ignatz Urban, of Berlin; Eugene Warming, of Copenhagen.

In Class III., Section 4 (Literature and the Fine Arts): —

Sir Sidney Lee, of London.

The following communication was given: —

Professor R. DeC. Ward. "The Weather Element in the Climate of New England."

The following papers were presented by title: —

"The Pathological Action of Radiant Energy on the Eye." By F. H. Verhoeff. Presented by Louis Bell.

"The Influence of the Magnetic Characteristics of the Iron Core of an Induction Coil upon the Manner of Establishment of a Steady Current in the Primary Circuit." By B. Osgood Peirce.

"Laboulbeniales Parasitic on Chrysomelidae." By Roland Thaxter.

OLIVER FAIRFIELD WADSWORTH.

BORN, BOSTON, APRIL 26, 1838.
DIED, BOSTON, NOVEMBER 29, 1911.

Fellow of the American Academy of Arts and Sciences
Class 2, Section 4, January 11, 1899.

AMONG Dr. Wadsworth's dominant characteristics were his fairness, his kindliness and his accuracy in observation and in statement; his fairness was exemplified in the justice of his appreciation of the work of his fellows in school, in college, and in professional life; his kindliness in the numberless human contacts which were a part of the experience of his hospital and private practice, extending over nearly half a century, and his accuracy was traceable to a parentage which made exactitude a daily practice and a continuous obligation.

Graduating from Harvard College in 1860, and not altogether satisfied with his own estimate of the value of his work there, although he had excelled in some of his studies and been always a joyous participator, and one of the leaders, in athletics, desirous of strenuous effort at accomplishment, he departed from accustomed and easy ways and made out into the then new west, taking up land near Denver and entering upon a farming proposition including the construction of an irrigating system sufficient in extent to occupy the major part of his two years of residence there. To the planning and execution of this work there must have been applied the inherent traits exhibited by his father in the admirable surveys, which are a synonym for reliability, faculties which showed themselves later in Dr. Wadsworth in other ways. The journey westward, a part of it in the most primitive of conveyances, and the self enforced residence with a stated task, was penitential in that it sought to wrest something tangible from his graduate days in compensation for what he deemed to have been lack of success in his undergraduate career. That this opinion was born of Dr. Wadsworth's moderate estimate of himself, as contrasted with his sense of duty and his aspirations, is set forth by the opinion of his classmates and, more authoritatively, in the recorded word of the then president of Harvard College, in a letter bearing date June 8, 1860.

> "I have great pleasure in stating that Mr. Oliver F. Wadsworth, a member of the present senior class, is much esteemed by the faculty of the college as a gentleman of high and honorable character, and worthy of the confidence and respect of any community in which he may establish himself. He has been faithful to his duties, agreeable in his manners and amiable in temper. I have much satisfaction in commending him to the associates he may be connected with in his new residence. I am sure they will find him not only a well-educated young man, but what is better still, a gentleman of honor and integrity, and deserving their high regard."
>
> (signed) C. C. Felton,
> President of Harvard College.

Whatever else the adventure into the west may have accomplished, it afforded a new consideration of the uses of life and an opportunity for maturing reflection which led him, upon the basis of a predilection, to turn to the study of medicine as a means for the expression of his desire in life and he entered the Harvard Medical School in March, 1862.

During the summer of that year he assisted in removing northward sick and wounded prisoners of the Peninsular Campaign under the auspices of the Sanitary Commission. In 1864 he was made a house officer in the Massachusetts General Hospital and graduated from the Harvard Medical School to be immediately commissioned Assistant Surgeon of the Fifth Massachusetts Cavalry. In July of the same year he was detailed for special duty at headquarters Twenty-Fifth Army Corps and remained in service until the mustering out of his regiment at the close of the war, being breveted captain in recognition of his fidelity to duty and the care which he bestowed upon the details of his work.

Deciding to devote himself to ophthalmology he went abroad to study in February 1869, returning in November 1870 to take up the practice of his chosen specialty. Almost immediately upon his return he was made Ophthalmic Surgeon to the Boston City Hospital and in 1874 Ophthalmic Surgeon to out-patients of the Massachusetts General Hospital, in 1881 he was appointed clinical instructor and in 1895 professor in the Harvard Medical School and, in the same year, Ophthalmic Surgeon to the Massachusetts Charitable Eye and Ear Infirmary.

In all these positions he displayed the same care, patience, tact, and skill, and his reputation as a practitioner and diagnostician was enhanced by his clarity as a teacher for, scrupulously careful as he was himself in all his observations, he was equally patient in the demonstration of his ascertained facts and in the training of his students. He was an active member and, for a time, president of the New England Ophthalmological Society and both there and in his papers before the Boston Society of Medical Sciences displayed in the discussion of mathematical subjects an ability which won him the confidence of his compeers. As a member of the American Ophthalmological Society he was a valuable contributor in its meetings, quite as much in the way of discussion of other papers as in the carefully prepared communications which he himself presented for he was strenuous in argument, keen in perception of failure in a logical sequence, and dogged in his determination to arrive at the scientific facts in any subject under consideration.

With all the work entailed by his hospital appointments and the demands of a large private practice, Dr. Wadsworth found time for study in his favorite subject, he was a close student of ophthalmologic literature and was almost encyclopedic in his ability to refer to and place a title or an author.

Of the forty-six original papers contributed by Dr. Wadsworth in the meetings of the various societies of which he was a member and to foreign journals, fifteen were upon conditions in the retina, choroid or optic nerve as might be expected of one who as an ophthalmoscopist was, as one of his associates has said, "the admiration and despair of his colleagues," nine papers treated of operations, four were upon anomalies of muscular balance and four were based upon original scientific research, including his description of the fovea centralis.

In addition to his special interests he found time to devote himself to the more general service of the members of his profession as assistant editor of the Boston Medical and Surgical Journal during the year 1868, and as one of the most valuable contributors of time, thought and labor to the upbuilding of the Boston Medical Library, a project originating with the late Dr. J. R Chadwick and now grown to be of great value not only in itself to this community, but as an example which has been followed in the foundation of similar libraries throughout the United States.

The estimate in which Dr. Wadsworth was held by those who had to deal with him in his daily life contains always expressions of appreciations of his balance, his judgment, his kindness, and, admiringly, of his power of concentration, and a gentleman who was intimately connected with him in both the Massachusetts General Hospital and the Massachusetts Charitable Eye and Ear Infirmary, says, "he was an extremely just, honorable, and high minded man, I have never known one who could do a kindness in so kindly a way, I never knew him to be hurried and, so far as one could judge by outward appearances, worried, as an illustration of the latter quality, after a long evening's work with him on statistics which he was compiling he remarked, just as I was leaving him, that one of his sons was very ill and that his recovery was extremely doubtful. Under such conditions to see a man his normal self through an entire evening and working over statistics with no indication of worry or anxiety impressed me very strongly. That he did worry and was anxious no one will doubt but he had the ability to an exceptional degree of being calm and composed in the face of an impending great calamity. Another quality which will come prominently to the minds of his friends was his love of argument, no one associated with him could fail to believe that there were two or more sides to every question, he never lost his temper, made use of sarcasm to rout his adversary or in any way showed contempt or want of consideration for the other opinion. He parted with the best of feeling and usually with a laughing regret that agreement was impossible.

In his chosen specialty Dr. Wadsworth held high rank, at medical meetings he was always prominent as a critic, a weak point in diagnosis or in argument rarely escaped his notice and his criticisms were so sound and well expressed that they were always received with exceptional interest and consideration. He was not a prolific writer, probably for the reason that much material which might have been placed on record did not pass his own criticism as to the accuracy and value when it came to be seriously considered and he has been known to spend much time and labor in preparing material for a paper that, in the end, was not thought worthy of being written and printed.

He was extremely successful as a practitioner and devoted much time and care to the study of his cases, as a diagnostician in opthalmoscopy he was exceptionally brilliant and it is doubtful if he had a superior in this branch of his specialty.

Another hospital associate of Dr. Wadsworth has said — "his skill in the use of the ophthalmoscope and the accuracy of his diagnoses were known to all his colleagues and were universally acknowledged by all who came in contact with him professionally. The careful study and patient investigation which he gave to each case that came under his care was never abridged by any consideration of time or personal convenience. His mental attitude until his diagnosis was made was always that of the impartial scientific observer." And still another, in the conclusion of a tribute to his memory, called him "the teacher of us all." That he was a teacher in example as in precept has come to the consciousness of the many who have known him even passingly; we judge a man by his attitude toward life with its buoyant activities and take the measure of him again as he faces death; with a physician's knowledge of the nature of his malady, in the midst of the prolonged pain which was a part of it, he was courageous so calmly that his courage did not ripple the surface of his courtly kindliness and a last visit to him, shortly before his death, is a memory of a pleasant familiar presence, a warm greeting and no mention of farewell.

DR. CLARENCE J. BLAKE.

Papers of Dr. O. F. Wadsworth.

Rupture of the sclerotic. *Boston M. & S. J.*, 1868.

Anaesthesia of the Retina. *Boston M. & S. J.*, 1872.

An unusual case of herpes zoster ophthalmicus. *Boston M. & S. J.*, 1875.

A case of ectropion treated by transplantation of a large flap without pedicle. *Boston M. & S. J.*, 1876.

A modification of the ophthalmoscope. *Boston M. & S. J.*, 1877.

Epithelioma of the limbus vorneae. *Boston M. & S. J.*, 1879.

Optico-ciliary neurotomy. *Boston M. & S. J.*, 1879.

Intra-ocular circulation; rhythmical changes in the venous pulse of the optic disk. (With Dr. J. J. Putnam.) *J. Nerv. & Ment. Diseases. Chicago*, 1878. Also *Tr. Am. Ophth. Soc., New York*, 1878.

Optico-ciliary neurotomy (case). *Boston M. & S. J.*, 1879.

Microscopic section of an epithelioma of the limbus corneae. *Boston M. & S. J.*, 1879.

Optico-ciliary neurotomy (2 cases). *Boston M. & S. J.*, 1879.

Peculiar affection of the ocular muscles. *Boston M. & S. J.*, 1880.

Color-Blindness. *Boston M. & S. J.*, 1880.

Optic neuritis after measles. *Boston M. & S. J.*, 1880.

Beiträge zur Opthalmologie, als Festgabe Wiesbaden Friedr. Homer's zur Feier d. 25 jahr. Jubiläums seiner Academ. Lehrthatigkeit gewidmet von M. Dufour, O. Haab, M. Knies, J. Michel, W. Schoen, und O. F. Wadsworth. 1881.

Circulation in the macula lutea retinae. *Boston M. & S. J.*, 1881.

Optico-ciliary neurotomy. *Tr. Am. Ophth. Soc., New York*, 1881.

The fovea centralis in man. Beitr. z. Ophth. *Wiesbaden*, 1882.

Optico-ciliary neurotomy. *Med. & Surg. Rep. Boston City Hospital*, 1882.

Phlyctenular disease of the eyes. *Boston M. & S. J.*, 1883.

Some cases of hysterical affection of vision. *Boston M. &. S. J.*, 1883.

Apparent curvature of surface caused by prismatic glasses. *Boston M. &. S. J.*, 1883.

Three cases of homonymous hemianopia. *Boston M. & S. J.*, 1884.

A case of myxoedema with atrophy of the optic nerves. *Boston M. & S. J.*, 1885. Also *Tr. Am. Ophth. Soc., Boston*, 1885.

A case of permanent zonular scotoma of traumatic origin; very small circle of central field with vision normal. *Am. J. Ophthal., St. Louis*, 1884–5.

Double optic neuritis and ophthalmoplegia from lead-poisoning; complicated by typhoid fever. *Boston M. & S. J.*, 1885. Also *Tr. Am. Ophth. Soc., Boston*, 1885.

Luxation of lens beneath Tenon's capsule. *Boston M. & S. J.*, 1885. Also *Tr. Am. Ophth. Soc., Boston*, 1885.

Lead-Poisoning complicated with probable typhoid fever. *Boston M. & S. J.*, 1885.

A case of recurrent paralysis of the motor oculi. *Boston M. & S. J.*, 1887. Also *Tr. Am. Ophth. Soc.*, 1887.

A case of congenital, zonular, grayish-white opacity around the fovea. Detachment of the retina in both eyes, with albuminuria of pregnancy; replacement of retina. *Boston M. & S. J.*, 1887. *Tr. Am. Ophth. Soc., Boston*, 1887.

A case of recurrent paralysis of the third nerve. *Boston M. & S. J.*, 1887.

The amblyopia of squint. *Boston M. & S. J.*, 1887.

Ophthalmoplegia externa. *Boston M. & S. J.*, 1888.

A case of extraction of a bit of steel from the vitreous by the magnet. *Boston M. & S. J.*, 1889.

Paralysis of the sphincter iridis. *Boston M. & S. J.*, 1889.

Spastic torticollis apparently due to faulty position of the eyes, and cured by tenotomy. *Boston M. & S. J.*, 1889. *Tr. Am. Ophth. Soc., Hartford,* 1889.

Two cases of extraction from the vitreous, of steel which had passed through the lens. *Boston M. & S. J.*, 1889. *Tr. Am. Ophth. Soc., Hartford,* 1889.

Mydriasis of one eye, with intact accommodation lasting four months after application of homatropine to both eyes. *Tr. Am. Ophth. Soc., Hartford,* 1889.

Plastic operation of the lower eyelid. *Boston M. & S. J.*, 1889.

Thrombus of arteria centralis retinae; large retinociliary artery; central vision unimpaired. *Boston M. & S. J.*, 1890.

A case of metastatic carcinoma of the choroid. *Boston M. & S. J.*, 1890.

Same, and Thrombus of arteria centralis retinae; large retinociliary artery; central vision unimpaired. *Tr. Am. Ophth. Soc., Hartford,* 1890.

Insufficiency of the Ocular Muscles. *Boston M. & S. J.*, 1890.

An adenoma of the Meibomian glands. *Tr. Am. Ophth. Soc., Hartford,* 1895.

Embolism of central artery; macula supplied by a cilio-retinal artery, retention of central vision. *Boston M. & S. J.*, 1896. *Tr. Am. Ophth. Soc., Hartford,* 1896.

Anomalies of muscular balance. *Boston M. & S. J.*, 1897. Also *Med. Communicat. Mass. Med. Soc.*, 1897.

Hemorrhage attending the extraction of cataract. *Boston M. & S. J.*, 1897. *Tr. Am. Ophth. Soc., Hartford,* 1897.

A model showing the position of the meridian of the eyeball in oblique direction of vision, as defined by Donders and Helmholtz, with some remarks on the misunderstanding of Helmholtz's statements. *J. Bost. Soc. Med. Sci.*, 1897–8.

In addition, Dr. Wadsworth wrote the Report on Ophthalmology for the Boston Medical and Surgical Journal for about fourteen years.

American Academy of Arts and Sciences

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(Corrected to July 1, 1914.)

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Class I.— *Mathematical and Physical Sciences.*— 169.

Section I. — *Mathematics and Astronomy.*— 34.

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Eliot Channing Clarke	Boston
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Charles Proteus Steinmetz	Schenectady, N. Y.
George Fillmore Swain	Cambridge
William Watson	Boston
George Chandler Whipple	Cambridge
Robert Simpson Woodward	Washington, D. C

CLASS II.— *Natural and Physiological Sciences.*— 119.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 29.

Cleveland Abbe	Washington, D. C.
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Class II., Section III.— *Zoölogy and Physiology.*— 35.

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Addison Emory Verrill	New Haven, Ct.

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Elmer Ernest Southard	Boston
Richard Pearson Strong	Boston
Ernest Edward Tyzzer	Boston
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CLASS III.— *Moral and Political Sciences.*— 130.

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Basil Lanneau Gildersleeve	Baltimore, Md.
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Alfred Louis Kroeber	Berkeley, Cal.
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Thomas Raynesford Lounsbury	New Haven, Ct.
David Gordon Lyon	Cambridge
Clifford Herschel Moore	Cambridge
George Foot Moore	Cambridge
Hanns Oertel	New Haven, Ct.
Charles Pomeroy Parker	Cambridge
Frederick Ward Putnam	Cambridge
Edward Kennard Rand	Cambridge
George Andrew Reisner	Cambridge
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John Williams White	Cambridge
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Irving Fisher	New Haven, Ct.
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Arthur Twining Hadley	New Haven, Ct.
Charles Homer Haskins	Cambridge
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge
Alfred Thayer Mahan	New York
Roger Bigelow Merriman	Cambridge
William Bennett Munro	Cambridge
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William Mulligan Sloane	New York
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George Pierce Baker	Cambridge
Arlo Bates	Boston
William Sturgis Bigelow	Boston
Le Baron Russell Briggs	Cambridge
George Whitefield Chadwick	Boston
Samuel McChord Crothers	Cambridge

Wilberforce Eames	New York
Henry Herbert Edes	Cambridge
Arthur Fairbanks	Cambridge
Arthur Foote	Brookline
Kuno Francke	Cambridge
Daniel Chester French	Stockbridge
Robert Grant	Boston
Henry Lee Higginson	Boston
Mark Antony DeWolfe Howe	Boston
George Lyman Kittredge	Cambridge
Gardiner Martin Lane	Boston
William Coolidge Lane	Cambridge
Albert Matthews	Boston
William Allan Neilson	Cambridge
Bela Lyon Pratt	Boston
Herbert Putnam	Washington, D. C.
Denman Waldo Ross	Cambridge
John Singer Sargent	London, Eng.
William Robert Ware	Milton
Herbert Langford Warren	Cambridge
Barrett Wendell	Boston
George Edward Woodberry	Beverly

FOREIGN HONORARY MEMBERS.— 54.

(Number limited to seventy-five).

CLASS I.— *Mathematical and Physical Sciences.*— 18.

SECTION I.— *Mathematics and Astronomy.*— 3.

Arthur Auwers	Berlin
Felix Klein	Göttingen
Émile Picard	Paris

CLASS I., SECTION II.— *Physics.*— 8.

Svante August Arrhenius	Stockholm
Oliver Heaviside	Torquay
Sir Joseph Larmor	Cambridge
Hendrik Antoon Lorentz	Leyden
Max Planck	Berlin
Augusto Righi	Bologna
John William Strutt, Baron Rayleigh	Witham
Sir Joseph John Thomson	Cambridge

CLASS I., SECTION III.— *Chemistry.*— 5.

Adolf, Ritter von Baeyer	Munich
Emil Fischer	Berlin
Fritz Haber	Berlin
Wilhelm Ostwald	Leipsic
Sir Henry Enfield Roscoe	London

CLASS I., SECTION IV.— *Technology and Engineering.*— 2.

Heinrich Müller-Breslau	Berlin
William Cawthorne Unwin	London

CLASS II.— *Natural and Physiological Sciences.*— 19.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 5.

Waldemar Christofer Brögger	Christiania
Sir Archibald Geikie	Haslemere, Surrey
Viktor Goldschmidt	Heidelberg
Julius Hann	Vienna
Albert Heim	Zurich

CLASS II., SECTION II.— *Botany.*— 5.

John Briquet	Geneva
Adolf Engler	Berlin
Wilhelm Pfeffer	Leipsic
Hermann, Graf zu Solms-Laubach	Strassburg
Ignatz Urban	Berlin

CLASS II., SECTION III.— *Zoölogy and Physiology.*— 4.

Ludimar Hermann	Königsberg
Sir Edwin Ray Lankester	London
Elie Metchnikoff	Paris
Magnus Gustav Retzius	Stockholm

CLASS II., SECTION IV.— *Medicine and Surgery.*— 5.

Emil von Behring	Marburg
Sir Thomas Lauder Brunton, Bart	London
Angelo Celli	Rome
Sir Victor Alexander Haden Horsley	London
Adam Politzer	Vienna

CLASS III.— *Moral and Political Sciences.*— 23.

SECTION I.— *Theology, Philosophy and Jurisprudence.*— 4.

Arthur James Balfour	Prestonkirk

Heinrich Brunner	Berlin
Albert Venn Dicey	Oxford
Sir Frederick Pollock, Bart	London

SECTION II.— *Philology and Archæology.*— 9.

Ingram Bywater	London
Friedrich Delitzsch	Berlin
Hermann Diels	Berlin
Wilhelm Dörpfeld	Athens
Henry Jackson	Cambridge
Hermann Georg Jacobi	Bonn
Sir Gaston Camille Charles Maspero	Paris
Alfred Percival Maudslay	Hereford
Eduard Seler	Berlin

SECTION III.— *Political Economy and History.*— 5.

James Bryce	London
Adolf Harnack	Berlin
John Morley, Viscount Morley of Blackburn	London
Sir George Otto Trevelyan, Bart	London
Pasquale Villari	Florence

SECTION IV.— *Literature and the Fine Arts.*— 5.

Georg Brandes	Copenhagen
Jean Adrien Aubin Jules Jusserand	Paris
Rudyard Kipling	Burwash
Sir Sidney Lee	London
Sir James Augustus Henry Murray	Oxford

STATUTES AND STANDING VOTES

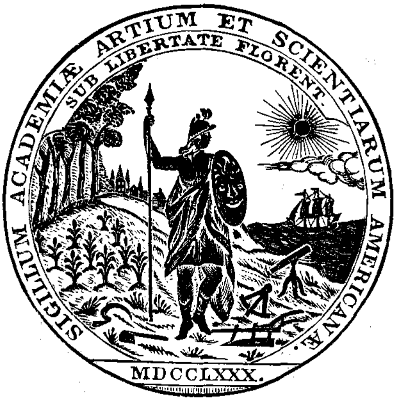
STATUTES

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913

CHAPTER I

THE CORPORATE SEAL

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ.
SUB LIBERTATE FLORENT.
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art. 2.

CHAPTER II

FELLOWS AND FOREIGN HONORARY MEMBERS AND DUES

ARTICLE 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

CLASS I. *The Mathematical and Physical Sciences*
  Section 1. Mathematics and Astronomy
  Section 2. Physics
  Section 3. Chemistry
  Section 4. Technology and Engineering

CLASS II. *The Natural and Physiological Sciences*
  Section 1. Geology, Mineralogy, and Physics of the Globe
  Section 2. Botany
  Section 3. Zoölogy and Physiology
  Section 4. Medicine and Surgery

CLASS III. *The Moral and Political Sciences*
  Section 1. Theology, Philosophy, and Jurisprudence
  Section 2. Philology and Archaeology
  Section 3. Political Economy and History
  Section 4. Literature and the Fine Arts

ARTICLE 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more than Two hundred in any one Class.

ARTICLE 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

ARTICLE 4. If any person, after being notified of his election as Fellow, shall neglect for two months to accept in writing and to pay his Admission Fee (unless he be at that time absent from the Commonwealth) his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for twelve months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii art. 2.

ARTICLE 5. Every Fellow hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x. art. 2.

ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to all the Fellows and Foreign Honorary Members.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three fourths of the Fellows present, out of a total attendance of not less than fifty, at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix. art. 1, 7; chap. x. art. 2.

CHAPTER III

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS

ARTICLE 1. Elections of Fellows and Foreign Honorary Members shall be by ballot, and only at the Stated Meetings in January and May. Three fourths of the ballots cast, and not less than twenty, must be affirmative to effect an election.

ARTICLE 2. Candidates must be proposed in writing by two Fellows of the Section for which the proposal is made. These signed nominations shall be sent to the Corresponding Secretary and shall be retained by him until the fifteenth of the following October or February, as the case may be, when all nominations then in his hands shall be immediately sent in printed form to every Fellow having the right to vote, with the names of the proposers in each case, and with a request to send to the Corresponding Secretary written comments on these names not later than the fifth of November or the fifth of March respectively.

All the signed nominations, with the comments thereon, received up to the fifth of November or the fifth of March shall be sent at once to the appropriate Class Committees, which shall report their decisions to the Council at a special meeting to be called to consider nominations, not later than two days before the meeting of the Academy in December and April respectively.

ARTICLE 3. All nominations approved by the Council shall be read to the Academy at a meeting in December or in April, or be sent to the

Fellows in print with the official notice of the meeting, and shall then be posted in the Hall of the Academy until the balloting.

Not later than two weeks after any nomination is reported to the Academy, the Corresponding Secretary shall send to every Fellow having the right to vote a brief printed account of the nominee.

See Chap. ii.; chap. vi. art. 1; chap. ix. art. 1.

CHAPTER IV

OFFICERS

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At the Annual Meeting in 1912 three Councillors, one from each Class, shall be elected by ballot to serve for one year, three for two years, three for three years, and three for four years. At each subsequent Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x. art. 1.

ARTICLE 2. If any office shall become vacant during the year, the vacancy may be filled by the Council in its discretion for the unexpired term.

ARTICLE 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and cause it to be sent to the Recording Secretary not later than four weeks before the Annual Meeting.

ARTICLE 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted, together with the list of nominees prepared by the Nominating Committee, in the call therefor, and shall be mailed to all the Fellows.

See Chap. vi. art. 2.

ARTICLE 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V

THE PRESIDENT

ARTICLE 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1; 2; chap. xi. art. 1.

CHAPTER VI

THE SECRETARIES

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

He shall post in the Hall a list of the persons nominated for election into the Academy; and after all elections, he shall insert in the Records the names of the Fellows by whom the successful candidates were nominated.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

ARTICLE 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII

THE TREASURER AND THE TREASURY

ARTICLE 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap. ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art.

CHAPTER VIII

THE LIBRARIAN AND THE LIBRARY

ARTICLE 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

ARTICLE 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

ARTICLE 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

ARTICLE 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

ARTICLE 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

Article 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

Article 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6; chap. x. art. 1, 2.

CHAPTER IX

The Council

Article 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and present to the Academy for action such of these nominations as it may approve by a majority vote of the members present at a meeting, of whom not less than seven shall have voted in the affirmative.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections of the same Class, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x. art. 1.

Article 2. Seven members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation.

They shall hold their respective positions during the pleasure of the Council.

ARTICLE 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

ARTICLE 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a Memoir for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii. art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X

STANDING COMMITTEES

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance*, to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap. vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee*, to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iii) *The Cyrus Moors Warren Committee*, to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publications*, to consist of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library*, to consist of the Librarian, *ex officio*, as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2; chap. ix. art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI

MEETINGS, COMMUNICATIONS, AND AMENDMENTS

ARTICLE 1. There shall be annually four Stated Meetings of the Academy, namely, on the second Wednesday of January, March, May, and October. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made, or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten

Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the second or the fourth Wednesday, or both, of each month not appointed for Stated Meetings, excepting July, August, and September; but no business shall be transacted at any meeting which may be held on the fourth Wednesday.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after nominations to Fellowship have been read and the result of the balloting for candidates has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii.; chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

STANDING VOTES

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. There may be chosen by the Academy, under the same rules by which Fellows are now chosen, one hundred Resident Associates. Not more than forty Resident Associates shall be chosen in any one Class.

The election of Resident Associates shall be for a term of three years with eligibility for reëlection.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be one-half that of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

RUMFORD PREMIUM

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American Islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

INDEX.

Academia de la Historia, Madrid, celebration of discovery of the Pacific, 656.

Ageratum, Revision of, 454.

Alkali-Granites and Porphyries of Quincy and the Blue Hills, Petrology of the, 201.

Allinson, F. G., elected Fellow, 670.

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Amory, Francis, extracts from the will of, 648, acceptance of gift of, 651.

Amphioxus, Studies on the Peripheral Nervous System of, 569.

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Arnold, W. R., elected Fellow, 653; accepts Fellowship, 654.

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Backlund, J. O., elected Foreign Honorary Member, 671.

Bacon, Roger, seventh centenary of, 647.

Baker, G. P., elected Fellow, 653; accepts Fellowship, 654.

Bartlett, Francis, death of, 647.

Beale, J. H., appointed member of Nominating Committee, 657.

Bell, Louis, Types of Abnormal Color Vision, 655.

Bigelow, R. P., elected Fellow, 653; accepts Fellowship, 654.

Bigelow, W. S., Details of the Buddhist System, 654; On the Method of Practising Concentration and Contemplation, by Chisho Daishi, a Monk of the Shuzenji Monastery of Tendai Mountain; translated from the Chinese by Okakura Kakuzo, 651; The Relation of Samaji to the Normal Waking Consciousness, 655.

Birkhoff, G. D., accepts Fellowship, 647. The Generalized Riemann Problem for Linear Differential Equations and the Allied Problems for Linear Difference and q-Difference Equations, 519.

Blake, C. J., Biographical notice of O. F. Wadsworth, 651, 673.

Blake, S. F., Contributions from the Gray Herbarium of Harvard University. New Series, No. XLI.— I. A Redisposition of the Species heretofore referred to *Leptosyne*. II. A Revision of *Encelia* and some related Genera, 333.

Bloomfield, Maurice, elected Fellow, 670.

Blue Hills, Mass., U. S. A., Petrology of the Alkali-Granites and Porphries of, 201.

Bogert, M. T., elected Fellow, 670.

Boston Rotary Club, tickets from, 652.

Bouton, C. L., elected Fellow, 653; accepts Fellowship, 654.

Bowditch, C. P., Report of Treasurer, 659.

Bowles, F. T., elected Fellow, 653; accepts Fellowship, 654.

Brandes, Georg., 659.

Bridgman, P. W., Phase Changes under Pressure. I. The Phase Diagram of Eleven Substances with especial reference to the Melting Curve, 648; The Technique of High Pressure Experimenting, 625, 654; Thermodynamic Properties of Twelve Liquids between 20° and 80° and up to 12000 kgm. per Sq. Cm., 1.

Briquet, John, elected Foreign Honorary Member, 671.

British Hondurus, On the Graminaea collected by Prof. M. E. Peck in, 493.

Brøgger, W. C., elected Foreign Honorary Member, 654; accepts Membership, 656.
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Buddhism, Buddhaghosa's Treatise on, entitled The Way of Salvation: analysis of Part I., on Morality, 147.
Buddhist System, Details of, 654.
Bullard, C. *See* Parker, G. H., and Bullard, C.
Bullock. C. J., accepts Fellowship, 647.
Bumstead H. A., elected Fellow, 653.
Burr, W. H., elected Fellow, 653; accepts Fellowship, 654.

Cannon, W. B., The Antagonism of Emotional States and the Significance as suggested by Recent Physiological Researches, 657.
Carrel, Alexis, elected Fellow, 670.
Carson, H. A., elected Fellow, 653; declines Fellowship, 658.
Chadwick, G. W., accepts Fellowship, 647.
Chandler, S. C., death of, 652.
Channing, Edward, resigns Fellowship, 648.
Chapin, C. V., elected Fellow, 670.
Christian, H. A., accepts Fellowship, 647.
Chrysomelidae, Laboulbeniales Parasitic on, 671.
Circolo Matematico di Palermo, thirtieth anniversary of, 656.
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Clinton, G. P., elected Fellow, 670.
Color Vision, Types of Abnormal, 655.
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Conduction, On Electric, and Thermo-electric Action in Metals, 657.
Conklin, E. G., elected Fellow, 670.
Consciousness, The Relation of Samaji to the Normal Waking Consciousness, 655.
Coolidge, Baldwin, presents lithograph of Rumford House, 651.
Coolidge, H. C., appointed to confer with others, 657.
Coolidge, J. L., accepts Fellowship, 647.
Coolidge, W. D., Demonstration of a New X-ray Tube, devised by, 658; Rumford Premium awarded to, 668.
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Crew, Henry, accepts Fellowship, 647.
Cross, C. R., Report of the Rumford Committee, 665.
Crothers, S. M., accepts Fellowship, 647.
Current, Steady, in the Primary Circuit, The Influence of the Magnetic Characteristics of the Iron Core of an Induction Coil upon the Manner of Establishment of a, 671.
Curve, Melting, The Phase Diagram of Eleven Substances with especial reference to the, 648.
Cushing, Harvey, elected Fellow, 653; accepts Fellowship, 654.

Daishi, Chisho, a Monk of the Shuzenji Monastery of Tendai Mountain, On the Method of Practising Concentration and Contemplation, 651.
Daly, R. A., appointed Member of Nominating Committee, 657.
Day, H. L., Observations of the Volcano Kilauea (Hawaii) in action, 658.
De Normandie, James, elected Fellow, 670.
Deutsche Shakespeare Gesellschaft, fiftieth anniversary of, 656; 658.
Dewey, D. R., accepts Fellowship. 647.
Dexter, F. B., declines Fellowship, 647.
Differential Equations, Linear, and the Allied Problems for Linear Difference and q-Difference Equations, The Generalized Riemann Problem for, 519.
Drisko, W. J., elected Fellow, 670.
Duane, William, elected Fellow, 670.
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Emotional States, The Antagonism of, and the Significance as sug-

gested by Recent Physiological Researches, 657.
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Eupatorieae, A Key to the Genera of the *Compositae* —, 429.
Eye, The Pathological Action of Radiant Energy on the, 671.

Farnam, H. W., elected Fellow, 653; accepts Fellowship, 654.
Fellows deceased, (11) —
  Francis Bartlett, 647.
  S. C. Chandler, 652.
  R. H. Fitz, 647.
  G. W. Hill, 659.
  E. S. Holden, 658.
  S. W. Mitchell, 652.
  Alfred Noble, 659.
  Okakura-Kakuzo, 647.
  B. O. Peirce, 652.
  C. S. S. Peirce, 659.
  C. P. Putnam, 659.
Fellows elected, (81) —
  F. G. Allinson, 670.
  W. R. Arnold, 653.
  G. P. Baker, 653.
  R. P. Bigelow, 653.
  M. Bloomfield, 670.
  M. T. Bogert, 670.
  C. L. Bouton, 653.
  F. T. Bowles, 653.
  H. A. Bumstead, 653.
  W. H. Burr, 653.
  Alexis Carrel, 670.
  H. A. Carson, 653.
  C. V. Chapin, 670.
  G. P. Clinton, 670.
  E. G. Conklin, 670.
  Harvey Cushing, 653.
  James de Normandie, 670.
  W. J. Drisko, 670.
  William Duane, 670.
  H. W. Farnum, 653.
  J. D. M. Ford, 653.
  G. A. Gordon, 670.
  L. C. Graton, 670.
  J. H. Hammond, 653.
  J. W. Hammond, 670.
  Alfred Hemenway, 670.
  Rudolph Hering, 653.
  B. H. Hill, 670.
  H. M. Howe, 653.
  J. C. Hubbard, 653.
  A. C. Humphreys, 670.
  C. C. Hutchins, 670.
  J. E. Ives, 653.
  H. S. Jennings, 670.
  E. P. Kohler, 653.
  A. B. Lamb, 653.
  F. D. Lambert, 670.
  R. S. Lillie, 670.
  B. E. Livingston, 670.
  Jacques Loeb, 670.
  G. R. Lyman, 670.
  Nathan Matthews, 670.
  R. B. Merriman, 653.
  E. G. Merritt, 670.
  D. C. Miller, 670.
  E. F. Miller, 653.
  R. A. Millikan, 670.
  C. L. E. Moore, 653.
  H. N. Morse, 670.
  H. V. Neal, 670.
  W. A. Neilson, 653.
  F. H. Newell, 653.
  T. B. Osborne, 670.
  W. B. Parsons, 653.
  Bliss Perry, 653.
  E. D. Peters, 670.
  S. C. Prescott, 670.
  Alfred Rehder, 670.
  G. A. Reisner, 670.
  R. G. D. Richardson, 670.
  M. A. Rosanoff, 670.
  Ellery Sedgwick 670.
  F. C. Shattuck, 670.
  Alexander Smith, 653.
  E. F. Smith, 670.
  C. M. Spofford, 653.
  J. O. Stieglitz, 653.
  R. P. Strong, 653.
  W. H. Taft, 653.
  E. E. Tyzzer, 653.
  F. H. Verhoeff, 670.
  W. C. Wait, 670.
  Eugene Wambaugh, 670.
  G. C. Whipple, 670.
  H. P. Whitlock, 653.
  J. H. Wigmore, 670.
  C. H. Williams, 653.
  G. G. Wilson, 653.
  G. P. Winship, 653.
  Owen Wister, 670.
  J. H. Woods, 653.
Fellows elected, declining Fellowship, (4),—
  F. B. Dexter, 647.
  A. B. Hart, 651.
  C. S. Hastings, 647.
  J. T. Morse, Jr., 647.
Fellows resigned, (2) —

Edward Channing, 648.
R. S. Peabody, 658.
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Fessenden, R. A., Long Distance Submarine Signalling by Dynamo-Electric Machinery, 655.
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Fish, F. P., accepts Fellowship, 647.
Fitz, R. H., death of, 647.
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Fondation Henri Poincaré, 659.
Foote, Arthur, accepts Fellowship, 647.
Ford, J. D. M., elected Fellow, 653; accepts Fellowship, 654.
Foreign Honorary Members, elected (12),—
Johann Oskar Backlund, 671.
John Briquet, 671.
Waldemar Christofer Brögger, 654.
Viktor Goldschmidt, 671.
Fritz Haber, 671.
Sir Sidney Lee, 671.
Alfred Percival Maudslay, 654.
Sir James Augustus Henry Murray, 654
Walter Nernst, 671.
Ignatz Urban, 671.
Max Planck, 671.
Eugene Warming, 671.
Foreign Honorary Members, deceased, (2),—
Sir David Gill, 654.
Sir John Murray, 659.
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French, D. C., accepts Fellowship, 647.

Gay, E. F., accepts Fellowship, 647.
General Fund, 660; Appropriations from the Income of, 656.
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Goldschmidt, Viktor, elected Foreign Honorary Member, 671.
Gordon, G. A., elected Fellow, 670.
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Grandgent, C. H., accepts Fellowship, 647.
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Grant, Robert, accepts Fellowship, 647.
Graton, L. C., elected Fellow, 670.
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Gulick, C. B., accepts Fellowship, 647.

Haber, Fritz, elected Foreign Honorary Member, 671.
Hall, E. H., On Electric Conduction and Thermo-electric Action in Metals, 657.
Hammond, J. H., elected Fellow, 653; accepts Fellowship, 658.
Hammond, J. W., elected Fellow, 670.
Hart, A. B., declines Fellowship, 651.
Harvard University. *See* Gray Herbarium, Jefferson Physical Laboratory, Zoological Laboratory.
Harvard University, Department of Geology, Meeting in conjunction with, 658.
Haskins, C. H., accepts Fellowship, 647.
Hastings, C. S., declines Fellowship, 647.
Hayes, H. V., Government Ownership of Telephones in England, 654.
Heats, Specific, of Liquids, An Improved Method for Determining the, 171.
Hemenway, Alfred, elected Fellow, 670.
Hering, Rudolph, elected Fellow, 653; accepts Fellowship, 654.
High Pressure Experimenting, The Technique of, 625; 654.
Hill, B. H., elected Fellow, 670.
Hill, G. W., death of, 659.
Holden, E. S., death of, 658.
House Committee, Report of, 667.
House Expenses, Appropriations for, 656.
Howe, H. M., elected Fellow, 653; accepts Fellowship, 654.
Hubbard, F. T., On the *Gramineae* collected by Prof. M. E. Peck in British Honduras, 1905–1907, 493; A Taxonomic Study of Setaria italica and its immediate Allies, 648.
Hubbard, J. C., elected Fellow, 653; accepts Fellowship, 656.
Humphreys, A. C., elected Fellow, 670.

Huntington, E. V., accepts Fellowship, 647.
Hutchins, C. C., elected Fellow, 670.
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Hydrobromic Acid, An Improved Method for Determining the Specific Heats of Liquids, with Date concerning, 171.
Hydrochloric Acid, An Improved Method for Determining the Specific Heats of Liquids, with Data concerning, 171.

Induction Coil, The Influence of the Magnetic Characteristics of the Iron Core of an, upon the Manner of Establishment of a Steady Current in the Primary Circuit, 671.
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Ives, J. E., elected Fellow, 653; accepts Fellowship, 654.

von Jagemann, H. C. G., accepts Fellowship, 647.
Jardin Impériale Botanique de St. Petersburg, celebration of, 647.
Jefferson Physical Laboratory, Contributions from, 1, 115, 625.
Jennings, H. S., elected Fellow, 670.
Jewett, J. R., accepts Fellowship, 647.

Kent, N. A., accepts Fellowship, 647.
Kilauea, Observations of the Volcano, in action, 658.
Kohler, E. P., elected Fellow, 653; accepts Fellowship, 654.
Kutchin, H. C., Studies on the Peripheral Nervous System of Amphioxus, 659.

Laboulbeniales Parasitic on Chrysomelidae, 671.
Lamb, A. B., elected Fellow, 653; accepts Fellowship, 654.
Lambert, F. D., elected Fellow, 670.
Lanman, C. R., Buddhaghosa's Treatise on Buddhism, entitled The Way of Salvation: analysis of Part I, on Morality, 147.
Lawrence, Wm., accepts Fellowship, 647.
Lawrence Scientifique Association, Meeting in conjunction with, 655.
Lee, Sir Sidney, elected Foreign Honorary Member, 671.
Leptosyne, A Redisposition of the Species heretofore referred to, 333.
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Library Committee, Report of, 662.
Lillie, R. S., elected Fellow, 670.
Linear Differential Equations and the Allied Problems for Linear Difference and q-Difference Equations, The Generalized Riemann Problem for, 519.
Liquids, An Improved Method for Determining the Specific Heats of, 171.
Liquids, Thermodynamic Properties of Twelve, between 20° and 80° and up to 12000 kgm. per Sq. Cm., 1.
Lithium Hydroxide, An Improved Method for Determining the Specific Heats of Liquids, with Data concerning, 171.
Litters, On the Size of, and the Number of Nipples in Swine, 397.
Little, A. D., accepts Fellowship, 647.
Livingston, B. E., elected Fellow, 670.
Loeb, Jacques, elected Fellow, 670.
Lyman, G. R., elected Fellow, 670.
Lyman, Theodore, A Note on the Life of Victor Schumann, with some account of recent progress in the Extreme Ultra Violet, 655.

Magnetic Characteristics of the Iron Core of an Induction Coil, The Influence of, upon the Manner of Establishment of a Steady Current in the Primary Circuit, 671.
Magnetization in Iron, The Maximum Value of the, 115.
Mallory, F. B., accepts Fellowship, 647.
Massachusetts Institute of Technology, Department of Geology,

Meeting in conjunction with, 658.
Matthews, Nathan, elected Fellow, 670.
Maudslay, A. P., elected Foreign Honorary Member, 654; accepts Membership, 654.
Melting Curve, The Phase Diagram of Eleven Substances with especial reference to the, 648.
Merriman, R. B., elected Fellow, 653; accepts Fellowship, 654.
Merritt, E. G., elected Fellow, 670.
Metals, On Electric Conduction and Thermo-electric Action in, 657.
Miller, D. C., elected Fellow, 670.
Miller, E. F., elected Fellow, 653; accepts Fellowship, 654.
Millikan, R. A., elected Fellow, 670.
Mitchell, S. W., death of, 652.
Moore, C. L. E., elected Fellow, 653; accepts Fellowship, 654.
Morality, Buddhaghosa's Treatise on Buddhism, entitled The Way of Salvation: analysis of Part I., 147.
Morse, H. N., elected Fellow, 670.
Morse, J. T., Jr., declines Fellowship, 647.
Munro, W. B., accepts Fellowship, 647.
Murray, Sir J. A. H., elected Foreign Honorary Member, 654; accepts Membership, 656.
Murray, Sir John, death of, 659.
Museo Nacional de Aqueologia Historia y Ethnologia, felicitations from, 652.
Museum of Comparative Zoölogy at Harvard College. *See* Zoological Laboratory.

Napier, John, Logarithmorum Canonis Mirifici Descriptio, 656.
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Neal, H. V., elected Fellow, 670.
Neilson, W. A., elected Fellow, 653; accepts Fellowship, 654.
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New England, The Weather Element in the Climate of, 671.
Newell, F. H., elected Fellow, 653.
Nichols, E. H., accepts Fellowship, 647.
Nipples in Swine, On the Size of Litters and the Number of, 397.
Nitric Acid, An Improved Method for Determining the Specific Heats of Liquids, with Data concerning, 171.
Noble, Alfred, death of, 659.
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Noyes, W. A., accepts Fellowship, 647.

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Osborne, T. B., elected Fellow, 670.
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Parker, G. H., and Bullard, C., On the Size of Litters and the Number of Nipples in Swine, 397.
Parsons, W. B., elected Fellow, 653; accepts Fellowship, 654.
Pathological Action, The, of Radiant Energy on the Eye, 671.
Peabody, R. S., resigns Fellowship, 658.
Peck, M. E., On the *Gramineae* collected by, in British Honduras, 1905–07, 493.
Peirce, B. O., The Demagnetizing Factors of Cylindrical Rods in High, Uniform Fields, 657; The Influence of the Magnetic Characteristics of the Iron Core of an Induction Coil upon the Manner of Establishment of a Steady Current in the Primary Circuit, 671; The Maximum Value of the Magnetization in Iron, 115.
Peirce, B. O., death of, 652.
Peirce, C. S. S., death of, 659.
Pender, Harold, accepts Fellowship, 647.
Perchloric Acid, An Improved Method for Determining the Specific Heats of Liquids, with Data concerning, 171.
Periodicals purchased, list of, 663.
Perry, Bliss, elected Fellow, 653; declines Fellowship, 658.
Peters, E. D., elected Fellow, 670.
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Porphyries of Quincy and the Blue Hills, Mass., U. S. A., 201.
Phase Changes under Pressure. I. The Phase Diagram of Eleven Substances with especial reference to the Melting Curve, 648.
Physical Science, Absence of Finality in, 648.
Pierce, G. W., Report of Publication Committee, 666; Wireless Telephony, 651.
Planck, Max. elected Foreign Honorary Member, 671.
Politzer, Adam, accepts Foreign Honorary Membership, 647.
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Pratt, B. L., accepts Fellowship, 647.
Prescott, S. C., elected Fellow, 670.
Pressure, Phase Changes under, 648.
Pressure Experimenting, High, The Technique of, 625; 654.
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Rand, E. K., accepts Fellowship, 647; A Winter in Rome, 648.
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Rehder, Alfred, elected Fellow, 670.
Reisner, G. A., elected Fellow, 670.
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Rosanoff, M. A., elected Fellow, 670.
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Sedgwick, Ellery, elected Fellow, 670.
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Shattuck, F. C., elected Fellow, 670.
Sheldon, H. N., accepts Fellowship, 647.

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Smith, Alexander, elected Fellow, 653; accepts Fellowship, 654.
Smith, C. F., elected Fellow, 670.
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Wambaugh, Eugene, elected Fellow, 670.
Ward, R. DeC., The Weather Element in the Climate of New England, 671.
Warren, C. H., Petrology of the Alkali-Granites and Porphyries of Quincy and the Blue Hills, Mass., U. S. A., 201.
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Weatherby, C. A., Some new Combinations required by the International Rules, 492.
Whipple, G. C., elected Fellow, 670.
Whitlock, H. P., elected Fellow, 653; accepts Fellowship, 654.
Wigmore, J. H., elected Fellow, 670.
Williams, C. H., elected Fellow, 653; accepts Fellowship, 654.
Williams, F. H., Demonstration of a New X-ray Tube, devised by W. D. Coolidge, and a consideration of some Measurements relating to Quality as well as Quantity of X-light, 658.
Willson, R. W., appointed member of Nominating Committee, 657.
Wilson, E. B., The Motion of a Radiating Oscillator, 657.
Wilson, G. G., elected Fellow, 653; accepts Fellowship, 654.
Winship, G. P., elected Fellow, 653; accepts Fellowship, 654.

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PLATE 1.

All figures are from methylene-blue preparations of *Branchiostoma caribaeum;* the epithelium of the skin is not present.

FIGURE 1. Nerves of the right side of the rostrum. × 93.
FIGURE 2. Nerves of the left side of the rostrum. × 93.
FIGURE 3. Nerves of the right side of the rostrum. A few nerve branches in the ventral region are not shown. × 93.
FIGURE 4. Nerves of the left side of the rostrum. × 93.

PLATE 2.

All figures are from methylene-blue preparations.

FIGURE 5. Branchiostoma caribaeum. Nerves of the right side of the rostrum. The epithelium of the skin is not present. Numerous cells of Quatrefages, Q, Q, are shown. × 92.

FIGURE 6. B. caribaeum. Anterior dorsal portion of a transparent specimen, showing the exit of the anterior dorsal nerves from the neural tube on the right side of the body. The branches of the dorsal rami of these nerves are drawn. Nerve II shows two roots. The places of division of these nerves into dorsal and ventral rami are also shown. × 32.

FIGURE 7. B. caribaeum. The main branches of the dorsal nerves of the buccal region, on the left side of the body. The branches r, r, pass to the right side of the body; a, a, indicate places of anastomosis. The branches o, o, form part of the outer mouth plexus; those marked i, i, form part of the inner mouth plexus; v. band nerve of velum. × 34.

FIGURE 8. B. lanceolatum. Anterior is up. The innervation of the velum. The epithelium of the velum is not present. The velar tentacles are indicated by t^1, t^2; a, a, point where one velar nerve band was cut; b, b, similar point in another cut nerve band; b', the cut nerve b near the point where it diverges from δ. For nerves α, β, γ, δ see text p. 590. × 41.

FIGURE 9. B. lanceolatum. Anterior is up. A portion of the velar plexus, and the nerves of two velar tentacles; c, c, cells in the epithelial covering remaining attached to the nerves. Cells, or nuclei, in connection with the velar plexus are shown at n, n. × 123.

FIGURE 10. B. lanceolatum. A portion of the fine plexus of a gonadial pouch. × 343.

PLATE **3**.

All figures are from methylene-blue preparations.

FIGURE 11. Branchiostoma lanceolatum. The first and second (second and third of Hatschek) myomeres, m_1, and m_2, and the proximal portions of nerves I, II and III, of the *right* side. Nerve II shows two roots and perhaps a third (between II and II_1). × 84.

FIGURE 12. B. lanceolatum. The first and second myomeres, m_1 and m_2, and the proximal portions of nerves I, II and III, of the *left* side. Nerve II shows two roots (II and II_1); *o.* olfactory pit. × 84.

FIGURE 13. B. lanceolatum. The visceral rami of a dorsal nerve, viewed from the interior. The branches supplying a gonadial pouch (*brs. gon.*) are shown. The larger meshes of the plexus on the interior (median) surface of this pouch are drawn. The place of emergence of the visceral ramus on the internal surface of the side muscles is indicated at *a.* The branches *i, i,* from ramus visceralis descendens pass to the gonadial pouch. The ventral muscle border is indicated at *m. v.* × 123.

FIGURE 14. B. lanceolatum. A nerve plexus on a secondary branchial bar, not far from the ligamentum denticulatum, viewed from the external (lateral) surface of the branchial basket. Branchial nerves of the anterior part of the animal. × 254.

PLATE 4.

All figures, except 15a, from methylene-blue preparations.

FIGURE 15. Branchiostoma lanceolatum. The fan-like branches of an ascending visceral ramus of a dorsal nerve, viewed from the interior. The visceral ramus reaches the internal surface of the side muscles at *a*. The branches at *c* and *d* anastomose with those of adjoining ascending visceral rami. The branch at *e* bends toward the exterior. The branches at *b* are bent away from the muscle, i. e. toward the median plane of the animal. × 93.

FIGURE 15a. Diagram of part of a cross section of an Amphioxus, to show the course of a dorsal nerve and its branches.

FIGURE 16. B. lanceolatum. View of the inner (median, or atrial) face of the lateral muscles of right side (lower half of figure), and of outer (atrial) surface of the branchial wall (*p. w.*), cut ventrally and turned up (the upper half of figure). To show branches of an ascending visceral ramus passing to the pharynx. Portions of the ligamentum denticulatum are marked *l. d.* The plexus formed by the fan-like branches of an ascending visceral ramus is also shown. × 84.

FIGURE 17. B. lanceolatum. Similar view to that of Figure 16. Nerves entering the pharynx from the ligamentum denticulatum, and a portion of the plexus of the latter structure. The plexus in the "pocket" portions of the ligament is shown at *pl. d.* The plexus of a primary branchial bar is marked *b. p. 1*, and that of a secondary bar, *b. p. 2*. × 177.

FIGURE 18. Branchiostoma caribaeum. Anterior is up. A part of the outer plexus of the mouth border, on the *right* side of the body. × 93.

FIGURE 19. B. lanceolatum. Plexuses on a primary bar, a secondary bar, and a cross-bar of the pharynx, with nerve cells, *c*, *c*, in connection. This is a view of the exterior surface of a part of the pharynx which is near its ventral surface and near the anterior end of the body on the right side; *a*, and *b*, have the same meaning as in the following figure. × 254.

FIGURE 20. B. lanceolatum. A view of the dorsal border of a portion of the left side of the pharynx from the interior of the latter, showing a nerve plexus. The primary bars are marked *a*, *a*, and the secondary bar, *b*. × 254.

FIGURE 21. B. lanceolatum. A nerve cell on the external side of a secondary bar of the pharynx. The view is from the right side of the latter, near the anterior end of the body. × 700.

PLATE 5.

Figure 24 from gold-choride preparation; all others from Golgi preparations.

FIGURE 22. Branchiostoma caribaeum. Transverse section. Nerve fibers entering a dorsal root, in the middle region of the animal. × 238.

FIGURE 23. B. caribaeum. Anterior at the *left* (?). A frontal section. Nerves in the transverse muscles, toward the anterior end of the body, on the right side. × 238.

FIGURE 24. B. caribaeum. A view of the inner surface of skin stripped from the dorsal fin in the region of the 20th myomere from the posterior end of the animal, showing fibers and other structures in connection with a branch of the dorsal ramus of a dorsal nerve. Ordinary epithelial cells are outlined at *e*. Special cells in the epithelial layer are marked *g*, and certain cell-like structures in connection with fibers are shown at *c*, *c*. A star-like place of union of fibers is indicated at *s*. The bases of the epithelial cells lie just exterior to the fibers drawn. × 410.

FIGURE 25. B. lanceolatum. Dorsal aspect of frontal section through the 7th myomere of the right side showing a portion of the corresponding ventral nerve root. × 203.

FIGURE 26. B. lanceolatum. Dorsal aspect of frontal section, next ventral to the preceding one, showing a more ventral portion of the same nerve. × 203.

FIGURE 27. B. lanceolatum. Frontal section at the level of the dorsal margin of the notochord, directly ventral to the neural tube. The myomere of the figure is slightly posterior to the middle of the specimen, on the right side. Smooth motor fibers are shown, and a closely associated group of end-plates situated in the side muscles, near the posterior myoseptum. × 141.

FIGURE 28. B. lanceolatum. Frontal section through the middle of the notochord, showing long motor fibers and end-plates in the side muscles. These muscles are posterior to the middle region of the body. × 141.

FIGURE 29. B. lanceolatum. Dorsal aspect of frontal section, showing motor fibers in the side muscles of the 18th myomere of the right side, in the region of the neural tube just ventral to the place of exit of the dorsal nerve roots. The section is about 46 micra in thickness. × 99.

FIGURE 30. Branchiostoma caribaeum. A nerve cell in the ventral part of the pharynx, in connection with the nerve plexus of that region. The cell lies on the exterior of an endostylar plate. × 226.

PLATE 6.

All figures are from methylene-blue preparations.

FIGURE 31. Branchiostoma caribaeum. The same individual as that shown in Figure 33. The nerves of the posterior end, on the right side of the body. × 93.

FIGURE 32. B. lanceolatum. The interior surface of the ventral fin and side muscles bordering upon it, from the region between the atriopore and the anus. Posterior is up. The branches from the ventral rami of dorsal nerves which reach the interior are marked *a*, *a*, *a*. The branch a_1, bends toward the exterior. Part of a membrane, somewhat detached from the side muscles, is indicated at *b*. The bilateral asymmetry of the muscle segments is illustrated. × 84.

FIGURE 33. B. caribaeum. Nerves of the posterior end of the body, on the left side. The epithelium of the skin is not present. The end of the notochord is bent in the same manner as the terminal ampulla of the neural tube. × 93.

FIGURE 34. B. lanceolatum. Cutaneous plexuses between right dorsal nerves XLVI, XLVII and XLVIII, in the region of the side muscles. The coarser, deeper plexus is marked p_1 (center of figure), the finer, more superficial one p_2 (upper left quarter of figure). A cell-like structure is shown at *c*, in connection with cutaneous nerve fibers and at *a*, a nerve passing through myoseptum to inner face of muscle. × 41.

FIGURE 35. B. lanceolatum. Cutaneous plexuses between right dorsal nerves LIII and LIV, and between LV and LVI, in the region of the side muscles. The letters correspond to those of Figure 34. The branch at *x* joins nerve LVII. × 41.

FIGURE 36. B. lanceolatum. Cutaneous plexuses between right dorsal nerves LIV and LV, in the region of the side muscles. The letters correspond to those of Figure 34. × 84.

PLATE 7.

Figures 37–39 and 41 from methylene-blue preparations; figure 40 from Golgi, and figure 42 from gold-chloride (after Ranvier) preparations.

FIGURE 37. Branchiostoma lanceolatum. The exteriorly directed cutaneous branchlets of left dorsal nerve XVI. The epithelium is not present. × 41.

FIGURE 38. B. lanceolatum. Cutaneous plexuses distributed over the side muscles, formed from connecting branches of left nerves XI, and XII. The coarser plexus is marked p_1, and the finer, more superficial plexus, p_2. A visceral nerve branch is indicated at *a*. × 84.

FIGURE 39. B. caribaeum. Cutaneous plexuses between left dorsal nerves XXXV, XXXVI and XXXVII, in the region of the side muscles. The letters correspond to those in preceding figure. × 17.5.

FIGURE 40. B. lanceolatum. Anterior is up. Frontal section. Nerve fibers entering the root of right dorsal nerve XXI. × 238.

FIGURE 41. B. lanceolatum. Dorsal is at the *left*. A plexus formed by the breaking up, for a short distance, of the main stem of the ventral ramus of right dorsal nerve XXVII. × 84.

FIGURE 42. B. caribaeum. A view of the outer surface of skin taken from the region over the side muscles, near the dorsal fin. Special cells are indicated at *g*, and modified cells at *s*. Apparent exudations from the special cells are marked *e*. The exudations lie at higher focus in their distal portions. × 1050.

PLATE 8.

All figures are from Golgi preparations, and all except 50 are from Branchiostoma lanceolatum and are dorsal aspects of frontal sections.

FIGURE 43. Section (anterior at the right) through the middle of the notochord (*ch.* sheath of notochord), showing part of a myomere in the middle region of the animal. Long, single motor fibers are drawn, the one bearing an end-plate traversing the side muscles for nearly their entire width. × 141.

FIGURE 44. Section (anterior at the left) through the 24th myomere of the left side, showing motor fibers and an end-plate. A slight impregnation of the muscle fibers adjoining the end-plate is drawn. × 203.

FIGURE 45. Section showing motor end-plates in the side muscles dorsal to the neural tube, on the left side. × 277.

FIGURE 46. Section (anterior at the left) showing long, smooth, closely associated motor fibers in the middle region of the 20th myomere of the left side of the body. The section passes through the notochord. × 141.

FIGURE 47. Section (anterior at the right) through the region of the notochord, showing long, smooth motor fibers near a myoseptum. × 141.

FIGURE 48. Section (anterior at the left) through the region of the 21st myomere of the left side of the body, showing a branched motor fiber near the anterior myoseptum. × 203.

FIGURE 49. Frontal section through the region of the 19th myomere of the left side, showing branched motor fibers near a myoseptum. × 203.

FIGURE 50. B. caribaeum. Transverse section through a ventral nerve near the middle region of the body, showing a nerve fiber bearing a structure (*c*) resembling a bipolar nerve cell. An apparent nucleus is shown in the cell-like structure. × 271.

FIGURE 24.

FIGURE 28.

Folder I. The thermal expansion, $\left(\frac{\partial v}{\partial \tau}\right)_p$, against pressure. In Figures 20 to 31 the thermal expansion for the separate liquids at 20°, 40°, 60°, and 80° is shown against pressure. In Figure 32 the average expansion between 20° and 80° is plotted against pressure for all twelve liquids in a single diagram. In the curves for the separate liquids, the curves cross and recross so many times that it would be confusing to try to determine from the diagrams alone what curve belongs to each temperature. The following detailed description is given, therefore, of the order of the curves at intervals of 2000 kgm., reading from below up.

FIGURE 20. Methyl Alcohol. The order of the curves, reading from below up, is as follows; at 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 40°–60°–80°–20°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 60°–40°–20°–80°; and at 12000 kgm., 80°–20°–40°–60°.

FIGURE 21. Ethyl Alcohol. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 20°–40°–60°–80°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 60°–80°–40°–20°; at 10000 kgm., 60°–40°–80°–20°; and at 12000 kgm., 20°–80°–40°–60°.

FIGURE 22. Propyl Alcohol. The order of the curves, reading from below up, is as follows; at 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 20°–40°–60°–80°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 40°–60°–20°–80°; and at 12000 kgm., 40°–20°–60°–80°.

FIGURE 23. Isobutyl Alcohol. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 60°–40°–20°–80°; at 4000 kgm., 80°–20°–40°–60°; at 6000 kgm., 80°–60°–20°–40°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 80°–60°–40°–20°; and at 12000 kgm., 80°–60°–40°–20°.

FIGURE 24. Amyl Alcohol. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 60°–40°–80°–20°; at 4000 kgm., 60°–80°–40°–20°; at 6000 kgm., 60°–40°–80°–20°; at 8000 kgm., 60°–40°–20°–80°; at 10000 kgm., 20°–40°–60°–80°; and at 12000 kgm., 80°–20°–40°–60°.

FIGURE 25. Ether. The order of the curves, reading from below up is as follows. At 1 kgm., 20°; at 2000 kgm., 20°–40°–60°–80°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 60°–80°–40°–20°; at 10000 kgm., 20°–40°–60°–80°; and at 12000 kgm., 20°–80°–40°–60°.

FIGURE 26. Acetone. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°; at 2000 kgm., 80°–60°–40°–20°; at 4000 kgm., 60°–80°–40°–20°; at 6000 kgm., 80°–60°–20°–40°; at 8000 kgm., 80°–60°–20°–40°; at 10000 kgm., all equal; and at 12000 kgm., 80°–60°–40°.

FIGURE 27. Carbon Bisulphide. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 60°–40°–20°–80°; at 4000 kgm., 80°–20°–40°–60°; at 6000 kgm., 80°–20°–60°–40°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 80°–60°–40°–20°; and at 12000 kgm., 20°–80°–40°–60°.

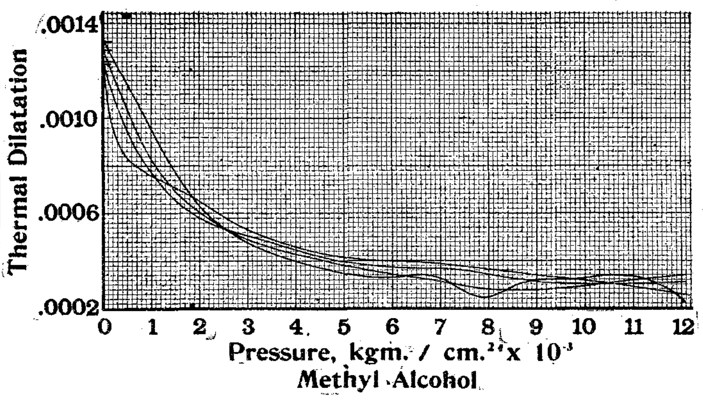
FIGURE 28. Phosphorus Trichloride. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 20°–40°–80°–60°; at 4000 kgm., 80°–20°–60°–40°; at 6000 kgm., 80°–20°–40°–60°; at 8000 kgm., all equal, at 10000 kgm., 80°–[20° and 60°]–40°; and at 12000 kgm., 80°–[20° and 60°]–40°.

FIGURE 29. Ethyl Chloride. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°; at 2000 kgm., 80°–20°–40°–60°; at 4000 kgm., 80°–[20° and 60°]–40°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 60°–40°–80°–20°; and at 12000 kgm., 80°–20°–40°–60°.

FIGURE 30. Ethyl Bromide. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°; at 2000 kgm., 80°–60°–40°–20°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 60°–40°–80°–20°; at 10000 kgm., 80°–60°–40°–20°; and at 12000 kgm., 80°–20°–40°–60°.

FIGURE 31. Ethyl Iodide. The order of the curves, reading from below up, is as follows. At 1 kgm., 20°–40°–60°–80°; at 2000 kgm., 80°–60°–40°–20°; at 4000 kgm., 80°–60°–40°–20°; at 6000 kgm., 80°–60°–40°–20°; at 8000 kgm., 80°–60°–40°–20°; at 10000 kgm., 60°–80°–40°–20°; and at 12000 kgm., 80°–20°–60°–40°.

FIGURE 32. The average dilatation between 20° and 80° for all twelve liquids. The numbers on the curves indicate the liquids in the same order as the diagrams for the separate liquids. In order to prevent overlapping the origin of each curve has been displaced one square with respect to the one next to it. The origin is so situated that the dilatation for all the liquids at 12000 kgm. is between 0.0002 and 0.0003. The scale is shown at the righ hand side. As an example of the use of the diagrams, the initial dilatati amyl alcohol is 0.001056.



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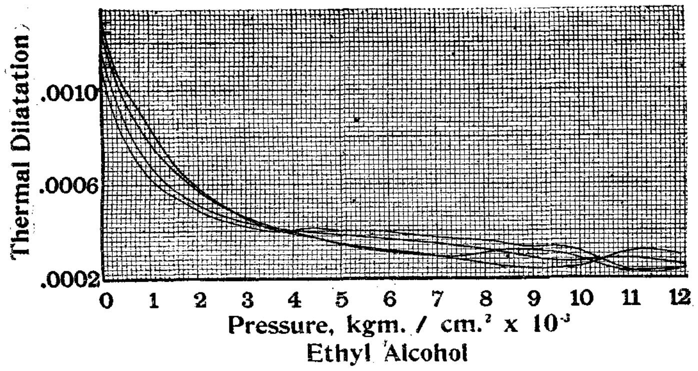

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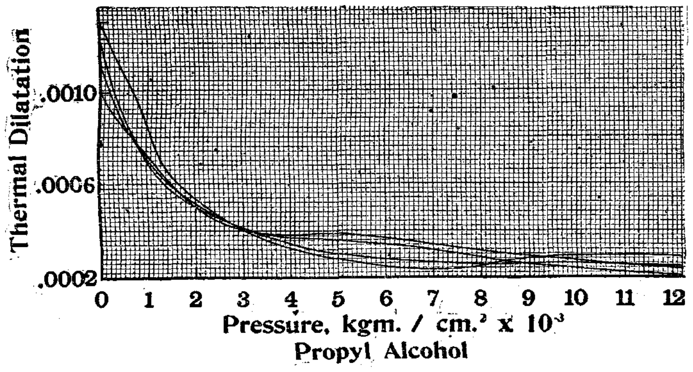

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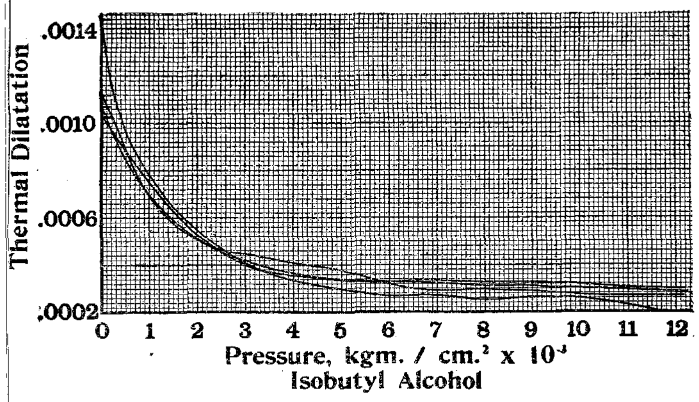

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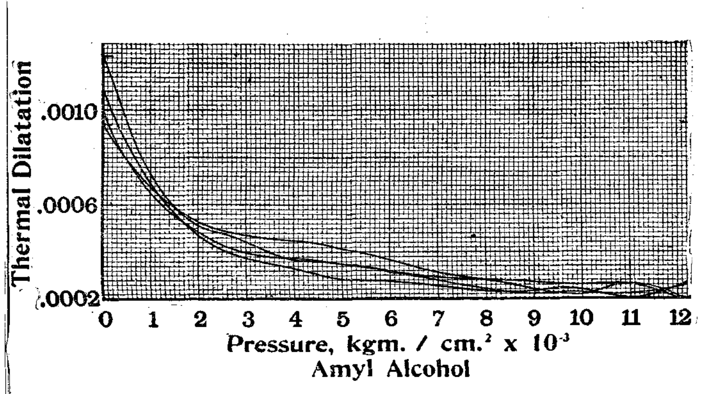

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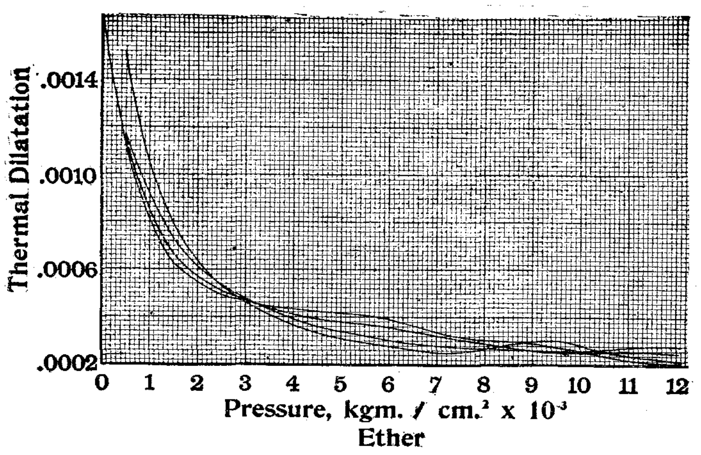

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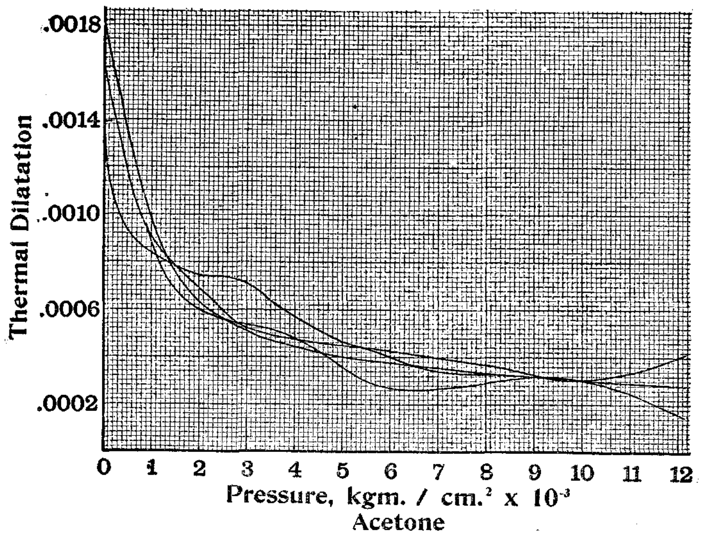

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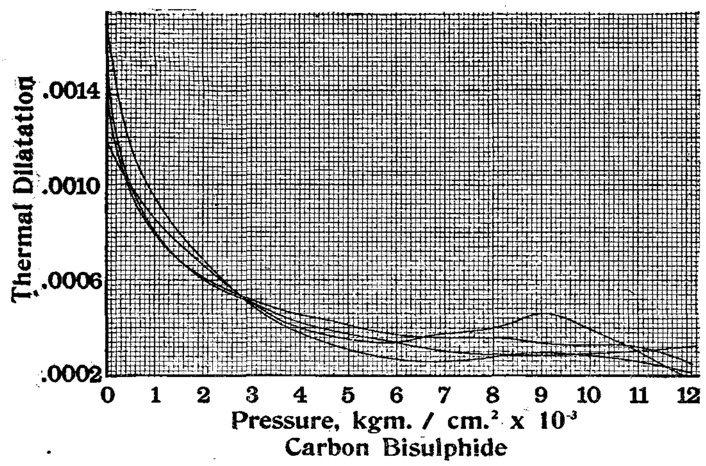

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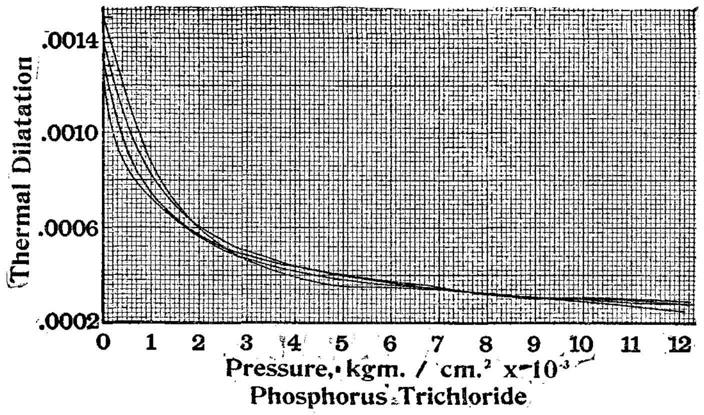
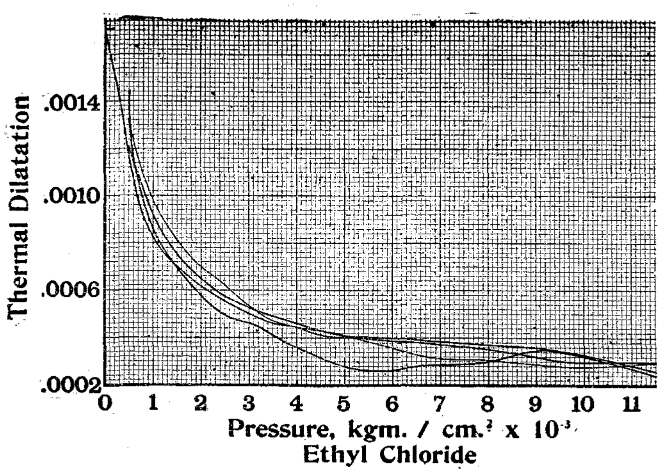


Figure 28.



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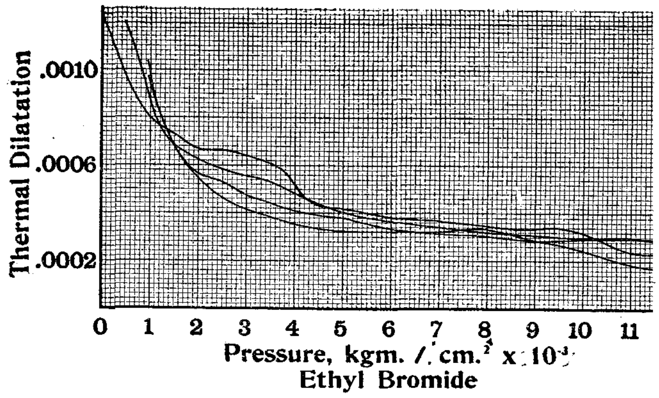


Figure 30.

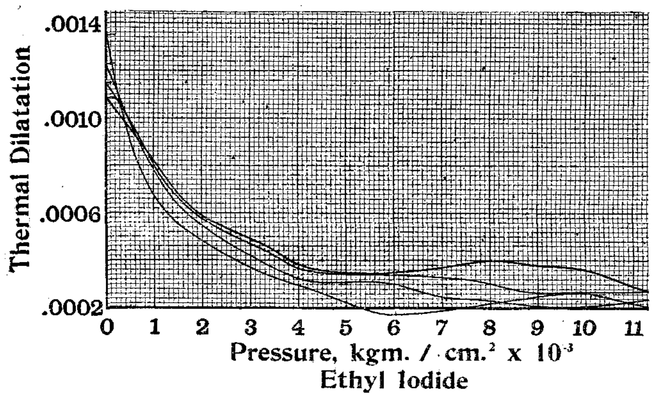


Figure 31.

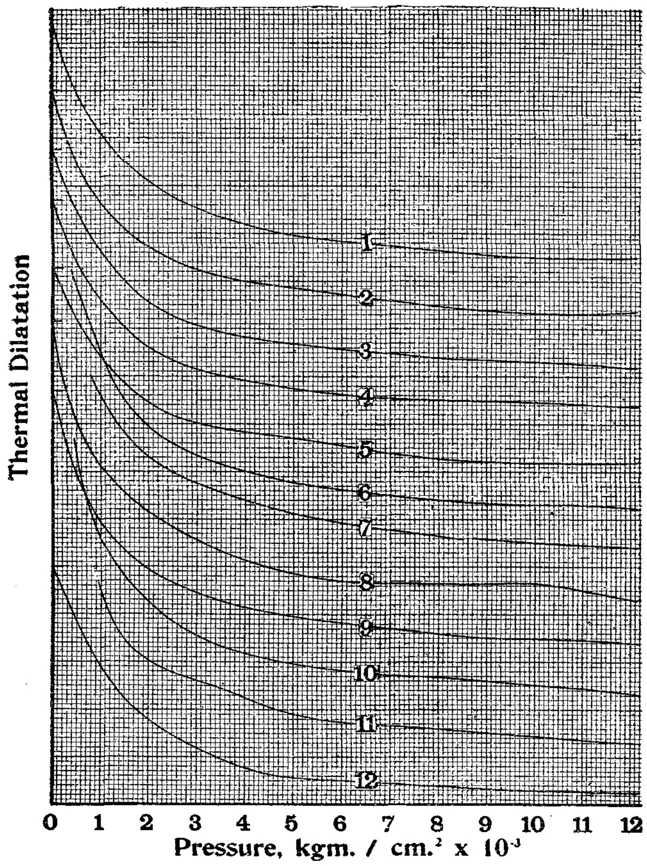


Figure 32.

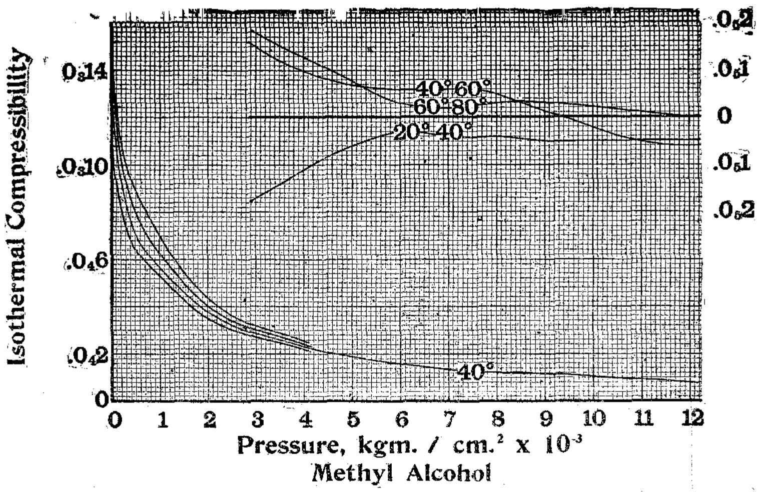


FIGURE 33.

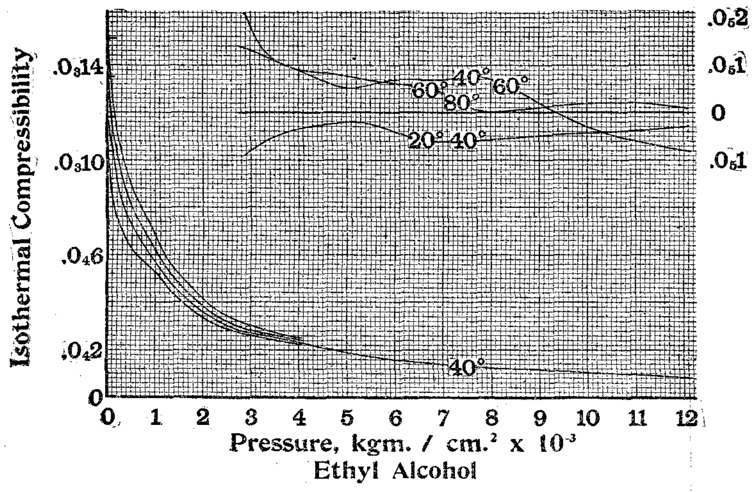


FIGURE 34.

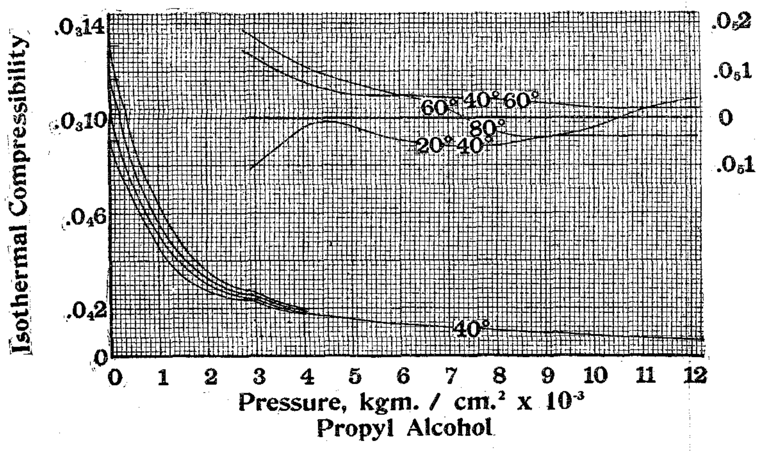


FIGURE 35.

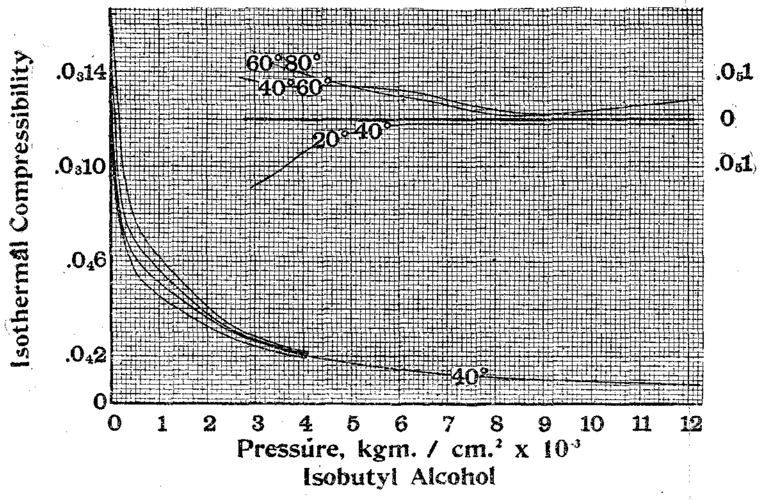


FIGURE 36.

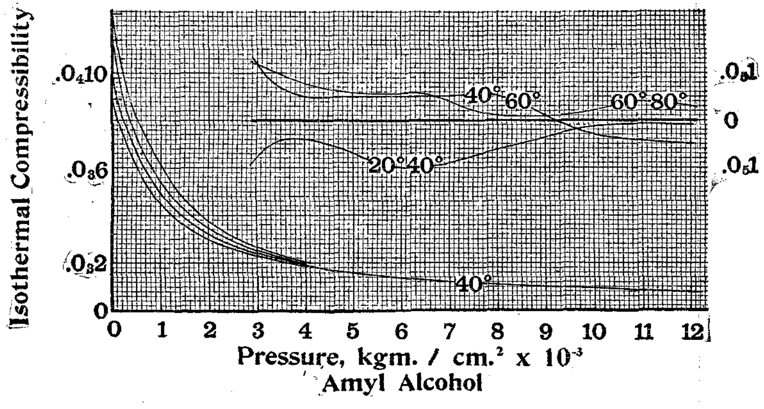


FIGURE 37.

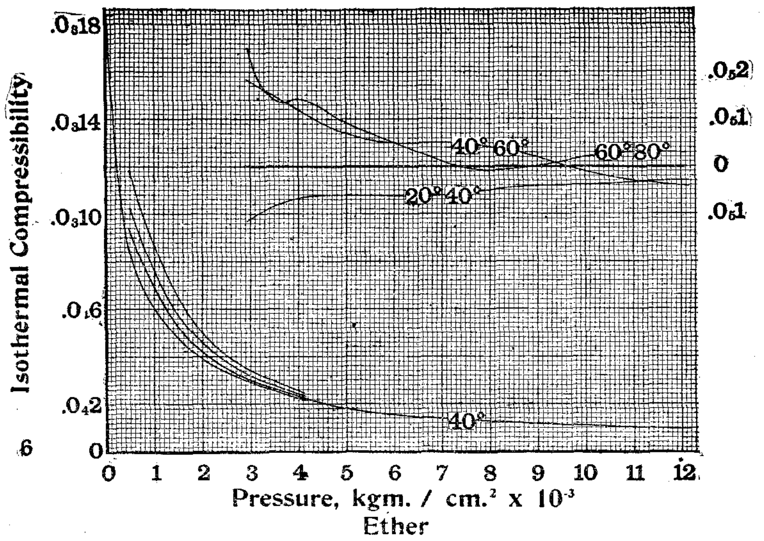


FIGURE 38.

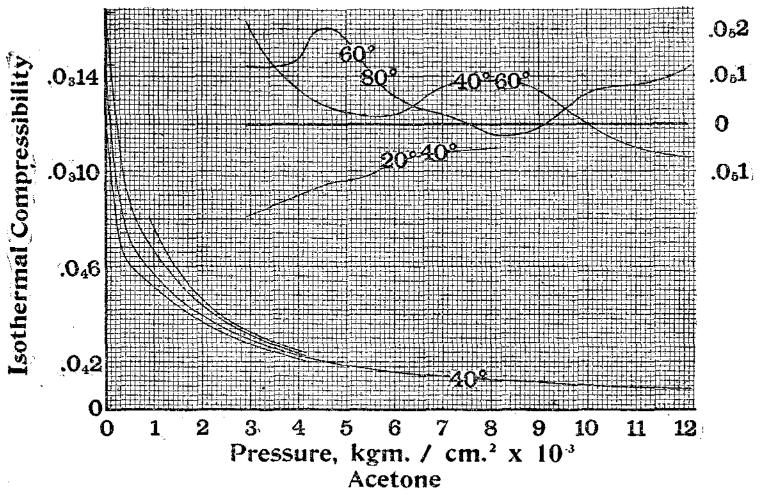


FIGURE 39.

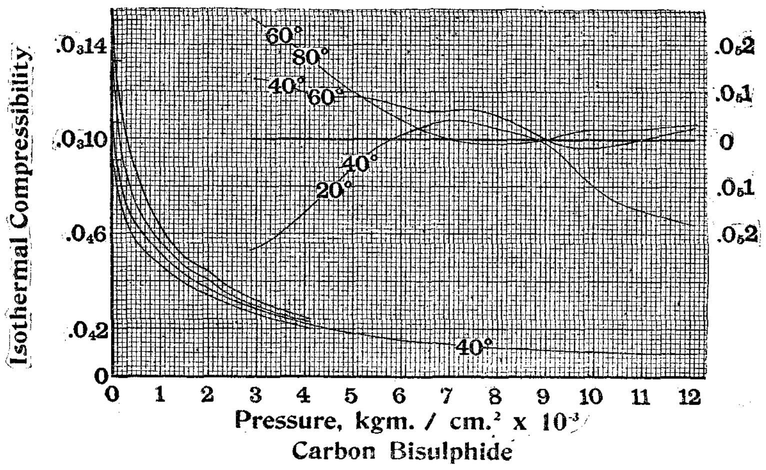


FIGURE 40.

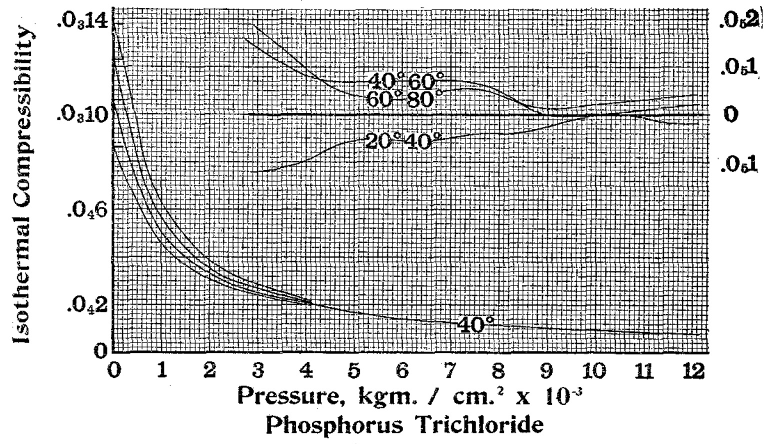


FIGURE 41.

Folder II. Isothermal compressibility, $\left(\frac{\partial v}{\partial p}\right)_\tau$, against pressure. Figures 33 to 44 give the compressibility for the liquids separately at intervals of 20°, and Figure 45 shows the average between 20° and 80° for all twelve liquids. In order to avoid confusion in the diagrams the following course is adopted. The compressibility at 40° is given over the entire pressure range. Up to 4000 kgm. the compressibilities at 20°, 60° and 80° are also shown on the same scale, but at the higher pressures, where the difference between the compressibilities becomes smaller, the difference for intervals of 20° is plotted in the upper right hand of the diagram on a larger scale. The scale of these difference curves is shown at the right hand side. A negative ordinate for the curve 20°–40° indicates that the compressibility is less at 20° than at 40°. The appearance of the difference curves is the same as would be presented if the curves of the lower part of the diagram were magnified twenty times and the curve for 40° straightened out into the heavy fiducial line. The order of the curves in the lower left hand part of the diagram is the same for all liquids, 20°–40°–60°–80°, reading from below up.

Figure 45 shows the average for the twelve liquids. In order to avoid overlapping, the origin of each curve is displaced one square with respect to the next one. The scale of the curves is shown at the right hand side. The origin is so situated that the compressibility for all twelve liquids is between $0.0_5 1$ and 0 at 12000 kgm. The numbers on the curves indicate the liquid in the same order as the curves for the separate liquids (Figures 33 to 44).

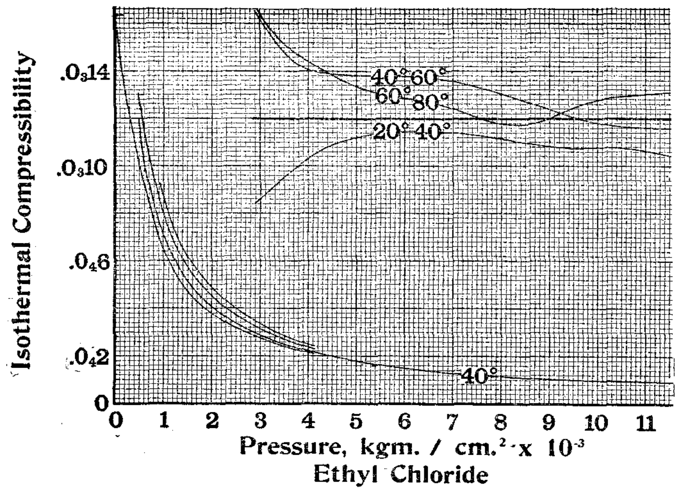


FIGURE 42.

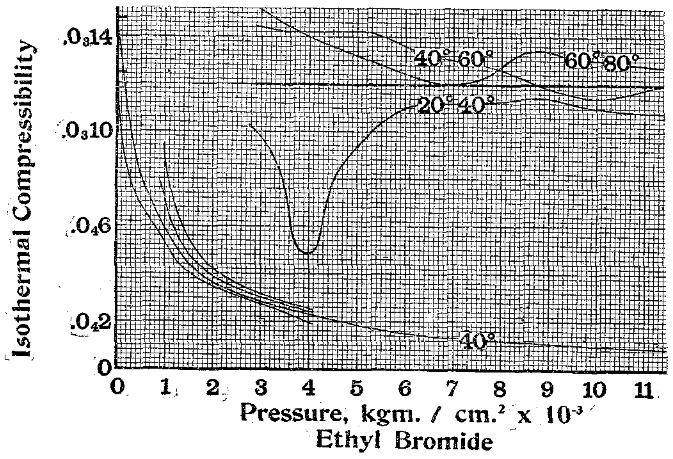


FIGURE 43.

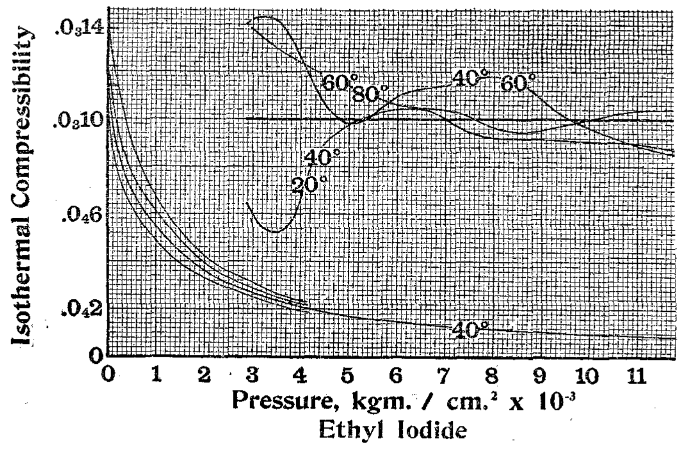


FIGURE 44.

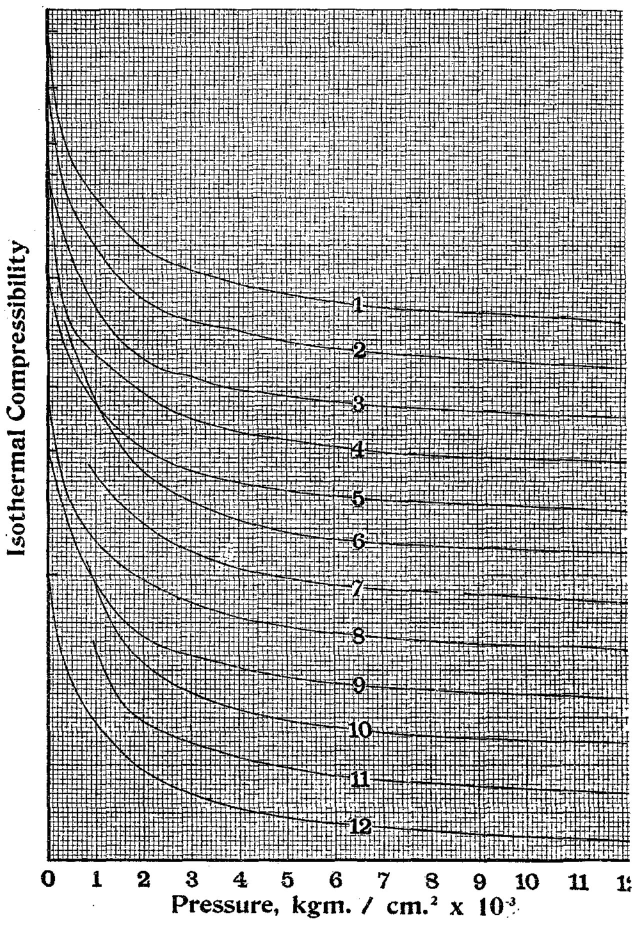
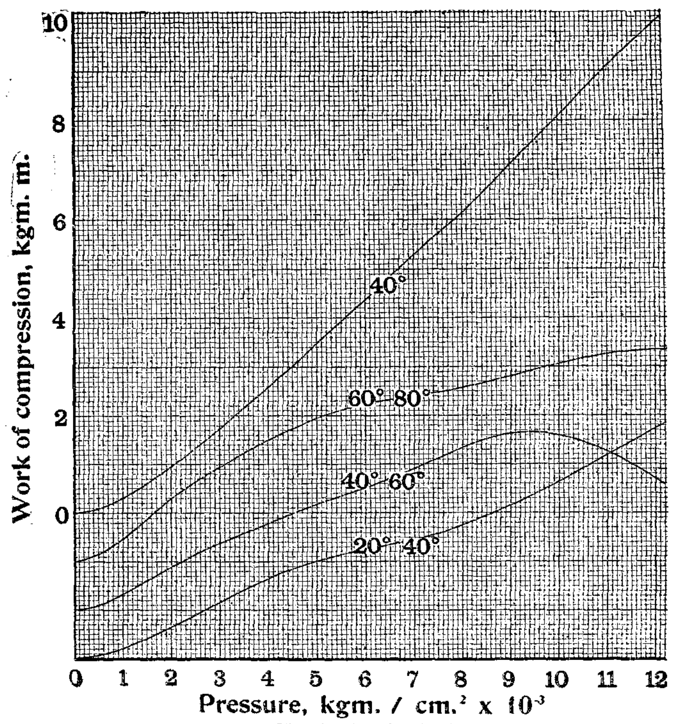
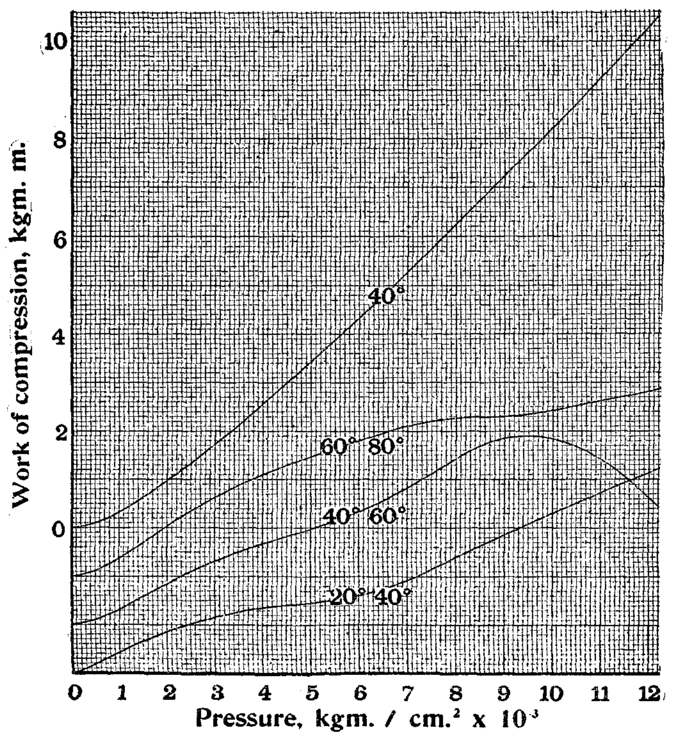


FIGURE 45.



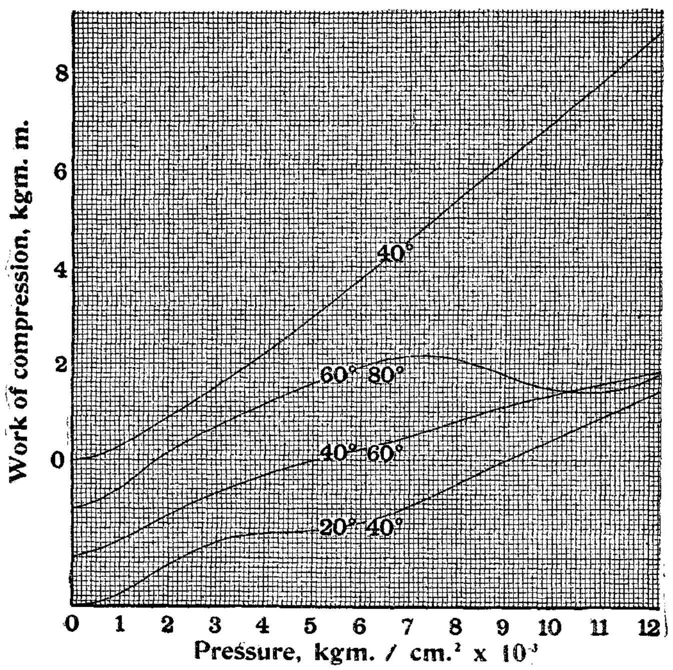
Methyl Alcohol

FIGURE 47.



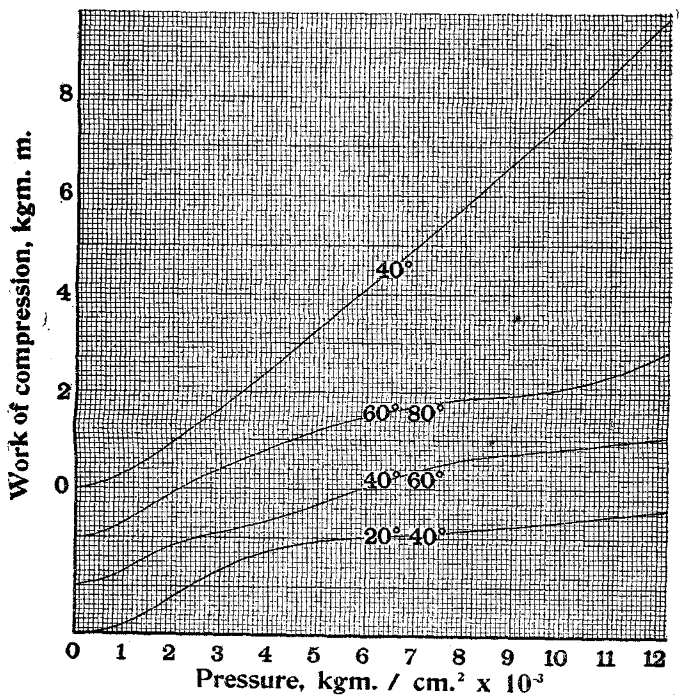
Ethyl Alcohol

FIGURE 48.



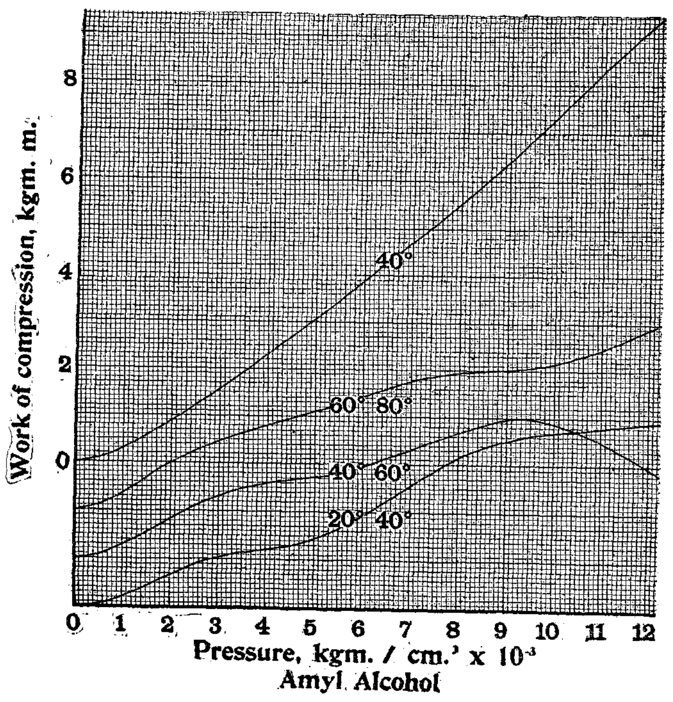
Propyl Alcohol

FIGURE 49.



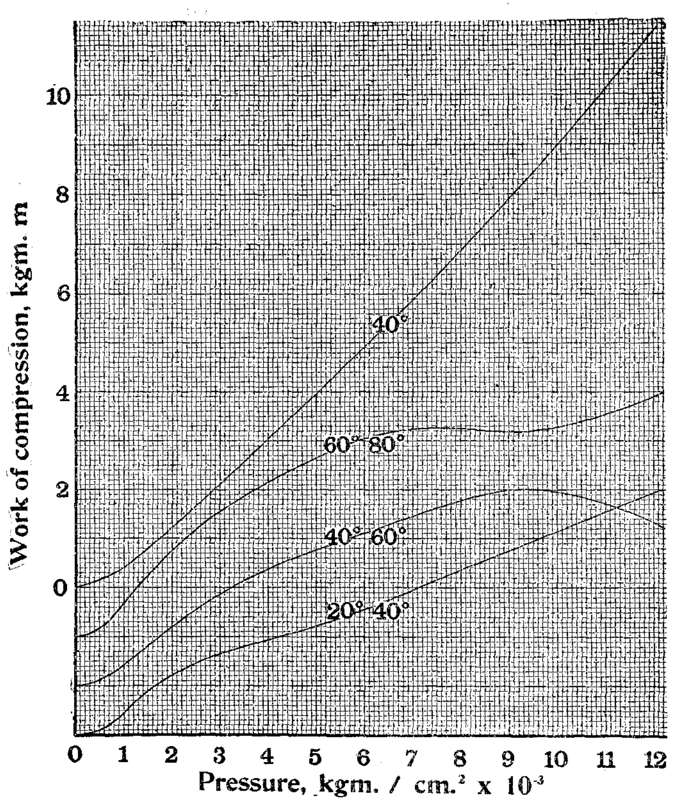
Isobutyl Alcohol

FIGURE 50.



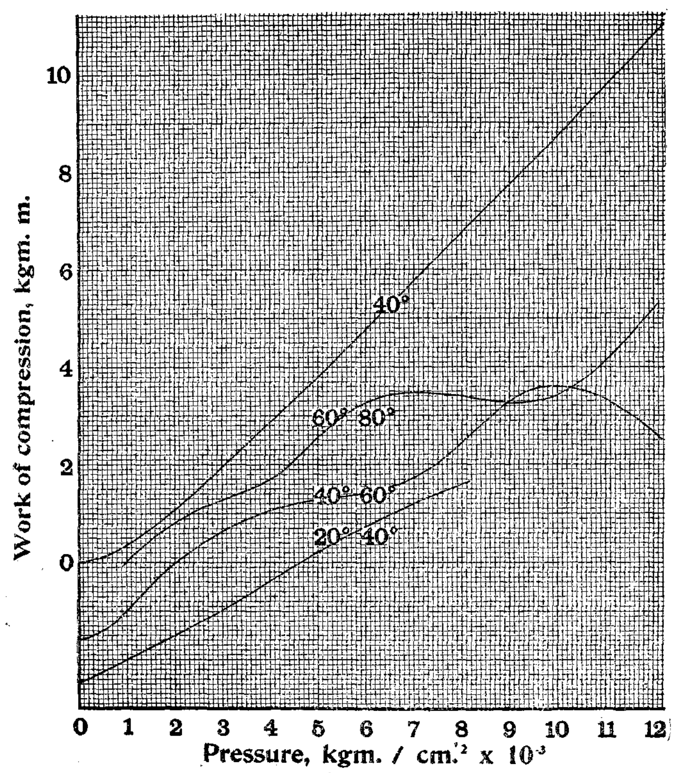
Amyl Alcohol

FIGURE 51.



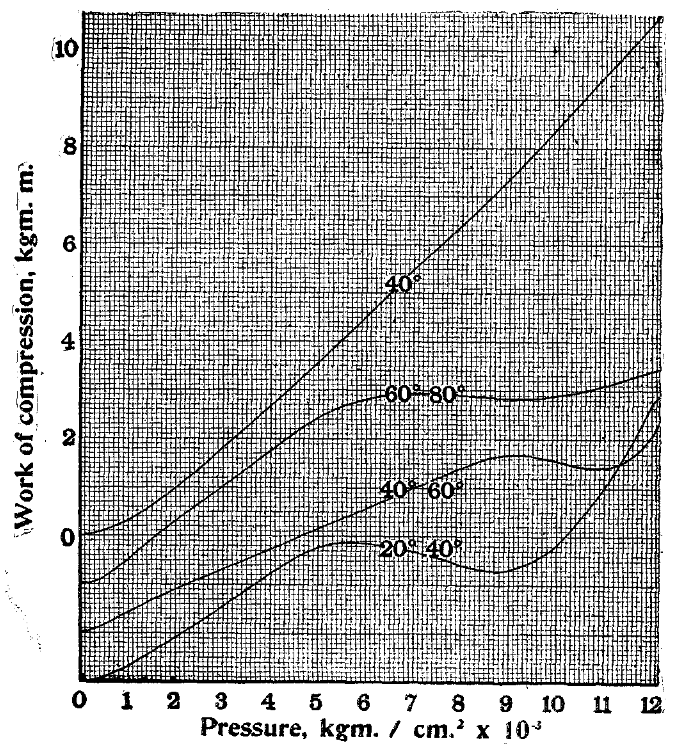
Ether

FIGURE 52.



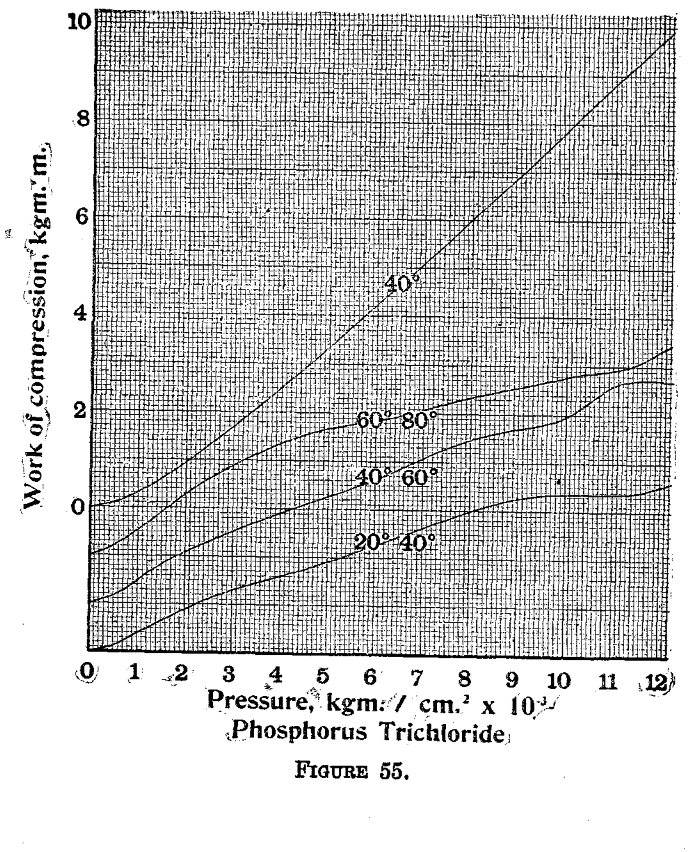
Acetone

FIGURE 53.



Carbon Bisulphide

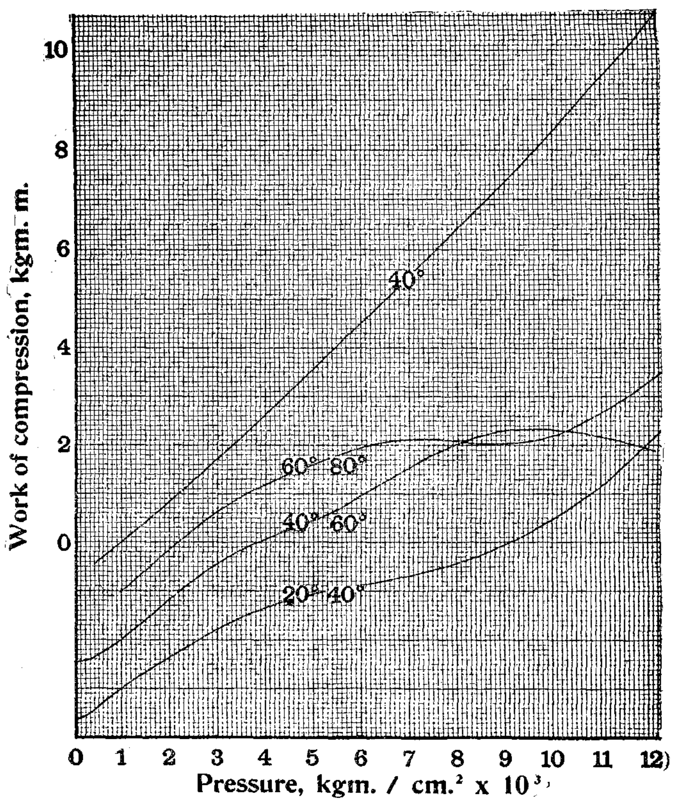
FIGURE 54.



Phosphorus Trichloride

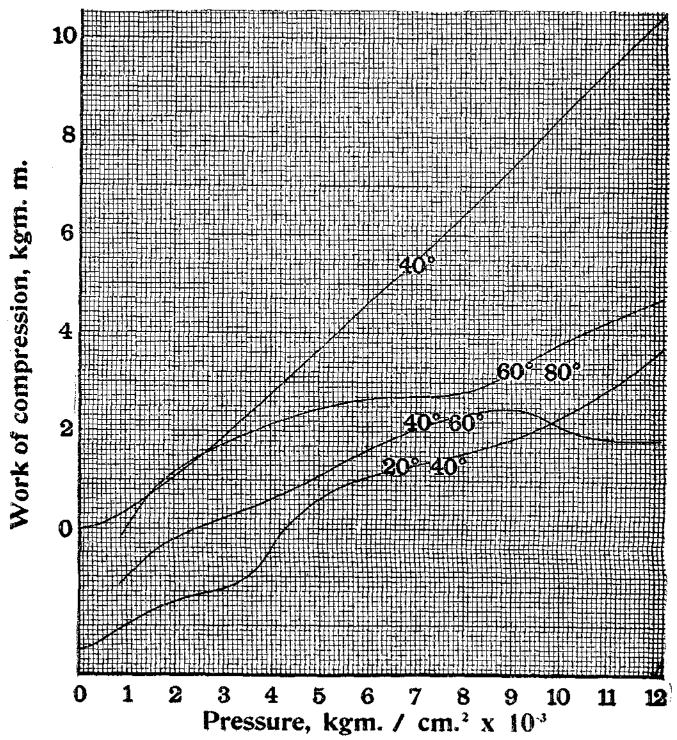
FIGURE 55.

Folder III. The work of compression $\left(W = \int p\, dv\right)$ against pressure. In Figures 47 to 58 W is shown for the liquids separately, and in Figure 59 the average between 20° and 80° for all twelve liquids. The work of compression varies very little with the temperature, so in order to avoid confusion, the complete curve for only 40° is given, and in addition, in the lower part of the diagram, on a scale one hundred times as large, the difference of the work at 20° intervals. Each of these difference curves starts from a new origin. Thus for example, the work of compression of ethyl iodide at 6000 kgm. and 80° is found to be 4.30 (the work of compression at 40°) plus 0.23 (the difference between the work at 40° and 60°), plus 0.30 (the difference between the work at 60° and 80°), or 4.83 kgm. m. in all. In Figure 59, giving the average work of compression over the entire temperature range, the numbers indicate the liquids in the same order as the immediately preceding figures for the separate liquids. In order to avoid confusion, the origin of each curve is displaced one square with respect to the next one. The break in the curve for acetone (7) is because of the freezing, which made it possible to take the average only from 40° to 80° over the upper part of the pressure range, instead of between 20° and 80° as at the lower pressures. The origin of the curve for ethyl chloride (10) is at 500 kgm. instead of atmospheric pressure as for the other liquids, because of the low boiling point of this substance at atmospheric pressure.



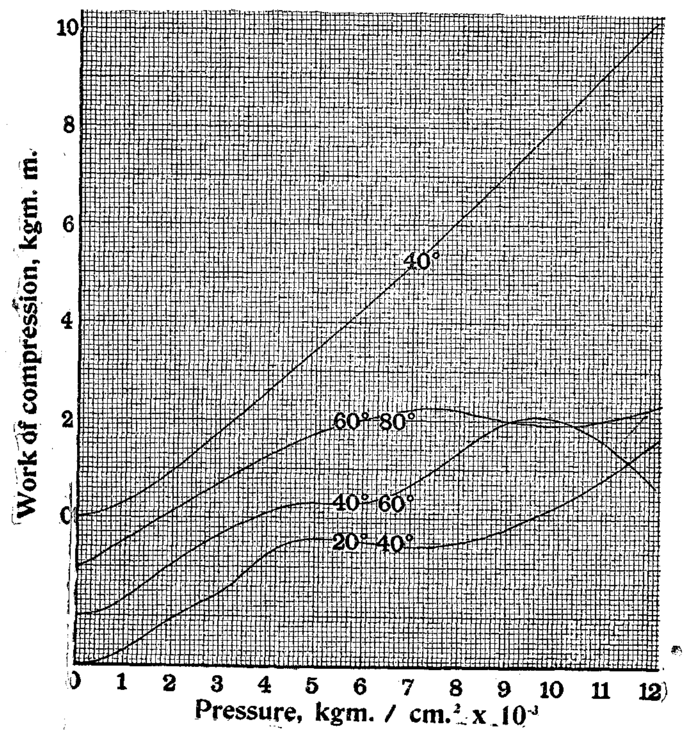
Ethyl Chloride

FIGURE 56.



Ethyl Bromide

FIGURE 57.



Ethyl Iodide

FIGURE 58.

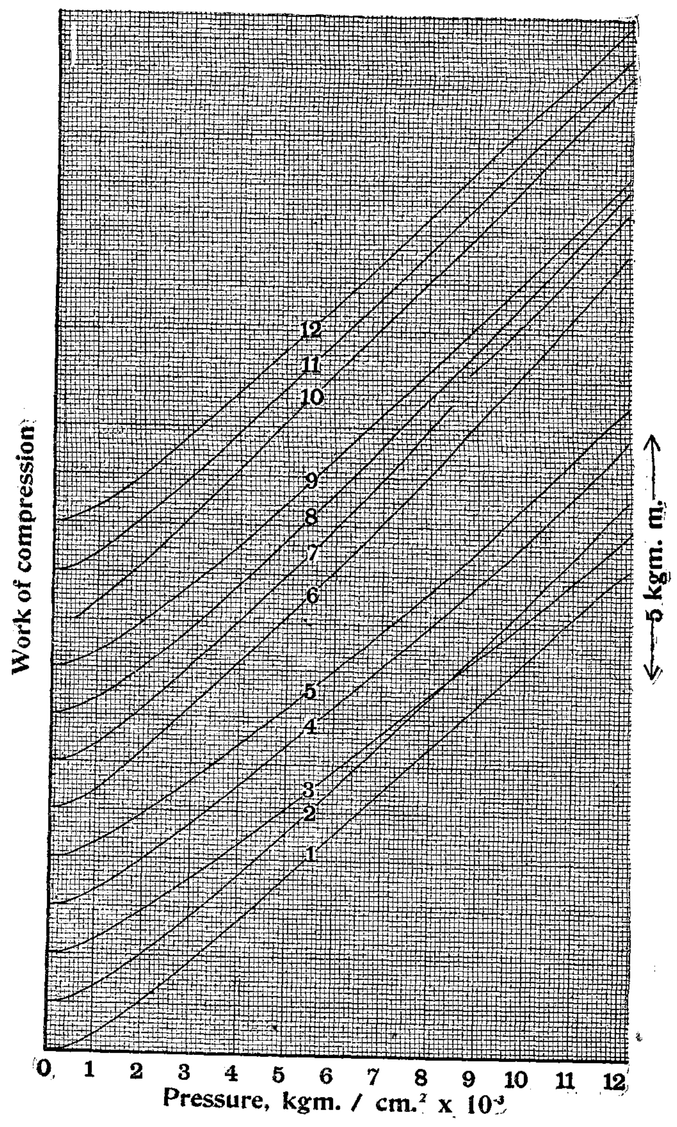
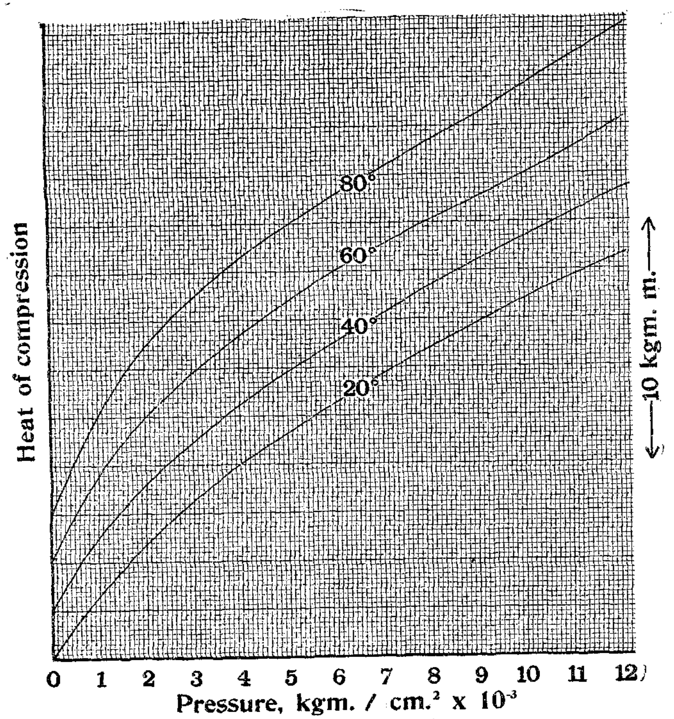
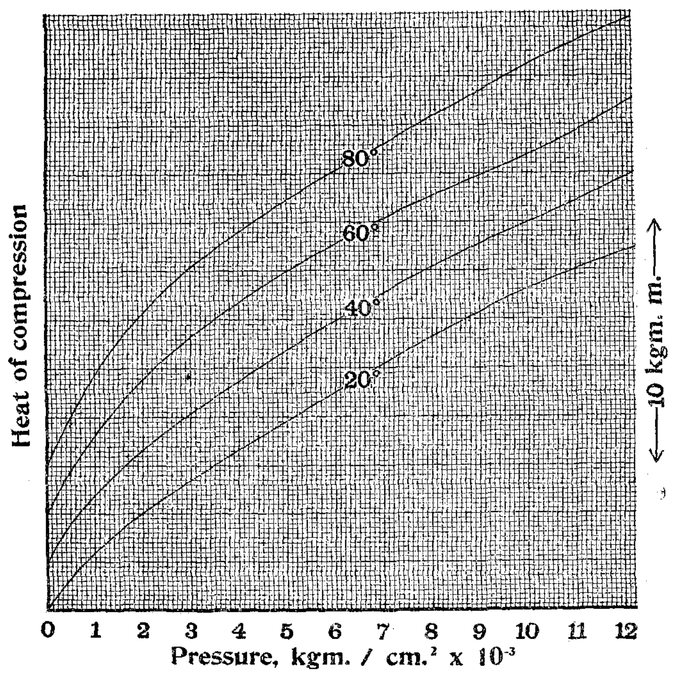


FIGURE 59.



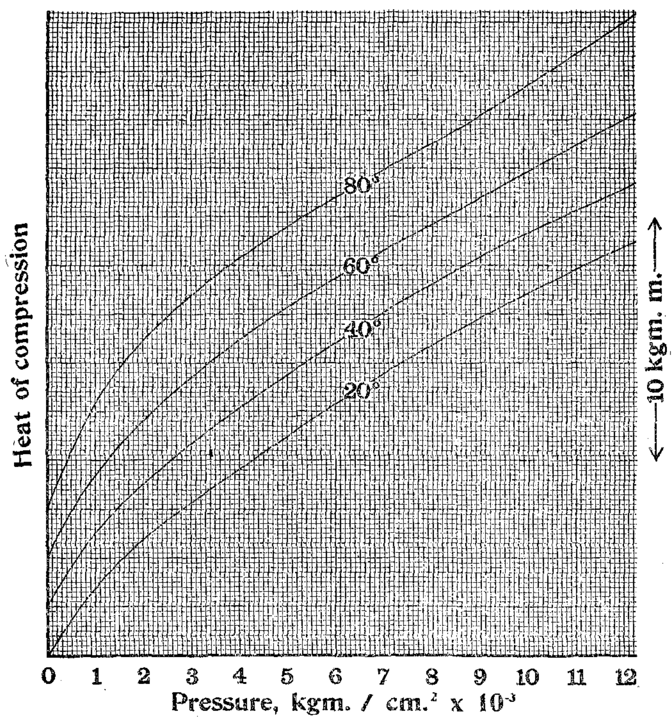
Methyl Alcohol

FIGURE 60.



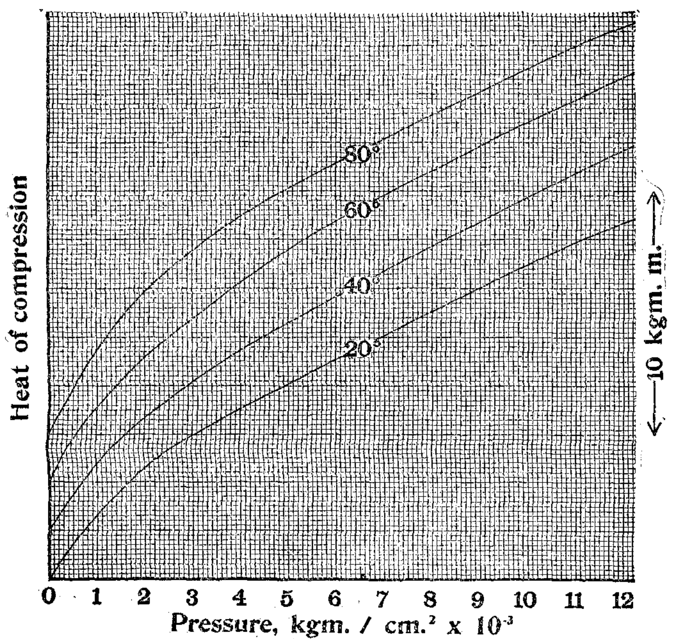
Ethyl Alcohol

FIGURE 61.



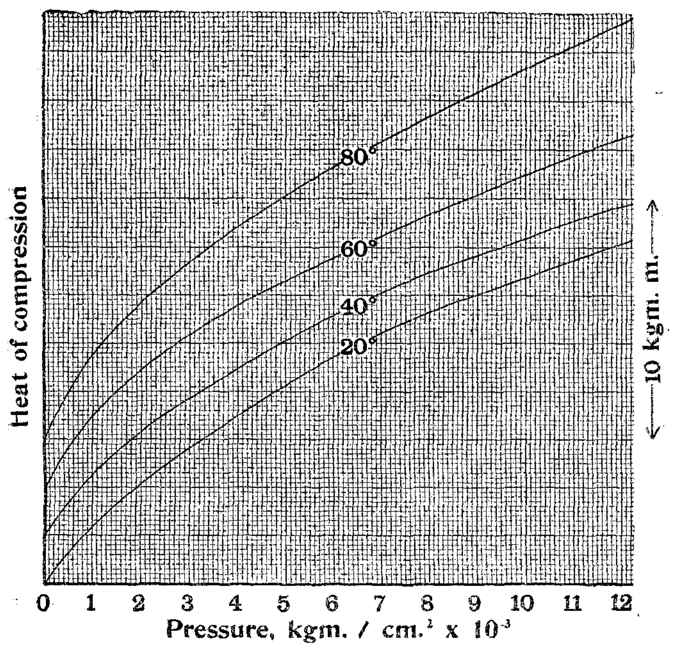
Propyl Alcohol

FIGURE 62.



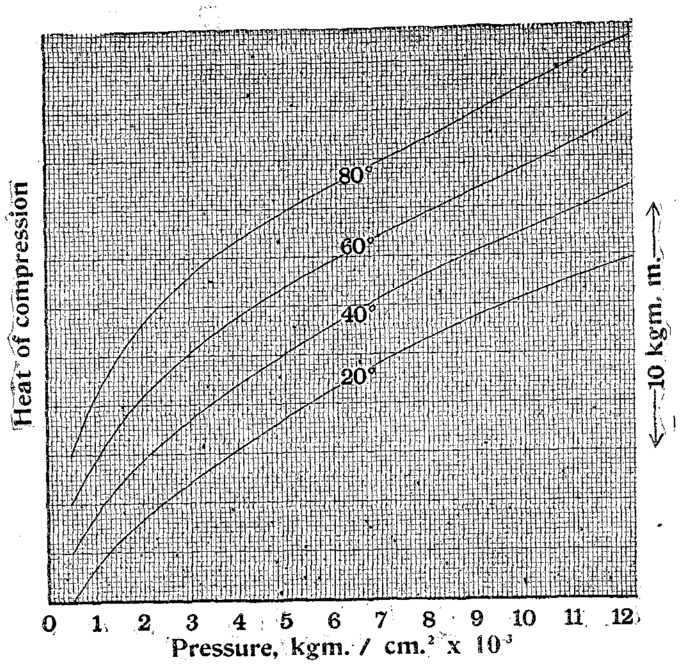
Isobutyl Alcohol

FIGURE 63.



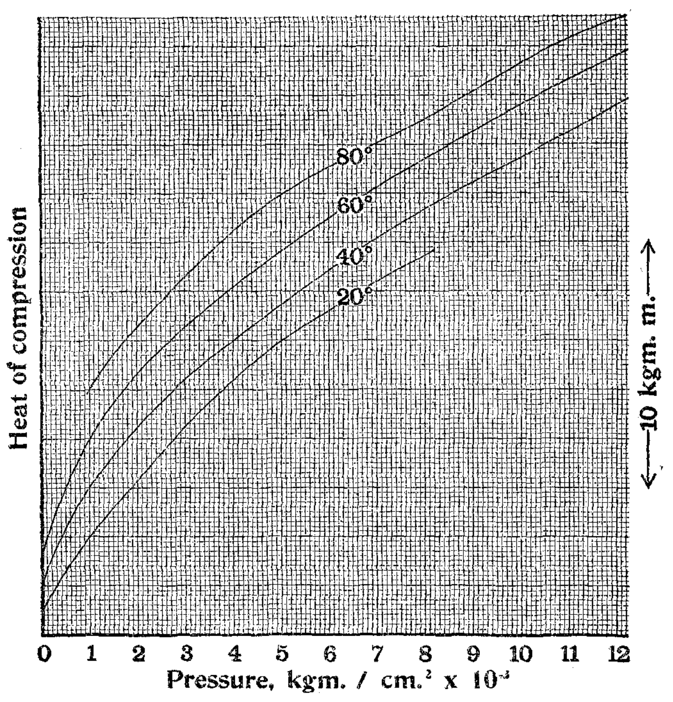
Amyl Alcohol

FIGURE 64.



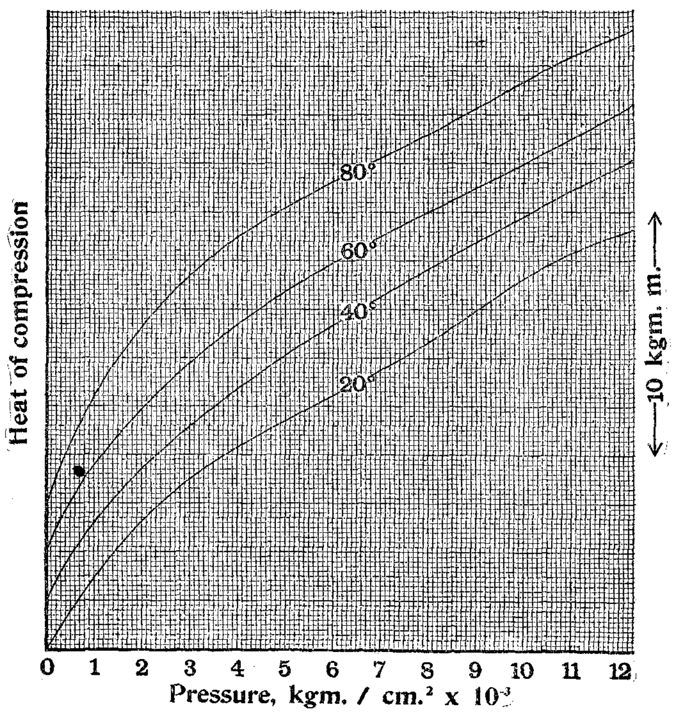
Ether

FIGURE 65.



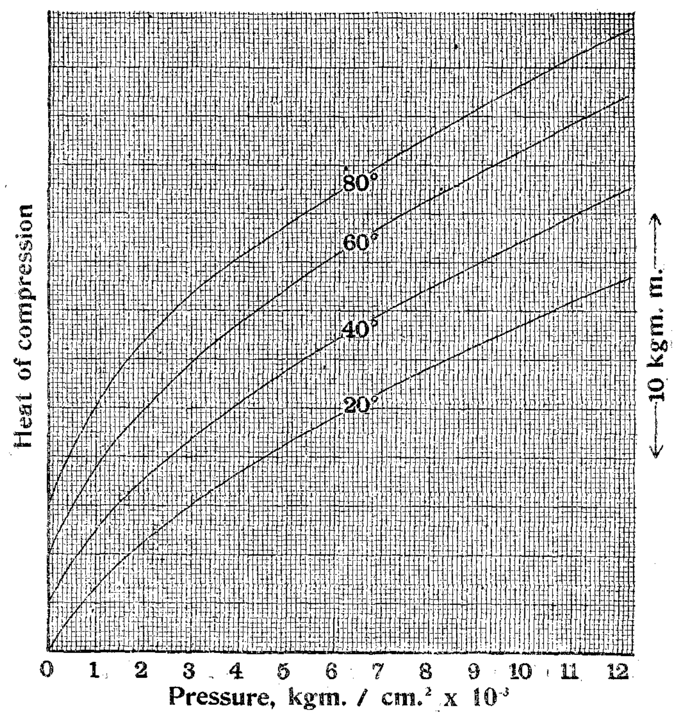
Acetone

FIGURE 66.



Carbon Bisulphide

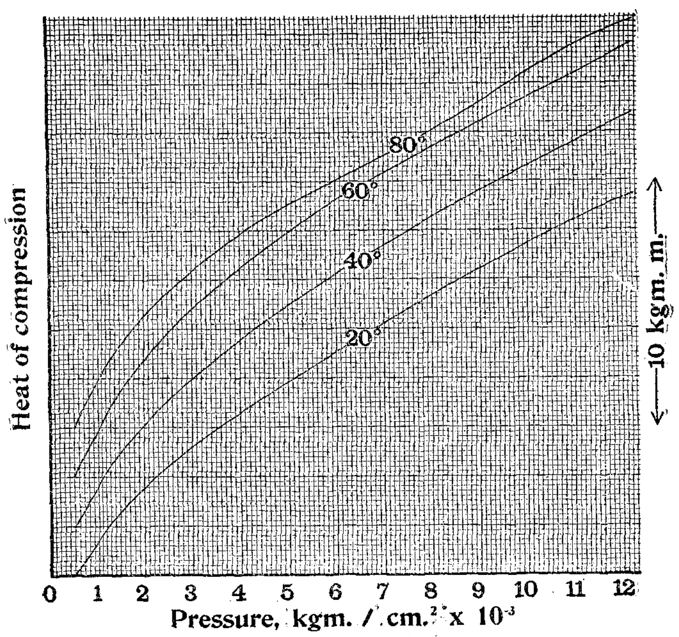
FIGURE 67.



Phosphorus Trichloride

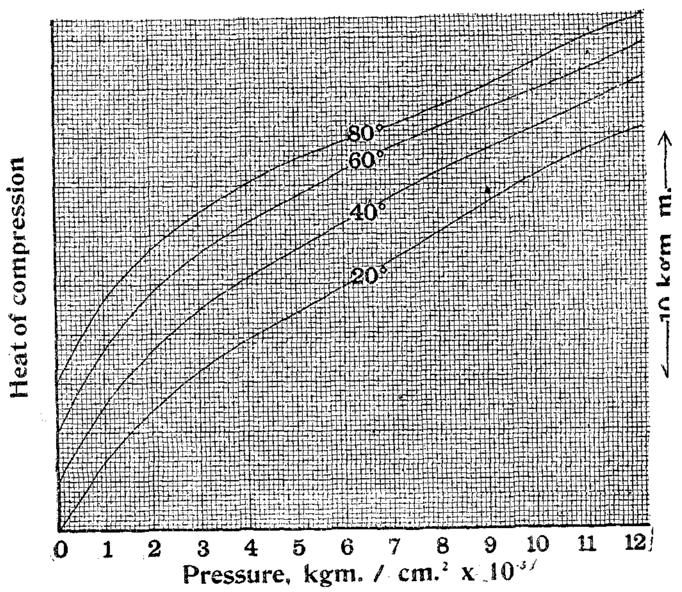
FIGURE 68.

Folder IV. The heat of compression $\left(Q = \int_T \left(\frac{\partial v}{\partial \tau}\right)_p dv\right)$ against pressure. In Figures 60 to 71 are shown the curves for the separate liquids at intervals of temperature of 20°. In order to avoid overlapping, the origin for different temperatures is not the same. The scale of the diagram is shown at the right hand side. Because of the low boiling point, some of the curves do not start from an origin at atmospheric pressure. Figure 72 shows the average heat of compression between 20° and 80° for all twelve liquids. The origin of each curve is displaced one square with respect to the next. Ethyl chloride (10) and ether (6) start from an origin at 500 kgm. because of the low boiling point. The curve for acetone (7) shows a break at 8000 kgm. because above 8000 the average had to be taken between 40° and 80° by reason of the freezing under pressure. The numbers in Figure 72 indicate the liquids in the same order as that of the separate diagrams.



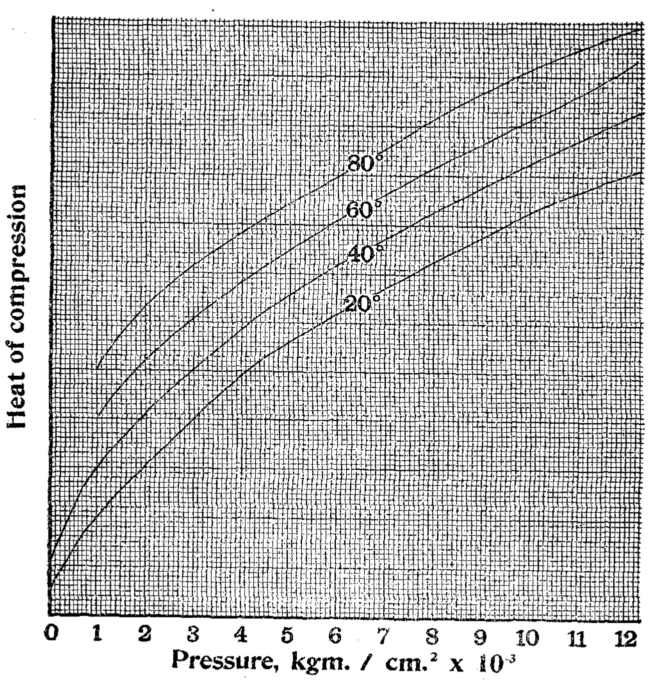
Ethyl Chloride

FIGURE 69.



Ethyl Iodide

FIGURE 70.



Ethyl Bromide

FIGURE 71.

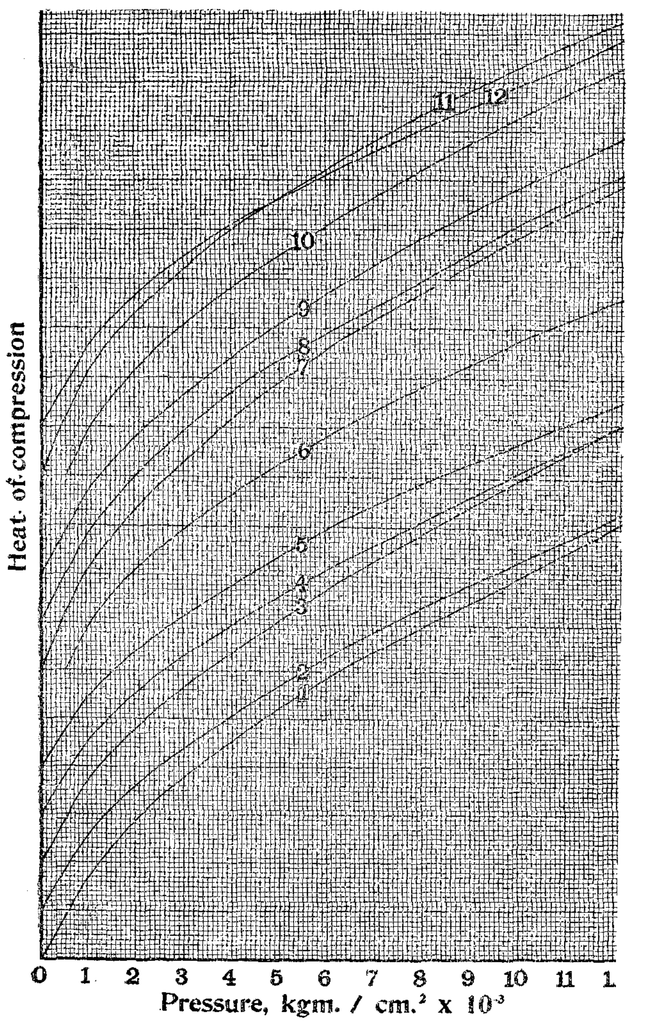


FIGURE 72.

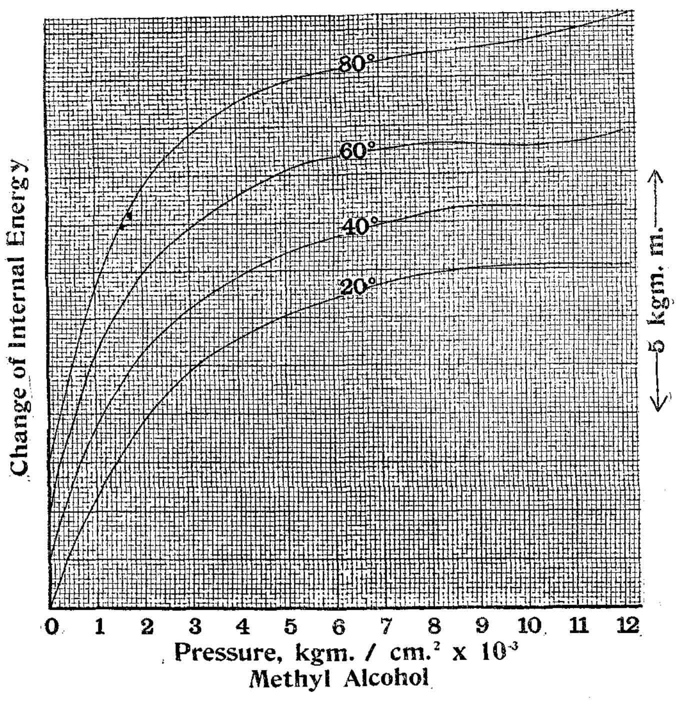


FIGURE 73.

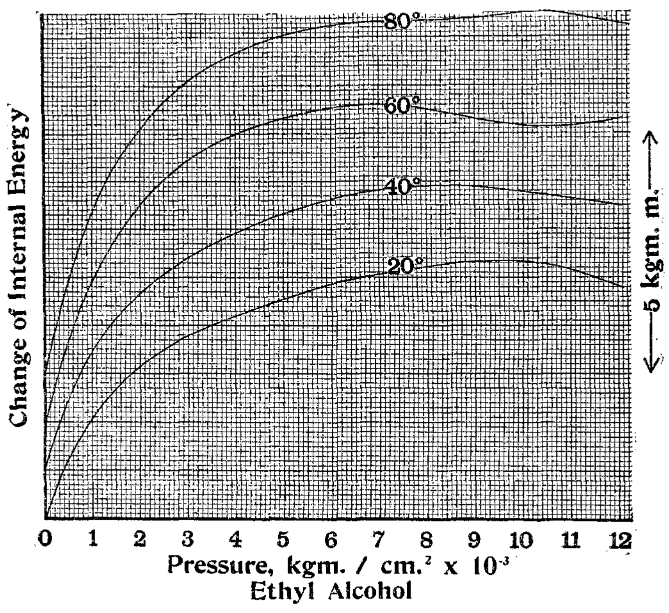


FIGURE 74.

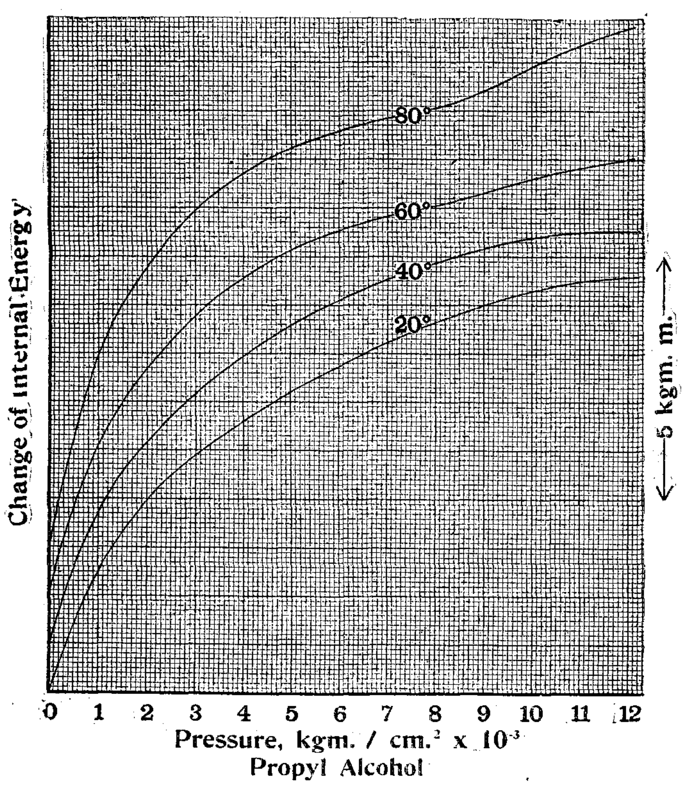


FIGURE 75.

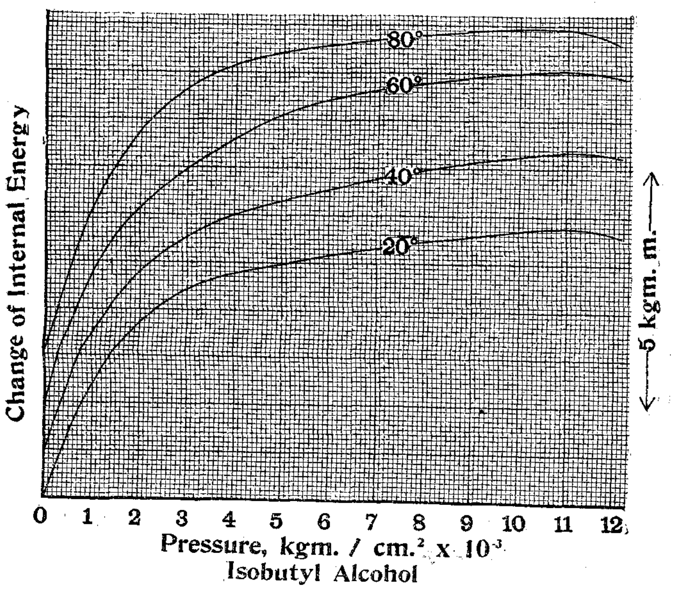


FIGURE 76.

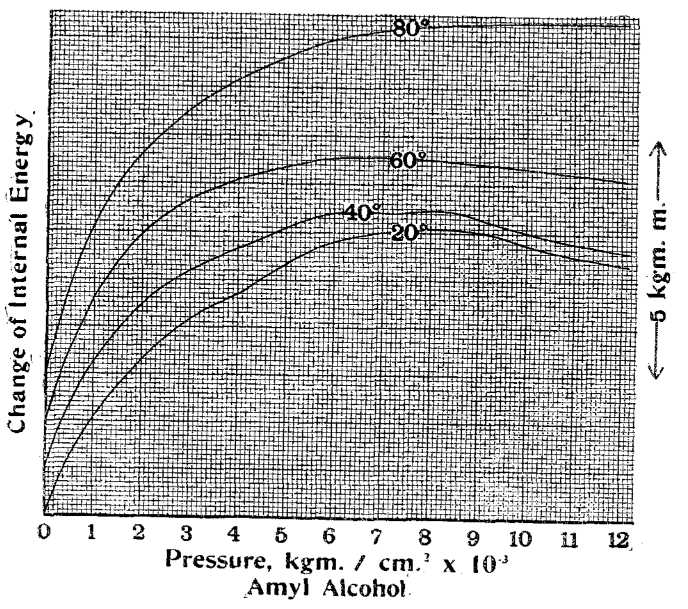


FIGURE 77.

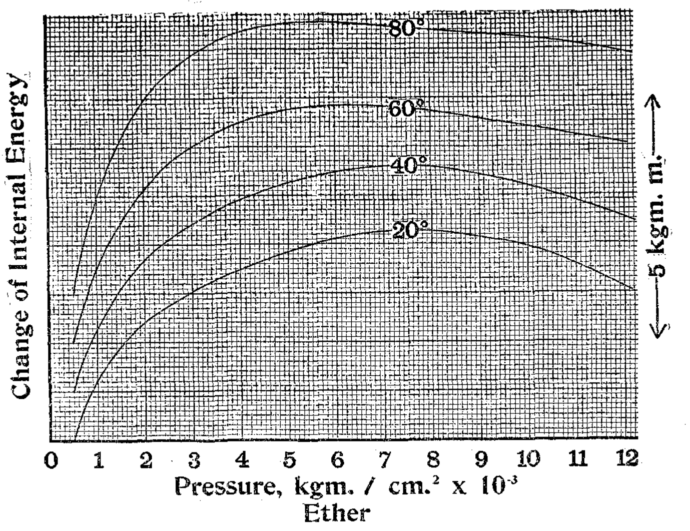


FIGURE 78.

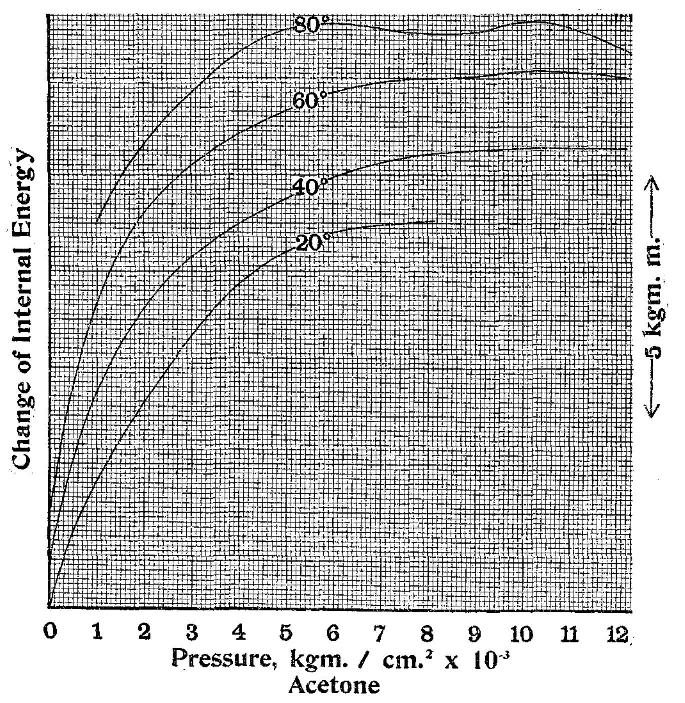


FIGURE 79.

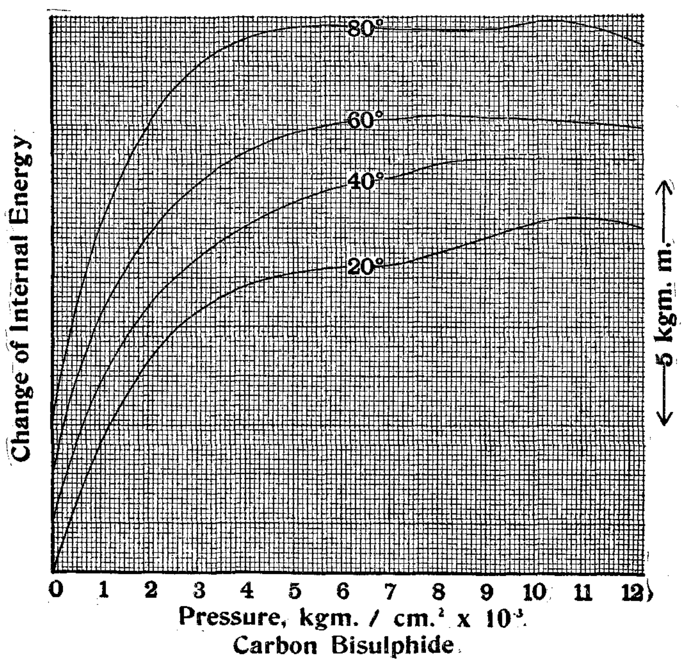


FIGURE 80.

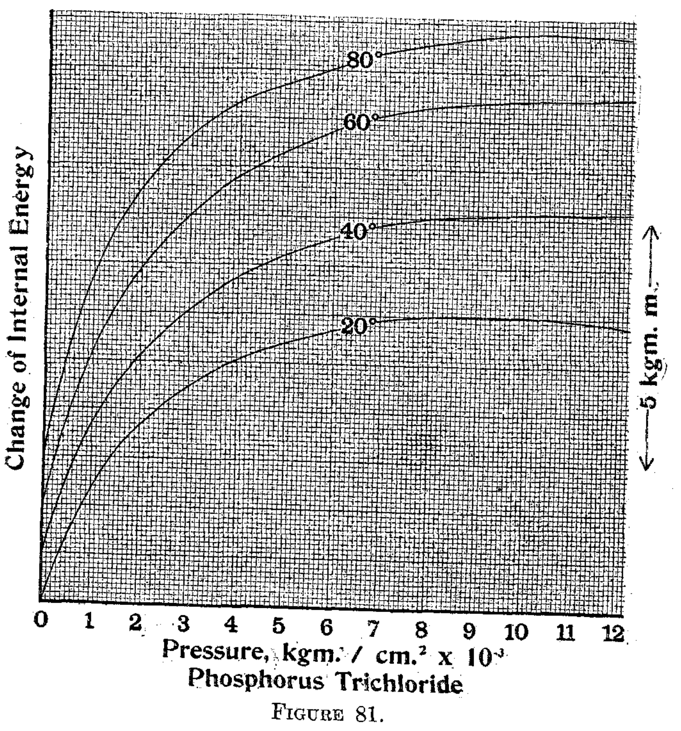


FIGURE 81.

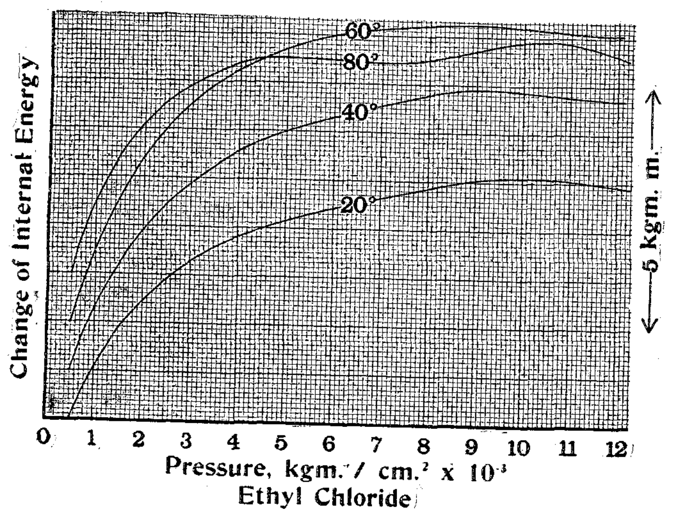


FIGURE 82.

Folder V. Change of internal energy, $\Delta E = Q - W$, against pressure. Figures 73 to 84 show the change of internal energy at intervals of 20° for the separate liquids. The change of energy plotted is the difference between the energy at atmospheric pressure and the pressure in question, so that a positive ordinate means that the internal energy has decreased with increasing pressure. In order to avoid confusion, the origin of the separate curves is displaced one square with respect to the next. The scale of the diagram is shown at the right hand side. The origin of some of the curves is not at atmospheric pressure, but at 500 or 1000 kgm. This is because the low boiling point makes it impossible to carry the liquid to atmospheric pressure at the temperature in question. In these cases, the change of internal energy shown is the difference between the internal energy at 500 or 1000 kgm. and the pressure in question. Figure 85 shows the average change of energy between 20° and 80° for the twelve liquids plotted in a single diagram against pressure, and in Figure 86, the same quantity against volume. The origin of these curves has been displaced in the usual way, and the liquids have been shown by numbers as is usual, (the order being the same as that of the separate diagrams.) The scale is shown at the right hand side.

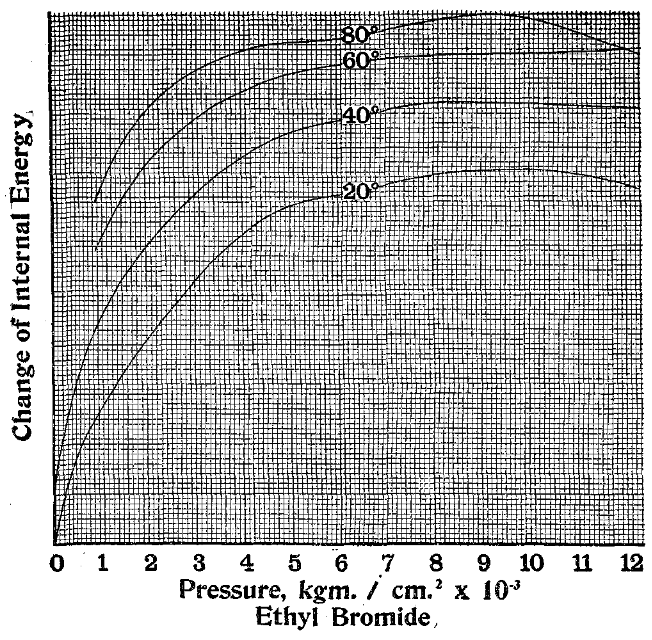


FIGURE 83.

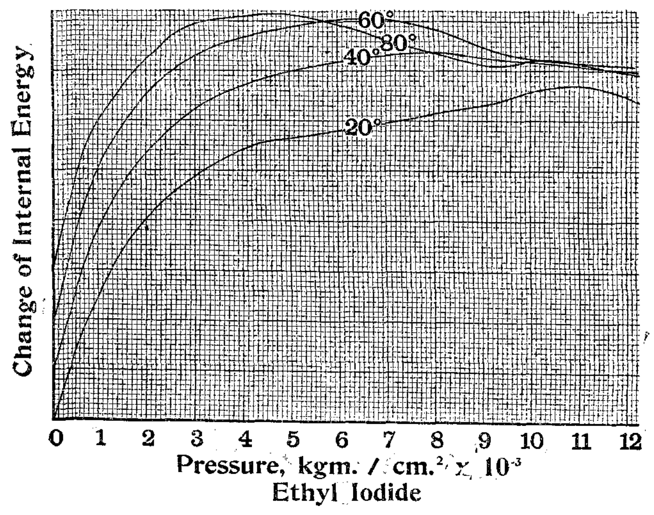


FIGURE 84.

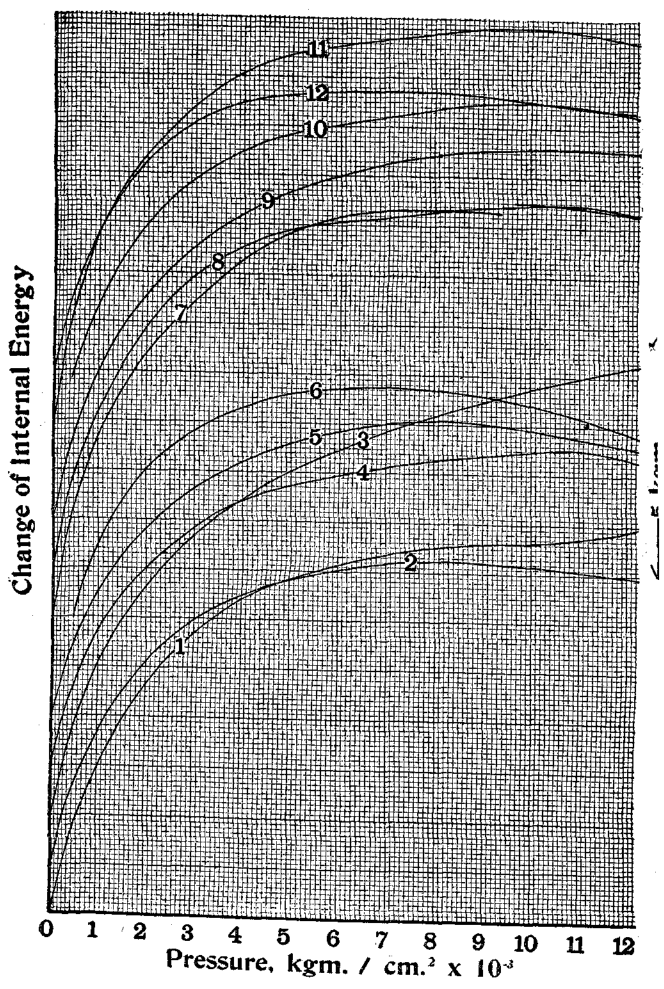


FIGURE 85.

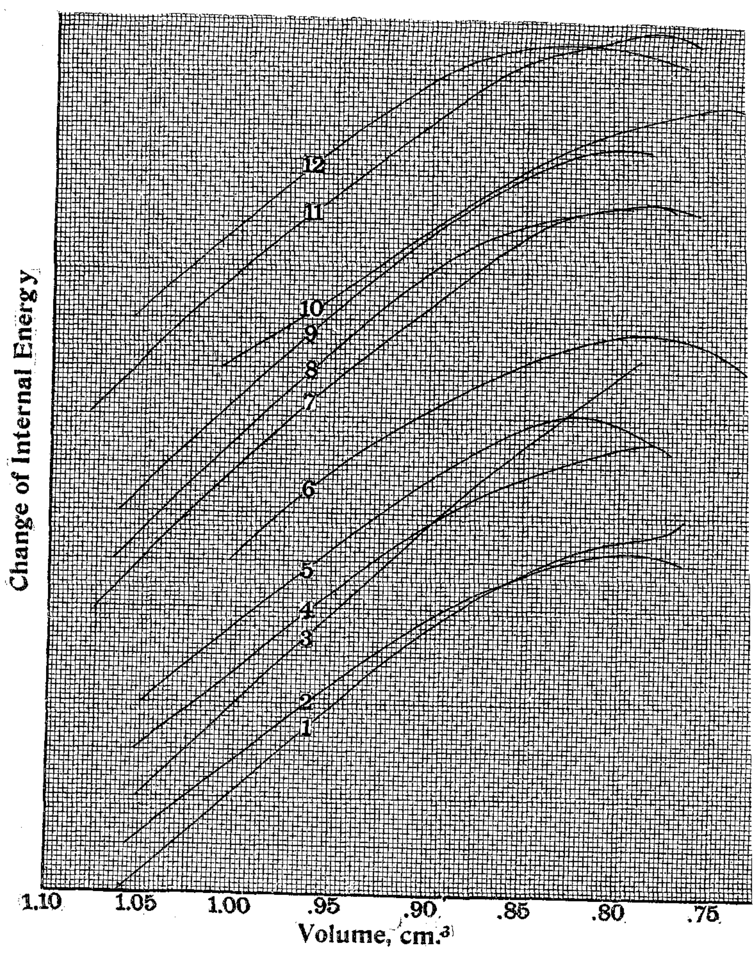


FIGURE 86.

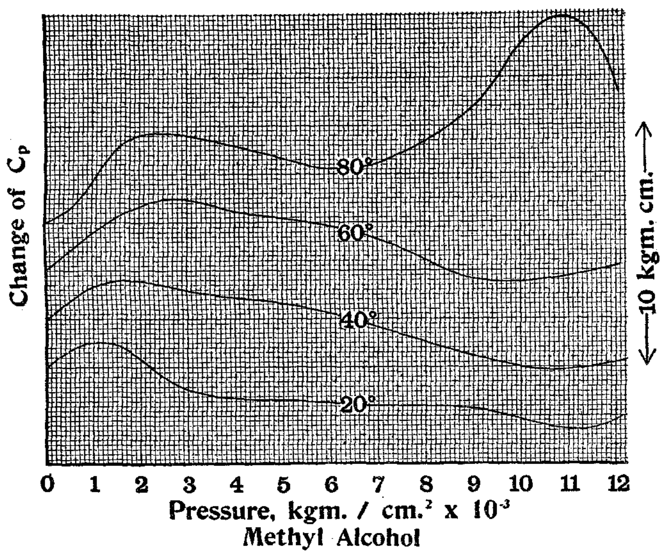


FIGURE 87.

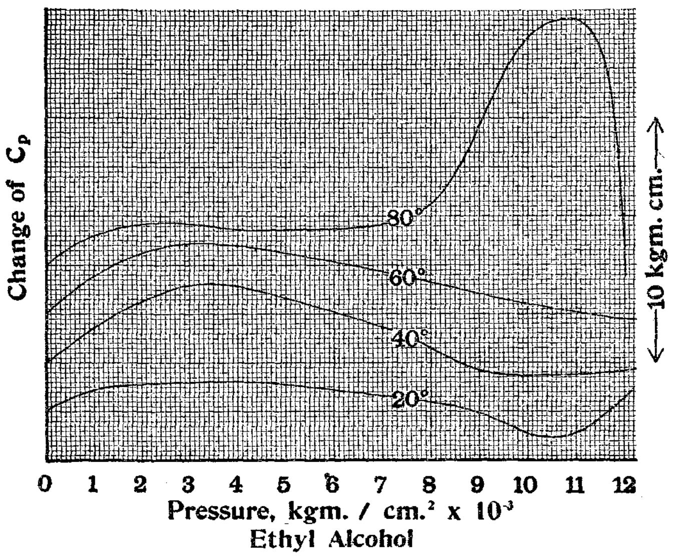


FIGURE 88.

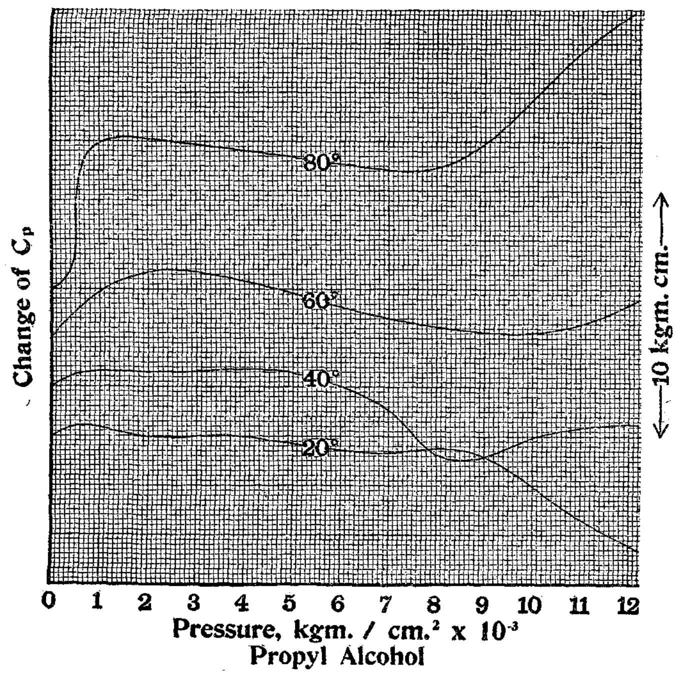


FIGURE 89.

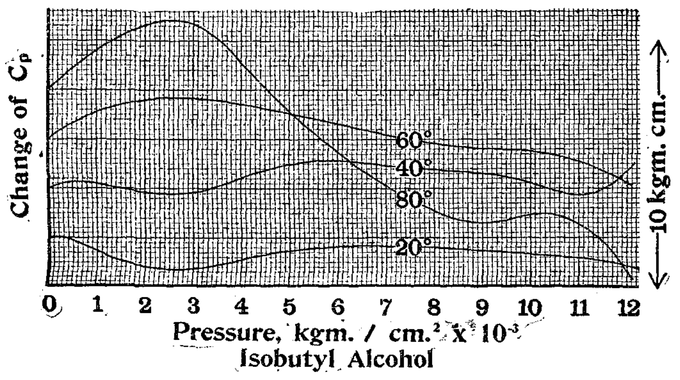


FIGURE 90.

Folder VI. The change of specific heat at constant pressure (C_p) against pressure. Note that the units in which C_p are measured are the number of kgm. cm. of work necessary to raise through one degree the quantity of liquid which at 0° *C.* and atmospheric pressure occupies one c.c. Figures 87 to 98 show the change of C_p at intervals of 20° for the liquids separately. A positive ordinate means that C_p is greater at atmospheric pressure than at the pressure in question. The origin of each of the curves has been displaced with respect to the next in the usual way, and the scale is shown at the right hand side. Because of the low boiling point, the origin for some of the curves is at 500 or 1000 kgm. In these cases, the difference shown is the difference between C_p at 500 or 1000 kgm. and the pressure in question. Figure 99 shows the average of C_p from 20° to 80° for all twelve liquids. The liquids are indicated by numbers, the origin of the separate curves has been displaced, and the scale is shown at the right hand side, in the usual way.

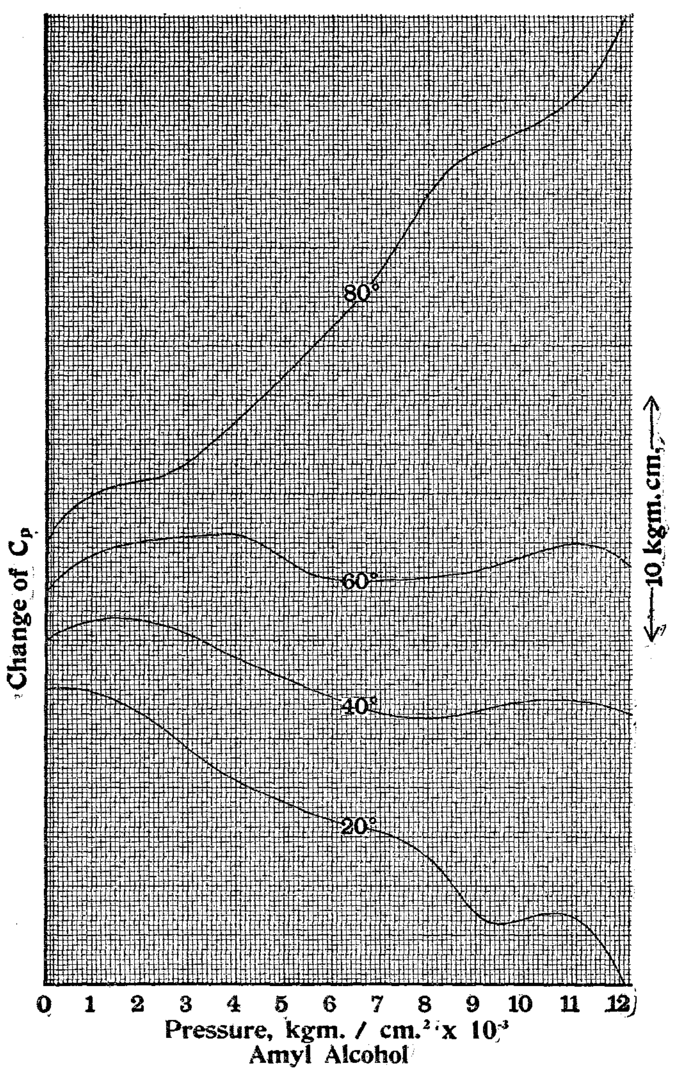


FIGURE 91.

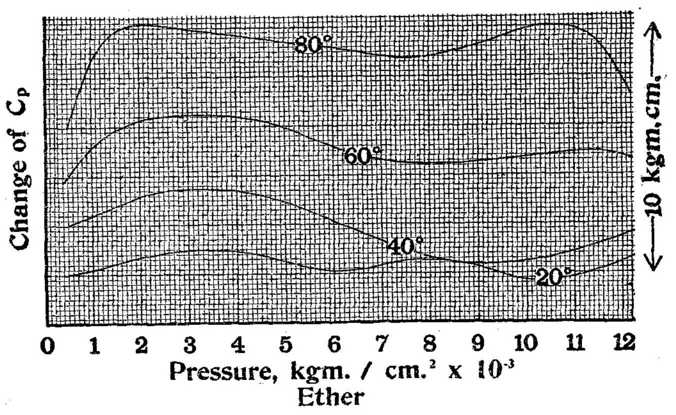


FIGURE 92.

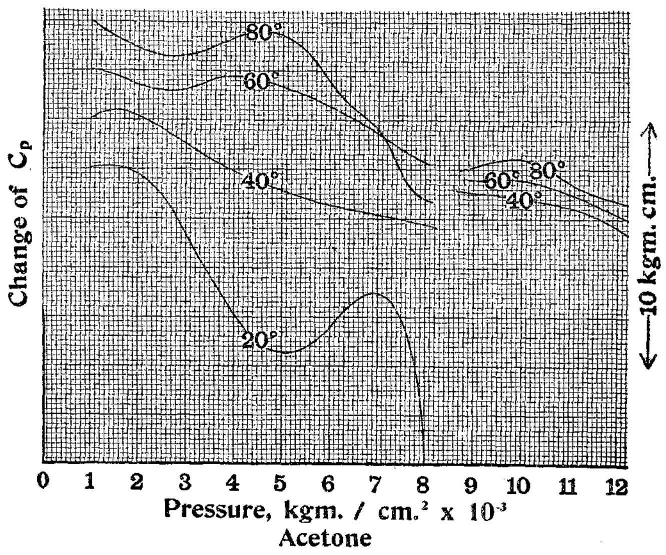


FIGURE 93.

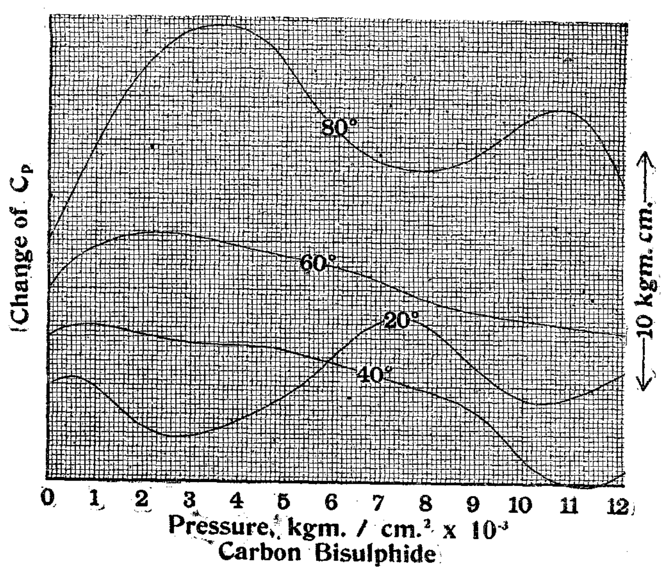


FIGURE 94.

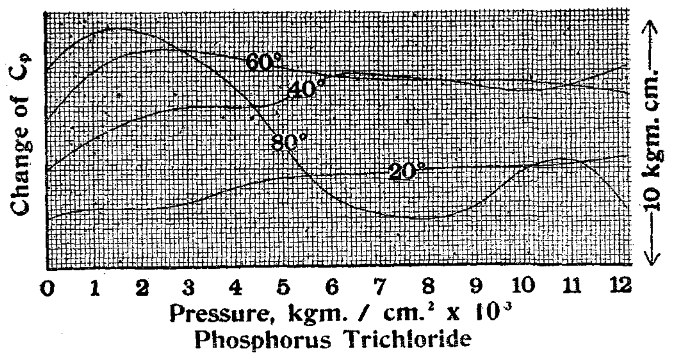


FIGURE 95.

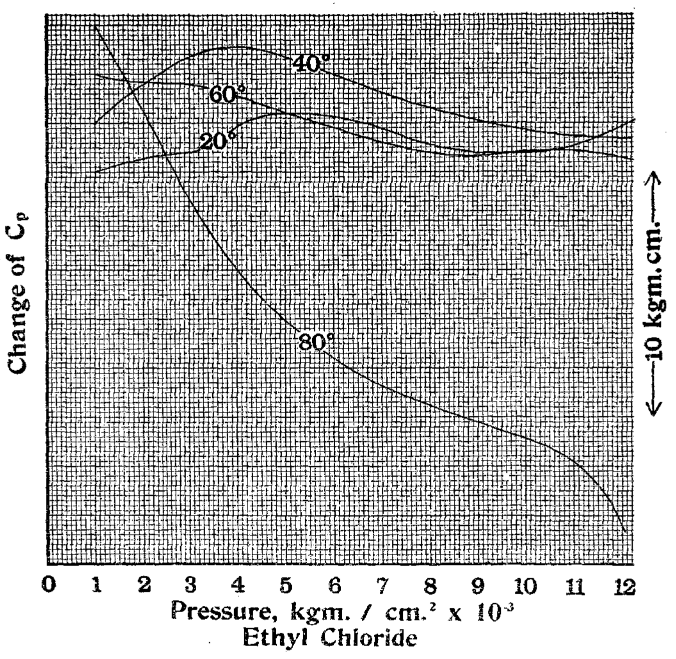


FIGURE 96.

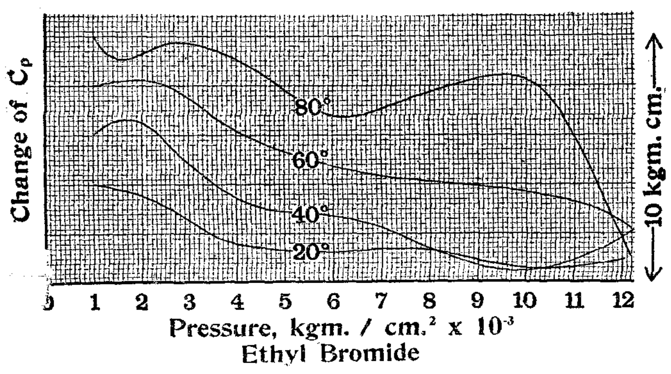


FIGURE 97.

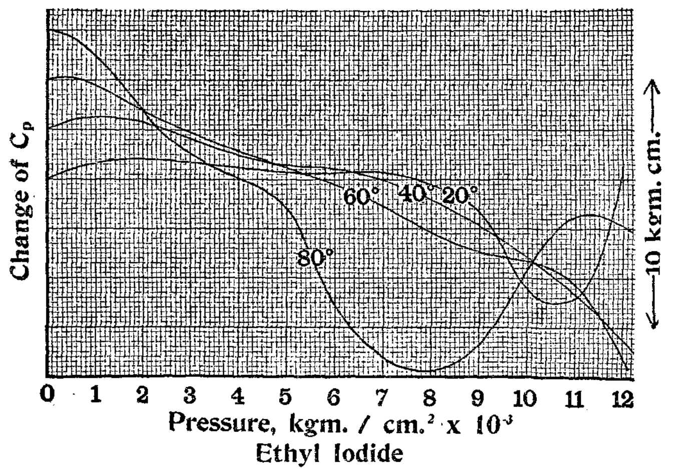


FIGURE 98.

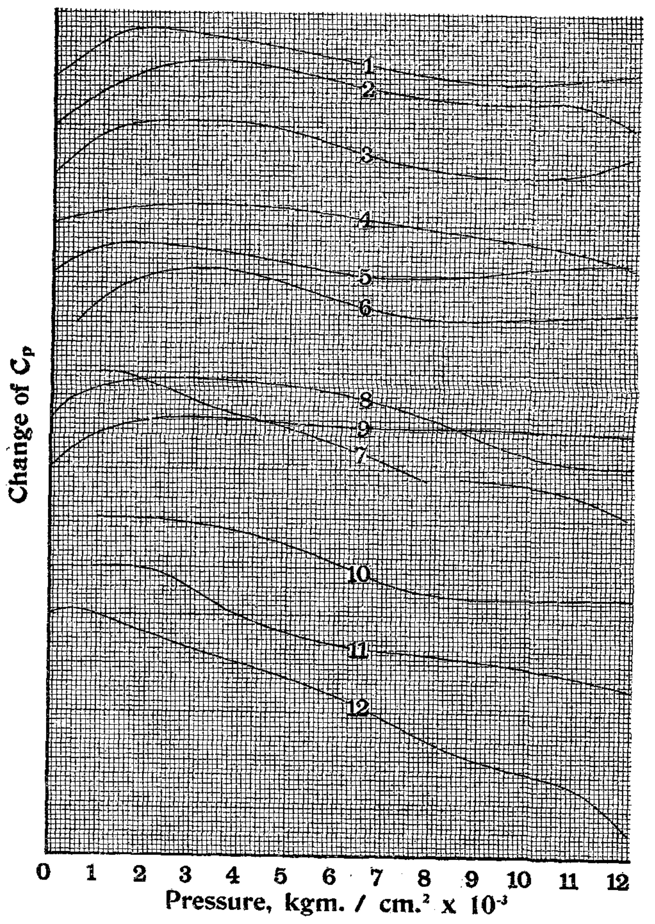


FIGURE 99.

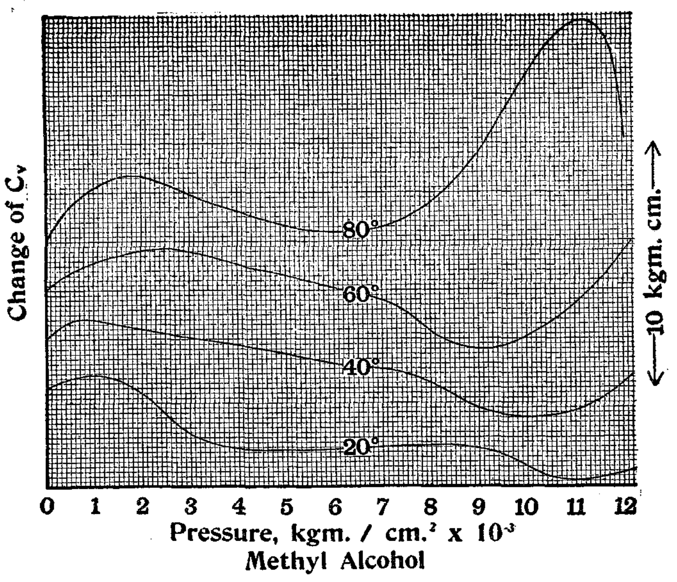


FIGURE 100.

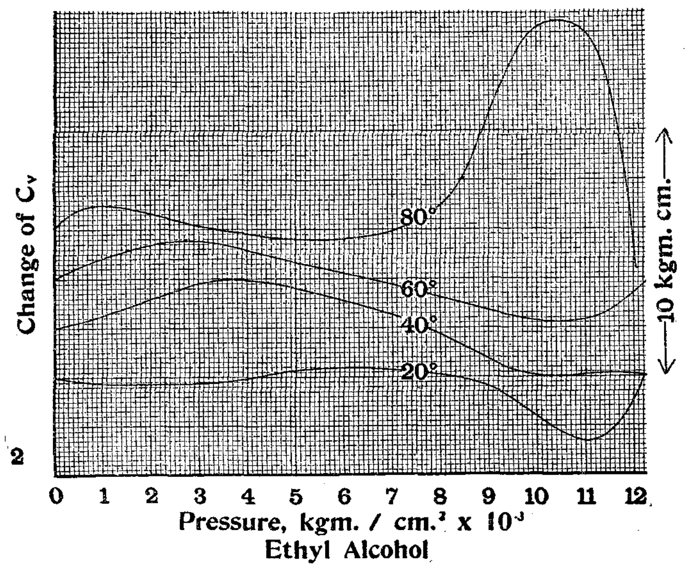


FIGURE 101.

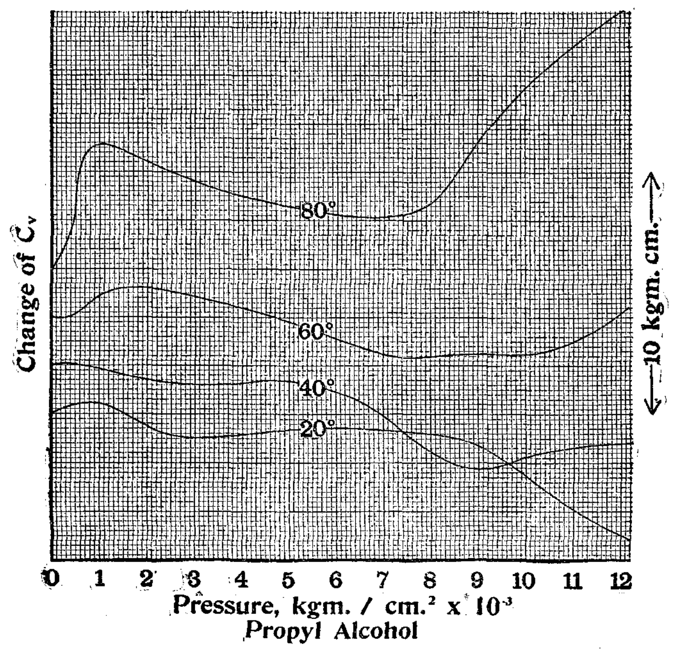


FIGURE 102.

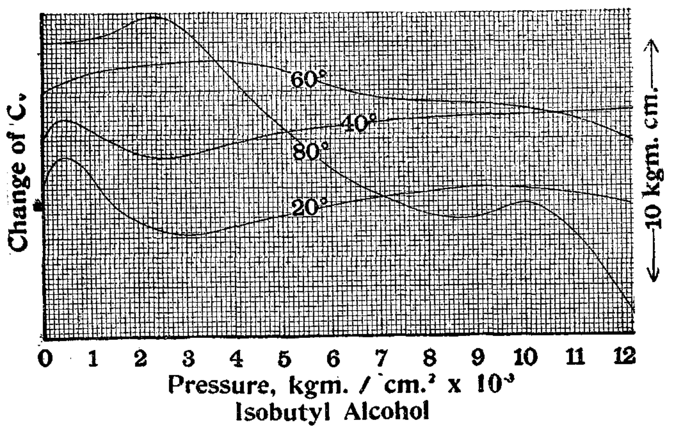
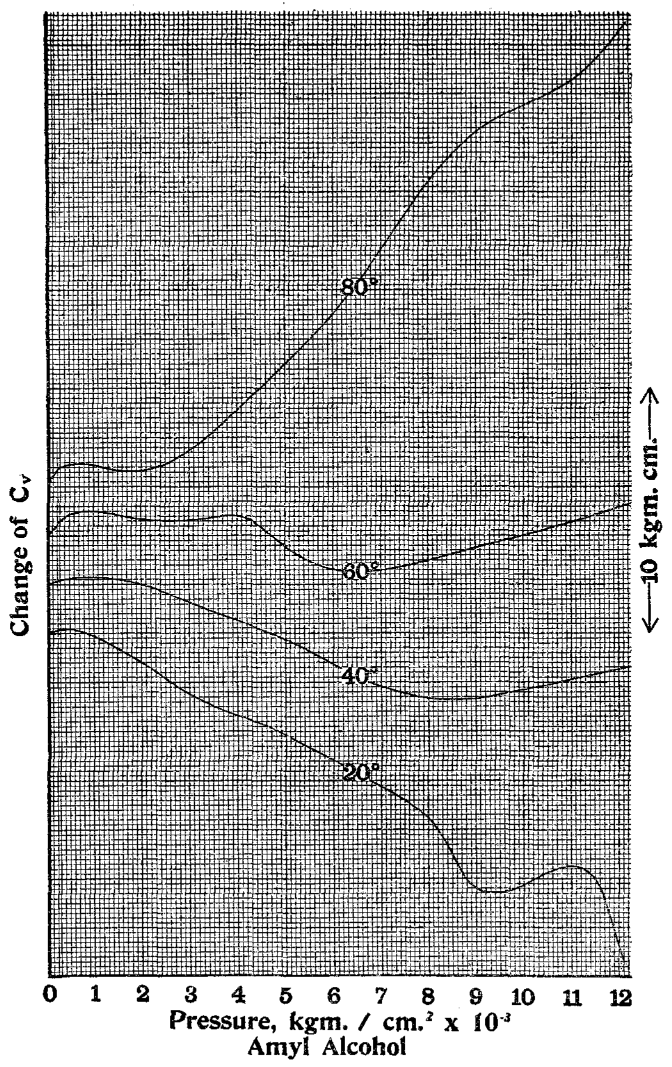


FIGURE 103.



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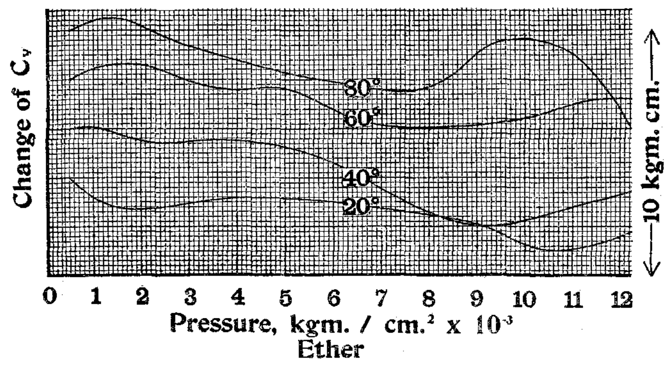

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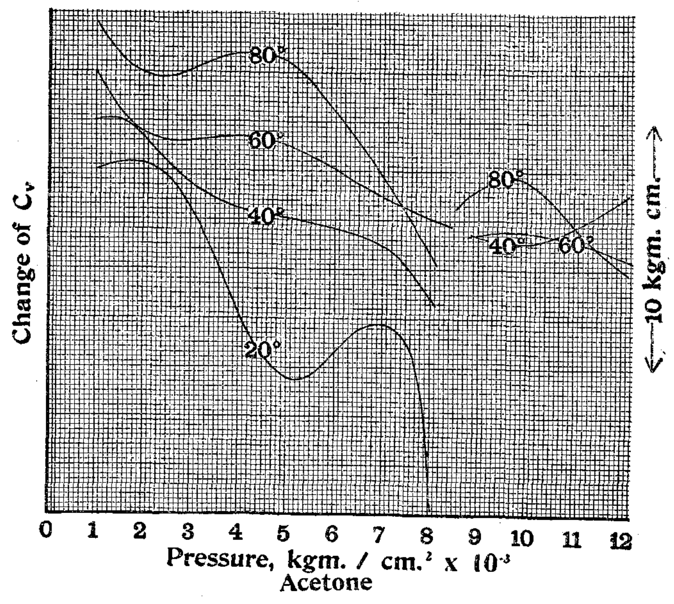

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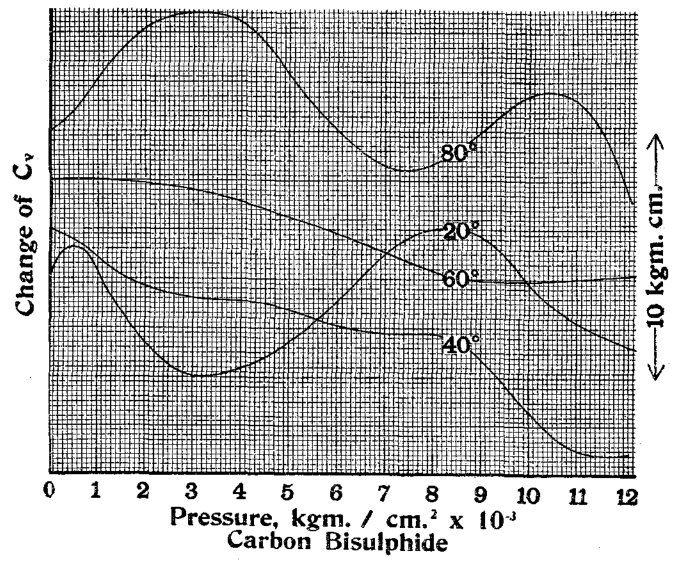


FIGURE 107.

Folder VII. The change of specific heat at constant volume (C_v) against pressure. Note that the units in which C_v is measured are the number of kgm. cm. of work necessary to raise through one degree the quantity of liquid which at 0° C. and atmospheric pressure occupies one c.c. Figures 100 to 111 show the change of C_v for intervals of 20° for the liquids separately. A positive ordinate means that C_v is greater at atmospheric pressure than at the pressure in question. The origin of each curve has been displaced with respect to the next in the usual way, and the scale is shown at the right hand side. Because of the low boiling point, the origin of some of the curves is at 500 or 1000 kgm. In these cases, the difference shown is the difference between C_v at 500 or 1000 kgm. and the pressure in question. Figure 112 shows the average of C_p from 20° to 80° for all twelve liquids. The liquids are indicated by numbers, the origin of the separate curves has been displaced, and the scale is shown at the right hand side, in the usual way.

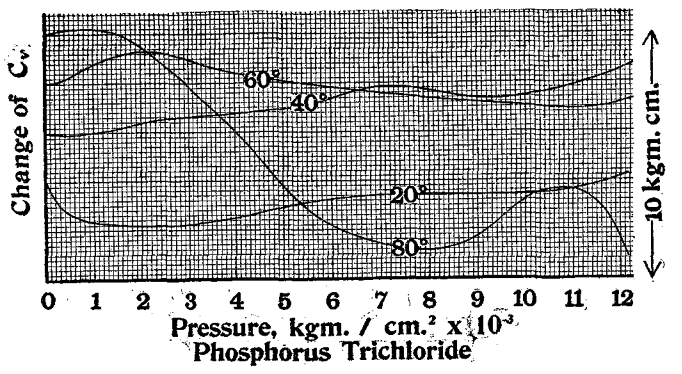


FIGURE 108.

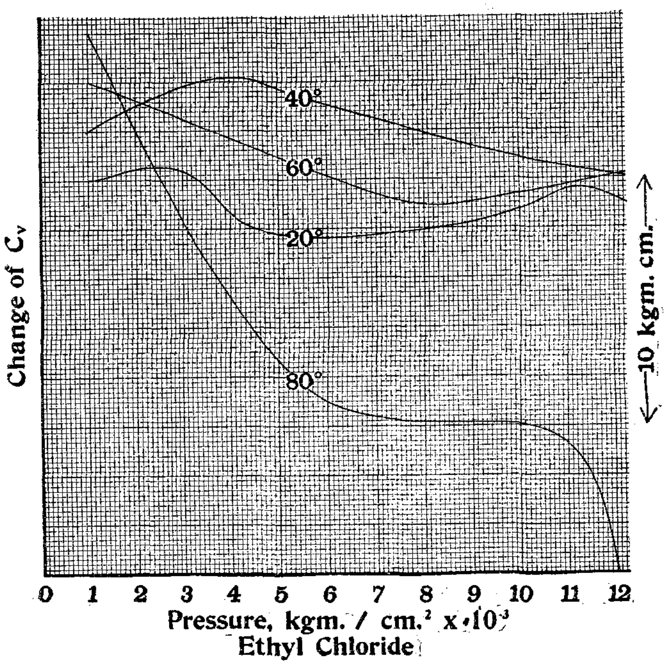


FIGURE 109.

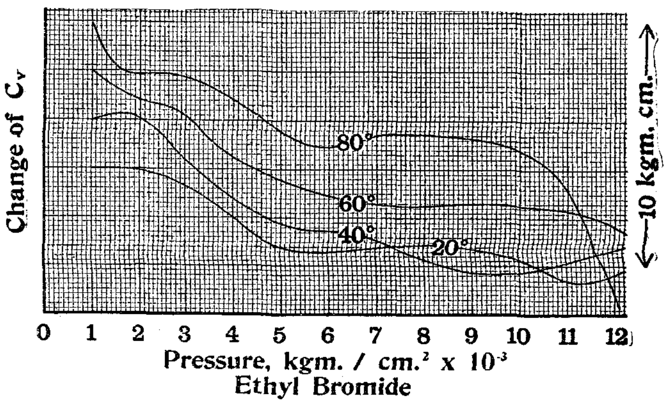


FIGURE 110.

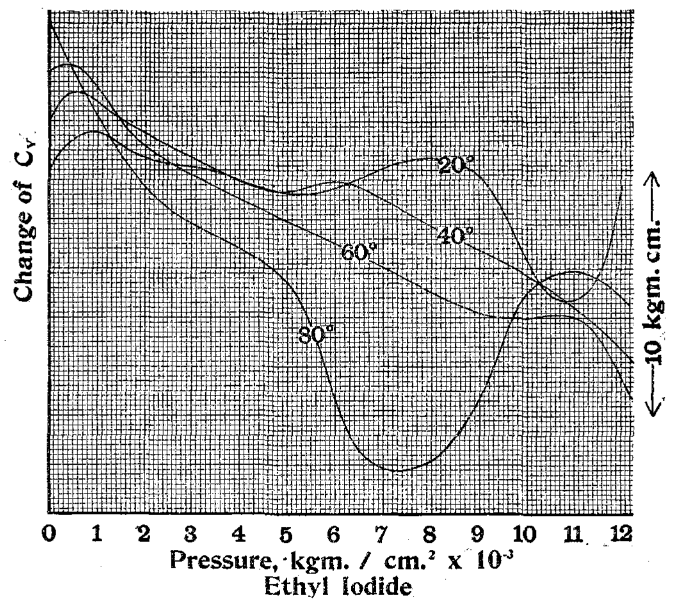


FIGURE 111.

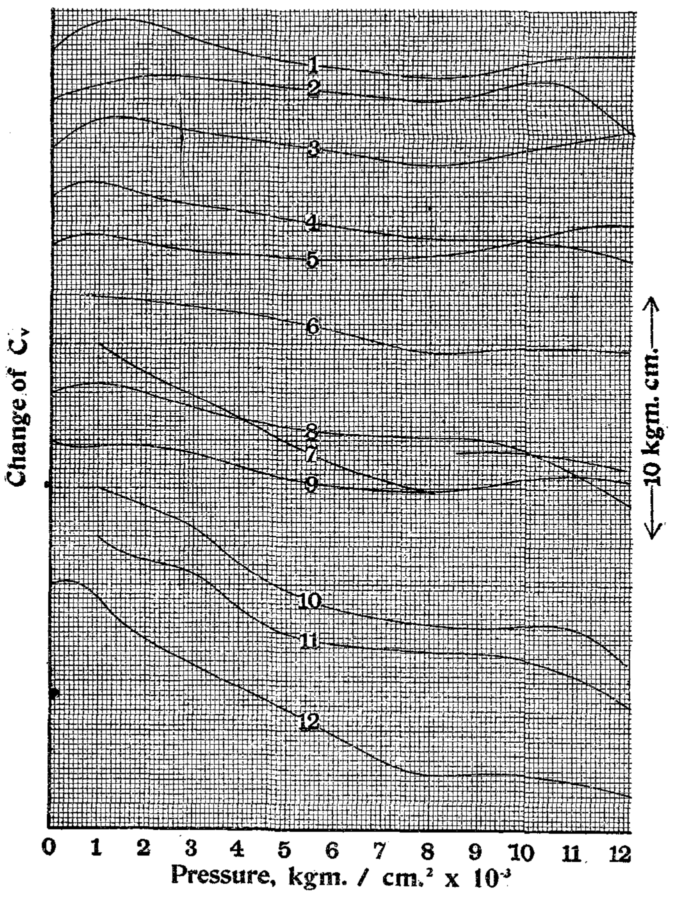
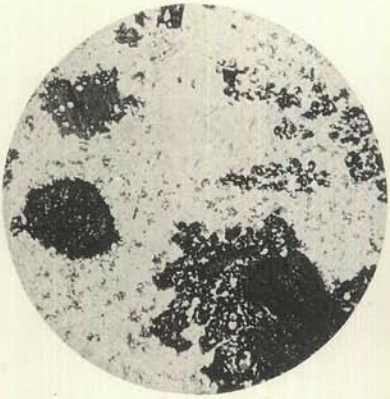


FIGURE 112.



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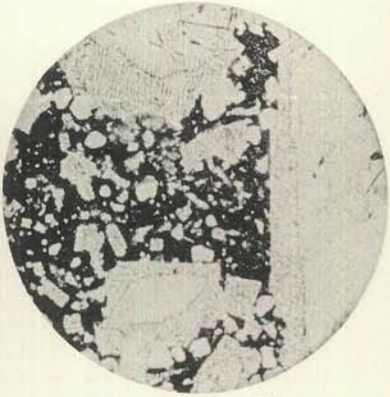

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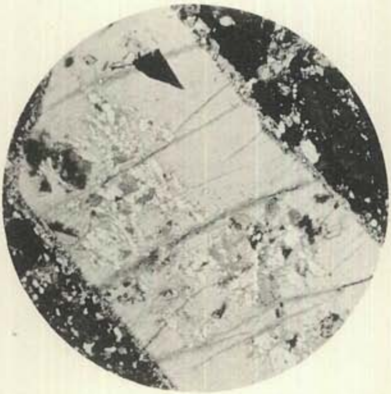

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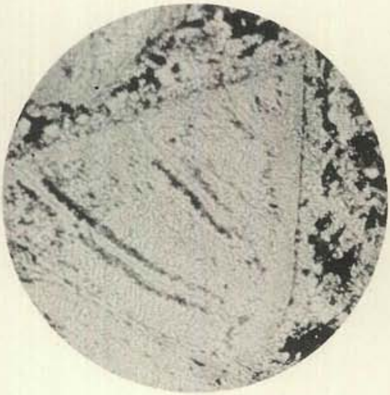

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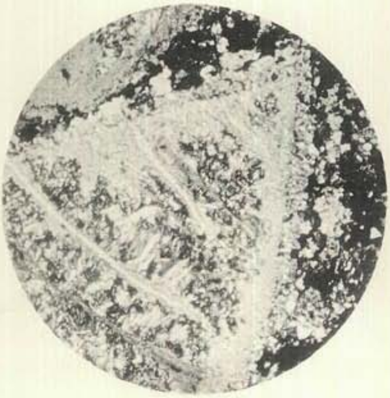

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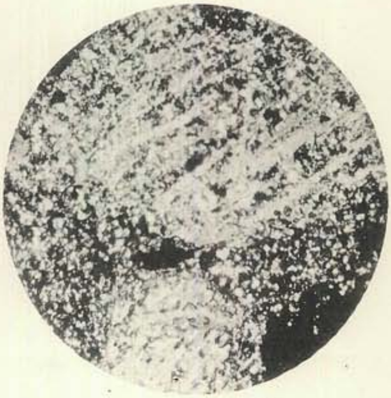

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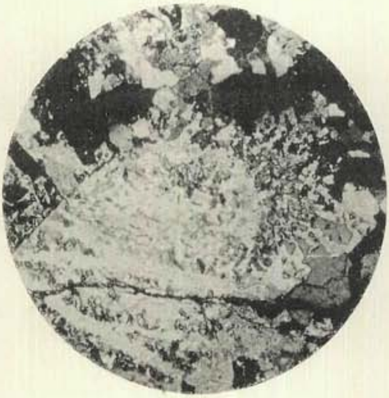

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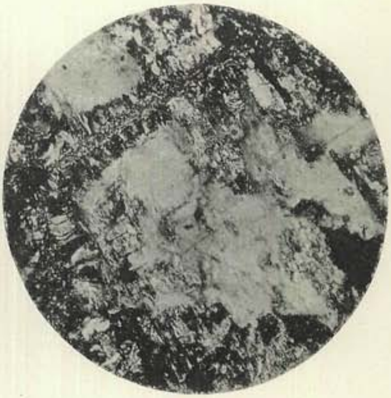

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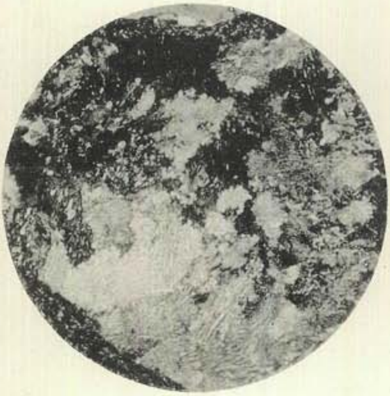

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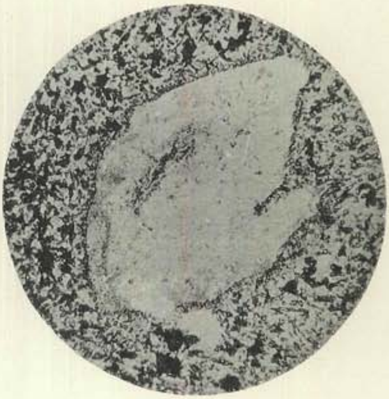

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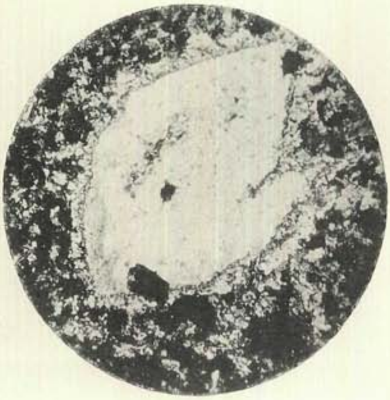

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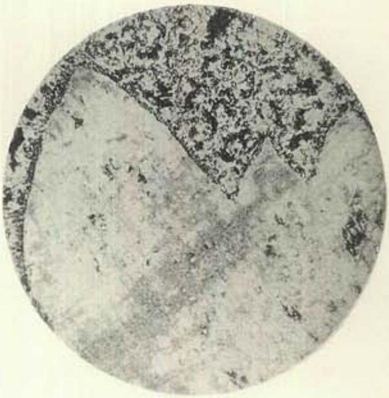

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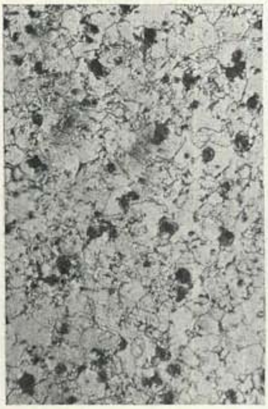

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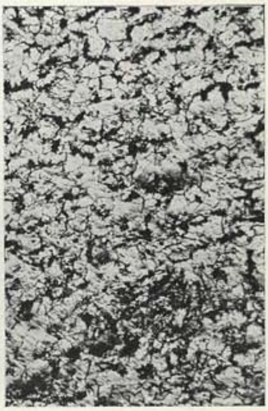

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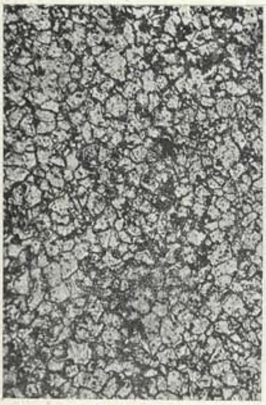

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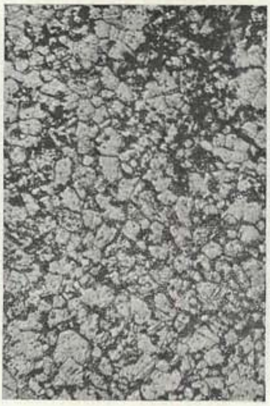

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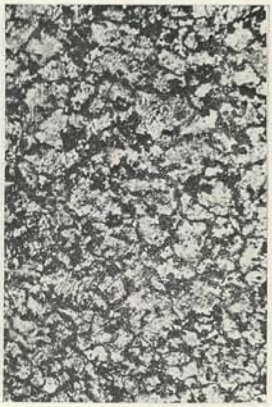

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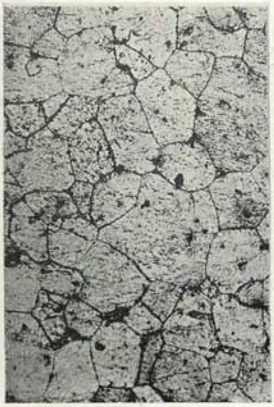

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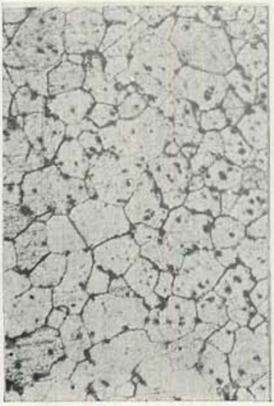

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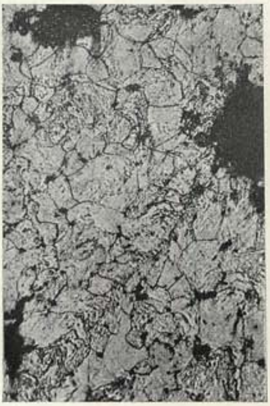

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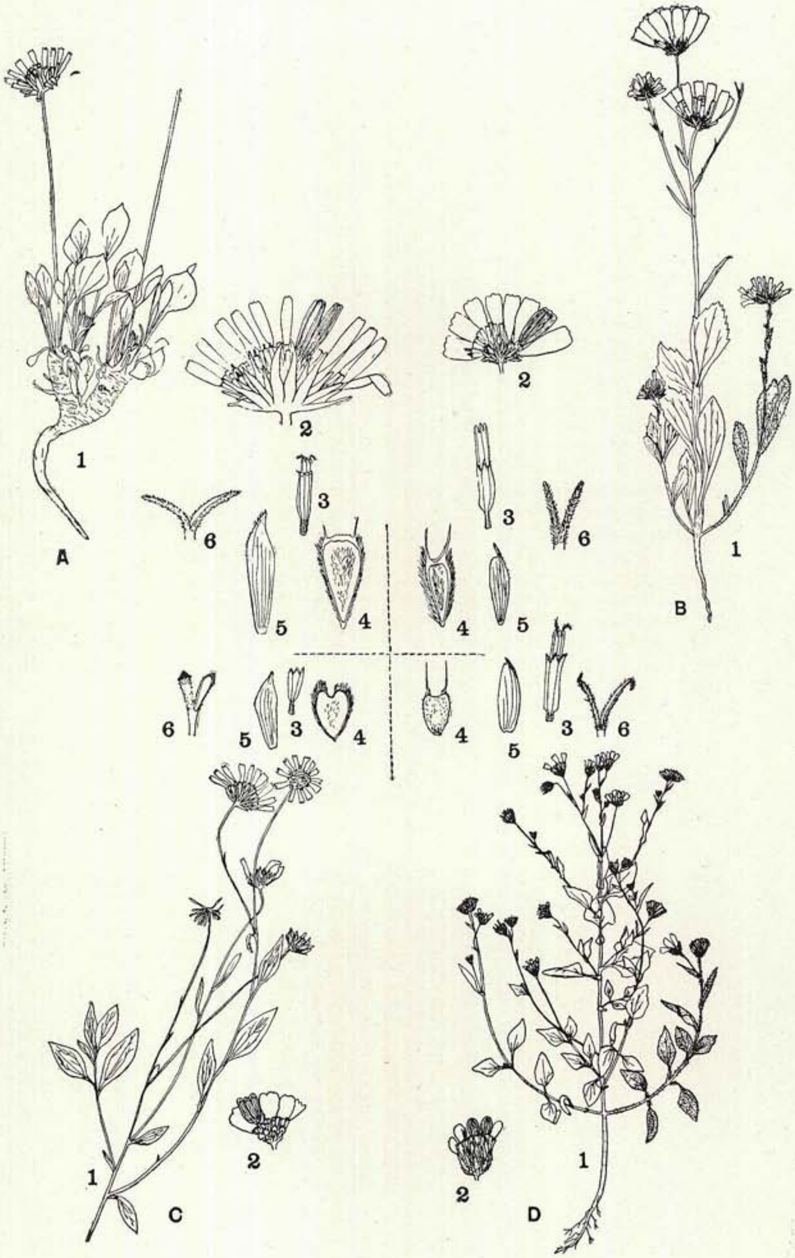
Magnification 120.

Blake. — Encelia and Related Genera.



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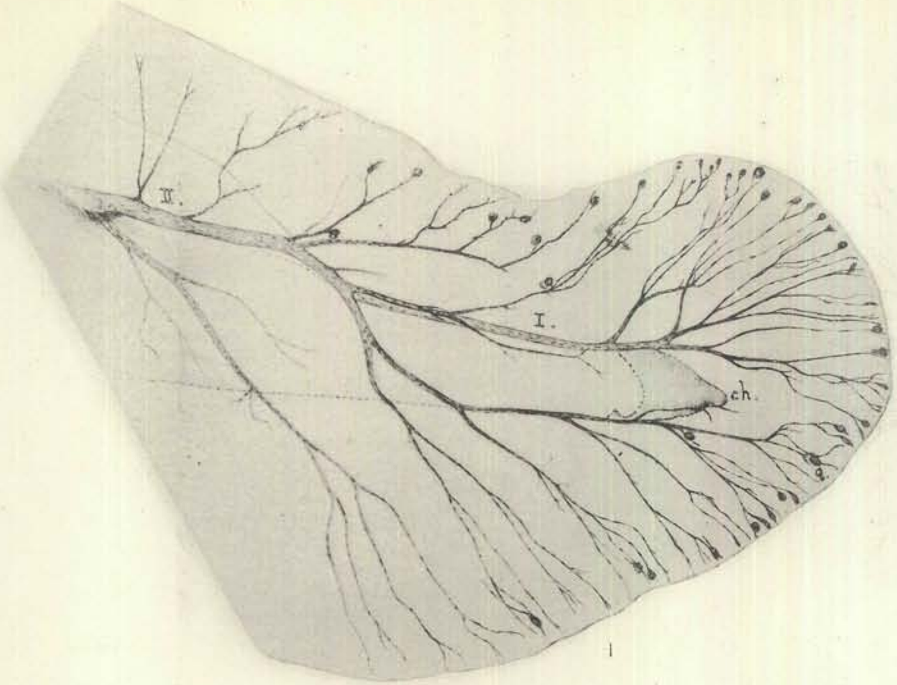
S. F. Blake, del.



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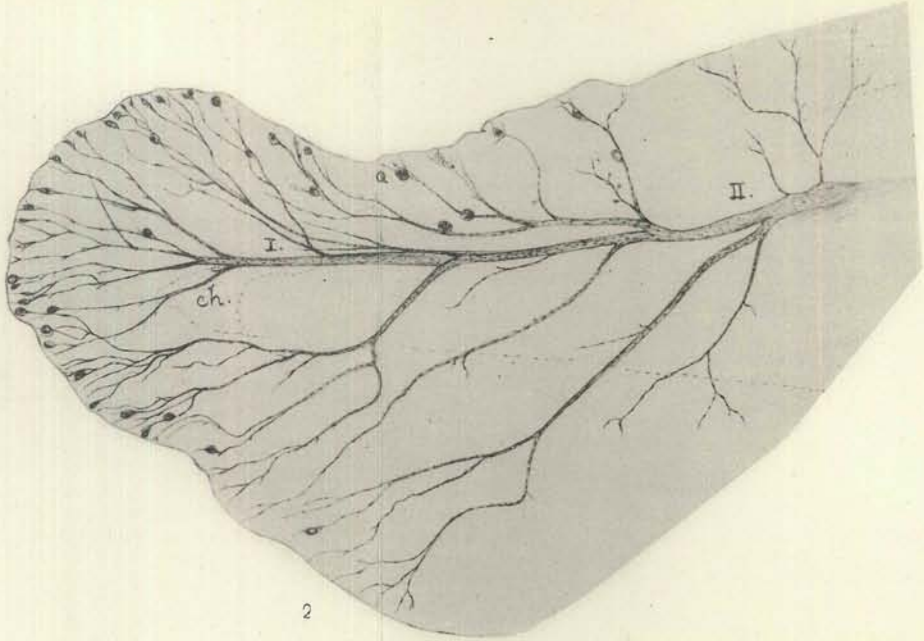

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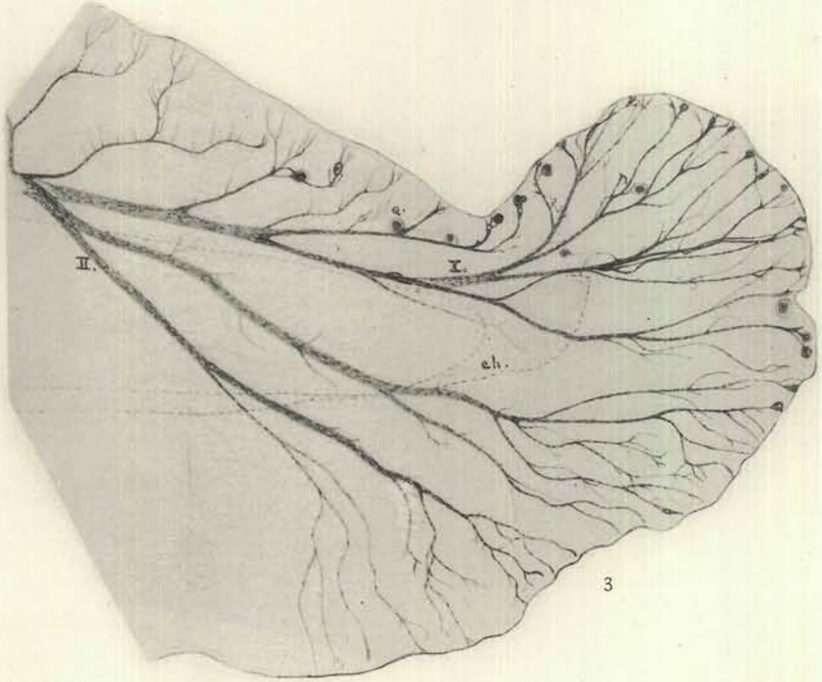

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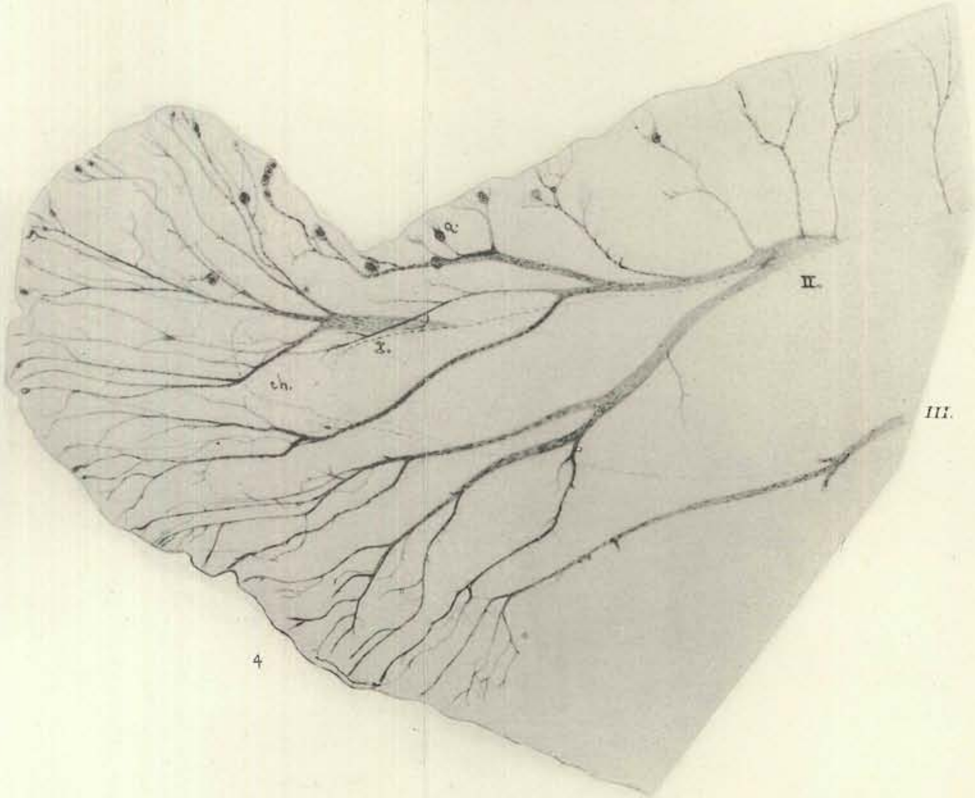

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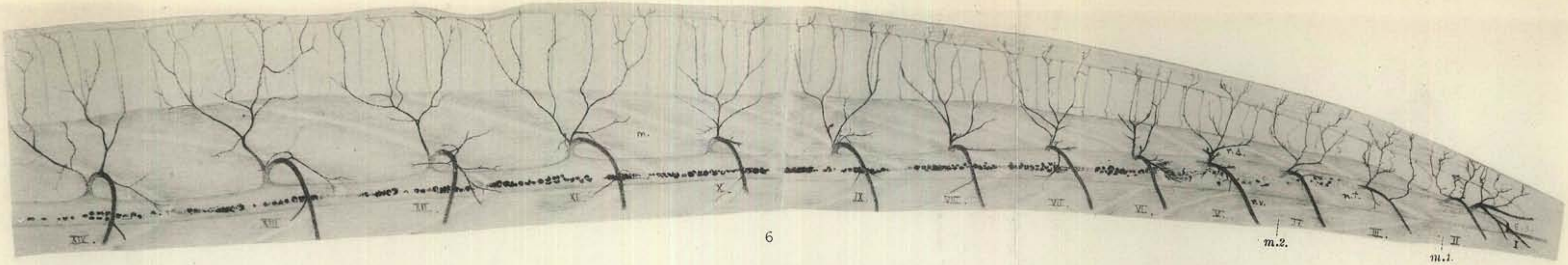
H. L. del.

HELIOTYPE CO., BOSTON.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XIV.
XIII
XII.
XI.
X.
IX
VIII.
VII.
VI.
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r.d.
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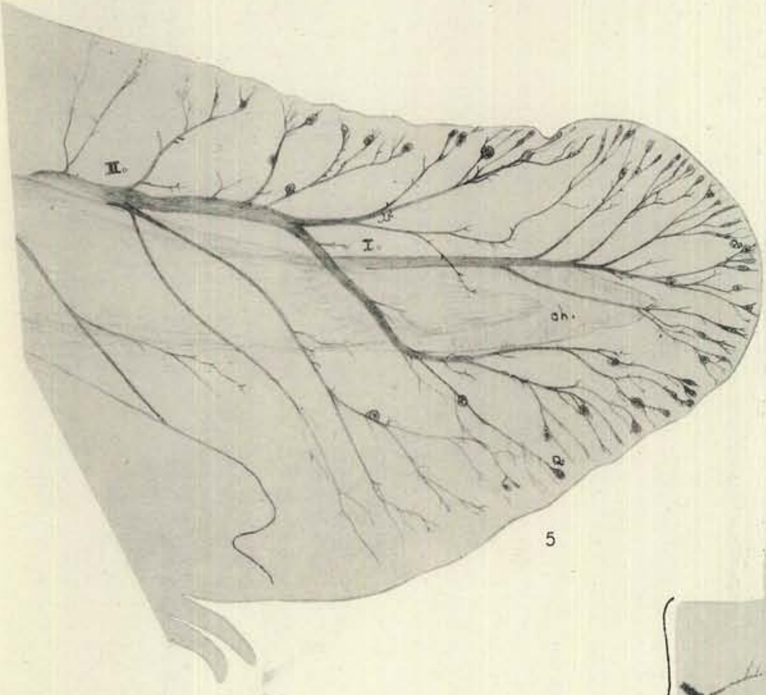
m.2.
m.1.

6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ch.

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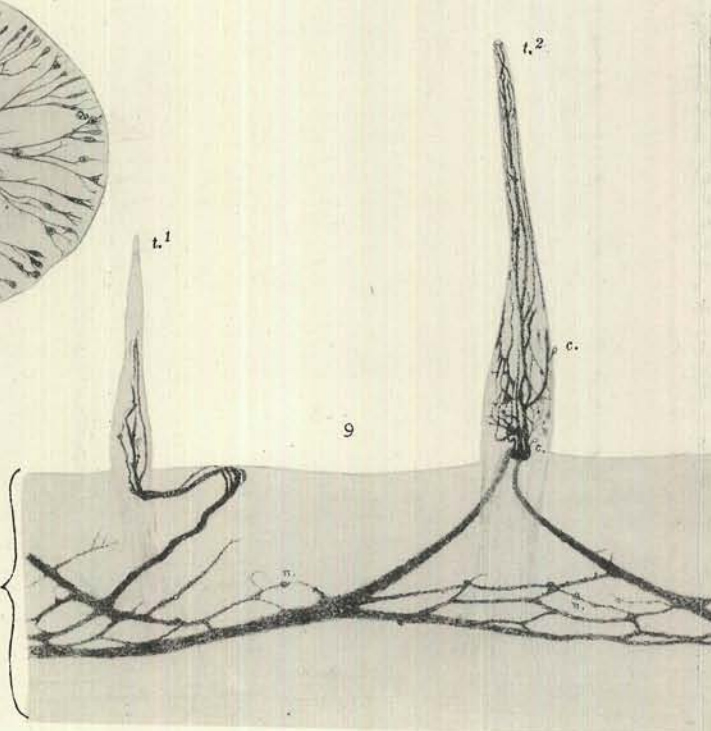
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v.
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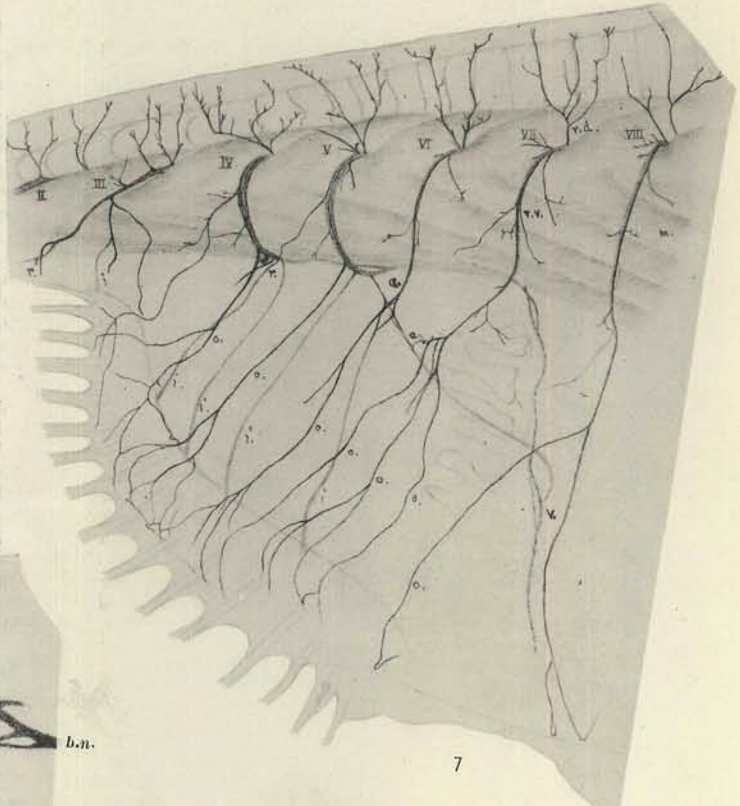
b.n.

9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VII
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m.
o.
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v.

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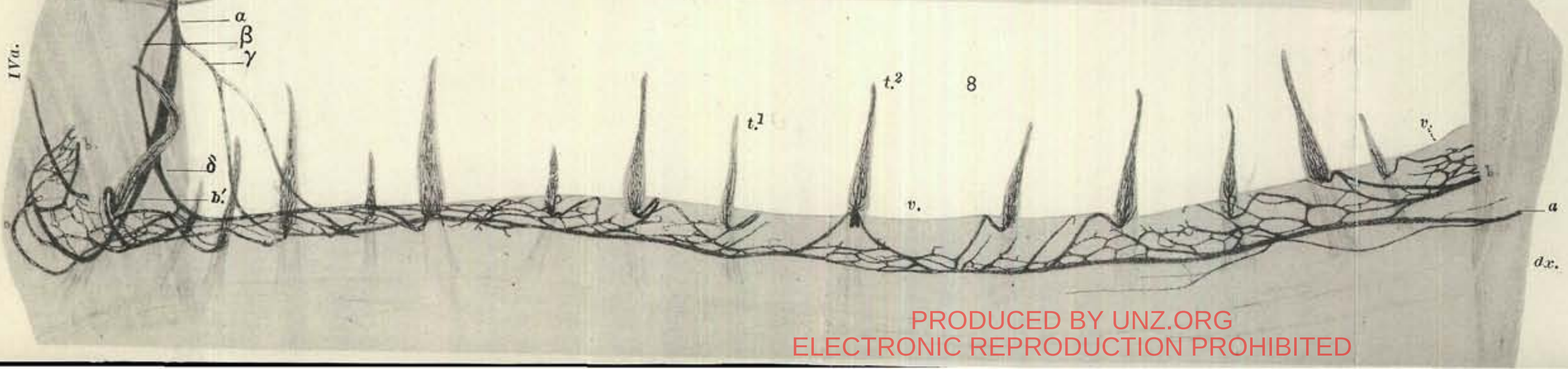
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IVa.
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b.
b'.
t.1
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dx.

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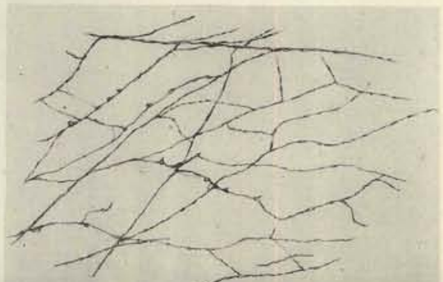

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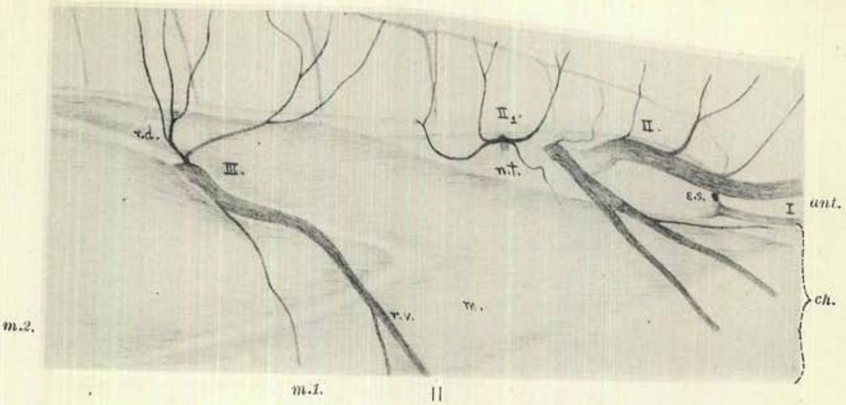
r.d.
III.
II₁.
n.t.
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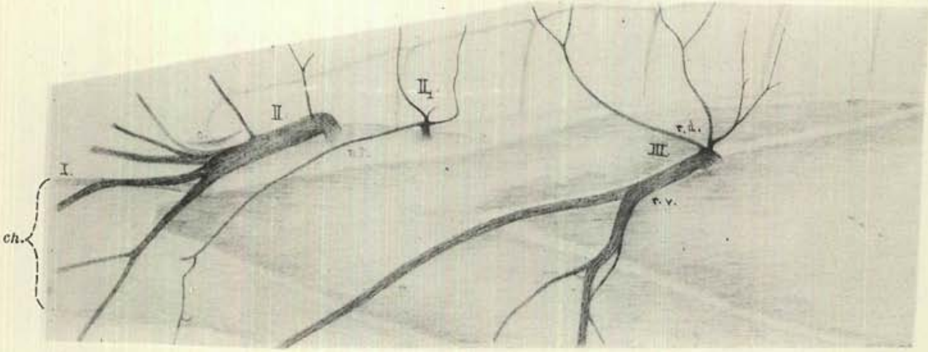
I
ant.
ch.
r.v.
m.
m.2.
m.1.

11


I.
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II
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II₁
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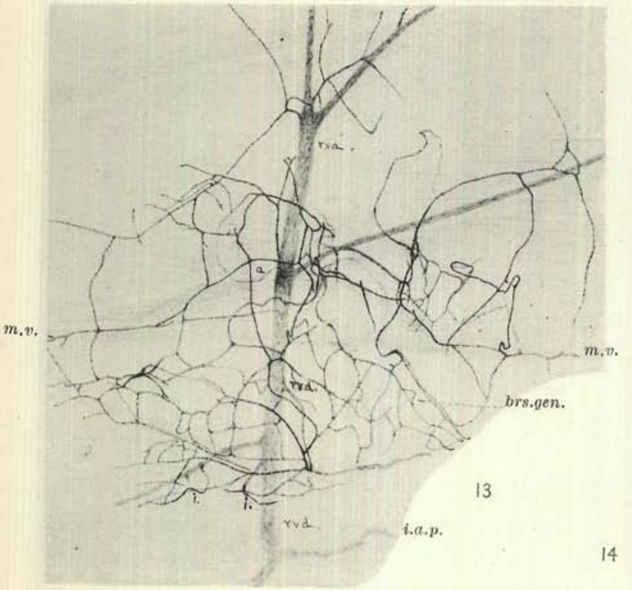
III
r.v.
m.1.
m.2.

12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r.v.d.
brs.gen.
m.v.
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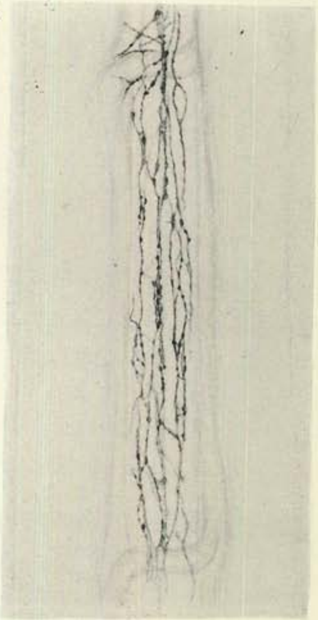
r.v.d.
i.a.p.

13



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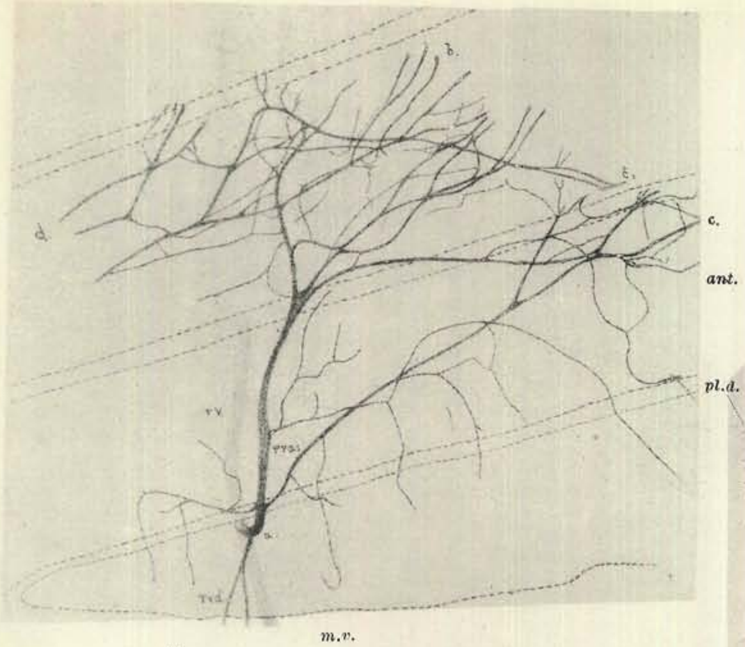
H.L. del.

HELIOTYPE CO., BOSTON.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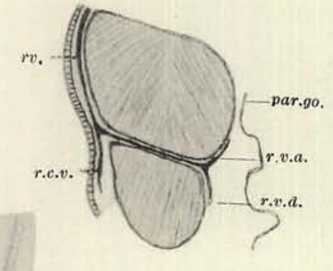
m.v.

15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par.go.
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r.c.v.
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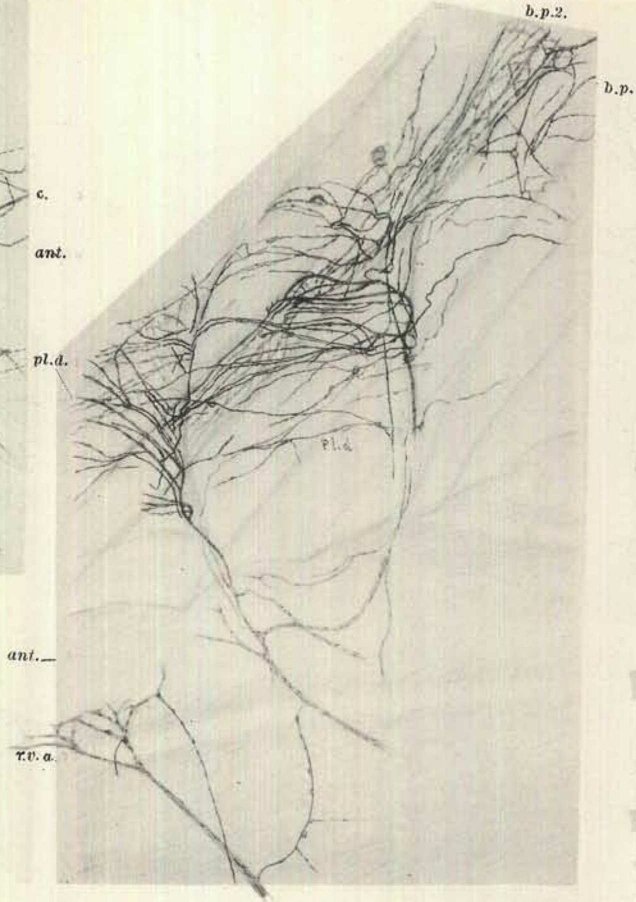
15a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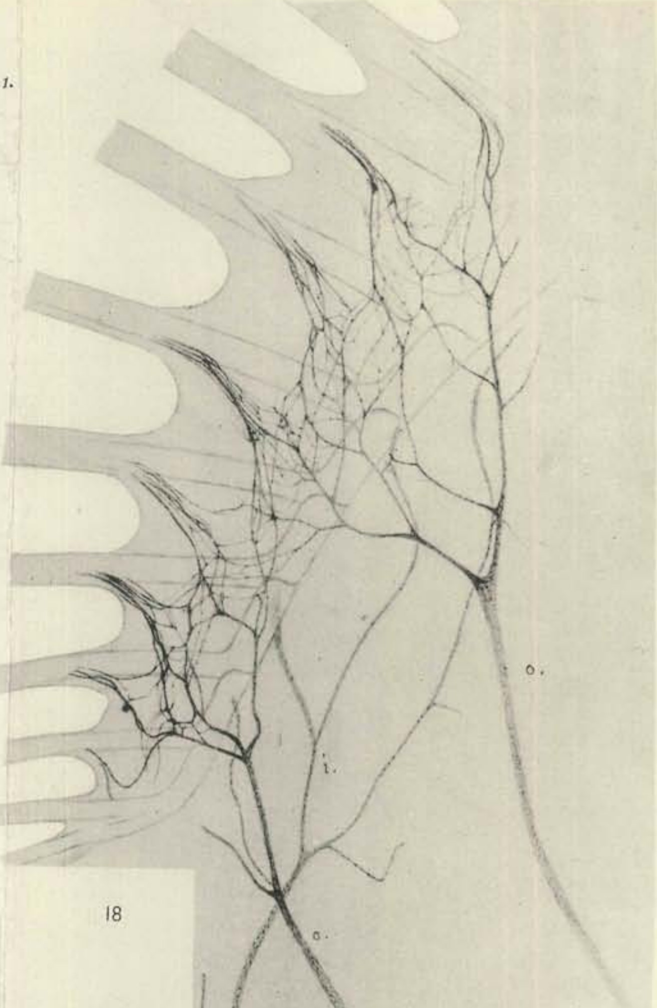
b.p.1.
ant.
r.v.a.
Pl.d.

17


o.
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IVsin.
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V

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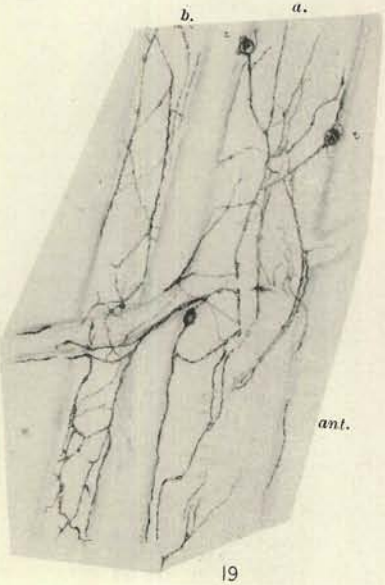

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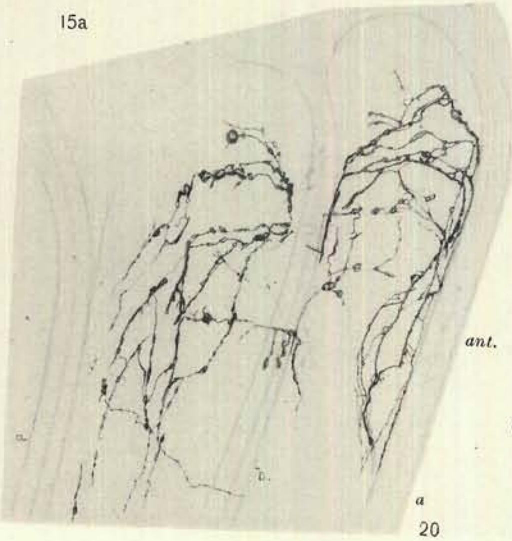

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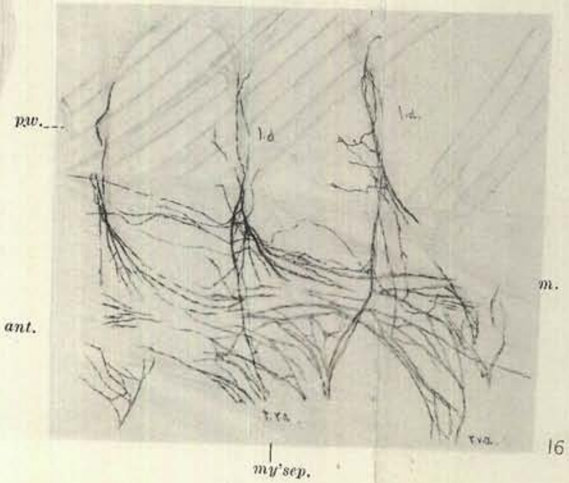
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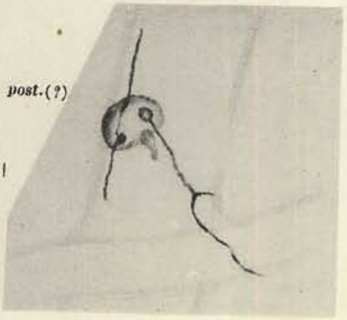

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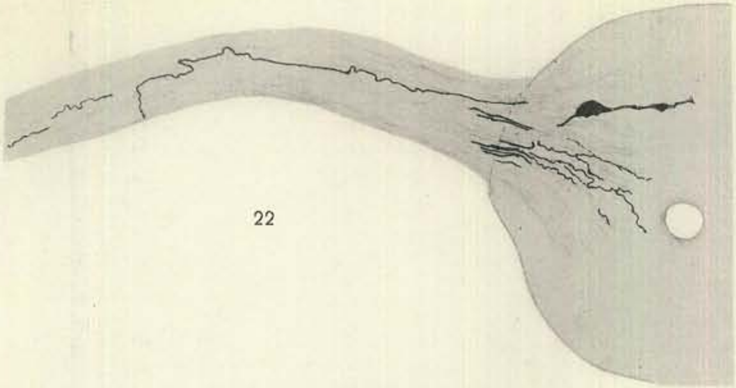
H. L. del.

HELIOTYPE CO., BOSTON.



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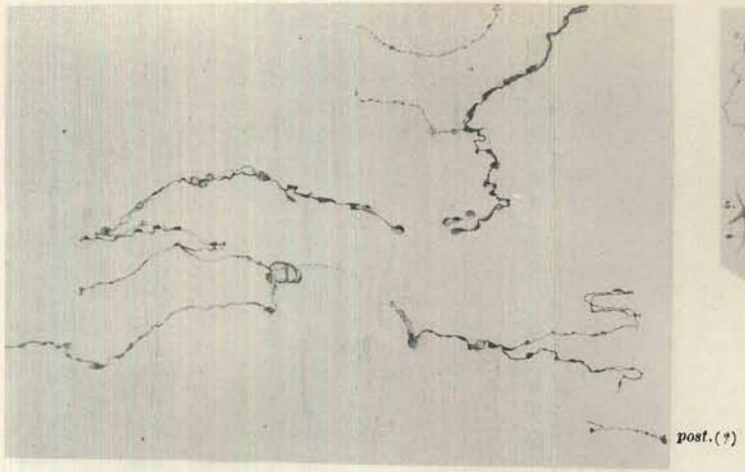

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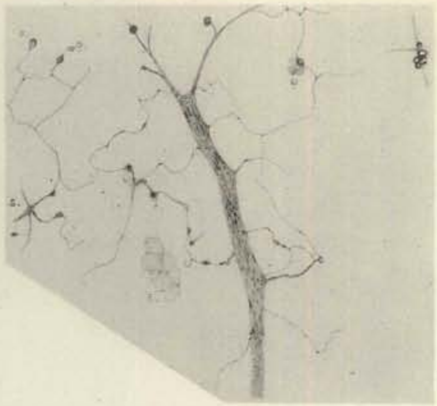

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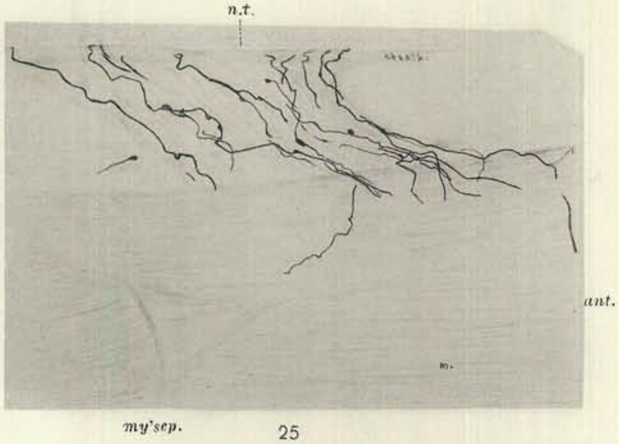
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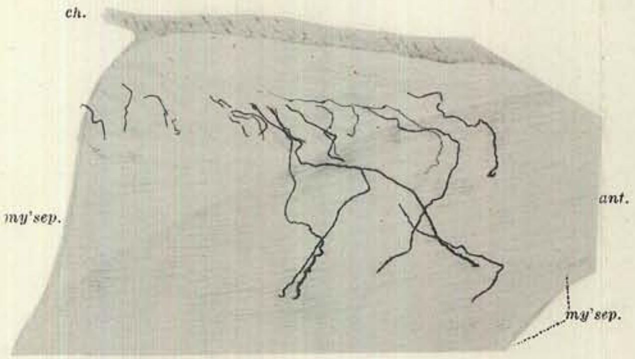
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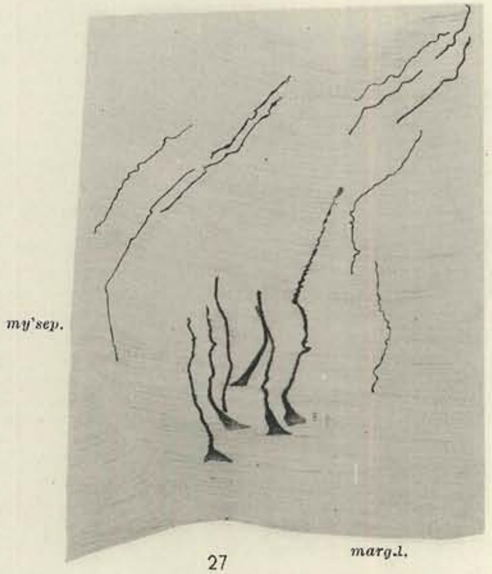
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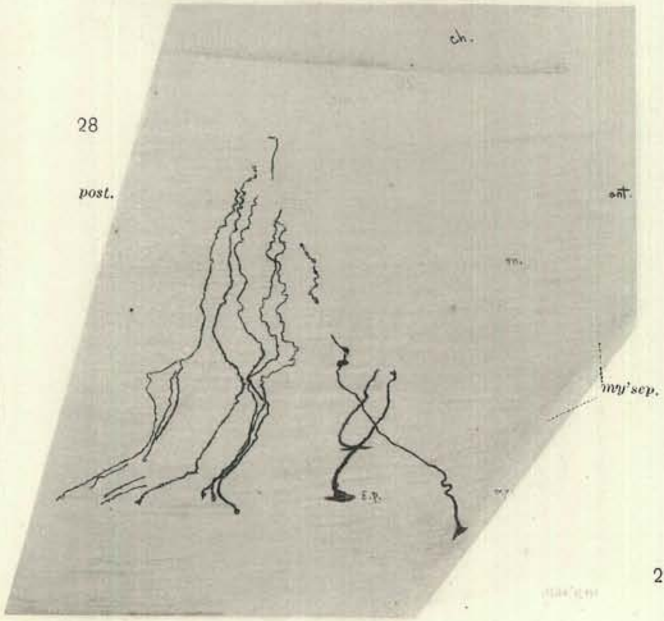
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s.p.

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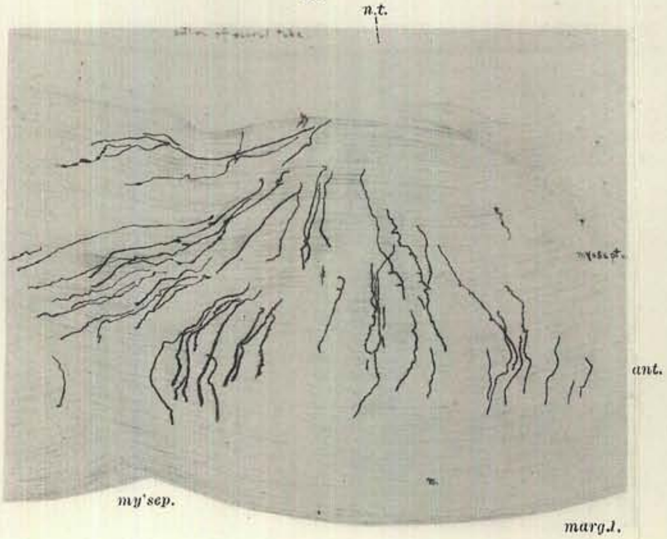
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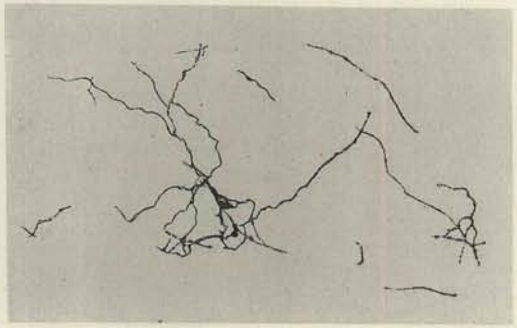
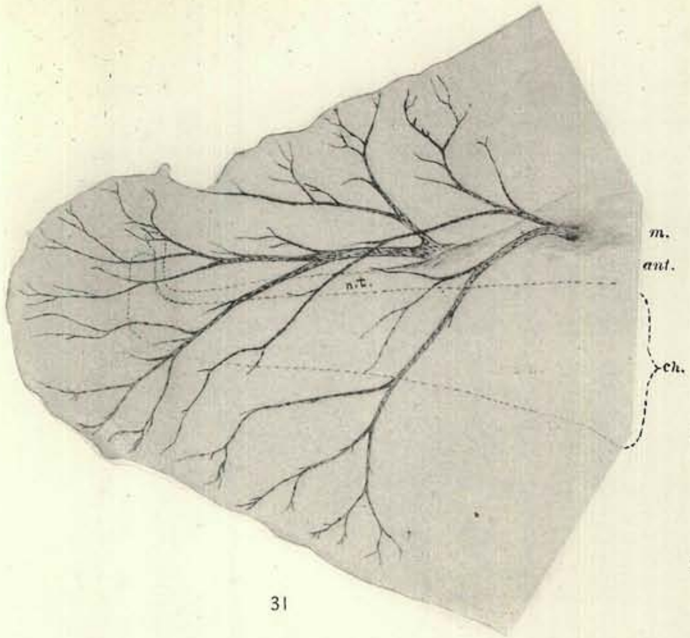
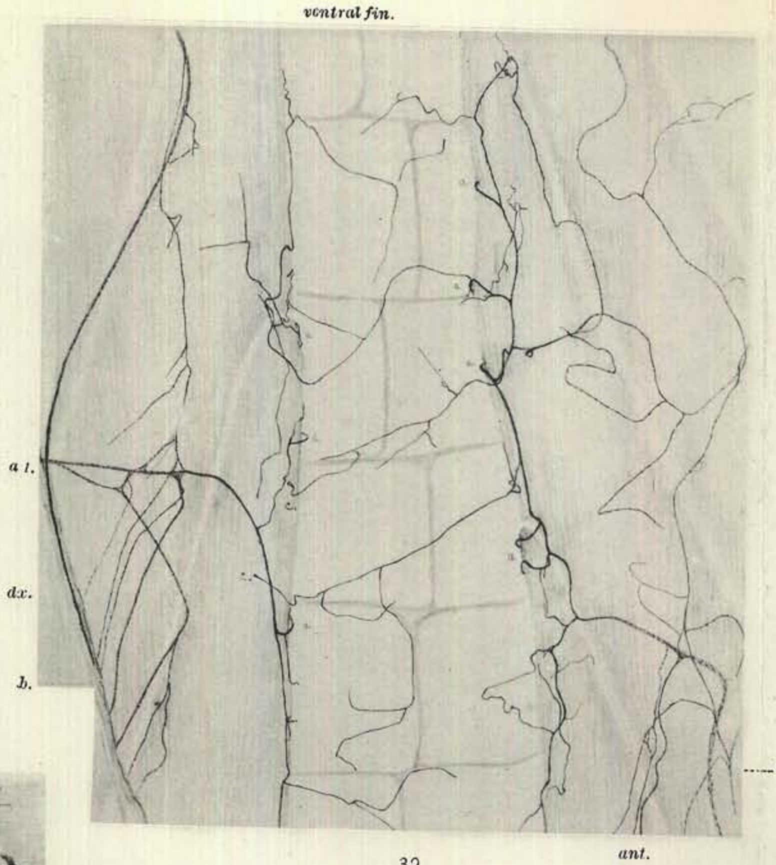


H. L. del.

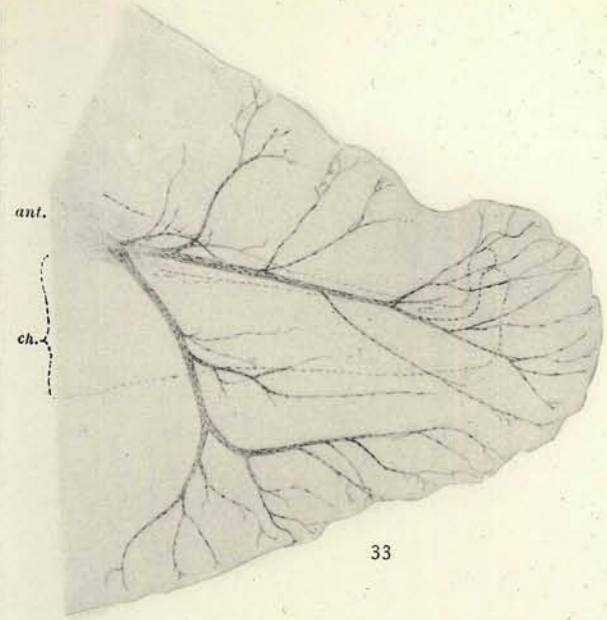
HELIOTYPE CO., BOSTON.



31


ventral fin.
a l.
dx.
b.
c.
a.
ant.
sin.

32



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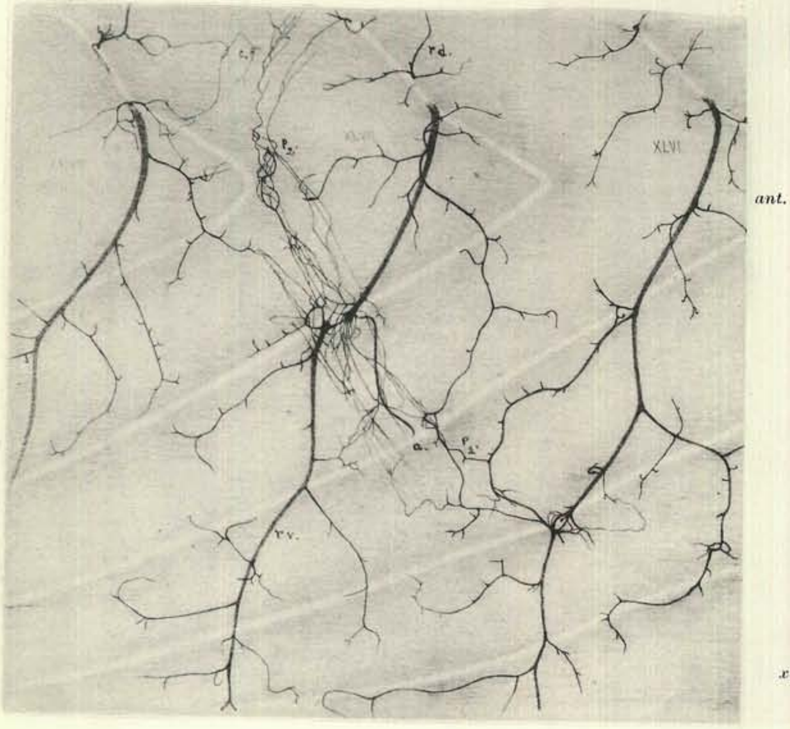
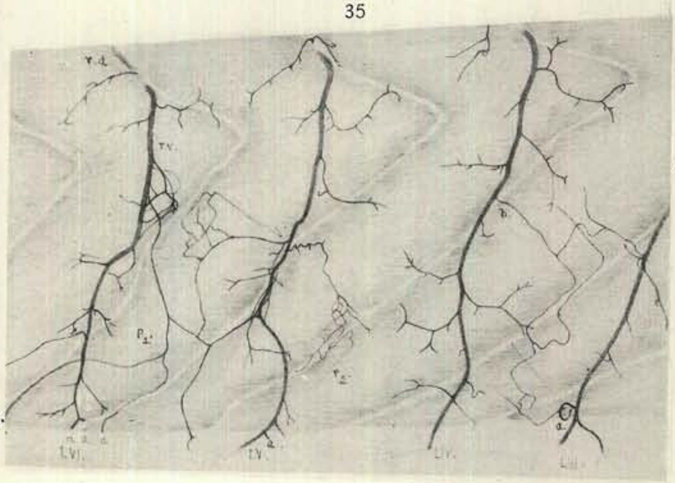

c.f.
r.d.
XLVII
XLVI
P2
a.
r.v.
ant.

35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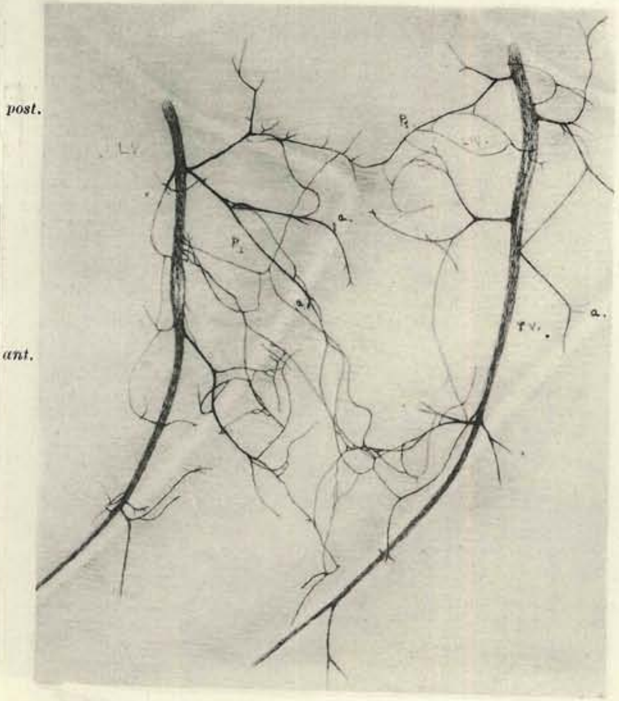
r.v.
P2
b.
x.
LVI
LV
LIV
a.

36


post.
ant.
LV
P1
a.
r.v.

H. L. del.

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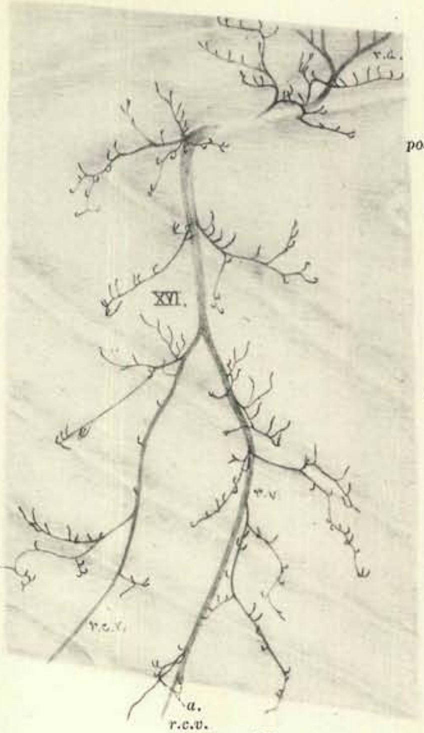
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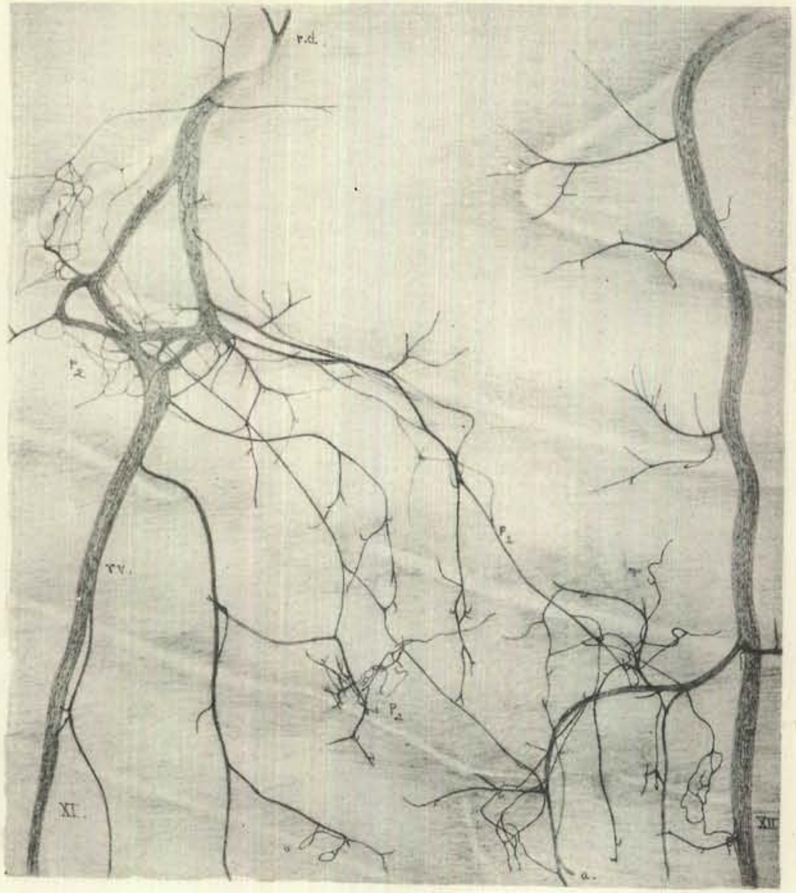
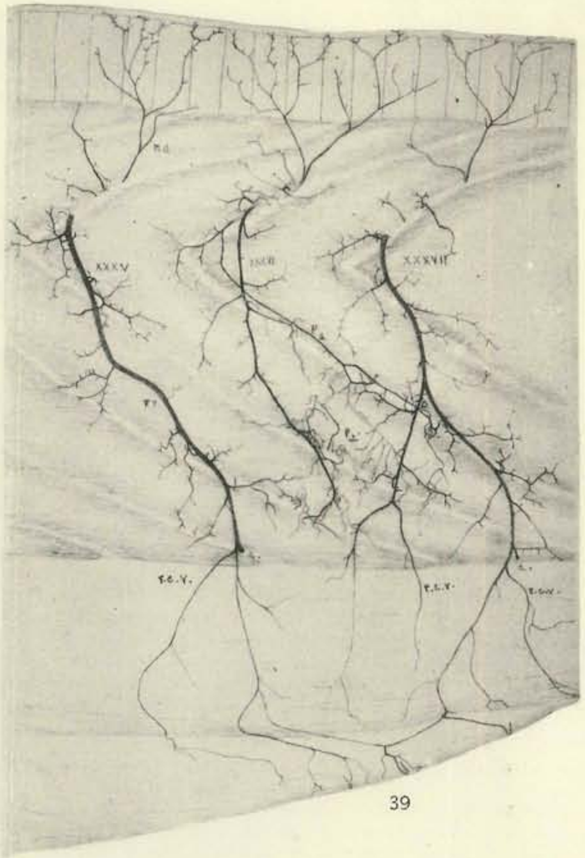
post.
XVI.
ant.
r.v.
r.c.v.
a.
r.c.v.

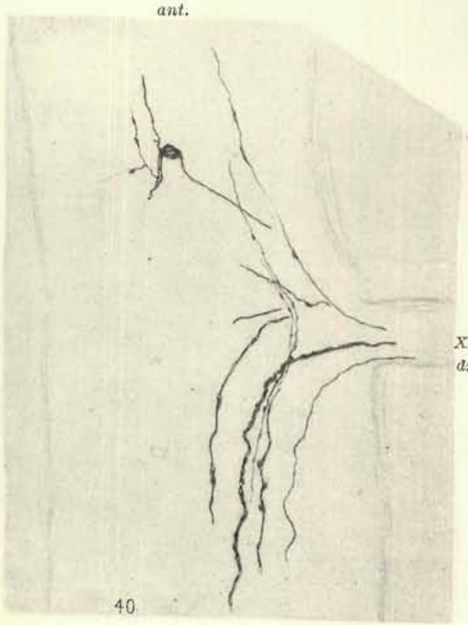
37


r.d.
r₂
r.v.
r₂
r₂
XI.
XII
a.

38


ant.
r.d.
XXXV
XXXVI
XXXVII
r₂
r₂
r.c.v.
r.c.v.
r.c.v.

39



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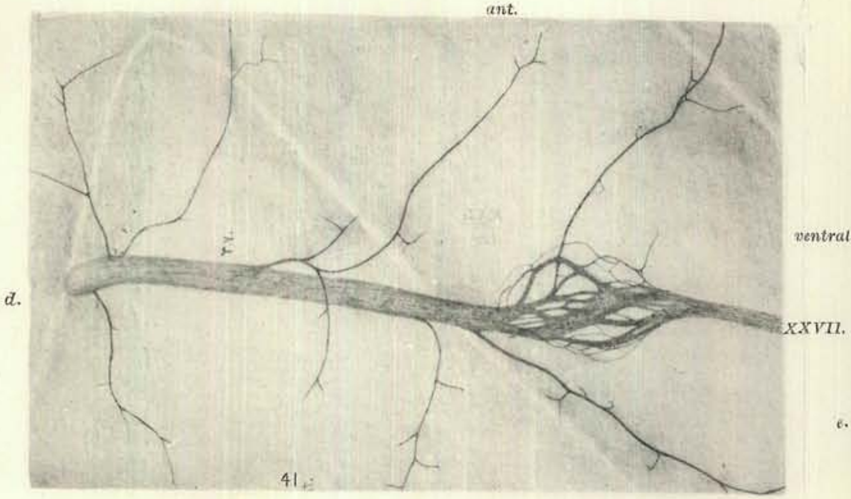
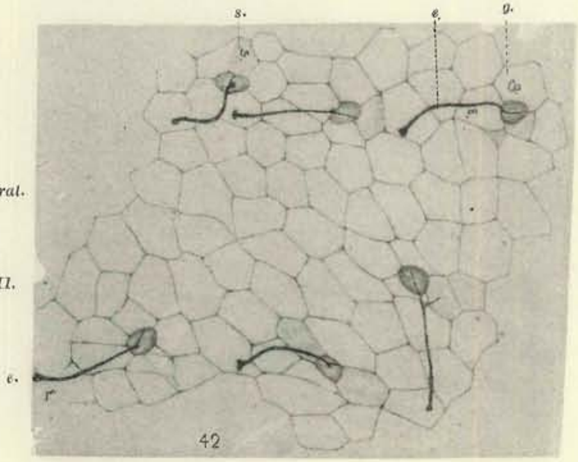

ant.
r.v.
d.
ventral.
XXVII.

41



42

H. L. del.

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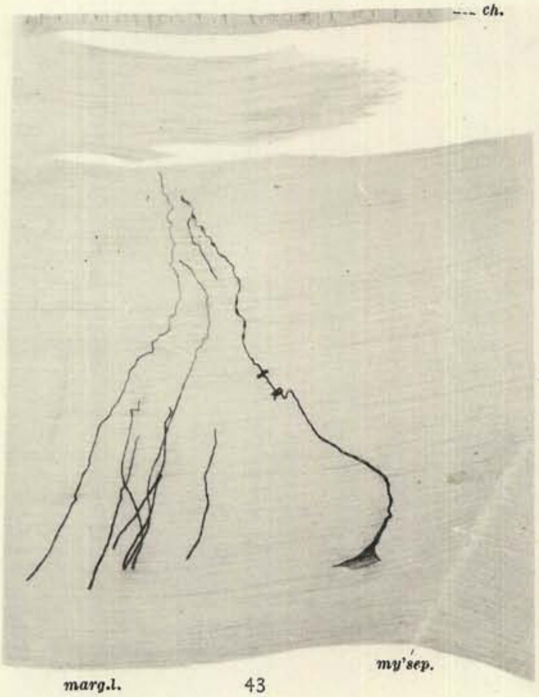
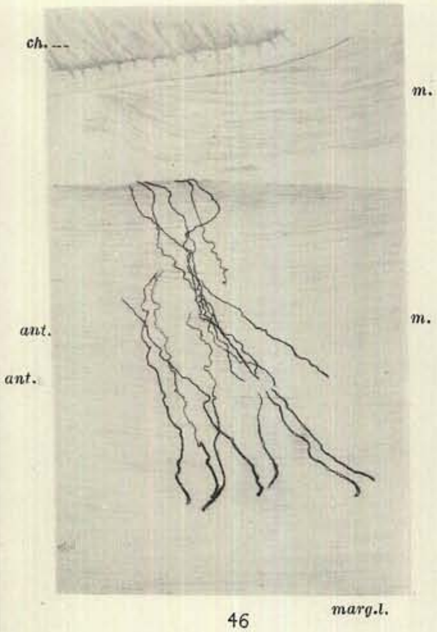
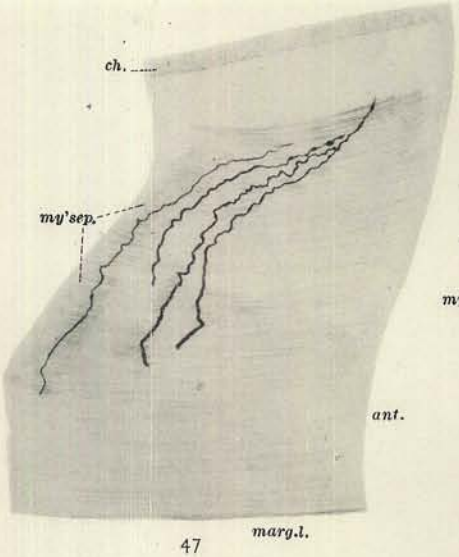

ch.
marg.l.
my'sep.

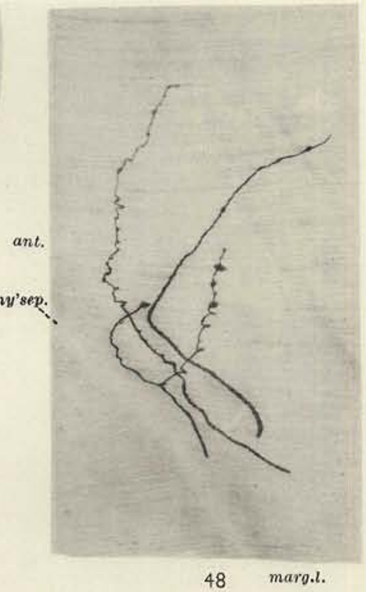
43


ch.
m.
m.
ant.
ant.
marg.l.

46


ch.
my'sep.
ant.
marg.l.

47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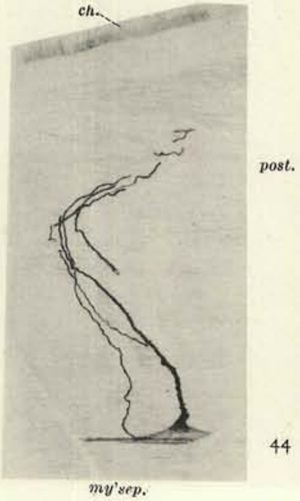
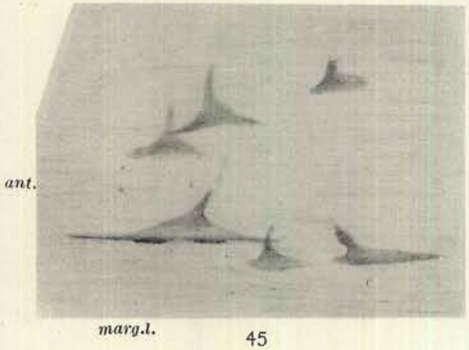
my'sep.
marg.l.

48


ch.
post.
my'sep.

44



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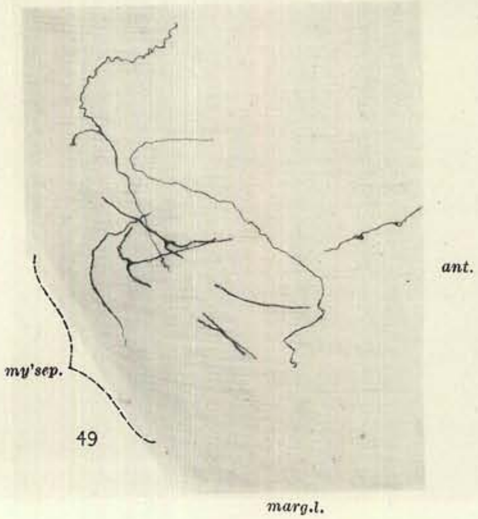
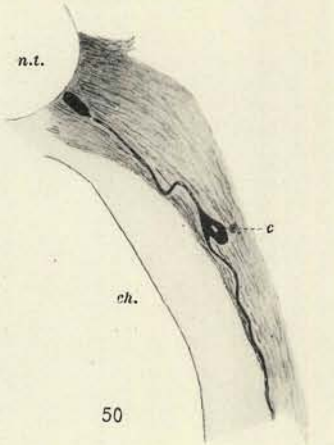

my'sep.
ant.
marg.l.

49



50

H. L. del.

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